DARK AND SINGULAR DISPERSIVE OPTICAL SOLITONS OF SCHRÖDINGER-HIROTA EQUATION BY MODIFIED SIMPLE EQUATION METHOD

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Abstract

This paper obtains dispersive dark and singular optical solitons, governed by Schrödinger-Hirota equation, in optical fibers. The integration algorithm is the modified simple equation method. Both Kerr law and power laws of nonlinearity are considered.

Keywords: solitons, Schrödinger-Hirota equation; modified simple equation method.

1 INTRODUCTION

Dispersive optical solitons are studied when group velocity dispersion (GVD) is small and needs to be supplemented with third order dispersion (3OD) [1-20]. The model that accurately describes this phenomena is the Schrödinger-Hirota equation (SHE) which stems from the familiar nonlinear Schrödinger's equation (NLSE) through Lie transform. This derivation is described in lot of earlier works [4-10]. The integration tool that will be applied to SHE is the modified simple equation method. This scheme will lead to the extraction of dark and singular optical solitons only. Both Kerr and power laws of nonlinearity are studied in this paper. It will also be proved that power law of nonlinearity will not support these soliton solutions. The constraint conditions for the existence of such solitons in Kerr law medium will be presented.

2 THE MODIFIED SIMPLE EQUATION METHOD (SUCCNCT REVIEW)

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \dots) = 0, \quad (1)$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [1, 19, 20].

Step-1: We use the transformation

$$u(x, t) = u(\xi), \quad \xi = x - ct, \quad (2)$$

where c is a constant to be determined, to reduce Eq. (1) to the following ODE :

$$Q(u, u', u'', \dots) = 0, \quad (3)$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step-2: We suppose that Eq. (3) has the formal solution.

$$u(\xi) = \sum_{l=0}^N a_l \left(\frac{\psi'(\xi)}{\psi(\xi)} \right)^l, \quad (4)$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3).

Step-4: We substitute (4) into (3), then we calculate all the necessary derivatives $u', u'', \dots$ of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi)$, $j = 0, 1, 2, \dots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (1) .

3 APPLICATION TO SHE

This section will implement modified simple equation method to SHE for two types of nonlinear media. The details are described in subsequent subsections.

3.1 KERR LAW NONLINEARITY

The dimensionless form of the perturbed SHE with STD and Kerr law nonlinearity is given by [4-6, 8-10, 12]

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i(\gamma q_{xxx} + \sigma |q|^2 q_x) = i\alpha q_x + i\lambda(|q|^2 q)_x + i\nu(|q|^2)_x q \quad (5)$$

where a represents the GVD and b is the spatio-temporal dispersion (STD) term. The inclusion of STD makes the model well-posed as suggested during 2012 [14, 15]. From perturbation terms, γ represents the 3OD coefficient, ν and σ represent the nonlinear dispersion, α the inter-modal dispersion and λ the self steepening term.

In order to solve this equation by using the modified simple equation method, the following solution structure is selected

$$q(x, t) = U(\xi)e^{i\Phi(x, t)}, \quad (6)$$

where the wave variable ξ is given by

$$\xi = k(x - vt). \quad (7)$$

Here, $U(\xi)$ represents the amplitude component of the soliton solution and v is the speed of the soliton, while the phase component $\Phi(x, t)$ is defined as

$$\Phi(x, t) = -\kappa x + \omega t + \theta, \quad (8)$$

where κ is the frequency of the soliton, ω is the wave number, while θ is the phase constant. Substituting (6) into (5) and decomposing into real and imaginary parts lead to

$$k^2 U''(a - bv + 3\gamma\kappa) - (a\kappa^2 + \alpha\kappa + \omega(1 - b\kappa) + \gamma\kappa^3)U + (c - \kappa\lambda + \kappa\sigma)U^3 = 0, \quad (9)$$

$$\gamma k^3 U'' - k(2a\kappa + \alpha + v(1 - b\kappa) - b\omega - 3\gamma\kappa^2)U + \frac{1}{3}k(\sigma - 3\lambda - 2\nu)U^3 = 0. \quad (10)$$

Balancing U'' with U^3 in Eqs. (9) and (10), then we get $N = 1$. Consequently we reach

$$U(\xi) = a_0 + a_1 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right), \quad a_1 \neq 0. \quad (11)$$

Substituting Eq. (11) in Eqs. (9) and (10) and then setting the coefficients of $\psi^{-j}(\xi)$, $j = 0, 1, 2, 3$, to zero, then we obtain a set of algebraic equations involving $a_0, a_1, k, \kappa, \gamma, \alpha, \nu, v$ and ω as follows:

ψ^{-3} coeff.:

$$a_1(\psi')^3(2k^2(a - bv + 3\gamma\kappa) + a_1^2(c + \kappa(\sigma - \lambda))) = 0, \quad (12)$$

$$\frac{1}{3}a_1 k(\psi')^3(a_1^2(-3\lambda - 2\nu + \sigma) + 6\gamma k^2) = 0, \quad (13)$$

ψ^{-2} coeff.:

$$3a_1\psi'(a_0 a_1 \psi'(c + \kappa(\sigma - \lambda)) - k^2 \psi''(a - bv + 3\gamma\kappa)) = 0, \quad (14)$$

$$a_1 k \psi' (a_0 a_1 \psi'(-3\lambda - 2\nu + \sigma) - 3\gamma k^2 \psi'') = 0, \quad (15)$$

ψ^{-1} coeff.:

$$-a_1 (\psi' (-3a_0^2(c - \kappa\lambda + \kappa\sigma) + a\kappa^2 + \alpha\kappa - b\kappa\omega + \gamma\kappa^3 + \omega) - k^2a - bv + 3\gamma\kappa\psi''') = 0, \quad (16)$$

$$ka_1 (\psi' (-2a\kappa + a_0^2(-3\lambda - 2\nu + \sigma) - \alpha + b\kappa v + b\omega + 3\gamma\kappa^2 - v) + k^2\gamma\psi''') = 0, \quad (17)$$

ψ^0 coeff.:

$$-a_0 (-a_0^2(c + \kappa(\sigma - \lambda)) + a\kappa^2 + \alpha\kappa - b\kappa\omega + \gamma\kappa^3 + \omega) = 0, \quad (18)$$

$$-\frac{1}{3}a_0k (3(2a\kappa + \alpha - b\kappa v - b\omega - 3\gamma\kappa^2 + v) + a_0^2(3\lambda + 2\nu - \sigma)) = 0. \quad (19)$$

Solving this system, we obtain

$$a_0 = \sqrt{\frac{(\sigma - 3\lambda - 2\nu) \left(b^2\kappa^2 \left(\frac{a}{\kappa} + a + \gamma\kappa \right) + a(1 - b\kappa)^2 \right) - (1 - b\kappa)^2 \left(3\gamma(c + 2\kappa(\lambda + \nu)) - \frac{b}{(1 - b\kappa)}(\sigma - 3\lambda - 2\nu)(2a\kappa + \alpha - 3\gamma\kappa^2) \right)}{\frac{1}{3}b(\sigma - 3\lambda - 2\nu)(3b(c - \kappa\lambda + \kappa\sigma) + (1 - b\kappa)(\sigma - 3\lambda - 2\nu))}}, \quad (20)$$

$$a_1 = \pm \sqrt{\frac{6\gamma k^2}{3\lambda + 2\nu - \sigma}}, \quad (21)$$

$$v = \frac{a(\sigma - 3\lambda - 2\nu) - 3\gamma(c + 2\kappa\lambda + 2\kappa\nu)}{b(\sigma - 3\lambda - 2\nu)} \quad (22)$$

$$\omega = \frac{1}{b(-3\lambda - 2\nu + \sigma)((b\kappa - 1)(-3\lambda - 2\nu + \sigma) - 3b(c - \kappa\lambda + \kappa\sigma))} (b(-3\lambda - 2\nu + \sigma) + ((-3\lambda - 2\nu + \sigma)(a\kappa^2 + \alpha\kappa + \gamma\kappa^3) - 3(2a\kappa + \alpha - 3\gamma\kappa^2)(c - \kappa\lambda + \kappa\sigma)) + (b\kappa - 1)(3a(-3\lambda - 2\nu + \sigma)(c - \kappa\lambda + \kappa\sigma) - 9\gamma(c + 2\kappa\lambda + 2\kappa\nu)(c - \kappa\lambda + \kappa\sigma) + 2\kappa\lambda + 2\kappa\nu)), \quad (23)$$

and

$$\psi'' = \pm \sqrt{-\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2}} \psi'. \quad (24)$$

$$\psi''' = -\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2} \psi'. \quad (25)$$

From Eqs. (24) and (25), we can deduce that

$$\psi' = \pm \sqrt{-\frac{3\gamma k^2}{2a_0^2(\sigma - 3\lambda - 2\nu)}} k_1 e^{\pm \sqrt{-\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2}} \xi}, \quad (26)$$

$$\psi = -\frac{3\gamma k^2}{2a_0^2(\sigma - 3\lambda - 2\nu)} k_1 e^{\pm \sqrt{-\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2}} \xi} + k_2. \quad (27)$$

where k_1 and k_2 are constants of integration. Substituting Eq. (26) and Eq. (27) into Eq. (11), we obtain following the following exact solution to Eq. (5).

$$q(x, t) = \left\{ a_0 + \frac{\frac{3\gamma k^2}{a_0(\sigma - 3\lambda - 2\nu)} k_1 e^{\pm \sqrt{-\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2}} (k(x - vt))}}{-\frac{3\gamma k^2}{2a_0^2(\sigma - 3\lambda - 2\nu)} k_1 e^{\pm \sqrt{-\frac{2a_0^2(\sigma - 3\lambda - 2\nu)}{3\gamma k^2}} (k(x - vt))} + k_2} \right\} e^{i(-\kappa x + \omega t + \theta)}. \quad (28)$$

where a_0 , v and ω are given by (20), (22) and (23) respectively.

If we set $k_1 = -\frac{2a_0^2(\sigma-3\lambda-2\nu)}{3\gamma k^2}e^{\pm\sqrt{-\frac{2a_0^2(\sigma-3\lambda-2\nu)}{3\gamma k^2}}\xi_0}$, $k_2 = \pm 1$, we obtain:

(i) When

$$\gamma(\sigma - 3\lambda - 2\nu) < 0,$$

we have

$$q(x, t) = \pm a_0 \tanh \left[\sqrt{-\frac{a_0^2(\sigma - 3\lambda - 2\nu)}{6\gamma k^2}}(k(x - vt) + \xi_0) \right] e^{i(-\kappa x + \omega t + \theta)}, \quad (29)$$

or

$$q(x, t) = \pm a_0 \coth \left[\sqrt{-\frac{a_0^2(\sigma - 3\lambda - 2\nu)}{6\gamma k^2}}(k(x - vt) + \xi_0) \right] e^{i(-\kappa x + \omega t + \theta)}, \quad (30)$$

These solutions represent dark soliton and singular soliton solutions respectively.

(ii) When

$$\gamma(\sigma - 3\lambda - 2\nu) > 0,$$

we have

$$q(x, t) = \pm i a_0 \tan \left[\sqrt{\frac{a_0^2(\sigma - 3\lambda - 2\nu)}{6\gamma k^2}}(k(x - vt) + \xi_0) \right] e^{i(-\kappa x + \omega t + \theta)}, \quad (31)$$

or

$$q(x, t) = \mp i a_0 \cot \left[\sqrt{\frac{a_0^2(\sigma - 3\lambda - 2\nu)}{6\gamma k^2}}(k(x - vt) + \xi_0) \right] e^{i(-\kappa x + \omega t + \theta)}, \quad (32)$$

These solutions are complex-valued.

3.2 POWER LAW NONLINEARITY

The dimensionless form of the perturbed SHE with STD and Power law nonlinearity is given by

$$iq_t + aq_{xx} + bq_{xt} + c|q|^{2n}q + i(\gamma q_{xxx} + \sigma|q|^{2n}q_x) = i\alpha q_x + i\lambda(|q|^{2n}q)_x + i\nu(|q|^{2n})_x q \quad (33)$$

Substituting the hypothesis (6) into (33), and decomposing into real and imaginary parts lead to

$$k^2 U''(a - bv + 3\gamma\kappa) - (a\kappa^2 + \alpha\kappa + \omega(1 - b\kappa) + \gamma\kappa^3)U + (c - \kappa\lambda + \kappa\sigma)U^{2n+1} = 0, \quad (34)$$

$$\gamma k^3 U'' - k(2a\kappa + \alpha + v(1 - b\kappa) - b\omega - 3\gamma\kappa^2)U + \frac{1}{(2n+1)}k(\sigma - (2n+1)\lambda - 2n\nu)U^{2n+1} = 0. \quad (35)$$

Set

$$U = V^{\frac{1}{n}}, \quad (36)$$

so that (34) and (35) transform to

$$k^2(a - bv + 3\gamma\kappa) \{nVV'' + (1 - n)V'^2\} - n^2(a\kappa^2 + \alpha\kappa + \omega(1 - b\kappa) + \gamma\kappa^3)V^2 + n^2(c - \kappa\lambda + \kappa\sigma)V^4 = 0, \quad (37)$$

$$n\gamma k^3 \{nVV'' + (1-n)V'^2\} - n^2k(2a\kappa + \alpha + v(1-b\kappa) - b\omega - 3\gamma\kappa^2)V^2 + \frac{n^2k}{(2n+1)}(\sigma - (2n+1)\lambda - 2n\nu)V^4 = 0. \quad (38)$$

Balancing VV'' with V^4 in Eqs. (37) and (38), then we get $N = 1$. Consequently we reach

$$V(\xi) = a_0 + a_1 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right), \quad a_1 \neq 0. \quad (39)$$

Substituting Eq. (39) in Eqs. (37) and (38) and then setting the coefficients of $\psi^{-j}(\xi)$, $j = 0, 1, 2, 3$, to zero, then we obtain a set of algebraic equations involving $a_0, a_1, k, \kappa, \gamma, \alpha, \nu, v$ and ω as follows:

ψ^{-4} coeff.:

$$a_1^2 (\psi')^4 (k^2(n+1)(a-bv+3\gamma\kappa) + a_1^2 n^2(c+\kappa(\sigma-\lambda))) = 0, \quad (40)$$

$$a_1^2 \gamma k^3 n(n+1) (\psi')^4 = 0, \quad (41)$$

ψ^{-3} coeff.:

$$a_1 (\psi')^2 (2a_0 n \psi' (k^2(a-bv+3\gamma\kappa) + 2a_1^2 n^2(c+\kappa(\sigma-\lambda))) - a_1 k^2(n+2)\psi''(a-bv+3\gamma\kappa)) = 0, \quad (42)$$

$$-\frac{1}{2n+1} a_1 k n (\psi')^2 (a_1 (a_1 n \psi' (\lambda + 2\lambda n + 2\nu n - \sigma) + \gamma k^2 (2n^2 + 5n + 2) \psi'') - 2a_0 \gamma k^2 n(2n+1)\psi') = 0, \quad (43)$$

ψ^{-2} coeff.:

$$-3a_0 a_1 k^2 n \psi' \psi'' (a-bv+3\gamma\kappa) - a_1^2 n^2 (\psi')^2 (-6a_0^2(c+\kappa(\sigma-\lambda)) + a\kappa^2 + \alpha\kappa - b\kappa\omega + \gamma\kappa^3 + \omega) + a_1^2 k^2 (n-1) (\psi'')^2 (a-bv+3\gamma\kappa) - k^2 n a_1^2 a - bv + 3\gamma\kappa \psi' \psi''' = 0, \quad (44)$$

$$\frac{1}{2n+1} k n a_1 (-3a_0 \gamma k^2 n(2n+1)\psi' \psi'' + a_1 (n((2n+1)(-2a\kappa - \alpha + v(b\kappa - 1) + b\omega + 3\gamma\kappa^2) - n((2n+1)(-2a\kappa - \alpha + v(b\kappa - 1) + b\omega + 3\gamma\kappa^2)))) = 0, \quad (45)$$

ψ^{-1} coeff.:

$$a_0 a_1 n (-2n \psi' (-2a_0^2(c-\kappa\lambda + \kappa\sigma) + a\kappa^2 + \alpha\kappa - b\kappa\omega + \gamma\kappa^3 + \omega) + k^2 a - bv + 3\gamma\kappa \psi''') = 0, \quad (46)$$

$$\frac{a_0 a_1 k n^2}{2n+1} ((-2(2n+1)(2a\kappa + \alpha - b\kappa v - b\omega - 3\gamma\kappa^2 + v) - 3a_0(\lambda + 2\lambda n + 2\nu n - \sigma))\psi' + k^2(2n+1)\gamma\psi''') = 0, \quad (47)$$

ψ^0 coeff.:

$$a_0^2 (-n^2) (-a_0^2(c+\kappa(\sigma-\lambda)) + a\kappa^2 + \alpha\kappa - b\kappa\omega + \gamma\kappa^3 + \omega) = 0, \quad (48)$$

$$-\frac{1}{2n+1} a_0^2 k n^2 ((2n+1)(2a\kappa + \alpha - b\kappa v - b\omega - 3\gamma\kappa^2 + v) + a_0(\lambda + 2\lambda n + 2\nu n - \sigma)) = 0, \quad (49)$$

Solving this system, we obtain

$$a_1 = 0, \quad \omega = \frac{\kappa(\kappa(a + \gamma\kappa) + \alpha) - a_0^2(c + \kappa(\sigma - \lambda))}{b\kappa - 1}, \quad (50)$$

$$v = \frac{1}{(2n+1)(b\kappa-1)^2}(-(2n+1)(\kappa(a(2-b\kappa) + \gamma\kappa(4b\kappa-3)) + \alpha) + a_0^2b(2n+1)(c + \kappa(\sigma - \lambda)) + a_0(b\kappa-1)(\lambda + 2\lambda n + 2\nu n - \sigma)) \quad (51)$$

These results lead to the trivial solutions. This is evident from the fact that dark and singular solitons of the type in Kerr law are not supported in power law, which has been proved earlier [6, 10].

4 CONCLUSIONS

This paper obtained dark and singular optical soliton solutions to SHE by the aid of modified simple equation method. Both Kerr and power laws of nonlinearity are studied. For power law, it was not possible to retrieve such soliton solutions, as expected. This is because dark and singular solitons of this type are not supported in power law medium. This integration scheme fails to obtain bright soliton solutions to SHE since it is an inherent drawback of the algorithm.

This problem will be further studied in future. SHE will be analyzed with stochastic as well as time-dependent coefficients. Moreover, the problem will be analyzed with fractional temporal evolution. Additional integration methodology will be employed to obtain further interesting results which will be reported elsewhere.

ACKNOWLEDGEMENTS

The fourth author (SPM) would like to thank the research support provided by the Department of Mathematics and Statistics at Tshwane University of Technology and the support from the South African National Foundation under Grant Number 92052 IRF1202210126. The fifth author (QZ) was funded by the National Science Foundation of Hubei Province in China under the grant number 2015CFC891. The seventh author (AB) would like to thank Tshwane University of Technology during his academic visit during 2016. The research work of eighth author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP 6-021-1-005. The authors also declare that there is no conflict of interest.

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