Fusing Landmark-based Features at Kernel Level for Face Recognition

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Abstract

Because of the dramatic intra-class variations in lighting, expression and pose of face images, no single feature is rich enough to capture all the discriminant information, fusing multiple features is an efficient way to improve performance for face recognition. But most of existing fusing methods use features sampling at fixed grid and manually set too many parameters, thus their performances are limited. In this paper, we first propose an improved landmark-based multi-scale LBP feature to address the dramatic pose and expression variations, which samples features around landmarks instead of fixed grid. Then we propose a novel model which fuses LBP feature and Gabor feature at kernel-level to capture the information of facial texture and facial shape, where the weighted coefficients

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between kernels, the discriminant projection matrix and the standard deviations of RBF kernel are simultaneously learnt by the proposed optimization algorithm. Experiments are done on LFW, AR and Extended Yale B datasets, and results show that not only does the proposed method get much better identification performance than some state-of-the-art methods, but it also achieves competitive result for verification task.

Keywords: Face recognition, Landmark-based feature, Feature fusion, Multi-kernel discriminant analysis, Face identification, Face verification.

1. Introduction

Face recognition has attracted much attention because of its wide range of applications in access control and video surveillance [1]. Because of the dramatic intra-class variations in facial expression, viewing angle, lighting conditions, occlusions and image quality, face recognition is still a challenging problem. One of the key important issue is to find some efficient features that are both discriminative for inter-class difference and invariant to intra-class variations. Because no single feature is rich enough to capture all the discriminative information, fusing complementary features has become an active research topic in face recognition [2, 3, 4].

To fuse multiple features, we first need to find some efficient features. In the past decades, many feature extraction methods have been proposed for face recognition. Local Binary Patterns (LBP), introduced by Ojala et al.[5] in 1996, is a good choice for coding fine details of facial appearance and texture. They later extend LBP to multiple scales and compress the LBP codes by the uniform patterns [6]. Because LBP can capture small texture details and is resistant to lighting changes, Ahonen et al. introduce an LBP based method for face recognition [7], which divides the face into a regular grid of cells and calculates the histogram of the uniform LBP's within each cell. Finally, the cell-level histograms are concatenated to produce a global descriptor vector. For LBP's impressive performance [8], many its variants have been proposed for face recognition [1, 9, 10].

Besides LBP, Gabor transform based feature extraction methods also have been an active research area in recent years. Gabor wavelets capture the local structure corresponding to specific spatial frequency, spatial locality, and selective orientation. Gabor feature has been demonstrated to be discriminative and

robust to illumination and expression changes. Liu et al.[11] introduce a Gabor-Fisher classifier for face recognition. Shen et al. [12] present the recognition results for Gabor features and general discriminant analysis. Histogram of Gabor phase feature is proposed by Zhang et al. [13] for face recognition. Xie et al. [14] propose learned local Gabor patterns for face representation and recognition.

There are other features that can be used in face recognition, such as Gradient-face [15], Scale Invariant Feature Transform (SIFT) [16], Histograms of Oriented Gradients (HOG) [17] and LEarning-based descriptor (LE) [18]. Although these features have shown promising performances on public face datasets collected in the controlled environment, these hand-crafted descriptors cannot work well for uncontrolled face recognition. Chen et al. [19] densely sample multi-scale descriptors centered at dense facial landmarks after accurate face alignment. For uncontrolled face verification, the resulting high-dimensional features achieve significant improvements over their low-dimensional version sampled at fixed grid.

Though the above mentioned features have been demonstrated their effectiveness respectively, for most of fusing models, combining all the features cannot get better performance than only combining a few features, but introduces much more computational cost. So, selecting some appropriate and complementary features is crucial for fusing model to get good performance. We find that LBP feature and Gabor feature are complementary because LBP captures small appearance details while Gabor feature reflects the facial shape. So we consider fusing the two types of features. In addition, because landmark-based feature can deal with the misaligned situation in face image better than grid-based feature, so we consider landmark-based feature for face recognition.

After selecting appropriate features, it is important to build a good fusing mod-

51 el. Because Kernel Discriminant Analysis (KDA) [20] can project the data onto
52 a non-linear discriminant subspace to suppress intra-class variations and maxi-
53 mize the gap between the images from different persons, many KDA-based fusing
54 models have been proposed. Tan et. al. [2] apply kernel discriminative common
55 vector for the combined feature vector to extract discriminant nonlinear features
56 for recognition. Lin et al. [3] fuse diverse image descriptors by the proposed
57 multiple kernel learning for dimensionality reduction. Ren et al. [4] present band-
58 reweighed Gabor kernel embedding for face representation and recognition, using
59 Multiple Orientation and Scale Transforms (MOST) with multiple Gabor band
60 features. But the above methods set too many parameters manually, including the
61 standard deviations of RBF kernel, thus their performances are limited. So we pro-
62 pose a novel model to fuse LBP feature and Gabor feature at kernel-level, where
63 the weighted coefficients between kernels, the discriminant projection matrix and
64 the standard deviations of RBF kernel are simultaneously learnt.

65 Most of papers concern only one sub-problem of face recognition, i.e., either
66 face identification or face verification. Face verification is to determine whether
67 a pair of face images are from the same person or not, which is a binary-class
68 problem. But in traditional face identification, we must identify which person a
69 face image belongs to, which is a multi-class problem. Face identification and face
70 verification are two sub-problems in face recognition [21]. Though some methods
71 can achieve impressive results in a sub-problem, they can't get high performance
72 in another sub-problem. In contrast, our method concerns both sub-problems, and
73 gets high performance in both sub-problems.

74 The main contributions of this paper are listed as follows:

- 75 • We propose an improved landmark-based multi-scale LBP feature, which

76 adopts different scales of LBP and different sizes of patch to generate mul-
77 ti-scale features. To control the resulting feature dimension, we reduce the
78 number of landmarks along with the increase of LBP scale. Because the
79 landmark-based features maybe lose the geometric information among the
80 landmarks, we also propose combining the fixed grid-based features to im-
81 prove the performance.

- 82 • We propose a novel model to fuse LBP feature and Gabor feature at kernel-
83 level, where the weighted coefficients between kernels, the discriminant
84 projection matrix and the standard deviations of RBF kernel are simulta-
85 neously learnt.
- 86 • We propose an optimization algorithm to solve the kernel-level fusing mod-
87 el, which alternatively calculates the three types of parameters.

88 The remainder of the paper is organized as follows: In section 2, we describe
89 the proposed method in detail. The experimental results are given in section 3.
90 Finally, we provide the conclusion in section 4.

91 **2. The proposed method**

92 In this section, we describe the proposed method in detail. In section 2.1, we
93 propose an improved landmark-based multi-scale LBP feature. In section 2.2, we
94 propose combining landmark-based feature and grid-based feature for face recog-
95 nition. Then a novel kernel-level fusing model and corresponding optimization
96 algorithm are proposed in section 2.4 and section 2.5, respectively.

Figure 1: The proposed landmark-based multi-scale LBP feature. The uniform LBP codes are computed at 5 scales, using 8 sampling points on a circle of radius 2, 3, 4, 5 and 6, respectively. Then a patch is extracted centering at each landmark of each scale, and is divided into 4×4 cells. Finally, we calculate the histogram of the LBP codes for each cell and concatenate all the histograms. Along with the increase of LBP scale, the number of landmarks is reduced, while the size of the patch is increased.

97 2.1. An improved landmark-based multi-scale LBP feature

98 Most of face recognition methods use grid-based feature [1] [7][11][22]. They
 99 extract some local features for a grid of locations in an aligned face image, and
 100 learn a subspace after concatenating all the local features. The features in the
 101 same location of two samples are compared with each other, no matter whether
 102 they are the same landmarks or not. Because of the dramatic variations for ex-
 103 pression and pose, face images can hardly be aligned accurately. The grid-based
 104 methods may fail on the misaligned condition. To address this problem, Chen et al.
 105 propose the High Dimensional LBP (HDLBP) feature [19], which extracts LBP
 106 feature in some patches centering at each landmark positions. In the landmark-
 107 based method, the features of the same landmarks of two samples are compared
 108 with each other, so HDLBP gets significant improvements for face verification
 109 over its low-dimensional version sampled at fixed grid. However, because the di-
 110 mension of HDLBP is very high, it needs much computational and storage cost. In

111 this subsection, we propose an improved landmark-based multi-scale LBP feature,
 112 denoted by **LMLBP**, to deal with this issue.

For each image, we first detect the facial landmark positions [23]. Then we apply similarity transformation using the landmark points to transform a face to a canonical frame, with size of 400×400 . After landmark detection and face normalization, we extract 5 scales of uniform LBP feature. To be specific, the LBP code of a pixel g_0 is attained by the following equation:

$$LBP_R = \sum_{p=0}^7 t(g_p - g_0)2^p, \quad (1)$$

113 where g_p is the p^{th} pixel of 8 equally spaced pixels on a circle of radius R centering
 114 at g_0 . if $g > 0$, then $t(g) = 1$, otherwise $t(g) = 0$. The original number of
 115 bins using 8 neighbors is 256, which can be compressed to 59 by the uniform
 116 patterns[6]. We set $R = 2, 3, 4, 5, 6$ for each scale, respectively, shown in the
 117 second row of Fig. 1.

Then we extract a patch centering at each landmark of each scale, and divide it into 4×4 cells. At the larger LBP scale R , the size of the patch S_R are bigger, i.e.,

$$S_R = S_2(\sqrt{2})^{R-2}, \quad (2)$$

118 where $S_2 = 40$ is the size of patch of the first LBP scale ($R = 2$). Note that
 119 at different LBP scales, the image sizes of are equal, while the patch sizes are
 120 different.

121 To control the resulting feature dimension, we reduce the number of landmarks
 122 along with the increase of LBP scale, shown in the first row of Fig. 1. Finally, we
 123 calculate the histogram of the uniform LBP codes for each cell and concatenate
 124 all the histograms to form the landmark-based multi-scale LBP feature, resulting

125 in a $59 \times 4 \times 4 \times (21 + 16 + 10 + 7 + 3) = 53808$ dimensional LBP feature.

126 Since the face images can hardly be aligned accurately for the dramatic intra-
127 class variations, the proposed landmark-based LBP feature can deal with the mis-
128 aligned condition for face recognition. The various LBP radiuses and histogram
129 schemes create the multi-scale feature, which fits human visual system.

130 It should mentioned that though our LMLBP feature and HDLBP feature [19]
131 are landmark-based feature, they are different as follows. First, HDLBP resizes the
132 original image to different scales to create multi-scale features, where the patch
133 sizes from different scales are the same. In contrary, LMLBP adopts different s-
134 cales of LBP and different sizes of patch to generate multi-scale features, which
135 can capture more abundant information. Second, for HDLBP, the numbers of land-
136 marks for all scales are the same and the resulting dimension of HDLBP feature is
137 very high (127440). For LMLBP, we reduce the number of landmarks along with
138 the increase of LBP scale to control the resulting feature dimension (53808 for
139 LMLBP). The feature dimension of LMLBP is much lower than HDLBP. More-
140 over, we will show in the experiment that the performance of LMLBP is better
141 than HDLBP.

142 2.2. Combining landmark-based feature and grid-based feature

Figure 2: Grid-based multi-scale LBP feature. The image of each scale is divided into a grid of cells and extract LBP descriptor for each cell. Note that the sizes of the cells are bigger at the larger scale.

143 Because the landmark-based features may lose the geometric information a-
144 mong the landmarks, we combine the grid-based multi-scale features to improve
145 the performance. We crop a center region after transforming a face to a canonical
146 frame. Similar to landmark-based feature, we first extract 5 scales of uniform LBP
147 feature. Then we divide the LBP image of each scale into a grid of cells. At the
148 larger LBP scale, the size of the cells are bigger, shown in the Fig 2. Finally, we
149 calculate the histogram of the uniform LBP codes for each cell and concatenate
150 all the histograms to form the grid-based multi-scale LBP feature.

151 After extracting the landmark-based multi-scale LBP feature and grid-based
152 multi-scale LBP feature, we simply concatenate the two features at feature level
153 to get the final LBP feature. We will show in the experiment that combining grid-
154 based feature and landmark-based feature can get higher performance.

155 2.3. Combining Gabor feature

156 Gabor wavelets can capture a number of salient visual properties including
157 spatial localization, orientation selectivity and spatial frequency selectivity quite
158 well, which can encode facial shape and appearance information. They have been
159 widely used in face recognition[11]. As many face recognition studies, we use 40
160 Gabor wavelets of 5 different scales and 8 orientations. By convolving the input
161 image with these 40 Gabor kernels, we get 40 Gabor images.

162 To get more useful information, we resize the input image with a scaling factor
163 of $1, \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2\sqrt{2}}, \frac{1}{4}$, respectively, to get five input images. Then we extract 40
164 Gabor images for each input image, and normalize each Gabor image to zero
165 mean and unit variance. We extract fixed-size patches centering at each landmark
166 in each Gabor image of each input image. Similar to the previous subsection, to
167 control the resulting feature dimension, we reduce the number of landmarks along

168 with the decrease of the size of input image. Then we downsample each image
 169 patch by 4×4 uniform lattices. Finally, we concatenate all the down-sampled
 170 patches to form the landmark-based Gabor feature.

171 To get the grid-based Gabor feature, the 40 Gabor images are directly down-
 172 sampled by 4×4 uniform grids. We construct a vector out of each downsampled
 173 responses, normalize it to zero mean and unit variance, and concatenate them into
 174 a single dimensional feature vector.

175 Similar to LBP feature extraction, after extracting the landmark-based Gabor
 176 feature and grid-based Gabor feature, we simply concatenate the two features at
 177 feature level to attain the final Gabor feature.

178 2.4. A novel kernel-level fusing model

179 In this subsection, we propose a novel kernel-level fusing model to fuse LBP
 180 and Gabor feature, where the weighted coefficients between kernels, projection
 181 matrix and kernel parameters are simultaneously learnt.

Suppose we have N training samples, $\{\mathbf{x}_i\}_{i=1}^N$, $\mathbf{x}_i = \{\mathbf{L}_i, \mathbf{G}_i\}$, where $\mathbf{L}_i$ is the
 obtained LBP feature of i^{th} training sample, and $\mathbf{G}_i$ is the obtained Gabor feature.
 Suppose there is an ensemble kernel k generated by linearly combining the base
 kernels $\{k_L, k_G\}$:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \beta_L k_L(\mathbf{x}_i, \mathbf{x}_j) + \beta_G k_G(\mathbf{x}_i, \mathbf{x}_j), \quad (3)$$

where

$$k_L(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(\frac{-\chi^2(\mathbf{L}_i, \mathbf{L}_j)}{\sigma_L^2}\right), \quad (4)$$

$$k_G(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(\frac{-\|\mathbf{G}_i - \mathbf{G}_j\|_2^2}{\sigma_G^2}\right). \quad (5)$$

182 Note that we use chi-square distance for LBP feature and l_2 norm distance for
 183 Gabor feature to further exploit different advantages of different features. We will
 184 show in the experiments that the chi-square distance is significantly better than l_2
 185 norm distance for LBP feature.

Let ϕ denote the non-linear mapping induced by the ensemble kernel k , where

$$\langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle = k(\mathbf{x}_i, \mathbf{x}_j). \quad (6)$$

Kernel based subspace methods can be exploited here as the basic criterion for feature extraction[20]. The objective function of KDA is:

$$\max_{\mathbf{A}} \frac{\text{Tr}(\mathbf{A}^T \mathbf{K} \mathbf{W} \mathbf{K} \mathbf{A})}{\text{Tr}(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A})}, \quad (7)$$

where $\mathbf{K}$ is the kernel matrix, $\mathbf{K}_{i,j} = k(\mathbf{x}_i, \mathbf{x}_j) = \beta_L k_L(\mathbf{x}_i, \mathbf{x}_j) + \beta_G k_G(\mathbf{x}_i, \mathbf{x}_j)$, and $\mathbf{W}$ is defined as:

$$\mathbf{W}_{i,j} = \begin{cases} 1/m_c, & \text{if } \mathbf{x}_i \text{ and } \mathbf{x}_j \text{ belong to the } c\text{-th class} \\ 0, & \text{otherwise.} \end{cases}$$

186 where m_c is the number of samples of class c .

187 Besides the projection matrix $\mathbf{A}$, it is also important to find the optimal fusing
 188 weights $\boldsymbol{\beta} = [\beta_L, \beta_G]^T$ and the kernel parameters $\boldsymbol{\sigma} = [\sigma_L, \sigma_G]^T$. Based on Eq.
 189 (7), we propose a new model to fuse LBP feature and Gabor feature, which finds
 190 the kernel-level fusing weights, the kernel parameters and the projection matrix,
 191 to suppress intra-class variations and maximize the gap between the images from
 192 different persons, i.e.:

$$\max_{\mathbf{A}, \boldsymbol{\beta}, \boldsymbol{\sigma}} \frac{\text{Tr} \left(\mathbf{A}^T \left(\sum_{i,j=1}^N \mathbf{W}_{i,j} \mathbf{K}_{(i)} \boldsymbol{\beta} \boldsymbol{\beta}^T \mathbf{K}_{(j)}^T \right) \mathbf{A} \right)}{\text{Tr} \left(\mathbf{A}^T \left(\sum_{i,j=1}^N \mathbf{K}_{(i)} \boldsymbol{\beta} \boldsymbol{\beta}^T \mathbf{K}_{(j)}^T \right) \mathbf{A} \right)}, \quad (8)$$

$$\text{where } \mathbf{K}_{(i)} = \begin{bmatrix} k_L(\mathbf{x}_1, \mathbf{x}_i) & k_G(\mathbf{x}_1, \mathbf{x}_i) \\ \dots & \dots \\ k_L(\mathbf{x}_N, \mathbf{x}_i) & k_G(\mathbf{x}_N, \mathbf{x}_i) \end{bmatrix} \text{ and } \beta \geq 0.$$

It should be mentioned that model (8) is a new model, which simultaneously optimizes three kinds of parameters, i.e., $\mathbf{A}$, β and σ .

2.5. Optimization algorithm

In this subsection, we propose an optimization algorithm to solve the kernel-level fusing model (8), which alternatively calculates $\mathbf{A}$, β and σ . Specifically, we first fix β and σ to calculate $\mathbf{A}$. Then we fix $\mathbf{A}$ and σ to calculate β . Last, we fix $\mathbf{A}$ and β to calculate σ . The process will be iteratively performed until some predefined conditions are achieved.

2.5.1. Optimize $\mathbf{A}$

When β and σ are fixed, the optimization problem is reduced to a standard KDA problem, which needs to solve the following eigen problem [20]:

$$\mathbf{K}\mathbf{W}\mathbf{K}\boldsymbol{\alpha} = \lambda\mathbf{K}\mathbf{K}\boldsymbol{\alpha}. \quad (9)$$

The columns of $\mathbf{A}$ are the eigenvectors corresponding to the top several eigenvalues. We use the Matlab code¹ developed by Cai to solve the KDA problem for its lower complexity.

2.5.2. Optimize β

When $\mathbf{A}$ and σ are fixed, the objective function becomes:

$$\max_{\beta} \frac{\beta^T \mathbf{S}_b^{\mathbf{A}} \beta}{\beta^T \mathbf{S}_t^{\mathbf{A}} \beta} \quad (10)$$

¹Matlab code: <http://www.cad.zju.edu.cn/home/dengcai/Data/DimensionReduction.html>

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$$\text{s.t. } \beta \geq 0$$

where $\mathbf{S}_b^{\mathbf{A}} = \sum_{i,j=1}^N w_{i,j} \mathbf{K}_{(i)}^T \mathbf{A} \mathbf{A}^T \mathbf{K}_{(j)}$ and $\mathbf{S}_t^{\mathbf{A}} = \sum_{i,j=1}^N \mathbf{K}_{(i)}^T \mathbf{A} \mathbf{A}^T \mathbf{K}_{(j)}$. The additional constraints $\beta \geq 0$ cause that the optimization problem can't be formulated as a generalized eigenvalue problem. It becomes a non-convex quadratically constrained quadratic programming problem which is known to be hard to solve. So we consider solving its convex relaxation by adding an auxiliary matrix $\mathbf{B}$:

208

$$\begin{aligned} & \max_{\beta, \mathbf{B}} \text{Tr}(\mathbf{S}_b^{\mathbf{A}} \mathbf{B}) \\ & \text{s.t. } \begin{cases} \text{Tr}(\mathbf{S}_t^{\mathbf{A}} \mathbf{B}) = 1 \\ \begin{pmatrix} 1 & \beta \\ \beta & \mathbf{B} \end{pmatrix} \succeq 0 \\ \beta \geq 0 \end{cases} \end{aligned} \quad (11)$$

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This is a semi-definite programming (SDP) relaxation of the non-convex problem, and can be efficiently solved by SDP. In the particular case of $\mathbf{B} = \beta \beta^T$, the above two problems are equivalent. We use the Matlab toolbox CVX² to solve the SDP problem. As a famous Matlab package for convex optimization, CVX is very fast and easy to use.

214

2.5.3. Optimize σ

When $\mathbf{A}$ and β are fixed, the objective function becomes

$$J(\sigma) \doteq \frac{\text{Tr}(\mathbf{A}^T \mathbf{K} \mathbf{W} \mathbf{K} \mathbf{A})}{\text{Tr}(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A})}, \quad (12)$$

To achieve the maximum of $J(\sigma)$, we propose an iterative method based on gradient descent. The kernel parameters σ are updated along the direction of

²Matlab code: <http://cvxr.com/cvx/>

maximizing the objective function:

$$\boldsymbol{\sigma}_{i+1} = \boldsymbol{\sigma}_i - \rho \nabla J(\boldsymbol{\sigma}_i), \quad (13)$$

where

$$\nabla J(\boldsymbol{\sigma}) = \left[\frac{\partial J}{\partial \sigma_L}, \frac{\partial J}{\partial \sigma_G} \right]^T, \quad (14)$$

$$\begin{aligned} \frac{\partial J}{\partial \sigma_L} = & \frac{Tr \left(\frac{\partial(\mathbf{A}^T \mathbf{K} \mathbf{W} \mathbf{K} \mathbf{A})}{\partial \sigma_L} \right) Tr(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A})}{(Tr(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A}))^2} \\ & - \frac{Tr(\mathbf{A}^T \mathbf{K} \mathbf{W} \mathbf{K} \mathbf{A}) Tr \left(\frac{\partial(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A})}{\partial \sigma_L} \right)}{(Tr(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A}))^2}, \end{aligned} \quad (15)$$

$$Tr \left(\frac{\partial(\mathbf{A}^T \mathbf{K} \mathbf{W} \mathbf{K} \mathbf{A})}{\partial \sigma_L} \right) = Tr \left(\mathbf{A}^T \frac{\partial \mathbf{K}}{\partial \sigma_L} \mathbf{W} \mathbf{K} \mathbf{A} + \mathbf{A}^T \mathbf{K} \mathbf{W} \frac{\partial \mathbf{K}}{\partial \sigma_L} \mathbf{A} \right),$$

$$Tr \left(\frac{\partial(\mathbf{A}^T \mathbf{K} \mathbf{K} \mathbf{A})}{\partial \sigma_L} \right) = Tr \left(\mathbf{A}^T \frac{\partial \mathbf{K}}{\partial \sigma_L} \mathbf{K} \mathbf{A} + \mathbf{A}^T \mathbf{K} \frac{\partial \mathbf{K}}{\partial \sigma_L} \mathbf{A} \right),$$

$$\frac{\partial \mathbf{K}_{i,j}}{\partial \sigma_L} = \beta_L \frac{\partial}{\partial \sigma_L} k_L(\mathbf{x}_i, \mathbf{x}_j) = \beta_L \frac{2\chi^2(\mathbf{L}_i, \mathbf{L}_j)}{\sigma_L^3} \exp \left(\frac{-\chi^2(\mathbf{L}_i, \mathbf{L}_j)}{\sigma_L^2} \right).$$

215 $\frac{\partial J}{\partial \sigma_G}$ can be computed similar to $\frac{\partial J}{\partial \sigma_L}$. Then we alternatively calculate $\mathbf{A}$, β
 216 and $\boldsymbol{\sigma}$, which can be repeated until some predefined conditions are achieved. The
 217 detail algorithm is summarized as Algorithm 1.

218

219 **Algorithm 1** Optimization algorithm for the fusing model.

220 **Require:**

221 The training set $\mathbf{X}$ and corresponding class label; Iteration times T ; Iteration
 222 coefficient ρ .

223 **Ensure:**

- 224 The projection matrix $\mathbf{A}$, the fusing weights β and the kernel parameters σ .
- 225 1: Let $\beta = [0.5, 0.5]^T$ and σ be the mean distance of all pairs of training samples.
- 226 2: Optimize $\mathbf{A}$ according to Eq. (9).
- 227 3: Optimize β according to Eq. (10).
- 228 4: Optimize σ according to Eq. (13).
- 229 5: If $t < T$, set $t = t + 1$, goto step 2.
- 230 6: **return** $\mathbf{A}$, β and σ .
-

231 2.6. Complexity analysis

232 In the training stage, optimizing the projection matrix $\mathbf{A}$ by Eq. (9) is an
233 eigen-decomposition problem. The classical solution algorithm requires $O(N^3)$
234 operations [20, 24], where N is the number of training samples. However, Cai et
235 al. presented an algorithm for KDA, called Spectral Regression Kernel Discrimi-
236 nant Analysis (SRKDA) [24], and designed an incremental SRKDA to solve the
237 eigen-decomposition problem, which only requires $O(N^2)$ operations. Optimiz-
238 ing the fusing weights β by Eq. (11) requires $O(n^2m)$ operations, where n is the
239 number of variables and m is the number of constraints, n is linear to the number
240 of the fusing descriptors, and m is quadratic to that one. Optimizing the kernel
241 parameters σ by Eq. (13) only needs to calculate the gradient direction once. Be-
242 cause we only consider two kinds of descriptors, the computational complexity
243 focuses on solving eigen-decomposition problem Eq. (9).

244 In the testing stage, we first need to project the testing sample by KDA, which
245 requires $O(Nd)$ operations, where N is the number of training samples and d
246 is the feature dimension. Then we use nearest neighbor classifier to classify the

247 testing sample, which needs $O(Np)$ operations, where p is the reduced feature di-
248 mension by KDA. In summary, the testing process only requires $O(N)$ operations.

249
250 Because the training process requires $O(N^2)$ operations, and the testing pro-
251 cess only requires $O(N)$ operations, the proposed method is scalable on large
252 datasets. Especially, the testing process of the proposed method is much faster
253 than some sparse representation-based classifiers, which will be proved in the ex-
254 periment.

255 3. Experimental results

256 In this section, we perform experiments on three datasets: Labeled Faces in
257 the Wild (LFW) [25], AR [26] and Extended Yale B [27], to demonstrate the
258 performance of the proposed method. In Section 3.1, we present the results for
259 face identification on LFW dataset. We first compare the proposed method with
260 some state-of-the-art methods. Then the Gabor-based methods and the LBP-based
261 methods are also compared with the proposed method. Last, we give the role of
262 each step of the proposed method. In Section 3.2, the results of face verifica-
263 tion under unrestricted protocol are presented. Finally, we give the results for AR
264 database and Extended Yale B database in Section 3.3 and Section 3.4 respective-
265 ly.

266 The proposed method **F**uses **L**andmark-based **L**BP feature and **G**abor feature
267 at kernel level, thus we call it **FLLG**.

268 3.1. Face identification on LFW dataset

269 LFW dataset contains 13233 images of 5749 people downloaded from the we-
270 b, which is designed for unconstrained face recognition with dramatic variations

Figure 3: Some examples of one person in LFW dataset.

271 of pose, illumination, expression, misalignment and occlusion, and so on. Some
 272 examples of one person are shown in Fig. 3. As [10], a subset of LFW are used in
 273 the identification experiments, which consists of 5425 images of 311 subjects with
 274 no less than 6 samples per subject. We randomly select 5 samples per subject for
 275 training, and the rest for testing. We repeat the experiments 10 times, and report
 276 some statistics such as the mean accuracy and standard deviation.

277 3.1.1. Comparison of state-of-the-art methods

278 We compare the proposed method with some state-of-the-art methods, such
 279 as Gradientface[15], SRC[28], ESRC[29], CESR[30], SLF-RKR[10], MOST[4]
 280 and Joint Bayesian (JB)[21], and we will see that the performance of the proposed
 281 method is much better than the state-of-the-art methods. For SRC and ESRC, we
 282 first project the images to a PCA subspace with 1000 dimensions for the sake of
 283 computation efficiency, while almost not losing any useful information. The Dual
 284 Augmented Lagrange Multiplier(DALM)[31] method is used to solve all the l_1
 285 minimization problems, where the sparsity coefficient $\lambda=0.001$. For JB, we also
 286 use the HDLBP feature[19] and project the features to a PCA subspace with 400
 287 dimensions proposed in the original paper. When we increase the PCA dimensions
 288 to 1000, the performance of JB is decreased dramatically. For Gradientface, we
 289 use the Cosine function to calculate the similarity between Gradientfaces. For
 290 MOST, we use the Euclidean distance for the extracted features. For SLF-RKR,
 291 we use the default optimal parameters in the original paper. For the proposed

292 fusing model, we simply let iteration times $T = 10$ and iteration coefficient ρ be
293 the mean distance of all pairs of training samples.

294 Fig. 4 shows the boxplot of the identification accuracies for state-of-the-art
295 methods and the proposed method on LFW dataset. The box has lines at the lower
296 quartile, median, and upper values of 10 recognition accuracies. The whiskers are
297 lines extending from each end of the box to show the extent of the rest of the data.
298 According to the results, we can draw the following conclusions:

- 299 • Gradientface, SRC, ESRC and CESR all get poor identification perfor-
300 mance on the database, with the mean identification accuracies of 27.89,
301 38.77, 45.90 and 36.09 percent, respectively.
- 302 • SLF-RKR is a LBP-based kernel method, whose performance is better than
303 SRC and ESRC. MOST is based on Gabor feature and multiple kernel learn-
304 ing, and get better result.
- 305 • HDLBP+JB is also get good performance with a mean accuracy of 84.62
306 percent, which has shown very good result for face verification[19]. We will
307 see in next sub-section that JB only get a mean accuracy of 67.61 percent
308 based on gird-based LBP feature, which is even much lower than KDA
309 that gets 76.19 percent. It proves that JB can't get as good result for face
310 identification as that for verification.
- 311 • The proposed FLLG significantly improves the performance compared with
312 the other state-of-the-art methods, geting a mean accuracy of 89.57 percent,
313 which clearly shows the effectiveness of the proposed method in dealing
314 with uncontrolled face identification.

Figure 4: Comparison of state-of-the-art methods and the proposed method on LFW dataset.

Table 1: Comparison of training time and classification time for state-of-the-art methods.

Methods	Training time(s)	Classification time(s)	Mean accuracy(%)
SRC	-	2.2	38.77
ESRC	-	7.9	45.90
CESR	-	24.1	36.09
SLF-RKR	-	18.2	58.99
MOST	490	0.20	69.78
HDLBP+JB	580	0.22	84.62
FLLG	672	0.28	89.57

Table 1 shows the comparison of training time and classification time for state-of-the-art methods. The recognition accuracy is the mean value of 10 random experiments. We can find that the sparse representation-based classifiers such as SRC, ESRC, CESR and SLF-RKR do not have any training process, but they take much more classification time, and get lower recognition accuracies. The classification of MOST is the fastest, but its recognition accuracy is not good enough. The classification time of FLLG is only a little more than HDLBP+JB, and the recognition accuracy of FLLG is the highest.

3.1.2. Comparison of Gabor-based methods

Because the proposed method uses Gabor feature, we compare it with some other Gabor-based methods, such as 1NN, PCA, LDA, SRC, KDA, JB. We use the grid-based Gabor feature for these methods. For PCA, we report the accuracy at 1000 dimensions. We first project the Gabor feature to a PCA subspace with 1000 dimensions for LDA and SRC, and 400 dimensions for JB.

Fig. 5 shows the Comparison of the Gabor-based methods and the proposed method on LFW dataset. According to the results, we can draw the following conclusions:

- 1NN and PCA get very low identification accuracies, because there are dramatic intra-class variations on LFW database though based on Gabor feature. The performance of LDA is better than PCA because LDA is a supervised method, but it is still poor.
- Based on Gabor feature, SRC is significantly better than the above methods and the one based on original image pixel, which demonstrates Gabor feature is good feature for face recognition, especially using SRC.

- The performance of JB is better than PCA and LDA, but is worse than SRC, which shows that though the binary classification method can get high performance for face verification, but it can't get high performance for face identification.
- The performance of the proposed FLLG is much better than all other methods, which clearly shows the superiority of the proposed method.

Figure 5: Comparison of Gabor-based methods and the proposed method on LFW dataset.

3.1.3. Comparison of LBP-based methods

Similar to the comparison of Gabor-based methods, we compare the proposed method with some traditional LBP-based methods. Besides the methods used in the above subsection, we also use the chi-square distance for 1NN, denoted by 1NN(X2), and the RBF kernel with chi-square distance for KDA, denoted by KDA(X2), to further exploit the advantage of LBP feature.

Fig. 6 shows the boxplot of the identification accuracies for the LBP-based methods on LFW dataset. As we can see, the relation of the results for 1NN,

353 PCA, LDA, SRC, KDA, JB and the proposed method is similar to those in Fig. 5.
 354 The identification accuracies of 1NN(X2) and KDA(X2) are significantly better
 355 than 1NN and KDA, respectively, which shows that the chi-square distance is
 356 better than l_2 norm distance for LBP feature. The performance of KDA is much
 357 better than SRC and JB by selecting an appropriate kernel, which proves that
 358 the kernel based method can efficiently model the highly non-linear problem for
 359 uncontrolled face recognition.

Figure 6: Comparison of LBP-based methods and the proposed method on LFW dataset.

3.1.4. The role of each step of the proposed method

361 Each step of the proposed method make a contribution to the high perfor-
 362 mance. Table 2 shows the role of each step of the proposed method. According to
 363 the results, we can draw the following conclusions:

- 364 • The identification accuracy of the grid-based multi-scale LBP feature is
 365 higher than grid-based LBP feature with only one scale, and that of landmark-
 366 based Multi-scale LBP is better than landmark-based LBP too, which proves
 367 that the multi-scale feature can improve the performance, especially for the
 368 landmark-based feature.

Table 2: The role of each step of the proposed method.

Method	Mean acc. $\pm$ std. dev.
Grid-based LBP	76.19 ± 2.0
Grid-based multi-scale LBP	81.27 ± 1.3
Landmark-based LBP	80.44 ± 1.8
Landmark-based multi-scale LBP	86.12 ± 1.5
Grid-landmark-based multi-scale LBP	86.69 ± 1.4
Grid-based Gabor	72.24 ± 1.6
Landmark-based Gabor	81.77 ± 1.7
Grid-landmark-based Gabor	82.12 ± 1.8
HDLBP	83.59 ± 1.1
FLLG$^{\sigma}$	87.89 ± 1.0
FLLG$^{\beta}$	88.73 ± 1.2
FLLG	89.57 ± 1.0

- 369 • The landmark-based features significantly outperform the grid-based fea-
370 tures, no matter using LBP or Gabor feature, which shows that the landmark-
371 based features are good at misaligned condition or pose variation on uncon-
372 trolled datasets.
- 373 • Simply combining the grid-based and landmark-based multi-scale LBP at
374 the feature level, i.e., Grid-landmark-based multi-scale LBP or Gabor fea-
375 ture, we can get higher performance, because the grid-based feature can
376 provide the geometric information among the landmarks.
- 377 • Our landmark-based multi-scale LBP feature outperforms HDLBP, though
378 the dimension of HDLBP is much higher than that of our feature. It proves
379 the effectiveness of our feature extraction method.
- 380 • Fusing the LBP feature and the Gabor feature by the proposed kernel-level
381 fusing model, FLLG gets the highest performance. It increases about an
382 average of 8.3 percent than grid-based multi-scale LBP, 3.4 percent than
383 landmark-based multi-scale LBP, 7.4 percent than Grid-landmark-based Ga-
384 bor, and 6.0 percent than HDLBP feature, which clearly shows the proposed
385 method can effectively fuse the LBP and Gabor features.

386 To examine the contribution of each component of the proposed fusing model,
387 we also consider the following two situations. In the first case, denoted as $FLLG^\sigma$,
388 the standard deviations are not automatically learnt, i.e., we simply let the standard
389 deviation be the mean distance of training samples and learn the weighted coef-
390 ficients between kernels and the projection matrix. In the second case, denoted
391 as $FLLG^\beta$, the weighted coefficients between kernels are not automatically learnt,
392 i.e., we simply set the same weighted coefficient for each kernel, and learn the

Table 3: Comparison of different alignment methods for grid-based feature.

Methods	Original	VGG	TCDCN	SDM
Mean acc. $\pm$ std. dev.	67.03 ± 1.9	74.43 ± 2.2	75.12 ± 2.3	76.19 ± 2.0

projection matrix and the standard deviations. We can find that the performance of FLLG is significantly better than that of $FLLG^\sigma$, which proves the efficiency of the learning model for the standard deviation. FLLG outperforms $FLLG^\beta$ too, which shows the necessity of the learning model for the weighted coefficients.

3.1.5. Comparison of different alignment methods for grid-based feature.

For grid-based methods, the features in the same location of two images are compared with each other, so the face images require to be aligned in a canonical space as accurately as possible. There are many methods to align images, such as Oxford Visual Geometry Group baseline face processing (VGG) [32], Supervised Descent Method (SDM) [23] and Tasks-Constrained Deep Convolutional Network (TCDCN) [33]. Table 3 shows the comparison of different alignment methods for grid-based feature, where the recognition accuracy is the mean value of 10 random experiments, and the ‘‘Original’’ method uses the original LFW images provided by the official website. We find that VGG, TCDCN and SDM get similar results, and is much better than the original images.

Because the accuracy of SDM is the highest, we use SDM to align the original images. Specifically, for each image, we first run the Viola Jones face detector [34] to locate the face. Then we use SDM to detect the facial landmark positions. Finally, we apply similarity transformation using these landmark points to transform a face to a canonical frame, and crop a center region of size 120×100 for

Figure 7: LBP features and Gabor features. The first three images are the LBP features with different scales, and the last three images are the Gabor features with different scales and orientations. We can find that LBP feature can encode fine details of facial appearance and texture, and Gabor feature can reflect facial shape and appearance information. Their complementary nature makes them good candidates for fusion.

413 further feature extraction.

414 3.1.6. *Comparison of fusing different kinds of features.*

415 For the proposed fusing model, selecting appropriate and complementary com-
 416 ponent features is crucial for good performance. LBP feature is a good choice for
 417 coding fine details of facial appearance and texture. In contrast, Gabor feature can
 418 encode facial shape and appearance information. Both representations can deal
 419 with illumination. Their complementary nature makes them good candidates for
 420 fusion, as shown in Fig. 7.

421 Besides LBP feature and Gabor feature, we can fuse more features. Recently,
 422 some learning-based features, such as LEarning-based descriptor (LE) [18] and
 423 Discriminant Face Descriptor (DFD) [1], can improve the classification results.
 424 Here we consider fusing more features, including the learning-based features and
 425 SIFT feature [16]. Table 4 shows the comparison of fusing different kinds of
 426 features, where the SIFT, LE and DFD features are created with multi-scale and
 427 landmark-based type, just like the proposed landmark-based multi-scale LBP fea-
 428 ture. We can find that when fusing two types of features, LBP+Gabor gets better
 429 performance than LBP+SIFT, LBP+LE and LBP+DFD. When fusing more than

Table 4: Comparison of fusing different kinds of features.

Features	Mean acc. $\pm$ std. dev.
LBP+Gabor	89.57 ± 1.0
LBP+SIFT	88.18 ± 1.1
LBP+LE	87.64 ± 1.2
LBP+DFD	87.56 ± 1.3
LBP+Gabor+SIFT	89.59 ± 1.1
LBP+Gabor+LE	89.53 ± 1.2
LBP+Gabor+DFD	89.32 ± 1.1
LBP+Gabor+SIFT+DFD	89.61 ± 1.1

two types of features, the recognition accuracy cannot be significantly improved compared with LBP+Gabor, but much computational cost is required. So we only consider to fuse LBP feature and Gabor feature for face recognition.

In recent years, deep learning has been proved to be able to learn the natural patterns in images. We also combine these deep learning-based features with the proposed fusing model to improve the performance. Table 5 shows the performance of combining two deep learning-based features, i.e., DeepID2 [35] and GoogleNet [36], with the proposed fusing model. We can find that combining DeepID2 and GoogleNet with the proposed fusing model can improve the recognition accuracy.

3.2. Face verification on LFW dataset

LFW specifies a number of evaluation protocols for face verification. We follow the unrestricted protocol and only use training data provided by LFW. There

Table 5: Combining deep learning-based features with the proposed fusing model.

Features	Mean acc. $\pm$ std. dev.
DeepID2	98.55 $\pm$ 0.3
GoogleNet	98.58 $\pm$ 0.2
DeepID2+GoogleNet	99.05 $\pm$ 0.2

are more than 10000 training samples of more than 5000 people, but most of people have only one or two samples. For the proposed method, to learn more representative features, we select the top 300 persons which have the most number of images, where there are more than 4000 samples, to train the projection matrix $\mathbf{A}$ and the fusing weights β . Then we project all training samples into the learned space of 299 dimensions. In order to get better performance for face verification, we use ITML[37] to learning a Mahalanobis distance function for the binary-class problem in the projected space of lower dimension, where 10000 positive pairs and 10000 negative pairs are uniformly sampled to learn the Mahalanobis matrix.

We compare against the best results listed in the LFW official web-site, such as Combined multishot [38], LBP multishot [38], LBP PLDA [39], Joint Bayesian [21], HDLBP+JB [19], Fisher vector [40], ConvNet-RBM [41], SLBP [42], VisionLabs [43], Aurora [44] and HPEN + HDLBP + JB [45], and the deep learning-based methods with outside training data such as ConvNet-RBM [41], DeepFace [46], DeepID [47], DeepID2 [35] and DeepID3 [48].

Table 6 lists mean verification accuracies and standard deviations for state-of-the-art methods on LFW dataset under unrestricted protocol without outside

461 training data. Our method achieves 94.03% face verification accuracy, which is
462 better than HDLBP+JB [19] and Fisher vector [40], and is only lower than H-
463 PEN+HDLBP+JB without outside training data.

464 Though the performances of the deep learning-based methods are better than
465 the proposed method, they come with some challenges. First, it remains unclear
466 how to design a “good” CNN architecture to adapt to a specific classification task
467 due to the lack of theoretical guidance [49]. Second, the training data is very
468 important for the performance of face recognition. To achieve better results, we
469 need to add more faces which are collected in the same situation as the evaluation
470 dataset [50]. Finally, because of the requirement for large amounts of training
471 data, it needs high computational cost during the training process [51]. So our
472 method which is less costly to train and evaluate is still competitive for face veri-
473 fication.

474 Fig. 8 shows the ROC curves averaged over 10 fold cross validation without
475 outside training data. For more explicit, we only show the figure where false
476 positive rate is from 0 to 0.5 and true positive rate is from 0.5 to 1. It can be
477 found that the performance of our method is as good as the best results, which
478 demonstrates that our method can not only get good identification performance
479 but also do well in verification task.

480 3.3. *Face identification on AR database*

481 In this section, we perform experiments on AR face databases, and compare
482 the proposed method with several state-of-the-art methods, such as LDA[22], Gra-
483 dientface (GF)[15], SRC[28], ESRC[29], SLF-RKR[10], MOST[4] and JB[21].

484 The AR face database[26] consists of over 4,000 facial images from 126 sub-
485 jects (70 men and 56 women). These images suffer different facial variations,

Table 6: Mean verification accuracies and standard deviations for state-of-the-art methods on LFW dataset under unrestricted protocol.

	Method	Mean acc. $\pm$ std. dev.
without outside training data	Combined multishot	89.50 ± 0.51
	LBP multishot	85.17 ± 0.61
	LBP PLDA	87.33 ± 0.55
	Joint Bayesian	90.90 ± 1.48
	HDLBP+JB	93.18 ± 1.07
	Fisher vector	93.03 ± 1.05
	ConvNet-RBM	91.75 ± 0.48
	SLBP	90.00 ± 1.33
	VisionLabs	92.90 ± 0.31
	Aurora	93.24 ± 0.44
	HPEN + HDLBP + JB	94.87 ± 0.38
	FLLG	94.03 ± 1.16
with outside training data	ConvNet-RBM	92.52 ± 0.38
	DeepFace	97.35 ± 0.25
	DeepID	97.45 ± 0.26
	DeepID2	99.15 ± 0.13
	DeepID3	99.53 ± 0.10

Figure 8: ROC curves averaged over 10 fold cross validation under LFW unrestricted protocol without outside training data.

including various facial expressions, illumination variations and occlusion by sun-
glasses or scarf. In our experiment, we use a subset consisting 119 subjects. For
each subject, 14 images with only illumination changes and expressions are se-
lected and taken into two sessions. The gray-scale images are cropped and resized
to 112×92 .

To ensure that our results will not depend on any special choice of the training
data, for each subject, seven images from the two sessions are randomly selected
for training and the rest seven images were for testing. We repeat the experiment
10 times. Fig. 9 shows the boxplot of the identification accuracies for state-of-
the-art methods on AR dataset. It can be found that the proposed method achieves
the best result with a mean of 98.76 percent, which is 8.8, 22.9, 5.4, 2.4, 2.5, 0.2
and 5.3 percent higher than those of LDA, Gradientface, SRC, ESRC, SLF-RKR,
MOST and JB, respectively. The standard deviation of FLLG is the smallest,

499 which shows the stability of the proposed method.

Figure 9: Boxplot of the identification accuracies for state-of-the-art methods on AR dataset.

500 3.4. Face identification on Extended Yale B database

501 In this section, we perform experiments on Extended Yale B face databases,
502 and compare the proposed method with several state-of-the-art methods. Extend-
503 ed Yale B database[27] contains about 2,414 frontal face images 38 individuals.
504 We use the cropped and normalized 96×84 face images, which are taken under
505 extreme illumination variation. For each subject, we randomly select some im-
506 ages for training respectively, and the rest for testing. We repeat the experiment
507 10 times in each condition.

508 Fig. 10 shows the boxplot of the identification accuracies for state-of-the-art
509 methods on Extended Yale B dataset with 5 samples per person. It can be found
510 that Gradientface gets high performance on the dataset with extreme illumination
511 variation. The proposed method achieves the best result with a mean of 98.79
512 percent, which is 4.7, 6.5, 12.1, 5.1, 8.2, 6.1 and 14.5 percent higher than those of
513 LDA, Gradientface, SRC, ESRC, SLF-RKR, MOST and JB, respectively.

Figure 10: Boxplot of the identification accuracies for state-of-the-art methods on Extended Yale B dataset with 5 samples per person.

4. Conclusion

In this paper, we propose an improved landmark-based feature and a novel fusing model for face recognition that can not only get good identification performance but also do well in verification task. The proposed landmark-based multi-scale LBP feature samples features around landmarks instead of fixed grid and adopts multi-scale histogram to get more robust representation. The multi-scale fixed grid sampling features are also combined, which can provide the geometric information among the landmarks. The proposed fusing model fuses LBP feature and Gabor feature at kernel-level, where the weighted coefficients between kernels, the discriminant projection matrix and the standard deviations of RBF kernel are simultaneously learnt. To solve the kernel-level fusing model, an optimization algorithm is also proposed, which alternatively calculates the three types of parameters. Experiments are done on several datasets, giving very promising results for the proposed method in both identification and verification.

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