Propagation properties of partially coherent high-order cylindrical vector beams through a turbulent atmosphere

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On the basis of the extended Huygens–Fresnel integral formula and the unified theory of coherence and polarization of partially coherent beams, analytical formulae of the elements of the beam coherence-polarization (BCP) matrix for a partially coherent high-order cylindrical vector (HCV) beam propagating through a turbulent atmosphere are derived under a high topological charge condition. The propagation of a radially polarized partially coherent beam through a turbulent atmosphere can be treated as a special case of this general result. Numerical results indicate that the intensity distribution and the polarization evolution of the beam are dependent on the propagation distance, structure constant, transverse coherence length and topological charge. It was also found that initial intensity distribution with a hollow shape gradually changes into Gaussian distribution, and the polarization structure will be slowly degenerated during propagation through a turbulent atmosphere. Interestingly, with increase of the topological charge, a partially coherent HCV beam can effectively mitigate the turbulence effect on its scintillation. These results may find applications in connection with free-space optical communication (FSO) and remote sensing.

Keywords: Atmospheric propagation; Cylindrical vector beams; Polarization; Atmospheric turbulence

1. Introduction

In recent years, the particular focusing property [1-3] and the propagation characteristics of the cylindrical vector (CV) beam, whose polarization distribution is cylindrically symmetric and rotates through $2\pi m$ (m is the topological charge) in one circuit of the beam axis, have attracted widespread attention due to intriguing applications in high resolution imaging [4-6] and particle trapping [7-9]. When $m > 1$, the vector beam, called the high-order cylindrical vector (HCV) beam, can generate discrete multiple focused spots in the focal plane, which is often used in optical tweezers [10]. Meanwhile, diverse methods for generating HCV beams are also widely put forward [11-14].

On the other hand, as a very promising means for data transmission, free-space optical communication (FSO) benefit people in many ways. However, an inevitable question will be faced regarding the effect of atmospheric turbulence on the laser signal which can cause deterioration of the reliability of the communication and greatly limit the FSO application. For this reason, various methods have been proposed to alleviate atmospheric turbulence scintillation. Liu et al. proposed that elliptical vortex beams can suppress scintillations when the topological charge and the ratio of minor axis to major axis are properly chosen [15]. Eyyuboğlu et al. reported that the scintillations of Bessel–Gaussian beams are smaller than those of Gaussian beams at large source sizes and

large width parameters [16]. Gu et al. reported that nonuniformly polarized beams have less scintillation than comparable uniformly polarized beams [17]. By exploring the mean squared width of partially coherent beams in random media, Gbur et al. found that partially coherent beams are less affected by atmospheric turbulence [18]. In addition, it is interesting that vector singularity, compared with phase singularity, is more stable in a turbulent atmosphere [19], which indicates that vector beams may enable further mitigation of the turbulence effect during their propagation. Therefore, combining the propagation advantages of partially coherent beams and vector beams, it is very practical and important to study the propagation characteristics of a partially coherent HCV beam in a turbulent atmosphere. So far, the propagation characteristics of a partially coherent radially polarized beam in the turbulent atmosphere has been reported. However, the propagation performance of a partially coherent HCV beam through a turbulent atmosphere has not yet been studied in depth.

In this paper, we investigate the intensity and polarization properties of a partially coherent HCV beam through a turbulent atmosphere based on the extended Huygens-Fresnel method by adopting the beam coherence-polarization (BCP) matrix. Firstly, the element of the BCP matrix in the initial plane is derived by the electric field of a partially coherent HCV beam. Then we substitute it into the extended Huygens-Fresnel integral formula, and the analytical formulas for the BCP matrix of a partially coherent HCV beam propagating through a turbulent atmosphere are obtained after quadruple integral calculation. Finally, the intensity and polarization properties of a partially coherent HCV beam can be deduced from the BCP matrix in the output plane. All mentioned above is deduced in Section 2. We also discuss the influence of structure constant, transverse coherence length and topological charge on the intensity distribution and degree of polarization, and the corresponding propagation properties are illustrated by numerical calculation in Section 3. The conclusions that increasing topological charge can effectively reduce the level of scintillation caused by atmospheric turbulence are presented in Section 4.

2. Theoretical analysis

Assume that the initial plane lies in the (x,y) plane and the observation plane (ξ,η) is parallel to the (x,y) plane and at normal distance z from it. The statistical average of the electric field of a partially coherent HCV beam in the initial plane ($z = 0$) can be expressed as:

$$\begin{aligned}\vec{E}(x, y, 0) &= E_x(x, y, 0)\vec{e}_x + E_y(x, y, 0)\vec{e}_y \\ E_x(x, y, 0) &= E_0 \rho^m \cos(m\varphi + \varphi_0) \cdot \exp\left(-\frac{\rho^2}{\omega_0^2}\right) \exp(i\phi_x) \\ E_y(x, y, 0) &= E_0 \rho^m \sin(m\varphi + \varphi_0) \cdot \exp\left(-\frac{\rho^2}{\omega_0^2}\right) \exp(i\phi_y)\end{aligned}\quad (1)$$

where $\rho = \sqrt{x^2 + y^2}$, $\tan(\varphi) = y/x$ and $E_0 = \sqrt{2/(\pi|m|!)}$; m is topological charge; E_0 is the amplitude constant; ω_0 is waist radius; φ_0 is the initial phase angle; ϕ_i ($i=x,y$) denotes an randomly distributed phase. We assume that the statistical distribution of ϕ_i corresponds to a Gaussian-Schell correlator and is given by

$$\langle \exp(i\phi_i) \exp(-i\phi_j) \rangle = B_{ij} \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{ij}^2}\right] \quad (2)$$

where (x_1, y_1) and (x_2, y_2) are two arbitrary points in the initial plane (x, y) ; the angle brackets $\langle \rangle$ represent the ensemble average; δ_{ij} ($i, j=x, y$) is the transverse coherence length; the factor B_{ij} settles for the relation as follows: $B_{xx} = B_{yy} = 1$ and $B_{xy} = B_{yx}^*$.

The cross-spectral density matrix can be expressed as [20]

$$\overline{W}(x_1, y_1, x_2, y_2, z) = \begin{bmatrix} W_{xx}(x_1, y_1, x_2, y_2, z) & W_{xy}(x_1, y_1, x_2, y_2, z) \\ W_{yx}(x_1, y_1, x_2, y_2, z) & W_{yy}(x_1, y_1, x_2, y_2, z) \end{bmatrix} \quad (3)$$

where $W_{ij}(x_1, y_1, x_2, y_2, z) = \langle E_i(x_1, y_1, z) E_j^*(x_2, y_2, z) \rangle$; ($i, j=x, y$)

$E_x(x_1, y_1, z)$ and $E_y(x_1, y_1, z)$ are the components of the complex electric field in the x and y directions, respectively. The asterisk stands for the complex conjugate. For simplicity, we consider propagation of a partially coherent HCV beam with a single frequency. Thus, the cross-spectral density matrix can be replaced by the BCP matrix involving the information about polarization and spatial correlation.

Substituting Eqs. (1)-(2) into Eq. (3), the element of the BCP matrix of a partially coherent HCV beam in the initial plane ($z = 0$) can be expressed as

$$\begin{aligned} W_{xx}(x_1, y_1, x_2, y_2, 0) &= E_0^2 \rho_1^m \cos(m\varphi_1 + \varphi_0) \cdot \rho_2^m \cos(m\varphi_2 + \varphi_0) \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{xx} \cdot \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{xx}^2}\right] \\ W_{yy}(x_1, y_1, x_2, y_2, 0) &= E_0^2 \rho_1^m \sin(m\varphi_1 + \varphi_0) \cdot \rho_2^m \sin(m\varphi_2 + \varphi_0) \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{yy} \cdot \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{yy}^2}\right] \\ W_{xy}(x_1, y_1, x_2, y_2, 0) &= E_0^2 \rho_1^m \cos(m\varphi_1 + \varphi_0) \cdot \rho_2^m \sin(m\varphi_2 + \varphi_0) \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{xy} \cdot \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{xy}^2}\right] \\ W_{yx}(x_1, y_1, x_2, y_2, 0) &= E_0^2 \rho_1^m \cos(m\varphi_1 + \varphi_0) \cdot \rho_2^m \sin(m\varphi_2 + \varphi_0) \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{yx} \cdot \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{yx}^2}\right] \end{aligned} \quad (4)$$

where $\rho_1 = \sqrt{x_1^2 + y_1^2}$, $\tan(\varphi_1) = y_1 / x_1$, $\rho_2 = \sqrt{x_2^2 + y_2^2}$ and $\tan(\varphi_2) = y_2 / x_2$.

Based on the extended Huygens-Fresnel integral formula, the BCP matrix of the beam in the observation plane ($z > 0$) can be stated as [21-25]

$$W_{ij}(\xi, \eta, \xi, \eta, z) = \frac{1}{\lambda^2 z^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{ij}(x_1, y_1, x_2, y_2, 0) \exp\left\{-\frac{ik}{2z}[(\xi - x_1)^2 + (\eta - y_1)^2] + \frac{ik}{2z}[(\xi - x_2)^2 + (\eta - y_2)^2]\right\} \exp\left[-\frac{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}{\rho_0^2}\right] dx_1 dy_1 dx_2 dy_2 \quad (5)$$

where λ is the wavelength and $k=2\pi/\lambda$ is the wave number; $\rho_0=(0.545C_n^2 k^2 z)^{3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium with C_n^2 being the structure constant. For ease of description, the Huygens-Fresnel integral formula is denoted as $W_{ij}(\xi, \eta, \xi, \eta, z) = \Im[W_{ij}(x_1, y_1, x_2, y_2, 0)]$.

In order to solve the element of the BCP matrix, we define four intermediate variables ($V_{ij}^I, V_{ij}^{II}, V_{ij}^{III}, V_{ij}^{IV}$) as follows:

$$\begin{aligned} V_{ij}^I &= \Im[E_0^2 (x_1 + iy_1)^m (x_2 - iy_2)^m \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{ij} \cdot \exp\left(-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{ij}^2}\right)] \\ V_{ij}^{II} &= \Im[E_0^2 (x_1 - iy_1)^m (x_2 + iy_2)^m \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{ij} \cdot \exp\left(-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{ij}^2}\right)] \\ V_{ij}^{III} &= \Im[E_0^2 (x_1 + iy_1)^m (x_2 + iy_2)^m \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{ij} \cdot \exp\left(-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{ij}^2}\right)] \\ V_{ij}^{IV} &= \Im[E_0^2 (x_1 - iy_1)^m (x_2 - iy_2)^m \cdot \exp\left(-\frac{\rho_1^2 + \rho_2^2}{\omega_0^2}\right) \cdot B_{ij} \cdot \exp\left(-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{\delta_{ij}^2}\right)] \end{aligned} \quad (6)$$

We also can exactly get the following Eq. 7 according to the Euler's formula

$$\begin{aligned}
\rho_1^m \cos(m\varphi_1 + \varphi_0) \cdot \rho_2^m \cos(m\varphi_2 + \varphi_0) &= \frac{1}{4}(x_1 + iy_1)^m (x_2 - iy_2)^m + \frac{1}{4}(x_1 - iy_1)^m (x_2 + iy_2)^m + \frac{1}{4}\exp(2i\varphi_0) \cdot (x_1 + iy_1)^m (x_2 + iy_2)^m + \frac{1}{4}\exp(-2i\varphi_0) \cdot (x_1 - iy_1)^m (x_2 - iy_2)^m \\
\rho_1^m \sin(m\varphi_1 + \varphi_0) \cdot \rho_2^m \sin(m\varphi_2 + \varphi_0) &= \frac{1}{4}(x_1 + iy_1)^m (x_2 - iy_2)^m + \frac{1}{4}(x_1 - iy_1)^m (x_2 + iy_2)^m - \frac{1}{4}\exp(2i\varphi_0) \cdot (x_1 + iy_1)^m (x_2 + iy_2)^m - \frac{1}{4}\exp(-2i\varphi_0) \cdot (x_1 - iy_1)^m (x_2 - iy_2)^m \\
\rho_1^m \cos(m\varphi_1 + \varphi_0) \cdot \rho_2^m \sin(m\varphi_2 + \varphi_0) &= -\frac{1}{4i}(x_1 + iy_1)^m (x_2 - iy_2)^m + \frac{1}{4i}(x_1 - iy_1)^m (x_2 + iy_2)^m + \frac{1}{4i}\exp(2i\varphi_0) \cdot (x_1 + iy_1)^m (x_2 + iy_2)^m - \frac{1}{4i}\exp(-2i\varphi_0) \cdot (x_1 - iy_1)^m (x_2 - iy_2)^m
\end{aligned} \tag{7}$$

Based on Eq. (5) and Eq. (6), the element of the BCP matrix in the observation plane can be expressed the linear combination of four intermediate variables ($V_{ij}^I, V_{ij}^{II}, V_{ij}^{III}, V_{ij}^{IV}$) after substituting Eq. (7) into Eq. (4) as

$$\begin{aligned}
W_{xx}(\xi, \eta, \xi, \eta, z) &= \frac{1}{4}V_{xx}^I + \frac{1}{4}V_{xx}^{II} + \frac{1}{4}\exp(2i\varphi_0) \cdot V_{xx}^{III} + \frac{1}{4}\exp(-2i\varphi_0) \cdot V_{xx}^{IV} \\
W_{yy}(\xi, \eta, \xi, \eta, z) &= \frac{1}{4}V_{yy}^I + \frac{1}{4}V_{yy}^{II} - \frac{1}{4}\exp(2i\varphi_0) \cdot V_{yy}^{III} - \frac{1}{4}\exp(-2i\varphi_0) \cdot V_{yy}^{IV} \\
W_{xy}(\xi, \eta, \xi, \eta, z) &= -\frac{1}{4i}V_{xy}^I + \frac{1}{4i}V_{xy}^{II} + \frac{1}{4i}\exp(2i\varphi_0) \cdot V_{xy}^{III} - \frac{1}{4i}\exp(-2i\varphi_0) \cdot V_{xy}^{IV} \\
W_{yx}(\xi, \eta, \xi, \eta, z) &= [W_{xy}(\xi, \eta, \xi, \eta, z)]^*
\end{aligned} \tag{8}$$

Here, four intermediate variables ($V_{ij}^I, V_{ij}^{II}, V_{ij}^{III}, V_{ij}^{IV}$) will be derived firstly. V_{ij}^I is organized as follows

$$V_{ij}^I = C_{ij0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_2, y_2) \cdot \exp[-\beta_{ij}^2(x_2^2 + y_2^2) - \frac{ik}{2z}(\xi x_2 + \eta y_2)] \cdot (x_2 - iy_2)^m dx_2 dy_2 \tag{9}$$

$$f(x_2, y_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-\alpha_{ij}^2(x_1^2 + y_1^2) + (\frac{ik\xi}{z} + \frac{2x_2}{\delta_{ij}^2} + \frac{2x_2}{\rho_0^2})x_1 + (\frac{ik\eta}{z} + \frac{2y_2}{\delta_{ij}^2} + \frac{2y_2}{\rho_0^2})y_1] \cdot (x_1 + iy_1)^m dx_1 dy_1 \tag{10}$$

where $C_{ij0} = E_0^2 B_{ij} / (\lambda^2 z^2)$, $\beta_{ij}^2 = 1/\omega_0^2 + 1/\delta_{ij}^2 + 1/\rho_0^2 - ik/(2z)$ and

$$\alpha_{ij}^2 = 1/\omega_0^2 + 1/\delta_{ij}^2 + 1/\rho_0^2 + ik/(2z).$$

For a simplified calculation, Eq. (10) can be modified as the following expression [26]

$$f(x_2, y_2) = \frac{\pi}{2^m \alpha_{ij}^{2m+2}} \exp[-\frac{k^2(\xi^2 + \eta^2)}{4z^2 \alpha_{ij}^2}] \exp[\frac{1}{\alpha_{ij}^2}(\frac{1}{\delta_{ij}^2} + \frac{1}{\rho_0^2})(x_2^2 + y_2^2) + \frac{ik}{z\alpha_{ij}^2}(\frac{1}{\delta_{ij}^2} + \frac{1}{\rho_0^2})(\xi x_2 + \eta y_2)] \cdot [2(\frac{1}{\delta_{ij}^2} + \frac{1}{\rho_0^2})(x_2 + iy_2) + \frac{ik\xi - k\eta}{z}]^m \tag{11}$$

Substituting Eq. (11) into Eq. (9) and after simplification, we obtain the analytic expressions of V_{ij}^I [26]

$$V_{ij}^I = \frac{z^{2m} E_0^2}{k^{2m} \alpha_{ij}^{m+1}} \exp[-\frac{2(\xi^2 + \eta^2)}{a_{ij} \omega_0^2}] \cdot \sum_{n=0}^m (C_m^n)^2 [4(\frac{1}{\delta_{ij}^2} + \frac{1}{\rho_0^2})]^n \cdot n! [(1 + \frac{4z^2}{k^2 \omega_0^4}) \frac{k^2}{a_{ij} z^2} (\xi^2 + \eta^2)]^{m-n} \tag{12}$$

where $a_{ij} = 1 + \frac{4z^2}{k^2 \omega_0^2} (\frac{1}{\omega_0^2} + \frac{2}{\delta_{ij}^2} + \frac{2}{\rho_0^2})$.

Accordingly, we obtain the analytic expressions of the remaining intermediate variables ($V_{ij}^{II}, V_{ij}^{III}$ and V_{ij}^{IV})

$$\begin{aligned}
V_{ij}^{II} &= V_{ij}^I \\
V_{ij}^{III} &= \frac{z^{2m} E_0^2}{k^{2m} \alpha_{ij}^{m+1}} \exp[-\frac{2(\xi^2 + \eta^2)}{a_{ij} \omega_0^2}] \cdot [(1 + \frac{4z^2}{k^2 \omega_0^4}) \frac{k^2}{a_{ij} z^2} (\xi + i\eta)^2]^m \\
V_{ij}^{IV} &= \frac{z^{2m} E_0^2}{k^{2m} \alpha_{ij}^{m+1}} \exp[-\frac{2(\xi^2 + \eta^2)}{a_{ij} \omega_0^2}] \cdot [(1 + \frac{4z^2}{k^2 \omega_0^4}) \frac{k^2}{a_{ij} z^2} (\xi - i\eta)^2]^m
\end{aligned} \tag{13}$$

Substituting Eqs. (12) and Eq. (13) into Eq. (8), we can obtain the element of the BCP matrix in the observation plane. When $\delta_{ij} \rightarrow \infty$, the BCP matrix of a partially coherent HCV beam degenerates into the BCP matrix of completely coherent HCV beams. When $m=1$, $\varphi_0=0$ and

$\delta_{ij} \rightarrow \infty$, the BCP matrix of a partially coherent HCV beam degenerates that of a radially polarized beam, which has the same expression with Eqs.(10)-(12) of Reference [23]. One also finds two parameters, the transverse coherence length and topological charge, have the similar effect on elements of the BCP matrix. Besides, the stronger the turbulence is, the smaller ρ_0 is. So the turbulence degenerates the transverse coherence length of the beam. Similar effects of turbulence on coherence degradation have been studied earlier for coherent Gaussian beams [27, 28].

Based on Eq. (8), the average intensity distribution of PCHCV beams can be expressed as [29]

$$I(\xi, \eta, z) = W_{xx}(\xi, \eta, \xi, \eta, z) + W_{yy}(\xi, \eta, \xi, \eta, z) \quad (14)$$

To judge the changes of polarization of a partially coherent HCV beam propagating in a turbulent atmosphere, the degree of polarization for a partially coherent HCV beam at a point can be given as [20]

$$P(\xi, \eta, z) = \sqrt{1 - \frac{4 \det \bar{W}(\xi, \eta, \xi, \eta, z)}{[Tr \bar{W}(\xi, \eta, \xi, \eta, z)]^2}} \quad (15)$$

where $\det$ and Tr denote the determinant and the trace of the matrix, respectively.

3. Results and Discussion

In this section, the intensity distribution and the polarization distribution of a partially coherent HCV beam during propagation through the turbulent atmosphere are studied. In the following numerical simulations, we fix the parameters $\lambda=6\text{nm}$, $\omega_0=1\text{cm}$ while $\delta_{xx}=\delta_{yy}=\delta_{xy}$ and $B_{xy}=1$ are assumed for meeting the generation conditions of a partially coherent HCV beam.

3.1 Intensity distribution

To investigate the influences of the propagation distance on the evolution properties of average intensity under three different topological charge conditions, we calculate the normalized intensity distribution of a partially coherent HCV beam propagating in a turbulent atmosphere for different values of the propagation distance with $C_n^2=10^{-15}\text{m}^{-2/3}$ and $\delta_{xx}=\delta_{yy}=2\text{cm}$, as shown in Fig. 1. It is found from Fig. 1 that the initial beam profile of a partially coherent HCV beam disappear gradually with increase of propagation distance z . This result indicates that the light energy will leak into the central part of the beam from the outer ring during propagation. Eventually, the intensity distribution turns into a Gaussian shape in the far-field observational plane. Meanwhile, we also find the transition from the beam profile of the source into Gaussian profile becomes slower under the same propagation distance with increase of topological charge. This phenomenon can be explained by the fact that the size of the dark core in the middle of a partially coherent HCV beam is larger with the increase of topological charge. Thus, the longer propagation distance is needed for initial field distribution into a Gaussian shape.

Fig. 1 Cross line ($y=0$) of the normalized intensity distributions at several propagation distances in a turbulent atmosphere for three different topological charges.

Fig. 2 shows the influences of structure constant on the intensity distribution of a partially coherent HCV beam propagating through a turbulent atmosphere with $z=150\text{m}$ and $\delta_{xx}=\delta_{yy}=2\text{cm}$. When the value of the structure constant increases, the transition from the beam profile of the source into the Gaussian profile becomes quicker, and the beam spot spreads more rapidly. The larger the value of the topological charge, the more slowly the beam profile will degenerate. The primary reason is the same as that mentioned above.

Fig. 2 Cross line ($y=0$) of the normalized intensity distributions at several structure constants in a turbulent atmosphere for three different topological charges.

We also explore the influences of transverse coherence length on the intensity distribution with $z=150\text{m}$ and $C_n^2=10^{-15}\text{m}^{-2/3}$. One finds from Fig. 3 that with a smaller transverse coherent length, the transition into a Gaussian shape is quicker. It shows that the beam under this condition cannot endure the atmosphere turbulence for a longer propagation distance. Compared with the results shown in Fig. 2 and Fig. 3, the increase of structure constant and the decrease of transverse

coherence length achieve the same effect on the evolution of a partially coherent HCV beam, which is mentioned in section 2. On the other hand, the propagation distance for the beam profile into a Gaussian shape is longer with increase of the topological charge. This indicates the increase of the value of topological charge may enable further mitigation of the turbulence effect on beam propagation, which will be favorable for FSO systems.

Fig. 3 Cross line ($y=0$) of the normalized intensity distributions at several transverse coherence lengths in a turbulent atmosphere for three different topological charges.

3.2 Polarization property

We also explore the evolution of the degree of polarization when a partially coherent HCV beam propagates through a turbulent atmosphere. Based on Eq. (15), the influences of propagation distance on the degree of polarization are calculated in Fig. 4 with $C_n^2 = 5 \times 10^{-15} \text{m}^{-2/3}$ and $\delta_{xx} = \delta_{yy} = 2 \text{cm}$. We assume that all the points in the initial plane ($z=0$) with the degree of polarization equals 1. In other words, the input beam is fully polarized. As shown in Fig.1, when the partially coherent HCV beam propagates in the turbulent atmosphere, the degree of polarization shows a dip, and the polarization degree of the on-axis point becomes zero. The width of the dip increases as the propagation goes on. Thus, a conclusion can be drawn that the initial polarization structure of a partially coherent HCV beam will be degenerated during propagation, and the beam becomes partially polarized. In the meantime, the width of the dip will get wider for a beam with a bigger topological charge.

Fig. 4 Cross line ($y=0$) of the degree of polarization at several propagation distances in a turbulent atmosphere for three different topological charges.

Fig. 5 shows the influences of structure constant on the degree of polarization after propagating in a turbulent atmosphere with $z=150\text{m}$ and $\delta_{xx}=\delta_{yy}=2\text{cm}$. As shown in Fig. 5, when the partially coherent HCV beam propagates in the turbulent atmosphere, the width of the dip increases with increase of the structure constants. Obviously, we can see that the larger the value of the topological charge, the faster the partially coherent HCV beam degenerates into a partially polarized beam.

Fig. 5 Cross line ($y=0$) of the degree of polarization at several structure constants in a turbulent atmosphere for three different topological charges.

Finally, we study the influences of transverse coherence length on the degree of polarization after propagating in a turbulent atmosphere. The propagation distance $z=150\text{m}$ and structure constant $C_n^2=5\times 10^{-15}\text{m}^{-2/3}$ are assumed. As shown in Fig. 6, the width of the dip is wider for a beam with a smaller transverse coherent length. Meanwhile, the propagation distance for the beam profile to change into a partially polarized beam is shorter with increase of the topological charge.

Compared with a fully polarized beam, a partially polarized beam can reduce the level of scintillation caused by atmospheric turbulence [30]. Therefore, we can mitigate the turbulence effect on beam propagation by controlling the topological charge, which is also favorable for FSO systems.

Fig. 6 Cross line ($y=0$) of the degree of polarization at several transverse coherence lengths in a turbulent atmosphere for three different topological charges.

4. Conclusions

Based on the extended Huygens–Fresnel integral formula and the unified theory of coherence and polarization of partially coherent beams, the analytical expressions for the elements of the BCP matrix have been derived, and the intensity and polarization properties of the partially coherent HCV beam in a turbulent atmosphere have been studied. The studies show that the beam profile and the polarization distribution are closely related to the propagation distance, the structure constant, the transverse coherence length and the topological charge. Firstly, the intensity distribution of the partially coherent HCV beam in a turbulent atmosphere can turn into a Gaussian shape as the propagation distance z increases. The degeneration process will become faster based on the condition of longer propagation distance, stronger turbulence and smaller transverse coherence length. But with increase of the topological charge, the partially coherent HCV beam can effectively mitigate degeneration into a Gaussian shape. Secondly, we can also find that a dip appears in the distribution of the degree of polarization and the width of the dip increases gradually during propagation. Eventually the beam becomes partially polarized. The dip width becomes wider with stronger turbulence, smaller transverse coherence length and longer propagation distance. Similarly, with increase of the topological charge, the partially coherent HCV beam can effectively mitigate levels of scintillation caused by atmospheric turbulence. These results will enable more robust FSO systems and also promote the development of remote sensing.

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