Machine vision methods based particle size distribution of ball- and gyro-milled lignite and hard coal

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Abstract

1 Mechanized coal mining generates substantial amount of fines that represents significant eco-
2 nomic value. Coal fines had several applications, such as gasification, liquefaction, combustion,
3 and beneficiation, and also pose health and safety hazards. Particle size and particle size distri-
4 bution (PSD) literature on fine coal are highly limited. In this study, PSD of laboratory ball- and
5 gyro-milled lignite and hard coal using machine vision analyses was determined and compared
6 with standard mechanical sieving. The developed machine vision ImageJ plugin utilized length-
7 and a new width-based sieveless volume-based for PSD evaluation. The plugin automated PSD
8 analysis with high speed, and produced better insight of the fine coal ($-300\text{ }\mu\text{m}$) PSD charac-
9 teristics through textual and visual outputs. Method of analysis, coal, and mill types, produced
10 statistically significant differences between these variables more on cumulative undersize and
11 less on PSD parameters using pooled data. Ground fine coal PSD exhibited an unimodal normal
12 distribution, and can be classified poorly-graded, positively skewed and leptokurtic, very well-
13 sorted, symmetrical, and platykurtic to mesokurtic. Mechanical sieving PSD characteristics, in
14 general, fell between machine vision width- and length-based characteristics. Unlike limited
15 standard sieves of mechanical sieving, the machine vision methods, with limitless virtual sieves,
16 accurately analyzed the PSD with necessary number of sieves. The outlined machine vision
17 methods can be readily utilized to analyze similar particulate materials.

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1. Introduction

Coal has peerless characteristics that allow it to be utilized for multiple purposes including electricity generation, steel production, and conversion to liquids and gaseous fuels. Its low cost, abundance, and broad distribution make it appealing for power production [1]. According to IEA (2012) [2], coal consumption is projected to increase 24% by 2035, compared to 2009.

The fines content in the world coal production has been increasing over the years due to the increase in mechanized mining operations. Only in South Africa, it is generally accepted that about 12% of the 254 Tg of run-of-mine coal is smaller than 0.5 mm (defined as fine coal) and 2-3% is smaller than 0.1 mm (ultra-fine coal) [3].

Several incentives exist for fine coal cleaning [4]. Lost fines represent a significant economic value. Increasingly stringent environmental regulations necessitate the removal of fine particles from effluent streams, and the removal of fine particles from process water is desirable so that water can be recycled. An increasing development of specialty applications for fine coal utilization, such as gasification, liquefaction, and fuel injection is on the rise. In addition, fine coal processing may provide the only means of reducing sulphur levels of many coals by removing the more finely disseminated pyrite [5]. If ultra-fines left unbeneficiated they can lead to significant potential yield loss and significant slimes dam capacity requirements [6]. Since the high value of these fines had been recognized, it was either added to the product streams, or upgraded and sold. The economic potential of coal fines has led to developments in fine coal beneficiation processes, like spirals and froth flotation [7].

Flotation is the most common and widely accepted method of enrichment of fine coal below 0.5 mm and even below 1 mm. This method has been used to enrich several million tonnes of coking and power coal in the world. Particle size and particle size distribution (PSD) have a significant effect on kinetics [8], wettability [9] and recovery of coal flotation [10], combustion properties [11] and the rheology of coal-water slurries [12], and also performance of the flu-

idized bed coal combustor [13]. For example, the right particle size distribution of the coal is one of the key factors to an efficient burning process and thus an efficient combustion and low emissions of NO_x.

The particle size and PSD are perhaps the most important physical characteristics that influence several processes the ground coal passes through and products made. Literature review shows numerous publications relating to sizing of various materials generally their sizes vary from 20 to 200 μ m using image analysis. These are predominantly coarse particles, which were well studied, such as aggregates [14, 15], fine aggregates [16], crushed limestones [17, 18], coarse coal particles [19], and iron ore pellets [20]. However, there were only a few publications relating to PSD determination of fine ground particles particularly in machine vision methods. Reported studies include characterization of ball-milled chromite ores [21], development of a machine vision Σ Volume-based sieveless analysis PSD plugin and testing on ball-milled celestite [22] and on rod-milled coal [23].

This study aims to determine the PSD of fine particulate lignite and hard coal ground by ball- and gyro-mills using both length- and a new width-based machine vision approach using simple imaging device as well as using laboratory mechanical sieving. Results of this work will also find applications in various ground coal characterization (e.g., physical, heat and mass transfer), preparation, utilization processes (e.g., fluidization, floatation, beneficiation, combustion), and other health and safety related issues (e.g., respiratory disorders and spontaneous combustion).

2. Materials and methods

2.1. Sample preparation of coals for PSD measurement

Lignite and hard coal samples (18 $\times$ 50 mm ROM, run of mine) were obtained for the study from Manisa and Zonguldak, Turkey, respectively. Some relevant physical properties of coal samples tested are presented in Table 1. A jaw crusher was used to obtain the coal stock to prepare feed size fraction of $-800 +600 \mu$ m for ball- and gyro-mill grinding studies. The optimum mechanical sieving time using a 212 μ m standard laboratory sieve (Retsch GmbH, Haan, Germany) both for lignite and hard coal samples was determined as 22 min. Grinding tests for

both coal samples in laboratory ball-mill (195 mm diameter and 200 mm height; 75 rpm) and gyro-mill (200 mm diameter and 280 mm length) was presented in detail earlier [24].

After optimum grinding duration (16 min for ball-mill and 1–1.5 min for gyro-mill) the –300 μm size fractions sample stocks suitable for machine vision analysis both for lignite and hard coal were obtained. This size fractions were also suitable for coal flotation studies. Spinning riffler (Quantachrome Instruments, Florida, USA) was used for deriving representative samples of lignite and hard coal ground by ball- and gyro-mills for machine vision analysis. From this representative riffler samples, the standard “cone-and-quarter” method was used to obtain the low volume subsamples required for image acquisition using flatbed document scanner in machine vision PSD analysis. An example of the original coal sample (ball-milled lignite) used in the study is shown in Figure 1. It can be observed from the image that the particles were about < 0.5 mm in dimension. Even though larger particles are seen, the sample actually contained a range of finer particles — some were seen as minute dots representing these fine particles away from the sample mass and others contained in the sample heap (Fig. 1).

2.2. Mechanical sieving for PSD measurement

A set of 800, 600, 425, 300, 212, 150, 106, 75, 53, and 38 μm standard laboratory sieves (Retsch GmbH, Haan, Germany), suitable for –800 +600 μm feed size, was employed for the mechanical sieve analysis of both coal samples, and 22 min was used as the optimum sieving time. In the selected sieve set, a few sieves above the limiting size (–300 μm) of machine vision samples, were also selected to capture possible coarser particles. Mass of material retained on each sieve was measured and the log-normal cumulative PSD [22] and cumulative undersize PSD were derived for mechanical sieving and compared with machine vision methods.

2.3. Machine vision based PSD measurement

2.3.1. Sample preparation and image acquisition

Prepared ground fine coal samples stocks (–300 μm size fraction) from the two mills were subjected to “cone-and-quarter” method for low volume imaging samples preparation. Image acquisition procedure is similar to that described in detail elsewhere [22]. As the color of fine

coal particles is black, a clean white paper as cover provided the good contrast, and the scanner lid was closed before scanning. Scanner software “ScanGear” (Canon Scan Utility) custom DPI of 5080 = 5 μ m resolution) was used for acquiring images of size 50.8 mm $\times$ 63.8 mm (2” $\times$ 2.5”). Four image replications, with each corresponding to 17 ± 2 mg of ground coal, were obtained for the analysis.

2.3.2. Image preprocessing

Image preprocessing is essentially converting the original color image into binary image, which is essential for various processing and PSD analysis. An open source image processing software ImageJ on Fiji (Ver. 1.49v, 64-bit, Munich, Germany) platform was used to perform the preprocessing with standard commands. The flowchart (Fig. 2A) and the front panel of the developed plugin (Fig. 2B) both for length- and width-based PSD analysis shows the various processing stages involved and the options available for the analysis.

The preprocessing basically involves conversion of color image (6.5–13.4 MB at 5080 DPI) to grayscale first and subsequently to binary image using suitable threshold values (Fig. 3). Since the ground coal particles are very fine ($\sim 300 \mu$ m), the high resolution of scan produce colored edges (reddish on top and bluish on the bottom), and the shiny facets of the fine coal particles also produce lighter colored interior (Fig. 3A). The Fiji standard command ‘8-bit’ created the grayscale image, and the threshold command with ‘Default’ option delineated the regions (Fig. 3B), which can be adjusted if required, and created the binary image (Fig. 3C). The holes on the particle surface, if created by shiny facets, will be filled by making a mask using ‘Analyze Particles...’ command with ‘Include holes’ option. From the resulting binary image, the lumps representing a collection of several overlapping/touching particles, which occurred during sample layout, were filtered out by applying the area filter ‘Size (pixel2):’ as 0-1500. On the lumps filtered binary mask image, the ‘Watershed’ segmentation was applied to resolve touching particles. The watershed automatic segmentation produced successful results (correct segmentation) but on a smaller proportion also produced false negatives (over-segmentation), but its occurrences are relatively less [22]. Preprocessed images were saved in graphics interchange

format (GIF) files, which were smaller files (0.56–2.1 MB.) and read quickly for analysis.

2.3.3. Plugin development

The PSD evaluation methods employ particle grouping based on particles' measured distinct length and width groups separately. These groups simulated “virtual sieves” for measurement and evaluated PSD in a sieveless manner [25, 22]. The geometrical volume (ΣVolume) was again used as the weighting factor for the new machine vision width-based PSD plugin (Fig. 2B). Figure 4 illustrates the measurement strategy with two actual particles ($\mathbf{P}_1$ and $\mathbf{P}_2$) as examples. The Feret diameter ($\mathbf{F}$) that enclosed the whole object represents the length, and the ‘pixel-march’ [26] distance ($\mathbf{w}$) along the minor axis direction ($\mathbf{b}$) represented the width. Width can be observed as different from the standard minor axis dimension ($\mathbf{b}$), similarly the length and the major axis ($\mathbf{a}$) differed (Fig. 4).

The key operation after measurements, using ImageJ standard commands, is the data sorting. This is necessary to determine the distinct dimension groups (length or width) in descending order to simulate the virtual sieves. An efficient “modified mergesort” algorithm from the “Java Array” class [27] was employed for sorting. This algorithm offered a guaranteed $n \times \log(n)$ performance compared to the commonly followed $n \times n$ of bubble-sort.

The several improvements features of the plugin (Fig. 2) include: (1) Dynamic allocation of the key arrays, which held the properties of particles, based on the number of identified particles; (2) Handling larger number of particles (>50,000) and improved speed of analysis (>11,000 particles/s); (3) Inspection of the particle length or width in a specified range for visualization and quick decision; (4) Automatic generation of spreadsheet output in a format suitable for statistical analysis (e.g., SAS); and (5) A suite of shape factors based on the particle dimensions.

2.4. Log-normal cumulative distribution of particles

In general, with the particulate sample has finer material, a log-normal distribution of the particles is usually assumed [28]. A plot of PSD with log of particle dimension on the horizontal axis and cumulative percent undersize of particle volumes on the normal probability vertical axis

shows the normal distribution by a linear trend. The cumulative normal distribution transformation function $[F(x, \mu, \sigma)]$ of the particles with particle volume as the classification parameter [22] was used for the transformation; where, x - variable volume of individual particle groups (mm^3), μ - mean sample volume (mm^3), and σ - standard deviation (STD) of the sample volume (mm^3). These transformed volume data were the normalized cumulative undersize volumes and when plotted on a semi-log paper will simulate the log-normal probability plot. Geometric mean length and the associated STD of the particles were determined using the ASABE Standards [28] method. Several cumulative dimension properties, PSD parameters, and plugin performance parameters defined in different sources [29, 30, 31, 22] are again employed for the outputs.

2.5. Statistical analysis dimensions and PSD parameters

A SAS (Ver. 9.3, 2009, SAS Institute, Cary, North Carolina, USA) macro %mmaov based on PROC MIXED [32] was used to perform the mean separation analysis to study the effect of independent parameters affecting the PSD. Collected images served as the four sample replications.

3. Results and discussion

3.1. Machine vision basic dimension measurements

The replications of individual sample binary images were analyzed and the results of averages and standard deviations (STD) of length, width, and other related parameters are presented in Table 2. The issue of overlapping or touching particles, which increases the dimension of the particles incorrectly, was addressed by (1) eliminating the lumps (formed during particles layout) by filtering out particles of size $>1500 \text{ pixel}^2$, and (2) applying ‘watershed’ segmentation on the filtered images. Among coal samples (types and mills), the number of lumps that were filtered out on average was found to be $4.7 \pm 2.7\%$ of the original number of particles, and the increase in number of particles due to watershed segmentation was $23.8 \pm 2.7\%$ (data not shown). This observation indicates the added number of touching particles resolved by watershed algorithm is far greater than the lumps filtered out. In general, no statistical differences between the type of mill and type of fine coal were observed on these common parameters; and a few differences were observed only with STD of averages or mean (Table 2).

Overall, on these common dimension related parameters major differences were not evident. Gyro-mill tend to produce smaller particles than ball-mill for both coal types based on mean dimensions. For the $-300\ \mu\text{m}$ samples, on average, the geometric mean and STD on length was $0.158 \pm 1.511\ \text{mm}$ and on width was $0.092 \pm 1.501\ \text{mm}$ (Table 2). Length by width ratio of both arithmetic average and geometric mean dimensions were in the range of 1.7–1.8, which indicated the particles are elongated. The geometric mean dimensions were about 1.9–2.3 times greater than the arithmetic mean, indicating the proper weightage given to the particles based on their volume (ΣVolume approach) in the geometric mean [28] than based on numbers in arithmetic mean. Skewness of length (0.81–1.19) and width (0.78–1.15) of ground coal indicate that they were significantly moderate to highly “positively skewed” with longer right tail (critical value = 0.2; $\alpha = 0.01$; $df = 1000$). Kurtosis of length (0.24–1.33) and width (0.35–1.52) indicate that the particle distributions were “leptokurtic” having significant narrow peak (critical value = -0.15 to 0.17 ; $\alpha = 0.02$; $df = 5000$).

3.2. Graphical outputs of length- and width-based methods

The developed plugin produced graphical outputs (Fig. 2) help visualize the measurements and processed data directly and quickly. For illustration, the ball- and gyro-milled lignite analysis graphical outputs are presented in Fig. 6. The length and width of the particles (Fig. 6A) clearly show that the lengths were below 0.3 mm and width below 0.2 mm. Frequency distribution of sorted lengths and widths (Fig. 6B) shows that they differ a little and have slightly longer right tail (“positively-skewed”), but resembled normal distribution. The volume of the particles (Fig. 6C), evaluated based on prolate spheroidal geometry, were exactly the same as both dimensions (Fig. 2A) used common volume data.

Contrary to common belief, the number of virtual sieves based on lengths (≤ 907), which is a dominant dimension and expected to generate more distinct lengths, was much less than the virtual sieves based on widths ($\leq 10,872$; Fig. 6D). Actually, width-based virtual sieves retained more particles from the middle range of sieves while for length based sieves towards the end range. The cumulative undersize distributions in regular scales for lengths and widths (Fig. 6E)

show that the length curve was flatter than the width because of the increased size range of the lengths. However, both cumulative undersize distribution displayed the 'S' shape that is indicative of normal distribution, and similar variation can be observed with other samples.

3.3. Number of particles of PSD analysis

In the image space of 50.8 mm × 63.5 mm (2" × 2.5"), the particles occupied only a meager portion of 1.3% – 7.5% (data not shown) indicating a possibility of holding higher density of particles (Table 3). The density of particles in this layout, given the total number of particles (23,593–48,949), ranged between 7 and 15 particles/mm². Particle sizes smaller than 9 pixels, which is about 17 μm equivalent diameter or smaller, vary with coal and mill types. Lignite produced less fines (12%) than hard coal (21%), and gyro-mill produced consistently slightly less fines than ball-mill.

Number of concave particles, which are curved with centroid falling outside the object profile, were insignificant (≤0.42%) as ground particles are crystalline and convex in shape. Even though the size range of length (7 to 422 μm) was almost twice of width (5 to 210 μm); as indicated before, the number of distinct length groups (839–907) were much less than that of widths (5,349–10,872). This distinct dimension groups, considering the total number of particles were on average was $2.8 \pm 0.8\%$ for length and $22.3 \pm 0.9\%$ for width, respectively. This interesting observation is unexpected, but indicated the nature of breaking of coal particles leading to more variation along width than length.

3.4. Volume contribution of particles

Although the bottom two fine sieves (<37 μm) retained about $35.8 \pm 9.7\%$ of the particles for analysis, their volume contribution was only $1.2 \pm 0.9\%$, which showed the negligible effect of the fine coal particles on overall distribution (Table 3). Similarly, the top two active sieves that retained particles (>150 μm length; >75 μm width) represented only about $14.1 \pm 3.2\%$ of the particles, but they have a greater volume contribution of about $65.9 \pm 6.7\%$.

3.5. Cumulative PSD dimensional properties

Length- and width-based significant cumulative dimensions, such as D_5 through D_{95} (Table 3) define the PSD, and these dimensions provide useful information about the distribution directly through their individual values. For example, D_{75} represents a top dimension limit that contains 75% volume of the particulate volume; in other words, represents a sieve opening through which 75% of the volume of particles pass through. The D_{50} splits the distribution into equal halves, and because it used volume as the basis, its values (Table 3) resembled closely with the geometric mean than arithmetic mean (Table 2). For example, with ball-milled lignite, the D_{50} (0.166 mm) was closer to the geometric mean (0.156 mm) than the arithmetic mean (0.083 mm).

Cumulative undersize dimensions (D_5 to D_{95}) did not show difference between mill types and coal types, except for coal types ground by gyro-mill (Table 3). This indicated even though the values were numerically different, these individual cumulative dimensions were not affected by mill or coal types statistically ($p < .05$; $\alpha = .05$). However, the overall analysis using the combined data (Overall* column of Table 3) showed the length-based analysis results were statistically greater ($p < .05$) than width because of obvious difference in these dimensions. Furthermore, between coal types the milled hard coal dimensions results were statistically greater than that of lignite for the whole range; and between mill types the statistical difference was only observed in the finer ($\leq D_{25} = 0.125$ mm) particle sizes.

Cumulative undersize dimensions derived from mechanical sieving (D_{90} , D_{75} , D_{50} , D_{25} , and D_{10} ; no replications) fell in between the length- and width-based results, but were closer to width-based results as their principles of separation was similar. The trend of mechanical sieving cumulative undersize dimensions was closer to width-based (absolute difference: 3% to 95%) than length-based (absolute difference: 31% to 237%) machine vision analyses (Table 3).

3.6. Overall machine vision PSD analysis parameters

Several overall PSD parameters describe the particulate fine coal PSD (Table 3). Relative span ($[(D_{90}-D_{10})/D_{50}]$), indicated the location of the middle dimension (D_{50}) in the dimension span; and a value of 2 indicating that D_{50} falling in the middle, and <2 it moving towards the

upper limit D_{90} , and vice versa . Thus, the observed relative span of 0.94 to 1.05 reveal that the D_{50} tended toward D_{90} , which showed the dominant contribution of the larger particles in PSD.

Coefficient of uniformity ($C_u = D_{60}/D_{10}$), varying from 1.94 to 2.22 indicated a clear nonuniformity, as any positive deviation from 1.0 means nonuniformity among the particles. Coefficient of gradation ($C_g = D_{30}^2/[D_{10} \times D_{60}]$) also known as coefficient of curvature, varied from 0.97 to 1.12. For uniform particles as well as in some distribution with D_{10} and D_{60} values so aligned that their product matched with squared D_{30} the $C_g = 1$. With granular materials, such as sand, the criteria for “well-graded” material is $C_u \geq 6$ and $1 < C_g < 3$ [33]; hence, the finely ground coal particles produced by both mills are classified “poorly-graded.” Uniformity index ($[D_5/D_{90}] \times 100$), for perfectly uniform particles will be 100% and the values observed ranging from 27% to 32% clearly indicated the deviation from the uniform particles dimensions.

The size grade number ($D_{50} \text{ (mm)} \times 100$), adopted by fertilizer industry with a typical range of 160 to 280 for fertilizer granules, ranged from 9 to 18 showed the milled coal samples belong to very fine size compared to typical fertilizers. Size range variation coefficient ($[D_{84}-D_{16}]/2D_{50} \times 100$), a measure that clearly indicated the variation in particle dimensions, ranging from 37% to 43% clearly depicted the dimension variations and nonuniformity of particles.

Original logarithmic graphical measures, classifying the particulate material, outlined by Folk and Ward [29] and Blott and Pye [30] was used in the interpretation of PSD parameters obtained. Graphic mean of ground fine coal samples was 0.14 ± 0.04 and their inclusive graphic STD ($[(D_{84}-D_{16})/4]+[(D_{95}-D_5)/6.6]$) of 0.05 ± 0.01 classified ground fine coal as “very well sorted” (standard range: <0.35). Inclusive graphic skewness ($[(D_{84}+D_{16}-2D_{50})/2(D_{84}-D_{16})]+[(D_{95}+D_5-2D_{50})/2(D_{95}-D_5)]$) of 0.06 ± 0.04 classified ground fine coal as “symmetrical” (standard range: $+0.1$ to -0.1).

Graphic kurtosis ($[D_{95}-D_5]/2.44[D_{75}-D_{25}]$) of 0.91 ± 0.03 described ground fine coal samples as “mesokurtic” (standard ranges: 0.67–0.90 - platykurtic, and 0.90–1.11 - mesokurtic). Observed geometric STD of high region (D_{84}/D_{50}) and low region (D_{50}/D_{16}) were 1.42 ± 0.04 and 1.60 ± 0.06 , respectively, showed that the particle dimensions were not uniform as these ratios (>1.4) were greater than 1.0. This claim was naturally corroborated by the geometric STD of

total region ($\sqrt{D_{84}/D_{16}}$) values of 1.51 ± 0.04 .

3.7. Variations of PSD parameter means

The mean separation with width-based analysis on cumulative undersize dimensions, type of mills for coals, and type of coals for mills mostly did not produce significant differences ($p < .05$; $\alpha = .05$; Table 3); however, with length-based analysis some differences between mill and coal types were observed. The overall means analysis using combined data (irrespective of other variables; Overall*; Table 3) revealed that the length- and width-based analysis of cumulative dimensional properties, type of coal other than D₅, and type of mill $\leq D_{25}$ were significantly different. With the combined data it is interesting to note that for analysis and coal types, which had produced consistent significant differences with cumulative undersize dimensions (9/36 = 25% not different), have not produced similar differences among the PSD parameters (19/39 = 49% not different).

3.8. Developed plugin's performance

On average, for the analysis of $32,504 \pm 10,689$ particles, it took a CPU time of about 3 ± 0.7 s, giving an analysis speed of $9,197 \pm 1,021$ particles/s for a MacBook Pro (2.8 GHz Intel Core 2 Duo, 8 GB RAM) laptop (Table 3). These results are indicative of the improved performance of the plugin, when compared to the results of a previous study with celestite minerals [22] using same configuration that analyzed $51,868 \pm 48,506$ particles in ≈ 34 s, with a speed of $1,612 \pm 182$ particles/s, meant a 5.7 folds increase in the developed plugin. Analysis speed can further be increased with present affordable powerful multicore computers.

3.9. Comparison of PSD determination methods

Both length- and width-based four replications showed similar trends, with length-based replications characteristics were more closer than width-based (plots not shown). To observe the characteristics of the three PSD determination methods (including mechanical sieving) a superimposed regular and log-normal plot was made from the combined sample image (Fig. 7). In this comparison, rather than using any of the four image replications, a combined image input of

these four replications in a 2×2 tile were made using ImageJ commands (“Make Montage...”) and analyzed as usual. This combined image for each coal type and mill combinations was the most representative and had a size of 101.6×127 mm ($4'' \times 5''$); containing $130,014 \pm 46,183$ particles; and had $1,076 \pm 9$ distinct lengths and $27,319 \pm 10,032$ distinct widths. Overall, the combined plot shows the PSD of all the methods are similar. For normal distribution, the ‘S’ shaped distribution in regular cumulative undersize characteristics becomes a straight line distribution in log-normal transformation as seen in the results (Fig. 7).

The width-based machine vision regular and log-normal curves differed a larger extent while the length-based curves coincided. The reason rests in the values of mean sample volumes of the virtual sieves (μ) and their STD (σ) of the cumulative normal distribution function [22]. For width-based analysis ($\mu = 0.0011 \text{ mm}^3$; $\sigma = 0.0082 \text{ mm}^3$) as well as mechanical sieving ($\mu = 10.00\%$; $\sigma = 13.95\%$) the σ on average were 7.35 and 1.40 times the μ , respectively. However, for length-based analysis ($\mu = 0.0284 \text{ mm}^3$; $\sigma = 0.0262 \text{ mm}^3$) the σ/μ ratio was less than unity (0.92) making both the regular and log-normal characteristics to coincide (Fig. 7).

The regular length- and width-based machine vision curves ran together with a constant distance (a function of length/width ratio). With the log-normal PSD curves, width- and length-based machine vision curves seemed to sandwich the mechanical sieving curves. Machine vision PSD characteristics of ground fine coal showed a clear linear trend of cumulative undersize from approximately 10% to 95% indicating a normal distribution in this range.

Regular mechanical sieving PSD curves followed closely the width-based machine vision curve, which is expected because the mechanical sieving is a “width-based classification” method. To make the mechanical sieving results resemble the length-based machine vision method the length transformation using the length/width ratio [22]. Based on the fine particulate nature of ground coal and the availability of limited sieves in the range of 0.1 mm, and use of sieves with opening >0.1 mm (5 out of 10) did not capture significant amount of material (Table 2). This can be also seen from the PSD plots that above 90% cumulative undersize level half are more mechanical sieve data were recorded, but their volume contribution to the total distribution was insignificant. Thus, the lack of standard sieves ($<38 \mu\text{m}$) cannot capture the full details of the

PSD matching the machine vision methods.

The machine vision methods presented in this study opened up a great opportunity to have a closer and detailed look into the total profile of the ground fine coal, and provided the PSD of the whole spectrum of particle sizes. Machine vision methods, with limitless virtual sieves, analyzed the distribution and allotted automatically the necessary number of sieves, irrespective of the sample's dimensional range. The developed length- and width-based machine vision methods allowed visualizing the PSD from both basis (length and width) and produced two different PSD characteristics that provided better insight into the PSD.

4. Conclusions

The PSD of finely ground lignite and hard coal produced by ball- and gyro-mills were successfully analyzed using the developed machine vision plugins that used length- and a new width-based sieveless volume-based analysis and compared with mechanical sieving. For a $-300\ \mu\text{m}$ milled coal samples, on average the geometric mean and STD on length was $0.158 \pm 1.511\ \text{mm}$, and on width was $0.092 \pm 1.501\ \text{mm}$ using machine vision methods. Gyro-mill tend to produce smaller sized particles than ball-mill for both coal types. Length by width ratio of particulate fine coal were in the range of 1.7–1.8 indicating the particles are elongated. According to analysis the finely ground coals tested were classified as: poorly-graded, moderate to highly positively skewed and leptokurtic, very well-sorted, symmetrical, platykurtic to mesokurtic, and exhibited an unimodal normal distribution.

Despite the size range of length was almost twice of width, the number of distinct width groups that represents virtual sieves was about 8.5 times that of length. Despite finer sieves ($-37\ \mu\text{m}$) retained about $35.8 \pm 9.7\%$ particles, their volume contribution was only $1.2 \pm 0.9\%$, and coarser sieves produced the opposite ($+150\ \mu\text{m}$ length; $+75\ \mu\text{m}$ width; $14.1 \pm 3.2\%$ particles contributing $65.9 \pm 6.7\%$). Particles sizes were nonuniform and had shown variation in dimensions (37% to 43%). Method of analysis, coal, and mill types, produced significant differences among all variables (analysis, coal, and mill types) on several cumulative undersize and PSD parameters only on the pooled data. Overall, PSD plots of the ground fine coal were sim-

ilar for all samples produced and analyzed by different methods; and mechanical sieving PSD characteristics got sandwiched between width- and length-based machine vision characteristics.

Even with domestic document flatbed scanner, the machine vision methods can easily analyze particles of size below $-200\text{ }\mu\text{m}$, a size range important for flotation applications, while detailed PSD of fine particles ($-38\text{ }\mu\text{m}$) is not possible mechanical sieving due to sieves limitations. The machine vision methods, with limitless virtual sieves, analyzed the distribution, allotted the necessary number of sieves, automated the analysis, performed analysis rapidly (speed of $9,197 \pm 1,021$ particles/s), and helped with results visualization. The outlined inexpensive machine vision methods and can be readily utilized to analyze similar particulate materials and obtain detailed look into the whole spectrum of particles sizes.

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Table 1

Some relevant properties of lignite and hard coal (18×50 mm ROM) samples used.

Sample	Lignite	Hard coal
Total moisture (original basis)	15.50%	$5 \pm 2\%$
Total ash (dry basis)	17.70%	$14 \pm 2\%$
Total sulfur (dry basis)	1.04%	0.9%
Lower calorific value (kcal/kg, dry basis)	4545	6600
Density by pycnometer (kg/m^3)	1.52	1.6

Table 2

General dimension measurements of lignite and hard coal finely ground by ball- and gyro-mill using machine vision length- and width-based methods.

Basic dimension		Length (mm); length-based analysis			Width (mm); width-based analysis		
		Lignite		Hard coal		Lignite	
Coal type	Mill type	Ball-mill	Gyro-mill	Ball-mill	Gyro-mill	Ball-mill	Gyro-mill
Maximum		0.359 ± 0.012 aA	0.425 ± 0.102 aA	0.386 ± 0.034 aA	0.386 ± 0.020 aA	0.208 ± 0.002 aA	0.202 ± 0.005 aA
Arithmetic mean		0.083 ± 0.002 aA	0.081 ± 0.002 aB	0.080 ± 0.012 aA	0.071 ± 0.004 aA	0.048 ± 0.001 aA	0.047 ± 0.002 aB
Arithmetic mean STD		0.055 ± 0.001 aA	0.054 ± 0.004 aA	0.060 ± 0.001 bB	0.057 ± 0.001 aA	0.032 ± 0.001 aA	0.030 ± 0.002 aA
Geometric mean		0.156 ± 0.005 aA	0.151 ± 0.004 aA	0.163 ± 0.003 aA	0.163 ± 0.004 aB	0.093 ± 0.003 aA	0.089 ± 0.003 aA
Geometric mean STD		1.485 ± 0.013 aA	1.530 ± 0.040 aA	1.482 ± 0.048 bA	1.546 ± 0.016 aA	1.496 ± 0.010 aA	1.499 ± 0.040 aA
Skewness		0.884 ± 0.094 aA	0.987 ± 0.240 aA	0.887 ± 0.343 aA	1.239 ± 0.178 aA	0.953 ± 0.090 aA	0.882 ± 0.295 aA
Kurtosis		0.574 ± 0.201 aA	1.168 ± 0.736 aA	0.548 ± 0.631 aA	1.494 ± 0.552 aA	1.007 ± 0.216 aA	1.259 ± 0.993 aA

Note: Number of sample image replications used = 4; Number of particles in the replications = 12,934–54,097; STD - standard deviation; minimum length = 0.007 mm, and minimum width = 0.005 mm; lowercase mean groups compare type of mills with same coal type and uppercase mean groups compare type of coal with same mill type; unsimilar letter groups represent statistically significant difference ($p < .05$; $\alpha = .05$).

Table 3

Machine vision analysis based PSD results of fine lignite and hard coal ground by ball- and gyro-mill, mean comparisons, and plugin performance using machine vision methods.

Machine vision method										Overall*
Coal type		Length-based analysis				Width-based analysis				
Mill type		Lignite		Hard coal		Lignite		Hard coal		
		Ball-mill	Gyro-mill	Ball-mill	Gyro-mill	Ball-mill	Gyro-mill	Ball-mill	Gyro-mill	
Total number of particles										
Total number particles <9 pixels		48,949 ± 3,529	32,031 ± 14,952	23,593 ± 5,453	25,444 ± 8,851	48,949 ± 3,529	32,031 ± 14,952	23,593 ± 5,453	25,444 ± 8,851	–
Total number particles <9 pixels		11.98% ± 1.14%	14.09% ± 7.39%	21.43% ± 6.11%	20.72% ± 4.58%	11.98% ± 1.14%	14.09% ± 7.39%	21.43% ± 6.11%	20.72% ± 4.58%	–
Total number of non-convex particles		0.15% ± 0.02%	0.29% ± 0.26%	0.42% ± 0.18%	0.33% ± 0.13%	0.15% ± 0.02%	0.29% ± 0.26%	0.42% ± 0.18%	0.33% ± 0.13%	–
Total number of distinct particles		907 ± 31	842 ± 84	839 ± 43	841 ± 68	10,872 ± 1,204	7,319 ± 3,205	5,448 ± 1,524	5,349 ± 1,775	–
Number of particles (mesh size):										
Above 425 µm (above 40)		0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	–
Between 425 and 212 µm (40 to 70)		2.64% ± 0.52%	2.51% ± 0.36%	3.18% ± 0.52%	2.85% ± 0.24%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	–
Between 212 and 150 µm (70 to 100)		9.83% ± 0.31%	8.19% ± 1.13%	10.52% ± 2.57%	7.53% ± 0.90%	0.76% ± 0.20%	0.59% ± 0.13%	0.72% ± 0.29%	0.66% ± 0.15%	–
Between 150 and 75 µm (100 to 200)		35.97% ± 3.60%	36.97% ± 1.98%	32.71% ± 8.93%	27.29% ± 3.30%	17.36% ± 0.30%	14.71% ± 1.53%	18.13% ± 6.27%	12.47% ± 1.65%	–
Between 75 and 63 µm (200 to 230)		9.26% ± 0.69%	10.58% ± 1.88%	7.28% ± 0.78%	8.52% ± 0.40%	8.99% ± 1.17%	10.21% ± 1.26%	9.06% ± 3.53%	6.65% ± 1.08%	–
Between 63 and 37 µm (230 to 400)		18.31% ± 0.93%	18.16% ± 5.87%	15.34% ± 3.70%	19.48% ± 2.48%	32.89% ± 2.94%	37.32% ± 3.66%	27.12% ± 2.10%	28.82% ± 1.58%	–
Between 37 and 20 µm (400 to 632)		13.57% ± 2.08%	11.34% ± 1.74%	12.11% ± 3.25%	16.24% ± 2.32%	19.52% ± 1.45%	16.53% ± 5.69%	15.80% ± 4.22%	20.40% ± 2.93%	–
Below 20 µm (below 632)		10.43% ± 0.99%	12.28% ± 6.50%	18.86% ± 5.39%	18.09% ± 4.18%	20.49% ± 2.51%	20.64% ± 6.94%	29.17% ± 7.72%	31.00% ± 4.22%	–
Volumes of particles (mesh size):										
Above 425 µm (above 40)		0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	–
Between 425 and 212 µm (40 to 70)		23.55% ± 4.22%	24.21% ± 2.51%	27.44% ± 3.80%	31.29% ± 2.92%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	0.00% ± 0.00%	–
Between 212 and 150 µm (70 to 100)		36.32% ± 2.27%	30.90% ± 1.58%	35.99% ± 1.18%	32.73% ± 1.27%	10.49% ± 2.39%	9.33% ± 2.06%	10.22% ± 1.82%	11.94% ± 2.74%	–
Between 150 and 75 µm (100 to 200)		34.93% ± 3.92%	38.10% ± 3.13%	32.19% ± 4.32%	29.68% ± 3.85%	62.10% ± 2.08%	56.91% ± 1.97%	64.02% ± 1.97%	59.95% ± 2.99%	–
Between 75 and 63 µm (200 to 230)		2.66% ± 0.20%	3.76% ± 0.87%	2.19% ± 0.57%	3.00% ± 1.43%	10.55% ± 1.43%	13.02% ± 1.78%	10.74% ± 1.62%	9.84% ± 2.01%	–
Between 63 and 37 µm (230 to 400)		2.24% ± 0.03%	2.73% ± 1.01%	1.92% ± 0.78%	2.86% ± 0.36%	14.90% ± 1.59%	18.91% ± 1.87%	13.27% ± 1.93%	15.75% ± 2.63%	–
Between 37 and 20 µm (400 to 632)		0.30% ± 0.05%	0.29% ± 0.09%	0.26% ± 0.12%	0.41% ± 0.08%	1.81% ± 0.13%	1.69% ± 0.57%	1.60% ± 0.71%	2.28% ± 0.21%	–
Below 20 µm (below 632)		0.02% ± 0.00%	0.02% ± 0.01%	0.02% ± 0.01%	0.03% ± 0.01%	0.16% ± 0.03%	0.15% ± 0.01%	0.17% ± 0.08%	0.24% ± 0.03%	–
Cumulative undersize dimensions (mm):										
D ₅		0.267 ± 0.005 b A	0.274 ± 0.003 a A	0.278 ± 0.008 a B	0.283 ± 0.00 a B	0.165 ± 0.005 a A	0.162 ± 0.005 a A	0.163 ± 0.004 a A	0.167 ± 0.003 a A	a b; b a; a a
D ₅₀		0.248 ± 0.006 a A	0.253 ± 0.005 a A	0.258 ± 0.009 a A	0.264 ± 0.00 a B	0.151 ± 0.005 a A	0.147 ± 0.006 a A	0.150 ± 0.005 a A	0.154 ± 0.005 a A	a b; b a; a a
D ₈₄		0.229 ± 0.008 a A	0.233 ± 0.006 a A	0.238 ± 0.008 a A	0.248 ± 0.01 a B	0.140 ± 0.005 a A	0.135 ± 0.005 a A	0.138 ± 0.004 a A	0.142 ± 0.005 a A	a b; b a; a a
D ₈₀		0.219 ± 0.008 a A	0.223 ± 0.006 a A	0.228 ± 0.008 a A	0.238 ± 0.01 a B	0.133 ± 0.005 a A	0.129 ± 0.006 a A	0.132 ± 0.005 a A	0.135 ± 0.005 a A	a b; b a; a a
D ₇₅		0.209 ± 0.008 a A	0.210 ± 0.006 a A	0.217 ± 0.008 a A	0.226 ± 0.01 a B	0.126 ± 0.005 a A	0.122 ± 0.006 a A	0.125 ± 0.004 a A	0.128 ± 0.006 a A	a b; b a; a a
D ₆₀		0.182 ± 0.007 a A	0.179 ± 0.006 a A	0.189 ± 0.007 a A	0.195 ± 0.01 a B	0.108 ± 0.005 a A	0.102 ± 0.005 a A	0.108 ± 0.004 a A	0.111 ± 0.006 a A	a b; b a; a a
D ₅₀		0.166 ± 0.007 a A	0.160 ± 0.005 a A	0.172 ± 0.006 a A	0.177 ± 0.01 a B	0.098 ± 0.005 a A	0.091 ± 0.004 a A	0.098 ± 0.004 a A	0.101 ± 0.007 a A	a b; b a; a a
D ₄₀		0.134 ± 0.006 a A	0.124 ± 0.004 b A	0.140 ± 0.003 a A	0.139 ± 0.01 a B	0.078 ± 0.003 a A	0.072 ± 0.002 b A	0.079 ± 0.002 a A	0.078 ± 0.005 a A	a b; b a; a a
D ₂₅		0.125 ± 0.005 a A	0.115 ± 0.005 b A	0.131 ± 0.002 a A	0.128 ± 0.00 a B	0.073 ± 0.003 a A	0.067 ± 0.002 b A	0.074 ± 0.002 a A	0.072 ± 0.005 a A	a b; b a; a b
D ₁₆		0.107 ± 0.003 a A	0.097 ± 0.006 b A	0.112 ± 0.003 b A	0.106 ± 0.00 a B	0.062 ± 0.002 a A	0.058 ± 0.002 b A	0.064 ± 0.003 a A	0.060 ± 0.002 a A	a b; b a; a b
D ₁₀		0.092 ± 0.002 a A	0.084 ± 0.005 b A	0.096 ± 0.005 b A	0.088 ± 0.00 a A	0.053 ± 0.001 a A	0.052 ± 0.002 a A	0.055 ± 0.004 a A	0.051 ± 0.001 a A	a b; b a; a b
D ₅		0.075 ± 0.000 a A	0.071 ± 0.005 a A	0.079 ± 0.007 a A	0.071 ± 0.00 a A	0.045 ± 0.000 a A	0.045 ± 0.002 a A	0.047 ± 0.004 a A	0.044 ± 0.001 a A	a b; b a; a b
Mechanical sieving results (mm):										
D ₉₀		–	–	–	–	0.190	0.075	0.106	0.180	–
D ₇₅		–	–	–	–	0.106	0.067	0.095	0.106	–
D ₅₀		–	–	–	–	0.080	0.056	0.080	0.053	–
D ₂₅		–	–	–	–	0.058	0.045	0.064	0.040	–
D ₁₀		–	–	–	–	0.043	0.039	0.054	0.035	–
Other dimensional PSD parameters:										
Relative span		0.943 ± 0.029 b A	1.054 ± 0.043 a A	0.940 ± 0.052 a A	0.999 ± 0.02 a A	0.996 ± 0.030 a A	1.041 ± 0.069 a A	0.962 ± 0.050 a A	1.021 ± 0.033 a A	a a; a a; b a
Coefficient of uniformity		1.986 ± 0.071 a A	2.132 ± 0.156 a A	1.970 ± 0.164 b A	2.215 ± 0.11 a A	2.029 ± 0.053 a A	1.971 ± 0.139 a A	1.939 ± 0.155 a A	2.165 ± 0.130 a A	a a; a a; b a
Coefficient of gradation		1.073 ± 0.042 b A	1.017 ± 0.015 a A	1.077 ± 0.062 a A	1.119 ± 0.08 a B	1.050 ± 0.039 a A	0.965 ± 0.028 a A	1.041 ± 0.044 a A	1.053 ± 0.094 a A	a b; b a; a a
Uniformity index		30.221 ± 0.764 a A	28.068 ± 2.079 a A	30.682 ± 3.685 a A	26.811 ± 0.97 a B	29.945 ± 0.931 a A	30.795 ± 2.487 a A	31.607 ± 2.753 a A	28.329 ± 1.168 a A	a a; a a; b a
Size grade number		16.596 ± 0.722 a A	15.956 ± 0.527 a A	17.177 ± 0.592 a A	17.636 ± 0.76 a B	9.801 ± 0.477 a A	9.098 ± 0.384 a A	9.784 ± 0.371 a A	10.043 ± 0.715 a A	a a; b a; a a
Size range variation coefficient		36.941 ± 1.743 b A	42.715 ± 2.337 a A	36.838 ± 1.764 b A	40.364 ± 0.73 a A	39.577 ± 1.686 a A	42.268 ± 2.913 a A	37.717 ± 1.923 a A	41.043 ± 2.194 a A	a a; b a; a a
Graphic mean		0.168 ± 0.006 a A	0.163 ± 0.004 a A	0.174 ± 0.004 a A	0.177 ± 0.00 a B	0.100 ± 0.004 a A	0.095 ± 0.003 a A	0.100 ± 0.003 a A	0.101 ± 0.005 a A	a b; b a; a a
Inclusive graphic STD		0.060 ± 0.003 b A	0.065 ± 0.003 a A	0.062 ± 0.005 a A	0.068 ± 0.00 a A	0.037 ± 0.001 a A	0.037 ± 0.003 a A	0.036 ± 0.002 a A	0.039 ± 0.001 a A	a b; b a; a a
Inclusive graphic skewness		0.032 ± 0.038 a A	0.084 ± 0.036 a A	0.046 ± 0.040 a A	0.004 ± 0.05 a B	0.074 ± 0.038 a A	0.143 ± 0.039 a A	0.081 ± 0.038 a A	0.039 ± 0.074 a A	a b; a a; b a
Graphic kurtosis		0.937 ± 0.042 a A	0.878 ± 0.038 a A	0.942 ± 0.031 b A	0.889 ± 0.02 a A	0.928 ± 0.038 a A	0.883 ± 0.050 a A	0.933 ± 0.031 a A	0.890 ± 0.042 a A	a a; a a; b a
Geometric STD of high region		1.382 ± 0.029 b A	1.463 ± 0.023 a A	1.387 ± 0.007 a A	1.405 ± 0.03 a B	1.425 ± 0.035 a A	1.483 ± 0.037 a A	1.407 ± 0.010 a A	1.420 ± 0.056 a A	b a; b a; b a
Geometric STD of low region		1.557 ± 0.038 a A	1.647 ± 0.094 a A	1.542 ± 0.086 b A	1.676 ± 0.06 a A	1.581 ± 0.026 a A	1.572 ± 0.089 a A	1.536 ± 0.090 a A	1.672 ± 0.074 a A	a a; a a; b a
Geometric STD of total region		1.466 ± 0.025 b A	1.552 ± 0.050 a A	1.462 ± 0.040 b A	1.534 ± 0.02 a A	1.500 ± 0.019 a A	1.526 ± 0.052 a A	1.470 ± 0.043 a A	1.540 ± 0.026 a A	a a; a a; b a
Computer performance parameters:										
Total CPU time taken (s)		4.25 ± 0.50	3.00 ± 1.15	2.50 ± 0.58	2.50 ± 0.58	4.00 ± 0.82	2.75 ± 1.50	2.50 ± 0.58	2.50 ± 1.29	–
Number of particles analyzed per s		10,137 ± 394	8,683 ± 2,699	8,401 ± 1,279	8,546 ± 1,793	11,218 ± 1,179	9,403 ± 1,318	8,249 ± 805	8,938 ± 1,578	–

Note: Data and mean separation results were based on the original four image replications. Lowercase mean groups compare type of mills with same coal type and uppercase mean groups compare type of coal with same mill type; unsimilar letter groups represent statistically significant ($p < .05$; $\alpha = .05$).

* overall comparison of means between groups (length-based vs width-based analysis; lignite vs hard coal; ball-mill vs gyro-mill. For example: a b; a a = means length- and width-based analysis as well as type of coals are significantly different but not the type of mill with combined data).

Fig. 1. Original ball-milled lignite sample shown as one of the example test material used in the study.

Fig. 2. Flowchart of the length- and width-based Σ Volume machine vision plugin for PSD sieveless analysis of ground fine coals.

Fig. 3. A section of original color scan (A) and preprocessed threshold (B) and binary image (C) of fine ball-milled lignite fine coal samples ($2'' \times 2.5''$; 5080 DPI) used in the machine vision analysis. White background is required to provide contrast to the black fine coal particles during image scanning.

Fig. 4. Measurement strategy of length- and width-based particle dimensions for PSD machine vision analysis. P_1 and P_2 are two fine ground particles projection; a and b are major- and minor-axes of fitted equivalent area ellipse; C is the centroid; E is the fitted equivalent area ellipse; F is the Feret diameter enclosing the whole particle - measuring the “length”; and w is the “pixel-march” method distance along the ellipse’s minor-axis and through its centroid - measuring the “width”.

Fig. 5. A typical image of original color and preprocessed binary image of fine ball-milled lignite used in the machine vision analysis ($2'' \times 2.5''$ samples; 10,160 pixel $\times$ 12,700 pixel; 5080 DPI; and 0.005 mm measurement resolution).

Length-based particle size distribution

Ball-milled lignite

Gyro-milled lignite

Width-based particle size distribution

Ball-milled lignite

Gyro-milled lignite

Fig. 6. Length- and width-based machine vision analysis of fine ball- and gyro-milled lignite samples graphical outputs. A - particle lengths and widths variation; B - frequency distribution of particle lengths and widths; C - volume distribution of particles from length-based and width-based analysis; D - number of particles retained in virtual sieves based on lengths and widths; and E - cumulative undersize distribution of particle based on lengths and widths, respectively.

Fig. 7. Cumulative regular and log-normal under size PSD plots of fine lignite and hard coal ground by ball- and gyro-mills of combined sample image using machine vision methods compared with mechanical sieving.

Graphical abstract