Portfolio Analysis of Solar Photovoltaics: Quantifying the Contributions of Locational Marginal Pricing and Power on Revenue Variability

Katrina Kumpf^a, Seth Blumsack^a, George S. Young^b, Jeffrey R. S.
Brownson^{a,*}

^a*Department of Energy and Mineral Engineering, College of Earth and Mineral Sciences,
Pennsylvania State University, PA 16802, USA*

^b*Department of Meteorology, College of Earth and Mineral Sciences, Pennsylvania State
University, PA 16802, USA*

Abstract

For firms developing and managing portfolios of PV assets in utility, commercial, and residential markets the financial performance of the portfolio is a highly relevant business decision factor. Informative metrics are required to quantify the revenue variance of a spatially distributed portfolio of PV assets as well as the individual asset. Financial analysis uses a risk measure known as ‘beta’ to describe the movement of assets relative to a broader portfolio. We define a measure termed the ‘solar beta’ that describes the movement of solar PV revenues at a given site with that of a portfolio of sites. The solar beta incorporates correlation between a site and portfolio, and the volatility of a site relative to the portfolio. We also derive and discuss a method to decompose revenue variance of individual PV assets into components representing price, power, and the interplay among diurnal/seasonal cycles and prevailing weather conditions. This decomposition and the solar beta are illustrated using nine modeled sites following a N-S trend within the PJM Interconnection. We find that revenue variance of a PV asset depends more on diurnal, seasonal, and meteorological fluctuations than on price fluctuations at a particular site. Specifically, the contribution of power variance exceeds the contribution of price variance by roughly a factor of five. Changes in mean market price have a larger effect on revenue variance compared to

*Corresponding author. Tel: +814-867-4227

Email address: solarpower@psu.edu (Jeffrey R. S. Brownson)

a proportional change in mean power production. The solar beta was found to be near 1.0 for most sites, indicating strong covariance within the portfolio due in large part to high correlation rather than similar volatility ratios. Lower beta values were found for sites at the perimeters of the study region, due to change in climate regime and population-power consumption cycles, implying portfolio risk reduction when these sites are included.

Keywords: photovoltaics, locational marginal pricing, portfolio analysis, beta

1. Introduction

The amount of PV electricity generation and total capacity has increased dramatically over the past decade. In the USA, PV technologies exhibited a 4000-fold increase in net annual generation over a span of ten years, climbing from 2 GWh in 2003 to 8,300 GWh in 2013.[1] The observed exponential growth trend in net power generation has created opportunities for individual firms to own and manage multiple PV assets. Firms such as SunEdison and SolarCity are developing and managing large portfolios of PV assets for utility, commercial, and residential markets. Likewise, the solar project finance firm Mosaic enables smaller-scale investors to collaborate in the construction of PV asset portfolios. For such firms and other like them, the financial performance of the portfolio, in complement with the individual assets, is thus the relevant business decision factor. As such, they require informative metrics quantifying revenue variance of a spatially distributed portfolio of PV assets.

Applying the tools of portfolio analysis to multisite PV asset assessment provides a model for evaluating the contribution of specific sites to revenue variance over space and time. For individual PV assets, revenue variance depends on the variances of both price and power and their co-fluctuations. The decomposition of revenue variance into price and power variance and co-fluctuation terms offers detailed insight into the market and meteorological origins of this risk factor. While such decompositions are routine within fluid dynamics, they are not yet common in energy economics. This approach will be addressed in subsequent sections.

For a portfolio of PV assets, revenue variance depends on the covariance of the individual assets. In financial portfolio management the β metric is used as a measure of return variance for a particular asset relative to movements

in the general market.[2, 3] An equivalent formulation may be derived for revenue variance for PV portfolios, which we term the ‘solar beta.’ This version of beta captures the impact of PV assets progressively added to the portfolio on revenue variance.

These new approaches to financial portfolios in solar are distinct from the works of Hoff and Perez, who showed that a geographically-dispersed portfolio of variable renewables (both solar and wind) can reduce aggregate output power variability.[4, 5] This information is of critical importance to grid operators. In contrast, the new approaches described here quantify the contributors to revenue variance, information of importance to firms owning and managing portfolios of multisite PV assets.

The eastern portion of the PJM Interconnection has been used as a case study. PJM is a Regional Transmission Organization whose footprint covers all or part of 13 states and the District of Columbia. One relevant feature of the PJM market for our analysis is that its market prices are determined with high temporal and spatial granularity. These prices, known as locational marginal prices (LMP, here referred to as “market price”), are published on an hourly basis for thousands of locations within the PJM footprint, indicating the value of providing power to a specific location at a specific time. This research explores the convolved and separate effects of energy production and market price on the overall revenue of photovoltaic arrays in nine different locations throughout the eastern portion of the PJM footprint.

2. Methodology

Nine geographically distributed sites within the PJM Interconnection were used for this analysis. The operational zone for each site and their coordinates are shown in Table 1. The sites are also displayed as mapped locations in Figure 1. The common time step (hourly) reflects the fineset-grained time step at which PJM publishes its LMP data. A period of seven years was used, from 2003 to 2009, for which publicly available hourly solar GHI and DNI data corresponded with available LMP data for the chosen PJM zones.

The sites were spaced across the region so as to reflect a variety of solar availability and pricing conditions across the portfolio. Each site was located in a unique operational zone within the PJM footprint, and sites were spaced >60 km apart so that the the hourly step for irradiation data would not yield overlap among sites. Spacing was selected to preclude advection of clouds

64 from one site to another on the hourly time scale for which our PJM market
65 data is available. Several papers have already noted for cloud motion and
66 speed assessments, that typical wind speeds in the boundary layer tend to
67 be much less than ~ 60 km per 1 hour time step, which is why that spacing
68 is used in this study.[6, 7, 8]

Table 1: Selected locations for analysis within the PJM network of the eastern coast, reflecting the corresponding electricity zones and geospatial coordinates.

Site Name	Corresponding PJM Zone	Site Coordinates
PENLC	Pennsylvania Electric Co (PA)	41.95 N 77.45 W
PPL	PPL Electric Utilities (PA)	41.75 N 75.75 W
JCPL	Jersey Central Power Light (NJ)	41.25 N 74.75 W
PSEG	Public Services Electric & Gas (NJ)	40.55 N 74.65 W
PECO	PECO Energy (PA)	40.15 N 75.35 W
AE	Atlantic Electric Co (NJ)	39.55 N 74.55 W
BGE	Baltimore Gas & Electric (MD)	39.35 N 76.65 W
PEPCO	Potomac Electric Power Co (MD)	38.35 N 76.65 W
DPL	Delmarva Power & Light (VA)	37.35 N 75.95 W

69 The solar irradiation data were acquired from Solar Anywhere®, an on-
70 line database of solar irradiation.[9] The data sets provided hourly global
71 horizontal irradiance (GHI), direct normal irradiance (DNI), and diffuse hor-
72 izontal irradiance (DIF), all with units of watts per square meter (W/m^2).
73 Only GHI and DNI were needed for the power simulations described below.
74 The hourly irradiance values had a resolution of 10 km by 10 km.

75 The LMP data was downloaded directly from the PJM website at an
76 hourly timescale in units of dollars per megawatt-hour ($\$/\text{MWh}$).[10] Energy
77 information was available in real-time as well as day-ahead market. Real-
78 time data was used for this study in order to compare each hourly value to
79 its corresponding solar irradiation value. These data sets were downloaded
80 and then organized by zones for the nine different locations chosen (listed in
81 Table 1).

82 2.1. Power Simulations

83 The solar irradiation data (GHI and DNI) was used to simulate PV
84 production using the TRaNsient SYstems Simulation (TRNSYS) modeling
85 software.[11] TRNSYS is a robust dynamic simulation tool, allowing data to
86 be input and then modeled based on parameters for solar photovoltaics, solar

Figure 1: Map of locations for for analysis selected within the PJM Interconnection, within the PJM network of the eastern coast, reflecting the corresponding electricity zones and geospatial coordinates.

87 thermal, hydropower, and other renewable energies. Figure 2 demonstrates
 88 the graphical model components used for this experiment. The processors
 89 used have been labeled in the image according to their function. Details for
 90 the component specifications within the TRNSYS model can be found in the
 91 supplementary material, Appendix A.

92 A SunPower Corporation® X20 series 250W panel was chosen as the
 93 basis for the TRNSYS simulations. All of the necessary parameters and
 94 inputs required for the TRNSYS model were taken from the panel's spec
 95 sheet available from the manufacturer.[12] At each of the nine sites, an array
 96 of 100 panels was simulated to represent a commercial or small-scale utility
 97 system of approximately 25 kW_p nameplate capacity.

Figure 2: TRNSYS model of PV power production simulations at a single location

2.2. Individual PV Assets: Revenue Variance

The derivation and discussion below follow the notation conventions of atmospheric fluid dynamics, the field which inspired this revenue variance decomposition approach.[13] Revenue from a PV asset is the product of two variables: electricity price and power. The variance of revenue is therefore also a function of these two variables. We define A as Revenue (\$/h), B as Market Price (\$/MWh), and C as Average Hourly Power (MW). Thus:

$$A = B * C \quad (1)$$

captures the relationship between revenue, price and power.

A particular value of a random variable (X) equals the sum of that point's fluctuation from the mean (X') and the overall mean of the variable ($\bar{X}$). In other words: $B = \bar{B} + B'$ and $C = \bar{C} + C'$. Using this definition the variance of A can be expressed in terms of the mean and fluctuating components of B and C . Note that by this definition the mean of any fluctuation is zero and mean quantities can be factored out of averages.

Thus, by definition

$$Var(A) = \overline{(A - \bar{A})^2} = \overline{A'^2} \quad (2)$$

Using the definitions above, the $\overline{(A - \bar{A})^2}$ term in Eq. 2 can be decomposed as follows:

$$\begin{aligned} Var(A) = & \overline{\bar{B}^2 C'^2} + \overline{\bar{C}^2 B'^2} + 2\overline{\bar{B} B' C'^2} + 2\overline{\bar{C} B'^2 C'} \\ & + 2\overline{\bar{B} \bar{C} B' C'} + \overline{B'^2 C'^2} - \overline{B' C'}^2 \end{aligned} \quad (3)$$

Eq. 3 (derived in Supplemental document, Appendix B) allows the revenue variance to be expressed in terms of the means and co-fluctuations of price and power.

116 2.3. Portfolio Assessment: The Solar Beta

117 To measure the contribution of a specific PV site to the revenue variance
 118 of a geographically distributed portfolio of PV assets, we adapt the concept of
 119 ‘beta’ from finance theory. Beta is traditionally used as a metric to compare
 120 the variance of returns from a single asset (like a stock) with that of a portfolio
 121 of assets or the broader asset market. The solar beta differs from financial
 122 beta in that it is based on revenue (A), rather than return (since the physical
 123 assets in photovoltaic systems are not typically bought and sold in high
 124 volumes as would be the case for financial assets), and the solar beta is
 125 relative to a portfolio of p assets, rather than the entire market. Given A ,
 126 the solar beta is defined as:

$$\beta = \frac{\text{Covariance}(A_p, A_i)}{\text{Variance}(A_p)} = \text{Correlation}(A_p, A_i) \sqrt{\frac{\text{Variance}(A_i)}{\text{Variance}(A_p)}} \quad (4)$$

127 where A_p is the portfolio revenue, and A_i is the revenue from the i^{th} PV
 128 asset.

129 The solar beta is thus the product of two factors: 1) the correlation of
 130 A for a single asset with A for a portfolio of PV assets, and 2) the rela-
 131 tive volatility (the square root of the ratio of variances) of A at the i^{th} site
 132 compared to that for the portfolio of PV assets, i.e. the volatility ratio:
 133 $\sqrt{\text{Var}(A_i)/\text{Var}(A_p)}$. For example, the revenue from a PV asset with a beta
 134 value of +1 fluctuates in perfect synchronization with that from the portfolio,
 135 while conversely the revenue from an asset with a negative beta fluctuates in
 136 the opposite direction from that of the portfolio.[2, 3]

137 Since the beta metric is a function of both correlation and volatility ratio,
 138 a low solar beta value could have two different causes. Low solar beta could
 139 indicate either 1) the revenue from a PV asset has low correlation with that
 140 of the portfolio, or 2) the revenue from a PV asset has low variance relative to
 141 that of the portfolio. In either case, a PV asset with a low solar beta indicates
 142 that the individual asset is a potential “hedge” for the portfolio (i.e., adding
 143 the low-beta asset reduces revenue variance for the portfolio). The solar
 144 beta is calculated on a site-by-site basis, evaluating the contribution of each
 145 individuals site’s variability to the variability of the portfolio.

146 3. Results

147 3.1. Individual PV Assets: Revenue Variance

148 Table 2 displays the mean values of market price (B) and average power
 149 (C), while Table 3 displays the revenue variance and its decomposition for
 150 each of the sites, illustrating the contributions of market price and average
 151 power, both of which reflect prevailing weather conditions at the sites. The
 152 following decomposition terms are described in association with their respec-
 153 tive minimum/maximum percent contributions to the total variance among
 154 the portfolio of sites (listed in order of appearance in Eq. 3).

Table 2: Mean values for Market Price (B) and Average Hourly Power (C), expressed for each site according to evaluation period of seven years.

Location	Mean Price (\$/MWh)	Mean Power (MW $\times 10^{-3}$)
PENLC	48.14	2.60
PPL	52.78	2.58
JCPL	55.65	2.72
PSEG	57.21	2.78
PECO	54.18	2.78
AE	56.78	2.86
BGE	57.10	2.87
PEPCO	57.73	2.96
DPL	55.21	3.04

Table 3: Values for revenue variance (A) and the terms of its decomposition for each of the sites. Where B = Market Price, and C = Average Hourly Power. *Note:* variance units are defined as the square of the units of the variable, hence $Var(A)$ is $(\$/h)^2$, $Var(B)$ would be $(\$/MWh)^2$, and $Var(C)$ (MW) 2 .

Location	$Var(A)$	$\overline{B^2 C'^2}$	$\overline{C^2 B'^2}$	$2\overline{B B' C'^2}$	$2\overline{C B'^2 C'}$	$2\overline{B C B' C'}$	$\overline{B'^2 C'^2}$	$-\overline{B' C'}^2$
PENLC	0.082	0.043	0.006	0.010	0.002	0.005	0.017	-0.000
PPL	0.109	0.050	0.009	0.014	0.004	0.006	0.026	-0.001
JCPL	0.133	0.061	0.012	0.015	0.005	0.008	0.033	-0.001
PSEG	0.143	0.065	0.011	0.018	0.006	0.009	0.034	-0.001
PECO	0.128	0.059	0.011	0.014	0.006	0.008	0.031	-0.001
AE	0.163	0.067	0.015	0.019	0.010	0.010	0.044	-0.001
BGE	0.164	0.068	0.016	0.016	0.010	0.009	0.045	-0.001
PEPCO	0.184	0.072	0.018	0.019	0.012	0.011	0.053	-0.001
DPL	0.147	0.068	0.014	0.016	0.005	0.009	0.035	-0.001

155 The first two terms in the decomposition ($\overline{B}^2 \overline{C'^2}$ and $\overline{C}^2 \overline{B'^2}$) capture the
 156 individual contributions of power variance and price variance to the revenue
 157 variance. The remaining five terms capture the joint contribution of power
 158 and price fluctuations resulting from their co-fluctuation. The first two terms
 159 together explain only 49-59% of the revenue variance, though they tend to
 160 explain a higher proportion of revenue variance for the sites located in zones
 161 that frequently experience transmission congestion (and thus higher market
 162 price spikes than other zones in the PJM footprint). Even so, quantifying
 163 the effect of co-fluctuation between price and power fluctuations is essential
 164 to understanding the revenue variability of each individual PV asset.

165 Inspection shows the individual variance contributions are not of equal
 166 strength ($\overline{B}^2 \overline{C'^2}$: 39-52%; $\overline{C}^2 \overline{B'^2}$: 7-10%). Algebraic manipulation of Eq. 5
 167 reveals that the ratio of $\overline{B}^2 \overline{C'^2} / \overline{C}^2 \overline{B'^2}$ is equivalent to the square of the ratio
 168 of coefficients of price and power. Thus, the relative scale of these two terms
 169 results from power having a greater variability relative to price by a factor
 170 of 5 (this primarily reflects diurnal variations in solar PV system output).

$$\frac{\overline{B}^2 \overline{C'^2}}{\overline{C}^2 \overline{B'^2}} = \frac{\overline{C}^2 / \overline{C'^2}}{\overline{B}^2 / \overline{B'^2}} = \left(\frac{\text{coeff. of var. for power}}{\text{coeff. of var. for price}} \right)^2 \quad (5)$$

171 $\overline{B}^2 \overline{C'^2}$: This term is the square of the mean price times the variance of
 172 power, and explains 39-52% of observed revenue variance. The diurnal
 173 cycle observed in this term reflects that of PV power output as seen
 174 for the PECO site in Figure 3 (Subfigure B relative to Subfigure D).

175 $\overline{C}^2 \overline{B'^2}$: This term is the square of mean power times the variance of price,
 176 and explains 7-10% of observed revenue variance. The diurnal cycle
 177 reflected in this term is thus the diurnal price cycle, not the power cycle
 178 as seen for the PECO site in Figure 3 (Subfigure A relative to Subfigure
 179 C). This term also reflects price variability over longer time scales,
 180 where some years experience much higher price spikes than others (not
 181 shown). It was found that these high-price events are not confined to
 182 any particular season.

183 Each of the remaining terms reflects the effects of fluctuations in both
 184 power and price. Three of them include triple or quadruple products of these
 185 fluctuations making their interpretation nontrivial. Each triple product term
 186 is the covariance of one fluctuating variable with the square of the other. The

Figure 3: Plots of average diurnal behavior for A) $\overline{C^2 B'^2}$ and B) $\overline{B^2 C'^2}$, relative to C) Price (B), and D) Power (C), respectively.

187 quadruple product term is the covariance of the squares of two fluctuating
 188 variables. The other two terms are easier to interpret as they include only
 189 the covariance of price and power.

190 $2\overline{B B' C'^2}$: This triple product term is the mean price times the covariance
 191 of power fluctuations with the square of price fluctuations, and explains
 192 10-13% of observed revenue variance. Thus, the term's diurnal cycle
 193 (PECO site, Figure 4) has the same general shape as $\overline{B^2 C'^2}$, reflecting
 194 the solar diurnal cycle. The inclusion of price fluctuations in the triple
 195 product accounts for the elevated values in the evening and the negative
 196 values in the predawn hours. A time series plot of this term for the
 197 7 years (not shown) reveals that the positive daytime contribution is
 198 frequently larger than the negative nighttime contribution.

199 $2\overline{C B'^2 C'}$: This triple product term is the mean power times the covariance
 200 of the square of price fluctuations with the power fluctuations, and
 201 explains 3-9% of observed revenue variance. Thus, the term's diurnal

Figure 4: Plot of average diurnal behavior for the mean price times the covariance of power fluctuations with the square of price fluctuations ($2\overline{B} \overline{B'C'^2}$).

cycle (PECO site, Figure 5 A.) reflects the broad daytime into evening peak in price as modulated by the power fluctuation cycle (about the mean of Figure 5 B, for the PECO site.) which reaches a peak at noon and changes sign around 7 am and 6 pm.

$2\overline{B} \overline{C} \overline{B'C'}$: This term is just the mean price times the mean power times the covariance of price and power. It explains 6-7% of observed revenue variance. Thus, the term reaches a maximum around noon when price and power are both large (not shown). Even multiplied by mean price and power the covariance contribution is small. Thus, including variance term as a simplistic improvement on the sum of the two variance terms is not sufficient to provide a useful guide for decision making for revenue risk management.

$\overline{B'^2 C'^2}$: This quadruple product term is the covariance of the square of the price fluctuations with the square of the power fluctuations, and explains 21-29% of observed revenue variations. Thus, the diurnal cycle of this term reflects the midday peak of solar power, but with the maximum delayed by about one hour because of the afternoon increase in price.

$-\overline{B'C'}^2$: This term is the square of the covariance of price and power. It makes a very small contribution to revenue variance of approximately 1%.

223 3.2. Portfolio Assessment: Solar Beta

224 The annual values for beta, site correlation, and volatility ratio of revenue
225 are displayed in Figure 6. The results for the the revenue-based solar beta
226 yields values near 1.0 for most of the sites. This occurs because the correlation
227 of most sites with the portfolio is 0.9 while the volatility ratio is slightly
228 above 1.0. Thus, most sites neither add nor subtract from the volatility of
229 the portfolio. Sites at the Northern and Southern ends of the assessed region
230 exhibit lower beta values. Low beta values near the domain ends appear
231 to be influenced more by low correlation with the broader portfolio of sites,
232 although the northernmost and southernmost site also exhibit the lowest
233 volatility ratios relative to the portfolio of sites as a whole. Thus, differences
234 in timing of fluctuations at these sites are a somewhat large contributor to
235 low betas as compared to differences in volatility ratios.[14]

236 From a meteorological perspective, those stations with beta near 1.0 are
237 all found within the Mid-Atlantic temperate climate regime, and are close
238 enough to each other to be affected by the same weather systems. In con-
239 trast, the sites at the N-S perimeter of the portfolio region are influenced
240 by different climate regimes. The northern-most site is under the influence
241 of the Great Lakes, while the southern site experiences subtropical weather
242 during the warm season.

Figure 5: Plot of average diurnal behavior for the mean power times the covariance of the square of price fluctuations with the power fluctuations ($2\bar{C} \bar{B'^2 C'}$) relative to relative to fluctuations from mean Price.

Figure 6: Annual values for beta, site correlation, and volatility ratio for revenue across all site.

243 4. Conclusion

244 Revenue risk for a portfolio of photovoltaic arrays within a region has
245 been described in terms of variability in the total revenue, accounting for
246 seven years of data from PV power production and locational marginal pricing
247 in the PJM region. Here, the dispersion of observed outcomes about
248 expected values defines the risk for an individual site within a portfolio. The
249 respective roles of price and power variability can be addressed by variance
250 decomposition, so as to quantify the relative contributions of these input
251 parameters to revenue risk, or total revenue variance. The beta metric for
252 spatially distributed sites can be used to plan portfolios, so as to quantify
253 and minimize revenue variability (i.e. hedge the revenue stream).

254 While total revenue variance is straightforward to calculate directly, the
255 decomposition of terms for the revenue variance permits assessment of the
256 roles of price and power, their diurnal and seasonal cycles, and the interplay
257 of these cycles. Results in this paper demonstrate that revenue variance
258 of a PV asset depends more on the diurnal, seasonal, and meteorological
259 fluctuations than on the price behavior for a site. Furthermore, an increase
260 in mean price will have a larger effect on revenue variance compared to a
261 proportional increase in mean power production. The co-fluctuation of price
262 and power terms account for the remaining third of the total revenue variance.

263 A solar beta can be defined appropriate to this class of problem that
264 is analogous to the financial beta in portfolio analysis. Illustrating the solar
265 beta for the PJM Mid-Atlantic region, it was observed that reductions in beta
266 at the spatial perimeters of the study were due to change in climate regime
267 (Great Lakes, subtropical coastal) and population-power consumption cycles.
268 This suggests that structuring portfolios of PV assets across climate regimes
269 may be a risk-reduction strategy for investors, assuming that costs and other
270 site characteristics did not materially vary between sites. Furthermore, the
271 correlation, and not the variation of revenue (volatility ratios), drives the
272 beta behavior for sites.

273 The results of this study suggest that more benefit to the investor could
274 be gained if solar energy production sites are considered in light of electricity
275 market conditions and relationships with other sites rather than in isolation.
276 Locations that have not traditionally been thought of as having the most
277 advantageous conditions for peak power generation may have value as part of
278 a geographically diverse portfolio. We have illustrated this effect in the U.S.
279 Mid-Atlantic region but similar logic may hold in other locales (for example,

280 examining the contribution of solar energy in areas such as northern Germany
281 to a broader European portfolio).

282 **Acknowledgements**

283 The authors wish to thank Dr. Lucas Witmer for insight on data acqui-
284 sition and processing, as well as initial TRNSYS modeling training. Blumsack
285 acknowledges support from the National Science Foundation under grant
286 CNS-133176.

- 287 [1] USA Energy Information Administration, Electric power monthly, with
 288 data for december 2013, Tech. rep., US Department of Energy (February
 289 2014).
 290 URL <http://www.eia.gov/electricity/monthly/pdf/epm.pdf>
- 291 [2] J. C. Hull, Options, future and other derivatives, Pearson Education,
 292 Inc, 2009.
- 293 [3] J. C. Francis, D. Kim, Modern Portfolio Theory: Foundations, Analysis,
 294 and New Developments, John Wiley & Sons, Hoboken, N.J, 2013.
- 295 [4] T. E. Hoff, R. Perez, Quantifying pv power output variability, Solar
 296 Energy 84 (2010) 1782–1793.
- 297 [5] R. Perez, K. Zweibel, T. E. Hoff, Solar power generation in the us: Too
 298 expensive, or a bargain?
 299 URL <http://www.asrc.cestm.albany.edu/perez/2011/solval.pdf>
- 300 [6] J. L. Bosch, J. Kleissl, Cloud motion vectors from a network of ground
 301 sensors in a solar power plant, Solar Energy 95 (2013) 13–20.
- 302 [7] M. Lave, J. Kleissl, Cloud speed impact on solar variability scaling–
 303 Application to the wavelet variability model, Solar Energy 91 (2013)
 304 11–21.
- 305 [8] V. Fung, J. L. Bosch, J. Kleissl, Cloud shadow speed sensor, Atmos.
 306 Meas. Tech. 7 (2014) 1693–1700.
- 307 [9] SolarAnywhere, Solar data (2013).
 308 URL <https://www.solaranywhere.com/Public/SelectData.aspx>
- 309 [10] PJM, Pjm real-time energy markets (2013).
 310 URL [http://www.pjm.com/markets-and-operations/energy/real-](http://www.pjm.com/markets-and-operations/energy/real-time.aspx)
 311 [time.aspx](http://www.pjm.com/markets-and-operations/energy/real-time.aspx)
- 312 [11] S. A. Klein, W. A. Beckman, J. W. Mitchell, J. A. Duffie, N. A. Duffie,
 313 T. L. Freeman, J. C. Mitchell, J. E. Braun, B. L. Evans, J. P. Kummer,
 314 R. E. Urban, A. Fiksel, J. W. Thornton, N. J. Blair, P. M. Williams,
 315 D. E. Bradley, T. P. McDowell, M. Kummert, D. A. Arias, TRNSYS 17:
 316 A transient system simulation program (2010).
 317 URL <http://sel.me.wisc.edu/trnsys>

- 318 [12] SunPower Corporation, X-Series Solar Panels: Supplementary Technical
319 Specifications, document no. 505915 rev a Edition (April 2013).
320 URL <http://us.sunpower.com/support/datasheets/>
- 321 [13] H. A. Panofsky, J. A. Dutton, Atmospheric turbulence: models and
322 methods for engineering applications, John Wiley & Sons, New York,
323 1984.
- 324 [14] J. Rayl, G. S. Young, J. R. S. Brownson, Irradiance co-spectrum anal-
325 ysis: Tools for decision support and technological planning, Solar Ener-
326 gydoi:10.1016/j.solener.2013.02.029.

327 Appendix A. TRNSYS Component Specifications

328 The following list specifies all of the inputs, outputs, and parameters of
329 the various components of the TRNSYS simulation components used for this
330 study. Information that is exchanged between the components is indicated
331 by the arrowed links in Figure 2).

332 1. *Data Reader*

333 (a) Inputs

- 334 • GHI and DNI data accessed from outside source
- 335 • LMP data accessed from outside source

336 (b) Outputs

- 337 • GHI and DNI data to *Irradiation Processor*
- 338 • LMP data to *Calculator* and to *Plot and Print*

339 2. *Irradiation Processor*

340 (a) Inputs

- 341 • GHI and DNI from *Data Reader*

342 (b) Parameters

- 343 • Solar constant (4899.6 kJ/h)
- 344 • Ground reflectance (0.2 for all sites)
- 345 • Latitude (site specific - Table 1)
- 346 • Shift in solar time (site specific - Table A.4)
- 347 • Slope of surface (site specific - Table A.4)

348 (c) Outputs to *PV (w/ Inverter)*

- 349 • Total incident radiation on tilted surface
- 350 • Incidence angle on tilted surface
- 351 • Sky diffuse radiation on tilted surface
- 352 • Beam radiation on tilted surface
- 353 • Array slope

354 3. *4-parameter PV (w/ Inverter)*

355 (a) Inputs from *Irradiation Processor*

- 356 • Total incident radiation on tilted surface
- 357 • Incidence angle on tilted surface
- 358 • Sky diffuse radiation on tilted surface

359 • Beam radiation on tilted surface
360 • Array slope
361 (b) Parameters
362 • Load voltage (120 V)
363 • Module short-circuit current and open-circuit voltage (6.2 A,
364 21.6 V)
365 • Module maximum power current and voltage (5.84 A, 42.8 V)
366 • Number of cells (72)
367 • Number of modules in array (100)
368 • Module area (1.244 m²)
369 • Ambient temperate (293 K)
370 • Semiconductor bandgap
371 (Monocrystalline - 1.12)
372 • Maximum inverter power
373 (10000 W)
374 • Maximum inverter voltage
375 (1000 V)
376 (c) Outputs
377 • Maximum Power Point (W) to *Calculator*
378 • Maximum Power Point (W) to *Plot and Print*
379 4. *Calculator*
380 (a) Inputs
381 • LMP (\$/MWh) from *Data Reader*
382 • Maximum power (W) from *PV (w/ Inverter)*
383 (b) Calculations
384 • $\text{LMP (\$/MWh)} \times \text{Avg Power (W)} \times (1 \text{ MW}/1\text{E}6 \text{ W}) = \text{Rev-}$
385 $\text{enue (\$/h)}$
386 (c) Outputs
387 • Revenue (\$) to *Plot and Print*
388 5. *Plot and Print*
389 (a) Inputs/Outputs
390 • LMP (\$/MWh) from *Data Reader*
391 • Maximum Power Point (W) from *PV (w/ Inverter)*
392 • Revenue (\$/h) from *Calculator*

393 Note that two of the parameters for the *Irradiation Processor* are “site
 394 specific”—the shift in solar time and the slope of the surface. Table A.4 gives
 395 a list of these solar shifts and slope tilts for each location.

Table A.4: List of shift in solar hour and PV surface slope for all locations

Site #	Surface Slope	Solar Shift
1	34°	−2.5
2	34°	−0.8
3	33°	+0.3
4	33°	+0.4
5	32°	−0.4
6	32°	+0.5
7	31°	−1.7
8	30°	−1.7
9	29°	−1.0

396 Appendix B. Variance Derivation Proof

397 The following proof was developed to confirm the use of Eq. 3 for the
398 variance analysis.

Let: $A = B * C$ and, by definition:

$$Var(A) = \overline{(A - \bar{A})^2}$$

$$Var(A) = \overline{(B * C - \bar{B} * \bar{C})^2}$$

$$Var(A) = \overline{B^2 * C^2} - \overline{2 * B * C * \bar{B} * \bar{C}} + \overline{B * \bar{C}}^2$$

$$Var(A) = \overline{B^2 * C^2} - 2 * \overline{B * \bar{C}} * \overline{B * \bar{C}} + \overline{B * \bar{C}}^2$$

$$Var(A) = \overline{B^2 * C^2} - 2 * \overline{B * \bar{C}}^2 + \overline{B * \bar{C}}^2$$

$$Var(A) = \overline{B^2 * C^2} - \overline{B * \bar{C}}^2$$

399 Recalling that a particular value of a variable (X) equals the sum of
400 that point's perturbation from the mean (X') and the overall mean of the
401 variable ($\bar{X}$). In other words: $B = \bar{B} + B'$ and $C = \bar{C} + C'$. Using this
402 definition of the variance of A can be expressed in terms of the mean and
403 fluctuating components of B and C. Note that by this definition the mean of
404 any fluctuation is zero and mean quantities can be factored out of averages.

$$Var(A) = \overline{(\bar{B} + B')^2 (\bar{C} + C')^2} - \overline{(\bar{B} + B') (\bar{C} + C')}^2$$

$$Var(A) = \overline{(\bar{B}^2 + 2\bar{B}B' + B'^2)(\bar{C}^2 + 2\bar{C}C' + C'^2)} - \overline{\bar{B} \bar{C} - \bar{B}C' - B'\bar{C} - B'C'}^2$$

$$Var(A) = \overline{B^2 \bar{C}^2} + \overline{2\bar{B}^2 \bar{C}C'} + \overline{B^2 C'^2} + \overline{2\bar{B}B'\bar{C}^2} + \overline{4\bar{B}B'\bar{C}C'} + \overline{2\bar{B}B'C'^2} + \overline{B'^2 \bar{C}^2} + \overline{2B'^2 \bar{C}C'} + \overline{B'^2 C'^2} - (\bar{B} \bar{C} + \bar{B}'C')^2$$

$$\begin{aligned}
Var(A) = & \overline{B^2} \overline{C'^2} + \overline{C^2} \overline{B'^2} + 2\overline{B} \overline{B'C'^2} + 2\overline{C} \overline{B'^2C'} \\
& + 2\overline{B} \overline{C} \overline{B'C'} + \overline{B'^2C'^2} - \overline{B'C'}^2
\end{aligned}$$

405 This equation was used to calculate the contributions of power (MW) and
406 price (\$/MWh) to revenue (\$/h), given that revenue is a product of hourly
407 average solar irradiation power and locational marginal pricing.