A multi-objective assessment of the effect of solar PV array orientation and tilt on energy production and system economics

Joshua D. Rhodes^{a,*}, Charles R. Upshaw^b, Wesley J. Cole^c, Chris L. Holcomb^d, Michael E. Webber^{b,e,**}

^a*Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin, 301 E Dean Keeton St. Stop C1752, Austin, TX 78712-1094*

^b*Department of Mechanical Engineering, The University of Texas at Austin, 204 E. Dean Keeton Street, Stop C2200, Austin, Texas 78712-1591*

^c*Department of Chemical Engineering, The University of Texas at Austin, 200 E. Dean Keeton St. Stop C0400, Austin, Texas 78712-1591*

^d*The Butler Firm, 1601 Rio Grande St #331, Austin, TX 78701*

^e*Energy Institute – The University of Texas at Austin, 2304 Whitis Ave Stop C2400 Austin TX 78712-1718*

Abstract

This analysis considers the effect of the placement (azimuth and tilt) of fixed solar PV systems on their total energy production, peak power production, and economic value given local solar radiation, weather, and electricity market prices and rate structures. This analysis details a model that was used to calculate the output of solar PV systems across a range of azimuths and tilts to find the energetically and economically optimal placement. The result of this method, which concludes that the optimal placement can vary with a

*Corresponding author

**Principal corresponding author

Email addresses: joshdr@utexas.edu (Joshua D. Rhodes), crupshaw@gmail.com (Charles R. Upshaw), wesleycole@gmail.com (Wesley J. Cole), chris.lee.holcomb@gmail.com (Chris L. Holcomb), webber@mail.utexas.edu (Michael E. Webber)

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multitude of conditions, challenges the default due-south placement that is conventional for typical installations. We found that for Austin, TX the optimal azimuth to maximize energy production is about $187 - 188^\circ$, or $7 - 8^\circ$ west of south, while the optimal azimuth to maximize economic output based on the value of the solar energy produced is about $200 - 231^\circ$ or $20 - 51^\circ$ west of south. The differences between due south (which is the conventional orientation) and the optimal placement were on the order of 1–7%. Solar PV arrays that are not constrained on their placement, such as ground mounted arrays, could possibly increase overall production or the value of that production at no additional installation cost by a slight rotation from the traditional orientation of due south. The model is then extended to consider 1,020 locations across the United States that have Typical Meteorological Year (TMY) data to provide a spatial view of optimal solar placements across the country. For the rest of the US we found that for most locations the energetically optimal solar PV azimuth is within 10° of south. Considering the temporal value of the solar energy produced from spatially-resolved market conditions derived from local time-of-use rates, the optimal placement shifts to a west-of-south azimuth in attempt to align solar energy production with higher afternoon prices and higher grid stress times. There are some locations particularly on the west coast that have west-of-south energy optimal placements, possibly due to typical morning clouds or fog. Further solar considerations, such as maximum peak alignment also have the potential to influence solar placement. These results have the potential to be significant for solar PV installations: utilities might alter rate structures to encourage solar generation that is more coincident with peak demand, utilities might also use west-of-south

solar placements as a hedge against future wholesale electricity price volatility, building codes might encourage buildings to match roof azimuths with local optimal solar PV generation placements, and calculations of the true value of solar in optimal and non-optimal placements can be more accurate.

Keywords:

optimal solar placement, maximum value, peak power demand

1. Introduction

This analysis considers the effect of the placement (azimuth and tilt) of fixed solar PV systems on their total energy production, peak power production, and the economic value of that energy production. Solar energy production is important for a multitude of reasons including reduced carbon emissions; a fuel source that is free, renewable, domestic and distributed; its contributions for energy equity, and because the prices of solar panels are falling [1–6]. As a consequence of all these factors, in the US, solar installations are expected to grow significantly and outpace new fossil fuel power plants in the mid-future [7]. Thus, analytical methods – such as the one developed herein – that optimize the usefulness of those installations are valuable.

There have been many investigations into the optimal tilt for solar PV systems. These investigations were summarized in a recent review by Yadav and Chandel [8] and therefore will not be reproduced here. However, it is important to note that many of these analyses consider solar energy production assuming that a southern azimuth (in the northern hemisphere) is optimal for energy production. While the south-orientated rule-of-thumb

19 might be best for completely clear skies, non-uniform, temporal meteorolog-
20 ical conditions such as fog or clouds, environmental conditions such as smog,
21 and geographic features such as mountains can block solar radiation and re-
22 duce solar panel output at different times of the day and change the optimal
23 orientation of the panels [9, 10]. Additionally, solar PV power output is a
24 function of panel temperature [11], so dry bulb temperature fluctuations and
25 wind speed (because of convective heating or cooling) alter PV electricity
26 production. Lave and Kleissl [10] determined the optimal tilt and azimuth
27 angles for the continental United States using a high resolution grid (0.1° by
28 0.1°). They considered solar radiation effects, such as cloud cover, but not
29 temperature effects.

30 Another consideration for optimal PV orientation is the value of the elec-
31 tricity generated. Because solar energy production does not always precisely
32 align with maximum electricity grid load, even placements that might be
33 non-optimal from an energy production basis might be optimal on an eco-
34 nomic or peak power production basis [12–14]. Luoma et al. [15] used day-
35 ahead market electricity prices to determine optimal solar PV orientations
36 in California. Their work found that the market electricity prices shifted the
37 optimal orientation of some arrays further west of south.

38 This analysis extends the body of knowledge by estimating optimal solar
39 orientations for 1,020 U.S. locations while considering the effects of dry bulb
40 temperature, wind speed, and actual (i.e., not clear-sky) solar radiation. This
41 analysis also considers the value of solar energy produced by using time-of-
42 use rates throughout the continental US as a proxy for average local grid
43 conditions. This work differs from similar solar placement work [10] in that

we consider the production of AC electricity (after panel, inverter, and other derate losses) as the metric for optimal placement. This work also differs from other optimal economic placement work [15] by expanding to the entire US and (at least for Austin) considering multiple economic inputs.

2. Methods

This analysis uses available solar insolation and electricity price data to 1) determine the insolation on a given plane (with available solar radiation tools), 2) build a residential system-based solar PV production model, 3) estimate the total energy, power, and economic impacts of system azimuth and tilt (placement) for Austin, TX, 4) extend the analysis to other locations across the US, and 5) explore the peak power production and solar ramp rate implications of varying solar placements using the aforementioned datasets. The workflow is given in Figure 1.

Figure 1: This diagram shows the process of modeling the energy and the value of energy produced from various solar placements.

57 2.1. Determining incident solar insolation on a plane

58 Because the goal of this analysis is to determine the value of various
59 solar placements, it was necessary to calculate the incident solar radiation
60 for multiple azimuths and tilts. There have been many investigations into
61 models that simulate total solar radiation on a tilted plane from that of the
62 horizontal, and readers are directed to texts and reviews of those studies [16,
63 17]. There are three main components of solar radiation on a surface, 1) direct
64 beam, 2) diffuse from the sky, 3) and the reflected from the ground [16].
65 Many models use the same methods for calculating the direct beam and
66 reflected, but differ on methods of calculating incident diffuse from the sky.
67 This analysis utilized the *solaR* package of the *R* language [18], which uses
68 the diffuse model of Hay and Davies [19] with the the horizon brightness
69 correction proposed by Reind et al. [20]. The total effective radiation on a
70 tilted plane is given by Equation 1:

$$I_T = B_T + R_T + D_T \quad (1)$$

71 where I_T is the incident radiation on the tilted plane, B_T is the beam radia-
72 tion on the tilted plane given by Equation 2:

$$B_T = I_B R_B \quad (2)$$

73 where I_B is the beam radiation on the horizontal plane, R_B is the ratio of
74 beam radiation on the tilted plane to that on a horizontal surface given by

Equations 3–5 [17]:

$$R_B = \frac{\cos(\theta)}{\cos(\theta_z)} \quad (3)$$

$$\begin{aligned} \cos(\theta) = & \sin(\delta) \sin(\phi) \cos(\beta) - \sin(\delta) \cos(\phi) \sin(\beta) \cos(\gamma) \\ & + \cos(\delta) \cos(\phi) \cos(\beta) \cos(\omega) + \cos(\delta) \sin(\phi) \sin(\beta) \cos(\gamma) \cos(\omega) \\ & + \cos(\delta) \sin(\beta) \sin(\gamma) \sin(\omega) \end{aligned} \quad (4)$$

$$\cos(\theta_z) = \cos(\phi) \cos(\delta) \cos(\omega) + \sin(\phi) \sin(\delta) \quad (5)$$

where δ is the declination, ϕ is the local latitude, β is the tilt of the surface, γ is the surface azimuth angle, and ω is the hour angle.

$R_{T,i}$ is the reflected radiation on the tilted plane given by Equation 6:

$$R_T = \alpha I_0 \frac{1 - \cos(\beta)}{2} \quad (6)$$

where α is the albedo of the ground (assumed to be 0.2) and I_0 is the total incident radiation on the horizontal plane.

The diffuse component D_T from Hay and Davies [19] with the the horizon brightness correction proposed by Reindl et al. [20] is given by Equation 7:

$$D_{T,i} = D_0 \left[\frac{1}{2} (1 - \kappa) (1 + \cos(\beta)) \times \left(1 + \sqrt{\frac{B_0}{G_0}} \times \sin(\beta/2)^3 \right) + \kappa R_{B,i} \right] \quad (7)$$

where D_0 is the diffuse radiation on the horizontal plane, κ is the anisotropy index such that $\kappa = I_B/I_0$ where I_0 is the solar constant, B_0 is the beam irradiance on the horizontal surface, and G_0 is the global irradiance on the horizontal surface.

There are three sources of solar data used in this analysis, 1) actual measured data for Austin, 2) TMY data, and 3) clear-sky data. For Equations 1–7, in cases where the data were ‘measured’ data, the horizontal beam and the diffuse were directly measured from pyranometers located in Austin, TX. In cases where TMY data were utilized in this analysis, the diffuse horizontal data in the TMY data was used as the diffuse horizontal value in Equations 1–7 and the beam horizontal value used is the difference between the global and diffuse horizontal values in the TMY data. In the case where the data used is the clear-sky data, the values were directly calculated.

2.2. Solar PV System Energy Production Model

To estimate AC solar PV electricity production from solar radiation on a plane, a solar PV energy production model was built. The overall model is given in Equations 8 – 9:

$$P_{out,i} = \eta_{pv,i} \times \eta_{inv,i} \times \eta_{other} \times I_{T,i} \quad (8)$$

$$\eta_{pv,i} = \eta_{ref} \left[1 - \beta_{ref} \left[T_{a,i} - T_{ref} + (T_{NOCT} - T_{a,NOCT}) \frac{I_{T,i}}{I_{NOCT}} \right] \right] \quad (9)$$

where $P_{out,i}$ is the power output of the system in W/m² of PV array, $\eta_{pv,i}$ is the efficiency of the solar PV panels, $\eta_{inv,i}$ is the efficiency of the (DC–AC) solar inverter, η_{other} are other (in this case constant) derate factors

103 (mismatch, connections, wiring, etc., taken to be 0.93) associated with solar
 104 PV arrays, $I_{T,i}$ is the incident radiation on the tilted plane (Equation 1), η_{ref}
 105 is the efficiency of the PV panels (taken to be 12%), β_{ref} is the temperature
 106 coefficient of the PV panels (taken to be 0.0045 K^{-1}), T_a is the ambient
 107 temperature, T_{ref} is the reference temperature of the PV panels ($25 \text{ }^\circ\text{C}$),
 108 T_{NOCT} is the nominal operating cell temperature at operating test conditions,
 109 $T_{a,NOCT}$ is the ambient NOCT ($20 \text{ }^\circ\text{C}$) and I_{NOCT} is the incident radiation for
 110 the NOCT test, which is 800 W/m^2 , for an overview of the NOCT equations,
 111 see Skoplaki and Palyvos [21], all (besides constants) at time i . The efficiency
 112 of the modeled inverter $\eta_{inv,i}$ was modeled as a 6^{th} degree polynomial fit of
 113 a commercially available solar inverter (Power-One PVI-5000 [22]), scaled
 114 from a nominal 5 kW PV array of commercially available solar PV panels
 115 (Lumos LS250 [23]) to a per m^2 of array. Note that the constants used in this
 116 analysis are average and thus the output might differ for other technologies,
 117 for example, thin film modules are less affected by temperature, so they might
 118 have more western optimum placements as they are able to hold closer to
 119 their rated efficiency as their temperature increases.

120 *2.3. Calculating the value and effect of solar placement*

121 After construction of the solar PV model based on meteorological, astro-
 122 nomical conditions, and assumed PV characteristics, a second model was de-
 123 veloped to calculate the solar PV electricity produced from a solar PV system
 124 for any given placement, accommodating different tilts and azimuths. This
 125 model consisted of three steps. First, given a placement and the horizontal
 126 solar radiation values, it calculated the solar radiation on a plane. Second,
 127 using the solar PV model developed in Section 2.2 and weather data, it cal-

128 culated the energy produced at that placement. The last step calculated the
 129 value of the energy produced using either local market conditions from the
 130 Electric Reliability Council of Texas (ERCOT) or local utility rates. ERCOT
 131 settlement point prices (SPP) are actual wholesale electricity market rates,
 132 available on a 15-minute interval, for the ERCOT grid (about 85% of Texas).
 133 ERCOT SPP data used in this analysis included ERCOT 2011 (the actual
 134 prices from 2011, averaged on the hour; 2011 was considered to be a scarcity
 135 year with higher than usual prices), ERCOT AVG (the average ERCOT price
 136 for 2011 – 2013, averaged on the hour), ERCOT AUCT (the actual ERCOT
 137 prices for the time of the measured Austin solar radiation data). Local utility
 138 rates were collected from the OpenEI utility rate database [24].

139 All possible combinations of azimuths ranging from 90° (due east) to 270°
 140 (due west) and tilts from 0° (horizontal) to 45° were used to calculate the so-
 141 lar insolation on the respective plane. These data were fed into the Solar PV
 142 Energy Production Model (Section 2.2), along with weather data to quantify
 143 the amount of energy produced over one year for that particular configu-
 144 ration. These data were then multiplied by the temporally corresponding
 145 ERCOT electricity market price (for the Austin specific analysis) or TOU
 146 rate data (for the national analysis) and summed to calculate the value of
 147 the solar energy produced as per Equation 10:

$$Value = \sum_i^{1year} P_{out,i} \times \Delta t \times Price_{1,i} + P_{out,i} \times Price_{2,i} \quad (10)$$

148 where $P_{out,i}$ is the power output of the solar PV system in W, Δt is the time-
 149 step, $Price_{1,i}$ is the economic price (ERCOT SPP or TOU rate, \$/kWh), and
 150 $Price_{2,i}$ is the price associated with reduction in overall demand charges for

151 a commercial or industrial consumer that has the solar PV system behind
 152 the meter, all at time i . For industrial solar power plants, $Price_{2,i}$ could
 153 also be used to estimate ancillary service value or a capacity payment. For
 154 this analysis $Price_{2,i}$ was considered to be fixed at 0 (because Texas has
 155 an energy-only market), but it could be considered in another analysis that
 156 looked at markets with capacity payments or the ability of solar PV to reduce
 157 demand changes for arrays behind the meter (many commercial and indus-
 158 trial customers have demand changes in addition to energy charges). This
 159 price would be appropriate to consider on a case by case basis, but is beyond
 160 the scope of this more general analysis. This calculation was then completed
 161 for multiple radiation inputs (measured, TMY, and clear-sky), weather in-
 162 puts (measured and TMY), and pricing inputs (market and electric rate) for
 163 Austin.

164 *2.4. Optimal Solar Placement on a National Level*

165 The analysis was then generalized to a national level. TMY data were
 166 gathered and processed in the same way as described in Section 2.3 for 1,020
 167 locations across the US [25]. The data were run through the solar placement
 168 value (Section 2.3) program in a similar fashion. However, to speed up the
 169 process, an optimization routine (R function `optim`, with method of Byrd et.
 170 al. [26]) was used so that each location’s placements of both energy and value
 171 did not have to be directly computed. The optimization used a quasi-Newton
 172 method (L-BFGS-B), a version of the Broyden–Fletcher–Goldfarb–Shanno
 173 algorithm with box constraints [27]. The initial starting point was taken to
 174 be the local location’s rule of thumb placement. The optimization reduced
 175 the solving time from 12 hours to less than 1 hour on a 16 core HPC.

176 The expanded model considered both total energy produced, power pro-
 177 duced, and the value of the produced energy. The energy-only model is the
 178 same as for the more Austin-specific analysis. However, the value of energy
 179 model was somewhat different. In order to consider the regional differences
 180 in electricity markets, local TOU energy rates were used as a proxy for the
 181 temporal value of energy, as it was assumed that these rates would be de-
 182 signed such that times of higher costs would be typically associated with
 183 times of higher grid stress/demand. An attempt was made to obtain an
 184 electric utility TOU rate for each state from the OpenEI database [24]. Of
 185 the 50 US states, at least 37 states had TOU rates for at least one customer
 186 class. When each simulation was run, the amount of energy produced during
 187 a given interval was multiplied by the rate for that hour. For all locations
 188 within a given state, a single representative rate from a large city in that
 189 state was used, for states without TOU rates, the closest (shortest Euclidian
 190 distance) to the nearest rate (by lat/long of largest city in each state) was
 191 used. However, when making recommendations for a specific location, using
 192 exact local data would yield the best results.

193 *2.5. Optimal Placement for Summer Peak Reduction*

194 The last step of the analysis was to explore the effects of solar placement
 195 on summer peak power reduction. The summer peak times are defined as
 196 June – August, from 14:00 – 20:00 CST for Austin, TX [28]. These times
 197 are typically associated with high wholesale electricity prices and grid stress,
 198 mainly due to residential air-conditioning load [29]. For this analysis the
 199 same approach was taken as with Equation 10, except the $Price_{1,i}$ was given
 200 a value of 1 during summer peak hours and 0 otherwise.

201 3. Results and Discussion

202 The following section discusses the results of the model and presents a
203 discussion of the results and implications of this analysis for solar energy
204 production.

205 3.1. Calculating the value and effect of solar placement

206 To verify that the entire process was running correctly, the model was
207 first executed with clear-sky radiation. Because there is no clear-sky equiv-
208 alent weather information available, TMY weather data (temperature, wind
209 speed) were used. Using clear-sky as the radiation input should cancel out
210 all weather shading effects to solar panel production and should indicate
211 an optimal energy azimuth of due south and a tilt related to the local lati-
212 tude (about 30° for Austin). The model provided just that result, shown in
213 Figure 2.

214 Table 1 summarizes the results of the various cases for both total energy
215 production and the value of the energy produced in Austin, TX. For the cases
216 where the placement was optimized for maximum total energy generated, the
217 TMY case shifts the arrays about 8° west of south. For the cases where the
218 placement was optimized for maximum total economic value of the energy
219 generated, the cases shift the arrays about 20 to 51° west of south, depending
220 on the price considered. While the increase in the amount of energy generated
221 in the optimal cases was negligible, the increased economic values ($\$/\text{m}^2/\text{yr}$)
222 for shifting the solar PV arrays west of south were on the order of 1–7%.

223 Figure 2 shows the total number of kWh per year produced (normalized
224 for 1 m^2 of array) for every combination of azimuth and tilt, 90° – 270° and

Table 1: Cases for for analysis of multiple radiation, weather, and economic data inputs for Austin, TX.

Radiation Inputs	Weather Inputs	Pricing Data	Optimal Placement	Units	Optimal Value	Value at 180°/30°	Percent Change	Figure Reference
Clear Sky	TMY	–	180°/30°	kWh/m ² /yr	245.8	245.8	0.0	Figure 2
TMY	TMY	–	188°/28°	kWh/m ² /yr	156.3	156.0	0.2	Figure 3
Measured	Measured	ERCOT AUCT	204°/25°	\$/m ² /yr	5.94	5.85	1.6	Figure 4
TMY	TMY	ERCOT AVG	219°/29°	\$/m ² /yr	7.97	7.67	3.9	Figure 5
TMY	TMY	AE TOU Rate*	200°/25°	\$/m ² /yr	11.07	10.96	1.0	Figure 6
TMY	TMY	ERCOT 2011	231°/30°	\$/m ² /yr	10.24	9.58	7.0	Figure 7

*Austin Energy residential time-of-use rate

225 0°–45°, respectfully using clear-sky radiation and TMY weather data.

226 Figure 3 shows the effect of using TMY radiation and weather on optimal
227 placement. The number of kWhs overall are reduced compared to Figure 2
228 because this data include the effect of clouds on the amount of solar radiation
229 that reaches the earth’s surface. It is interesting to note that using ‘typical
230 meteorological’ data for Austin indicates that shifting the array west 8° and
231 2° towards the horizontal (from the 180°/30° rule-of-thumb) produces the
232 most amount of energy, about 0.2% more than the rule-of-thumb. These
233 results would suggest that meteorological events such as cloud cover in the
234 mornings and winter typically block the sun more than in the afternoon and
235 summer.

236 Figures 4 and 5 show the optimal azimuth for the value (\$/m²/year)
237 of electricity produced (Equation 10) for the 2012-2013 measured data and
238 coincident ERCOT prices and the TMY data with average ERCOT prices,
239 respectfully. Both figures show that placement is shifted west when optimiz-
240 ing based on market value.

Figure 2: Heat map of model results for clear-sky radiation and TMY weather showing an optimal energy azimuth of 180° and 30° tilt, as expected for Austin, TX. Contour lines show areas of percent of maximum energy in 5% increments.

Figure 3: Heat map of model results for TMY radiation and TMY weather showing an optimal energy azimuth of 188° and 28° tilt, indicating the due south azimuth might not be optimal for total energy generation in Austin, TX when typical meteorological conditions are considered. Contour lines show areas of percent of maximum energy in 5% increments.

Figure 4: Heat map of model results for measured 2012-2013 radiation and weather with coincident ERCOT prices showing an optimal value (\$/m²/year) azimuth of 204° and 25° tilt for Austin, TX. Contour lines show areas of percent of maximum values in 5% increments.

Figure 5: Heat map of model results for TMY radiation and weather with average ERCOT prices showing an optimal value (\$/m²/year) azimuth of 219° and 29° tilt for Austin, TX. Contour lines show areas of percent of maximum values in 5% increments.

241 Figure 6 shows the values associated with Austin TMY solar radiation
242 and weather with Austin Energy’s residential TOU rate and also shows shows
243 how azimuth and tilt are related under the TOU rate. For example, if a
244 solar PV array’s azimuth were constrained to 150° , its optimal tilt is not the
245 25° associated with the unconstrained array, but 18° , a 0.5% ($\$/\text{m}^2/\text{year}$)
246 difference.

247 Figure 7 shows the values associated with Austin TMY solar radiation
248 and weather with ERCOT prices from 2011.

249 In general, higher summer electricity prices drive azimuth west and tilt
250 towards the horizontal [10], as is seen in the Austin + TOU rate case. How-
251 ever, the later and the higher the electricity prices, the further the sun has
252 dipped in the sky and the steeper the tilt will need to be to capture the in-
253 cident radiation as seen in the TMY + ERCOT AVG and TMY + ERCOT
254 2011 cases. The TMY + ERCOT 2011 case is taken to be a scarcity pricing
255 scenario as that year the price cap ($\$3000/\text{MWh}$) in ERCOT was hit for 54
256 15-minute periods, most after 16:00 local time, versus just once in 2012 and
257 twice in 2013. However, the current price cap in ERCOT has been raised to
258 $\$5000/\text{MWh}$ with plans to further raise to $\$9000/\text{MWh}$. A high price cap
259 and more instances of scarcity pricing (during historically consistent times of
260 the day) could have an impact on the optimal placements of fixed solar PV
261 installations, namely further west with a steeper tilt. Utilities could incen-
262 tivize these solar placements as a hedge towards a more volatile wholesale
263 electricity market.

Figure 6: Heat map of model results for TMY radiation and weather with Austin Energy Residential TOU rate showing an optimal value ($\$/\text{m}^2/\text{year}$) azimuth of 200° and 25° tilt for Austin, TX. Contour lines show areas of percent of maximum values in 5% increments.

Figure 7: Heat map of model results for TMY radiation and 2011 ERCOT prices showing an optimal value (\$/m²/year) azimuth of 231° and 30° tilt for Austin, TX. Contour lines show areas of percent of maximum values in 5% increments.

264 *3.2. Optimal Solar Placement on a National Level*

265 *3.2.1. Optimal azimuth*

266 The same analysis was performed on data from 1020 locations in the US
267 that have TMY data. Figure 8 shows the results of the energy-only analysis.

Figure 8: Map of continental US showing the energetically optimal azimuth of solar PV systems. The red points indicate optimal solar azimuths east of south (less than 160°), orange points indicate southerly optimal solar azimuths ($160^\circ - 170^\circ$), green points indicate southerly optimal solar azimuths ($170^\circ - 190^\circ$), purple points indicate slightly west of south optimal solar azimuths ($190^\circ - 200^\circ$), and blue points indicate optimal azimuths west of south (greater than 200°).

268 Most of the locations in the US fall within the south-facing band ($170^\circ -$
269 190°) indicating that the rule-of-thumb approach might work for most of the
270 country. However there are some notable exceptions. The results indicate
271 that a band of locations from Wyoming, through Colorado, and into New

272 Mexico have optimal azimuths that face east of south. This result is probably
273 explained in that these locations are along the Rocky Mountains and have
274 prevailing summer afternoon thunderstorms. Another location of interest the
275 US west coast, where the model indicates an optimal west-facing azimuth for
276 almost the entire length of the coast. This effect is probably explained by
277 persistent cloudiness in the early part of the day. Similar results have been
278 observed in earlier studies as well [10, 15].

279 These findings are significant because Colorado ranks 10th (130 MW) in
280 the US for installed solar capacity and California ranks 1st at 2,051 MW
281 (Texas ranks 7th at 174 MW) [30]. While data on the actual placements of
282 solar PV systems are not readily available, if roughly half of California’s small
283 scale (< 25 kW capacity) solar output could be improved 1% via an optimal
284 placement, the result would be an additional production of approximately
285 15 million kWh/year at current capacity levels – about the total annual
286 electricity consumption of 2,200 California homes [31].

287 Figure 9 shows the results when considering the maximum economic value
288 of the solar energy produced for all considered solar placements. Again, the
289 value of the electricity produced is approximated by the structure of a utility
290 TOU pricing structure that is either in the state of the TMY data location,
291 or if the state doesn’t have a TOU rate available, the closest location with a
292 TOU rate was chosen.

293 Overall the economic consideration shifts the number of optimally south-
294 facing (170°–190°) array locations from 920 (pure energy analysis) to 477
295 locations. The number of slightly west-facing (190°–200°) array locations
296 increased from 62 to 234, the number of further west-facing arrays (greater

Figure 9: Map of continental US showing optimal azimuth of solar PV systems when considering the value of the solar energy produced. The red points indicate optimal solar azimuths east of south (less than 160°), orange points indicate southerly optimal solar azimuths ($160^\circ - 170^\circ$), green points indicate southerly optimal solar azimuths ($170^\circ - 190^\circ$), purple points indicate slightly west of south optimal solar azimuths ($190^\circ - 200^\circ$), and blue points indicate optimal azimuths west of south (greater than 200°).

297 than 200°) increased from 2 to 264, the number of slightly east-facing (160° –
 298 170°) array locations increased from 36 to 45. However, because the rates
 299 were not the same, the change of optimal economic placement is different
 300 for different locations. A significant portion of the western half of the US,
 301 including Texas shifted west. Even some arrays that had an optimal energy
 302 placement east of south in Colorado trended west. The eastern half of the
 303 US, which for energy only was almost all south-facing is more mixed. For
 304 example, the TOU rates of Virginia and South Carolina, while they have
 305 higher summer afternoon prices, also have high morning prices in the winter
 306 months, presumably due to a morning grid peak from electrical heating or
 307 activity demand. Other states, such as New York, had rate structures that
 308 reflect higher afternoon grid demand.

309 *3.2.2. Optimal Tilt*

310 While best practices would have tilts of solar systems determined by local
 311 solar data as is attempted in this analysis, many times the tilt is decided
 312 based on the local latitude alone [8]. Figure 10 shows the deviation from the
 313 rule of thumb (i.e. local latitude) for optimal energetic tilt in each location.

314 Figure 10 suggests that while accurate for parts of the southwest US, the
 315 optimal energy tilt is typically lower than the local latitude, especially in
 316 the states surrounding Tennessee and Kentucky. Lower optimal tilts would
 317 indicate the prevalence of more sunny days when the sun is higher in the
 318 summer sky. Figure 11 shows a map of the deviation from the rule of thumb
 319 tilts as determined by the maximum value ($\$/\text{m}^2/\text{year}$ based on local TOU
 320 electric rates) of the solar energy produced.

321 Figures 10 and 11 are very similar, except in situations where the local

Figure 10: Map of continental US showing deviation from the rule of thumb tilt (local latitude) based on total energy production. Negative values mean that the optimal tilt is below the local latitude.

Figure 11: Map of continental US showing deviation from the rule of thumb tilt (local latitude) based on the value of local energy production. Negative values mean that the optimal tilt is below the local latitude.

322 rates incentivize either more summer or winter production. For example,
323 in California, high summer afternoon electricity prices force the optimal tilt
324 lower to produce more during the summer peak.

325 3.3. *Optimal Azimuth and Tilt to Align with Summer Peak Demand*

326 To understand how solar PV systems should be placed if the goal were
327 to generate as many kWh during summer peak hours as possible, the anal-
328 ysis was run again where the value of solar was only considered during the
329 afternoon summer hours. This portion of the analysis restricted the time of
330 interest to June through August, and between 14:00 to 20:00 – a time period
331 typically associated with high electric grid stress times and higher wholesale
332 electricity prices. For Austin, as well as for most of the US, the optimal peak
333 array placement was shifted due west. The average peak optimal azimuth
334 was 266° with a standard deviation of 6.4° and the average optimal peak
335 tilt was 51° with a standard deviation of 4.6° . Figure 12 shows the average
336 generation curves for various solar placements in Austin using TMY data,
337 including optimal peak placement. The top part of Figure 12 shows the gen-
338 eration curves for the entire year and the lower part shows the curves for
339 only the summer months (June – August). Table 2 summarizes the differ-
340 ences in energy produced (area under the curves) from the placements shown
341 in Figure 12.

342 For Austin, the optimal energy and optimal value placements do not
343 differ much from south placements in terms of energy use. However, west-
344 facing and optimal peak placement generate about 14 and 20% less energy
345 throughout the year. In the summer, the optimal energy, optimal value, and
346 the west-facing array generate about the same amount of energy as the south-

Figure 12: Plot showing the average generation profiles of solar PV systems at various placements in Austin, TX and the average ERCOT SPP for the same time period (Section Appendix A).

Table 2: Percent change in amount of energy generated and the value of energy generated (considering Austin’s TOU rate) by various solar PV placements as compared to a south facing ($180^\circ/30^\circ$) array for an entire year, only the summer months (July – August), and for just the peak hours during the summer months (14:00 – 20:00) for Austin, TX.

Placement	Full year	Summer only	Summer peak hours only	Value of energy
South-Facing ($180^\circ/30^\circ$)	0.00	0.00	0.00	0.00
Optimal Energy ($188^\circ/28^\circ$)	0.17	1.13	5.00	0.71
Optimal Value Placement ($219^\circ/29^\circ$)	-1.86	0.88	16.44	0.93
West-Facing Placement ($270^\circ/30^\circ$)	-14.32	-1.34	23.04	-7.70
Optimal Peak Placement ($270^\circ/40^\circ$)	-19.55	-7.20	24.07	-11.12

347 facing array with the optimal peak array generating less. During the summer
348 peak demand hours, all placements generate more energy than south-facing
349 arrays with west and optimal peak placements generate 23 and 24% more
350 energy during peak hours, respectfully. The differences in the value of energy
351 for Austin’s TOU rate increase to the optimal value array, but then fall as
352 the placement moves further west. However, this assessment is contingent
353 on the economics considered and these numbers would differ for other rates
354 or ERCOT prices.

355 4. Conclusions

356 This analysis considers the the effect of various placements (azimuth and
357 tilt) of solar PV systems on energy generation and value of that energy gen-
358 eration for a yearly period with various environmental and economic inputs

359 in Austin, TX and many locations across the US. The results are generally
360 consistent with previous studies on optimal solar placement. However, this
361 analysis extends the body of knowledge by considering the AC electricity pro-
362 duced from a typical solar PV system and by considering electricity prices
363 on a national scale. These results show how local electricity markets (as
364 seen through TOU prices) affect the economic value of solar placement on a
365 national level.

366 These results have the potential to be significant for solar PV installa-
367 tions. However, the definition of optimal solar PV placement (especially the
368 optimal economic placement) might vary based on whom the definition is
369 asked. Large ground-mounted and flat roof arrays that require fixtures could
370 possibly take advantage of an optimal placement (or perceived future opti-
371 mal placement) at little to no additional cost than due south installations.
372 For a building of fixed orientation, a cost-benefit analysis would have to con-
373 sider possible less capacity (due to solar PV installations not being aligned
374 with the building lines) verses temporal generation revenue. Utilities can
375 encourage this optimal placement (which could be further west than arrays
376 designed to maximize energy production) by structuring rates that offset
377 their highest wholesale cost times and net-metering or similarly through a
378 ‘time-of-production’ buying rate for solar energy production. Also, these re-
379 sults could be used to influence roof azimuth and pitch in new construction
380 or retrofits to maximize the ability of installed solar PV systems to generate
381 energy.

382 While the solar tilt rule-of-thumb for total energy production might be a
383 good approximation for most of the US, it does not apply everywhere. No-

384 table examples include some locations east of the front range (Rocky Moun-
385 tains) and the majority of the west coast. Using local TOU electricity rates
386 as a proxy for local grid conditions further changes the optimal tilt. In many
387 locations, the optimal tilt is shifted down, particularly in locations that have
388 TOU rates with higher summer afternoon prices. Although in some cases,
389 rates shift the tilt steeper – particularly in places where TOU rates are high
390 in winter times.

391 In Austin, TX (for total energy production), TMY data indicate a 8° shift
392 west (188°) and a few degrees towards the horizontal (from the rule-of-thumb
393 30°) might be a better azimuth and tilt for energy production. Clear sky
394 radiation data reinforce the energy rule-of-thumb as expected. Considering
395 the value of energy produced, the optimal azimuth was pushed further west
396 ($\approx 20\text{--}51^\circ$) based on wholesale electricity market prices that are typically
397 higher in the mid to late afternoon hours. While the resulting improvements
398 might seem small, ($< 1 - 7\%$ difference), the improvement could be free
399 to implement during construction, and over the 25 year lifespan the excess
400 energy produced and revenue earned could be significant.

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416 **6. Disclosure of Affiliation**

417 In addition to research work on topics generally related to energy systems
418 at the University of Texas at Austin, some of the authors are equity partners
419 in IdeaSmiths LLC, which consults on topics in the same areas of interests.
420 The terms of this arrangement have been reviewed and approved by the
421 University of Texas at Austin in accordance with its policy on objectivity in
422 research.

423 **Appendix A. Data Collection and Preparation**

424 The data used in this analysis include 1) real-world solar insolation (global
425 and diffuse horizontal) and weather data, measured in 1-minute increments
426 in Austin, TX from July 2012 to June 2013, 2) simulated 1-minute clear-
427 sky radiation calculations for Austin (i.e. the amount of solar radiation that
428 would exist in the total absence of any type of weather that would block
429 or amplify actual radiation), 3) 1-minute solar PV output from residential
430 solar PV systems in Austin, 4) 15-minute electricity market Settlement Point

431 Price (SPP) data from the Electric Reliability Council of Texas (ERCOT),
432 the state-wide electric grid of Texas, 5) 1-hour Typical Meteorological Year
433 (TMY) solar and weather data, and 6) a representative sampling of utility
434 time-of-use (TOU) electric rates from across the US. The measured solar
435 radiation and weather data were collected at the SWEAT Lab (Solar, Wa-
436 ter, Energy and Thermal Lab) [32], on the roof a a 10-story building on
437 The University of Texas at Austin main campus [33]. The clear-sky radia-
438 tion measurements were calculated using Master’s equations [34]. The solar
439 PV output data are part of the Pecan Street Smart Grid Demonstration
440 Project [33], the archived pricing data are available from ERCOT [35], and
441 the utility rates were collected from the OpenEI utility rate database [24].
442 The TMY3 weather data are from files created by the National Renewable
443 Energy Lab (NREL) to serve as standardized input weather data for en-
444 ergy simulation programs and statistically represent a ‘typical’ year for the
445 specified location.

446 Because this analysis is temporal in nature, and many different datasets
447 were merged together, great care was taken to convert timestamps to a single
448 common form. For example, the measured solar radiation data were collected
449 with a UTC timestamp, the solar PV generation values were collected with
450 a ‘local time’ timestamp that accounted for daylight savings, TMY data are
451 collected in a ‘local time’ timestamp that does not account for daylight sav-
452 ings time, and utility rate data were collected with a ‘local time’ timestamp
453 that accounted for daylight savings. Also, accounting for the interval col-
454 lection method of each dataset was very important. For instance the solar
455 data are collected in an ‘interval beginning’ fashion such that the value for

456 14:00 is the average of all the data points collected from 14:00 – 14:59, and
457 the TMY data are in ‘interval ending’ such that the value for 16:00 is the
458 average of measurements collected in the 15:01 – 16:00 period. For simplicity
459 all timestamps were converted to Coordinated Universal Time (UTC) which
460 is the standard by which timezones are set.

461 Since even 1-hour can make a difference in the results of this analysis,
462 datasets that were only available with 1-hour increments (i.e. TMY and
463 utility rate data) were first corrected to not include daylight savings (if it
464 was accounted for) and then 30 minutes was subtracted or added from each
465 timestamp to ‘centralize’ the timestamp in the middle of its interval. The
466 1-minute data were not centralized because of their much higher reporting
467 frequency.

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