Design a Membrane System for Matrix Multiplication

Lian Ye^{a,b}, Ping Guo^{a,b}

^aCollege of Computer Science, Chongqing University, Chongqing 400030, China

^bChongqing Key Laboratory of Software Theory and Technology, Chongqing 400044, China

Abstract

Membrane systems are a framework of distributed parallel computing models, inspired by some basic features of biological membranes. In this paper, a membrane computing system named Π_{MM} is proposed to solve multiplication operation with matrices by its parallelism. It mainly includes three stages: firstly, construct new embedded membranes and input the values of two matrices; secondly, divide the membranes to generate new membranes according to the rows of matrix; finally, the solutions are obtained in each row membranes under the relatively calculation rules side by side. An instance is given to show the whole procedure how to solve the matrix multiplication by the Π_{MM} .

Keywords: Cell-like P systems, Matrix multiplication, Parallel computing, Membrane Computing

1. Introduction

Matrix operations have been widely used in the field of data mining, such as image processing, text mining, recommendation system, bioinformatics and so on. Matrix multiplication is one of important matrix arithmetic operations.

5 Membrane Computing is an emerging branch within Natural Computing[1], this new model of computation starts from the assumption that the processes taking place in the compartmental structure of a living cell can be interpret-

*Corresponding author

Email addresses: ylredleaf@cqu.edu.cn (Lian Ye), guoping@cqu.edu.cn (Ping Guo)

ed a computations. Membrane system is the computing system based on the principle of Membrane Computing, which consists of a hierarchical arrangement of membranes embedded in a skin membrane, and delimiting regions or compartments where multi-sets of objects and sets of evolution rules are placed. P systems are not used as a computing paradigm, but rather as formalism for describing the behavior of the system to be modeled.

On the one hand, it is commonly accepted the belief that living systems perform some information processing to keep them far from dynamical equilibrium as well as to accomplish several particular tasks[2] allowing them to react and to adapt to the environment they live in.

On the other hand, people look at the way nature computes with the purpose to abstract its information processing mechanisms in such a way to obtain new computational paradigms that can be put beside traditional ones to tackle specific problems, like NP problems in a feasible time. SAT problem was solved by the splitting rule of membrane system[3], and the HPP problem was also solved by the generating rule[4], A hybrid membrane evolutionary algorithm was proposed for solving constrained optimization problems[5].

In this paper, we propose a P system named Π_{MM} to solve matrix arithmetic, which is a new field in membrane computing. Of course, the new computing system is based on our previous works. We designed single-membrane arithmetic P system without priority rules to simplify the structure of the system greatly and improve the efficiency of computations at first[6]. Then we have designed an arithmetic expression evaluations system to provide an effective method for applications of numerical computation with membrane computing[7], and proposed different membranes for implementing primary Boolean and relational operations respectively for evaluating a logical expression[8]. In Ref.[9], a fraction arithmetic P system was presented for performing addition, subtraction, multiplication and division on fractions through designing the rules with priority, and then a method of fraction reduction was proposed[10]. The fraction arithmetic P system is considered as a calculator component in Π_{MM} , once the former operands are sending into the component, the operands of arithmetic

result will be given out.

40 The organization of the paper is as follows: the foundation of P systems and the standard notations used for matrices and definition of multiplication operation with matrices are introduced in section 2; the details of Π_{MM} , including the membrane structure and evolving rules are proposed in section 3. In section 4, an example is presented to show how the Π_{MM} works. The conclusions are
45 proposed in the final section.

2. Preliminaries

In membrane computing there are basically three ways to consider computational devices: cell-like systems[11, 12], tissue-like systems[13] and Neural-like Systems[14, 15]. The first one, using the biological membranes arranged hi-
50 erarchically, inspired from the structure of the cell; the second one using the biological membranes placed in the nodes of a graph, inspired from the cell inter-communications in tissues; the third one, Spiking neural P systems are a class of distributed parallel computing devices inspired from the way neurons communicate by means of spikes. Our work is focused on the cell-like systems.
55 So all the basic knowledge about cell-like systems and the description of matrix operation are given in this section.

2.1. Cell-like P systems

The main syntactic ingredients of a cell-like P system are the membrane structure, the multi-sets, and the evolution rules. A membrane structure con-
60 sists of several membranes arranged in a hierarchical structure inside a main membrane (the skin), and delimiting regions (the space in-between a membrane and the immediately inner membranes). Each membrane identifies a region inside the system. Regions defined by a membrane structure contain objects corresponding to chemical substances present in the compartments of a cell.
65 The objects can be described by symbols or by strings of symbols, in such a way that multi-sets of objects are placed in regions of the membrane structure.

The objects can evolve according to given evolution rules, associated with the regions. The semantics of the cell-like membrane systems is defined through a nondeterministic and synchronous model, by introducing the concepts of con-
70 figuration, transition step, and computation.

A basic transition P system of degree $m \geq 1$ is a tuple[1],

$$\Pi = (O, \mu, \omega_1, \dots, \omega_m, (R_1, \rho_1), (R_2, \rho_2), \dots, (R_m, \rho_m), i_o) \quad (1)$$

where,

(i) O is the alphabet of the system;

(ii) μ is a membrane structure consisting of m membranes, which are labeled
75 by numbers in the set $\{1, \dots, m\}$;

(iii) $\omega_1 \dots \omega_m$ are multi-sets, representing the objects initially present in the regions $(1, \dots, m)$ of the system;

(iv) $R_1, \dots, R_m$ are finite sets of evolution rules associated with the re-
regions $(1, \dots, m)$ of μ ; $(\rho_1, \dots, \rho_m)$ are strict partial order relations defined over
80 $(R_1, \dots, R_m)$ respectively, specifying a priority relation among the evolution
rules; The rule can be described as the form $(u \rightarrow v, \rho_i)$, where, $u \rightarrow v$ is
rewrite rule, and $\rho_i (1 \leq i \leq m)$ indicate the priority.

(v) i_o is indicating the output region.

2.2. Matrix Multiplication

85 More generally, it is possible multiply a matrix A times a matrix B if the
number of columns of A equals the number of rows of B. The first column of
the product is determined by the first column of B; that is, the first column of
AB is Ab_1 , the second column of AB is Ab_2 , and so on. Thus the product AB
is the matrix whose columns are $Ab_1, Ab_2, \dots, Ab_n$:

$$AB = (Ab_1, Ab_2, \dots, Ab_n) \quad (2)$$

90 The (i, j) entry of AB is the i_{th} entry of the column vector Ab_j . It is
determined by multiplying the i_{th} row vector of A times the j_{th} column vector
of B.

Definition 1. if $A=(a_{ij})$ is a $(m \times k)$ matrix and $B=(b_{ij})$ is a $(k \times n)$ matrix, then the product $AB=C=(c_{ij})$ is the $(m \times n)$ matrix whose entries are defined
95 by

$$C_{ij} = a_i b_j = \sum_{l=1}^k a_{il} b_{lj} \quad (3)$$

According to formula (3), there are $(m \times k \times n)$ times addition and multiplication, and the time complexity of the serial algorithm is $O(n^3)$.

2.3. Parallel computing

Parallel computing is a form of computation in which many calculations are
100 carried out simultaneously. Parallelism has been employed in high-performance computing, but it due to the physical constraints.

Analyzing the definition mentioned above, the calculation of each element of the matrix C is irreverent to each other. Therefore, the addition and multiplication could both be implemented parallel on a multiprocessor platform.

There are some classical algorithms, including DNS algorithm, Cannon al-
105 gorithm, Fox algorithm, algorithm, Systolic algorithm. All of them are based on the larger size of the matrix in accordance with the actual use of the processor into a number of small matrix. By solving the partial solution of the small matrix multiplication, the partial solution is finally obtained by accumulating
110 the solution of the original matrix.

The performance of a parallel matrix multiplication algorithm can be estimated by running time $t_p(n)$ and the number of processors $P(n)$.

Table 1 shows the performance of the classical algorithms[16].

Table 1: Performance of the classical algorithms				
Parameters	DNS	Cannon	Fox	systolic
$t_p(n)$	$\log n$	n	$n \log n$	n
$P(n)$	n^3	n^2	n^2	n^2

3. Π_{MM} for Matrix Multiplication

¹¹⁵ Π_{MM} is proposed as multi-core processors to solve matrix multiplication. In the system, the initial sequenced membranes denote the input and output membranes, and the generated zero membrane as a processor computes a value of product. The details of the system will be given in this section.

3.1. Definition

¹²⁰ According to formula (1), Π_{MM} to solve matrix arithmetic based on multiple membranes can be defined as:

$$\Pi_{MM} = (O, \mu, \Delta, \omega_1, \dots, \omega_m, R, i_o) \quad (4)$$

- (1) O is a finite and non-empty alphabet of objects, $O = \{\mu, \theta, \xi, \Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5\} \cup \{\mu_j, \beta_j, 1 \leq j \leq m\} \cup \{a_{i1}, a_{i2}, b_{i1}, b_{i2}, d_{i1}, d_{i2}, \gamma_i, 1 \leq i \leq k\} \cup \{c_{i1}, c_{i2}, 0 \leq i \leq k\} \cup \{+i, -i, +^i, -^i, 1 \leq i \leq k\}$
- ¹²⁵ (2) μ is the membrane structure composed of m membranes, each region labeled as $1, 2, \dots, m$, the number m is decided by the row of matrix A , shown in Fig.1. Each membrane has the fraction of scalar for a row.

Figure 1: Membrane structure

- (3) Δ is an embedded P system for completing fraction arithmetic operation and fraction reduction that we have been constructed in Ref. [9] and Ref. [10]. Assuming this system is existed in every membrane when it generated.
- ¹³⁰ (4) ω_i is multi-sets, representing the objects for i_{th} row of matrix initially present in the regions i . In initial state, $\omega_i = \Phi$.

(5) The rules in R should have a priority, and the explanations of some rules are given here, k indicates the priority:

- 135 • $[_h u \delta]_h \rightarrow v, k$
The membrane h is dissolved, and the object u is replaced of object v .
- $[_h u \rightarrow v]_h, k$
In the region named h , remove the multi-set of objects specified by u , and to introduce the objects specified by v .
- 140 • $[_h u \rightarrow (v, in)]_h, k$
The object v should be moved into the membrane k which is the upper membrane that has not been dissolved, and the object u will be removed from membrane h .
- $[_h u \rightarrow (v, out)]_h, k$
145 The object v will be moved to the region immediately outside membrane h .
- $[_h u \rightarrow [_h v]_h]_h, k$
The object u will be replaced of a new membrane containing object v .
- $[_{h1} u]_{h1} \rightarrow [_{h2} u]_{h2} [_{h2} u]_{h2}$
150 All objects and membranes in the original membrane will be duplicated and appear in the two new membranes.
- $u_j [_h]_h [_j]_j \rightarrow [_j v [_h]_h]_j$
The membrane will be filtered into j_{th} membrane by u_j , and u_j be replaced of symbol v .

155 The value of k is smaller, and the priority of the corresponding rule is higher. When $k = 1$, the corresponding rule will have the highest priority.

All of the rules that can be applied must be applied simultaneously. The marked output membranes are never dissolved.

3.2. Rule classification

160 All rules of the system can be classified into three categories composed of eight rule sets by different catalyst, because synchronization is guaranteed by

arranging some special objects in these rules.

$$R = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8\} \quad (5)$$

The rule set R_1 which control the whole computing procedure for guarantying synchronization is introduced here. The rule sets R_2 and R_3 are used to
 165 input the objects into membranes. R_4 is used to divide the generated membranes to obtain the copies of matrix. R_5 to R_7 are for calculation and R_8 is used to dissolve the membrane and output the solution.

Because the rules are applied according to non-determinism and maximal parallelism, the control objects are design for regulate the rules executing by
 170 special order. $\Psi_k (1 \leq k \leq 5)$ is flow control symbol for maximal parallelism, μ_j and $\beta_j (1 \leq j \leq m)$, are matching control symbol for non-determinism to activate particular objects.

$$\begin{aligned} R_1 = & \{r_1 : (\mu_j \mu \rightarrow \mu_j, 1 \leq j \leq m, 1)\} \cup \\ & \{r_2 : (\mu_j \rightarrow \mu_{j+1} \mu^k, 1 \leq j \leq m-1, 2)\} \cup \\ 175 & \{r_3 : (\theta \rightarrow \xi[], 2), r_4 : (\theta \xi \rightarrow \xi(\theta, in), 1), r_5 : (\xi \Psi_1 \rightarrow \xi(\Psi_1, in), 1), \\ & r_6 : (\Psi_1 \Psi_1 \rightarrow \Psi_2(\Psi_2, out), 1), r_7 : (\xi \Psi_2 \rightarrow \Psi_1)\} \end{aligned}$$

3.3. Input Initial Matrices

At first, each row of matrix A is inputted into initial membrane by rule set R_2 .

$$\begin{aligned} 180 \quad R_2 = & \{r_1 : (a_{i1} \rightarrow (a_{i1}, in_j)|_{\mu_j \mu}, 1 \leq i \leq k, 1 \leq j \leq m, 1)\} \cup \\ & \{r_2 : (a_{i2} \rightarrow (a_{i2}, in_j)|_{\mu_j \mu}, 1 \leq i \leq k, 1 \leq j \leq m, 1)\} \cup \\ & \{r_3 : (+_i \rightarrow (+_i, in_j)|_{\mu_j \mu}, 1 \leq i \leq k, 1 \leq j \leq m, 1)\} \cup \\ & \{r_4 : (-_i \rightarrow (-_i, in_j)|_{\mu_j \mu}, 1 \leq i \leq k, 1 \leq j \leq m, 1)\} \end{aligned}$$

Then generate the embedded membrane structure to store matrix B by rules
 185 r_3 and r_4 of R_1 . In other words, input the label θ to generate new membrane, the number of θ is n. Ψ_1 represents the end of membrane generating.

Next, input matrix B by columns, insert label Ψ_1 after a column. The rule set R_3 is for imputing matrix B.

$$\begin{aligned}
R_3 = & \{r_1 : (b_{i1} \rightarrow (b_{i1}, in)|_{\xi}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_2 : (b_{i2} \rightarrow (b_{i2}, in)|_{\xi}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_3 : (+_i \rightarrow (+_i, in)|_{\xi}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_4 : (-_i \rightarrow (-_i, in)|_{\xi}, 1 \leq i \leq k, 1)\}
\end{aligned}$$

3.4. Membrane division and permeation

Divide the generated membranes to obtain the copies of matrix B by Ψ_3 .
 The number of Ψ_3 is $\log_2 m$. The symbol β_j is used to send membrane into j_{th} membrane.

$$\begin{aligned}
R_4 = & \{r_1 : (\Psi_3[0]_0 \rightarrow [0]_0[0]_0, 1)\} \cup \\
& \{r_2 : \beta_j[0]_0 \rightarrow [j\beta[0]_0]_j, 1 \leq j \leq m, 1)\}
\end{aligned}$$

Figure 2: Division and permeation

3.5. Calculation

The rule set R_5 is used to send objects into inner membranes in each marked membrane.

$$\begin{aligned}
R_5 = & \{r_1 : (a_{i1} \rightarrow (a_{i1}, in)|_{\beta}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_2 : (a_{i2} \rightarrow (a_{i2}, in)|_{\beta}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_3 : (+_i \rightarrow (+_i, in)|_{\beta}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_4 : (-_i \rightarrow (-_i, in)|_{\beta}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_5 : (a_{i1} \rightarrow d_{i1}(a_{i1}, in)|_{\Psi_4}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_6 : (a_{i2} \rightarrow d_{i2}(a_{i2}, in)|_{\Psi_4}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_7 : (+_i \rightarrow +^i(+_i, in)|_{\Psi_4}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_8 : (-_i \rightarrow -^i(-_i, in)|_{\Psi_4}, 1 \leq i \leq k, 1)\} \cup
\end{aligned}$$

$$\begin{aligned}
& \{r_9 : (a_{i1} \rightarrow d_{i1}|_{\Psi_4}, 1 \leq i \leq k, 2)\} \cup \\
& \{r_{10} : (a_{i2} \rightarrow d_{i2}|_{\Psi_4}, 1 \leq i \leq k, 2)\} \cup \\
& \{r_{11} : (+_i \rightarrow +^i|_{\Psi_4}, 1 \leq i \leq k, 2)\} \cup \\
& \{r_{12} : (-_i \rightarrow -^i, in)|_{\Psi_4}, 1 \leq i \leq k, 2)\} \cup \\
& \{r_{13} : (\beta \rightarrow \beta(\beta, in), 2), r_{14} : (\beta\Psi 2 \rightarrow \Psi_4, 1), r_{15} : \beta \rightarrow \lambda, 3\} \\
215 \quad & \text{The rule set } R_6 \text{ is used to realize formula } \frac{c_{i1}}{c_{i2}} = \frac{b_{i1}}{b_{i2}} \times \frac{d_{i1}}{d_{i2}} \text{ for multiplication.} \\
& R_6 = \{r_1 : (b_{i1} \rightarrow (\xi_1 t_1, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_2 : (b_{i2} \rightarrow (\xi_1 t_2, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_3 : (d_{i1} \rightarrow (\xi_2 t_1, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_4 : (d_{i2} \rightarrow (\xi_2 t_2, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
220 \quad & \{r_5 : (+_i \rightarrow \gamma_i(+\times, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_6 : (-_i \rightarrow \gamma_i(-\times, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_7 : (+^i \rightarrow \gamma_i(+\times, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_8 : (-^i \rightarrow \gamma_i(-\times, in_\Delta)|_{\Psi_4}, 1 \leq i \leq k, 3)\} \cup \\
& \{r_9 : (t_1 \rightarrow c_{i1}|_{\gamma_i}, 1 \leq i \leq k, 1)\} \cup \\
225 \quad & \{r_{10} : (t_2 \rightarrow c_{i2}|_{\gamma_i}, 1 \leq i \leq k, 1)\} \cup \\
& \{r_{11} : (+\gamma_i \rightarrow +_i, 1 \leq i \leq k, 1)\} \cup \\
& \{r_{12} : (-\gamma_i \rightarrow -_i, 1 \leq i \leq k, 1)\}
\end{aligned}$$

After multiplication, the rule set R_7 is used to sum up.

$$\begin{aligned}
& R_7 = \{r_1 : (c_{i1} \rightarrow (\xi_1 t_1, in_\Delta)|_{\Psi_4}, 0 \leq i \leq 1, 4)\} \cup \\
230 \quad & \{r_2 : (c_{i2} \rightarrow (\xi_1 t_2, in_\Delta)|_{\Psi_4}, 0 \leq i \leq 1, 4)\} \cup \\
& \{r_3 : (c_{i1} \rightarrow (\xi_1 t_1, in_\Delta)|_{\Psi_4}, 2 \leq i \leq k, 4)\} \cup \\
& \{r_4 : (c_{i2} \rightarrow (\xi_1 t_2, in_\Delta)|_{\Psi_4}, 2 \leq i \leq k, 4)\} \cup \\
& \{r_5 : (+_i \rightarrow \gamma(++ , in_\Delta)|_{\Psi_4}, 0 \leq i \leq 1, 4)\} \cup \\
& \{r_6 : (-_i \rightarrow \gamma(-+ , in_\Delta)|_{\Psi_4}, 0 \leq i \leq 1, 4)\} \cup \\
235 \quad & \{r_7 : (+_i \rightarrow (++ , in_\Delta)|_{\Psi_4}, 2 \leq i \leq k, 4)\} \cup \\
& \{r_8 : (-_i \rightarrow (-+ , in_\Delta)|_{\Psi_4}, 2 \leq i \leq k, 4)\} \cup \\
& \{r_9 : (t_1 \rightarrow c_{01}|_\gamma, 1), r_{10} : (t_2 \rightarrow c_{02}|_\gamma, 1)\} \cup \\
& \{r_{11} : (+ \rightarrow \mu +_0 |_\gamma, 1), r_{12} : (- \rightarrow \mu -_0 |_\gamma, 1)\} \cup \\
& \{r_{13} : (\mu^k \rightarrow \Psi 5, 1), r_{14} : (\gamma^{k-1} \rightarrow \mu, 1)\}
\end{aligned}$$

240 The rule set R_8 is used to dissolve the inner membrane and output the

solution in the marked membrane.

$$\begin{aligned}
R_8 = & \{r_1 : (c_{i1} \rightarrow (c_{(i+1)1}, out)|_{\Psi_5}, 0 \leq i < k, 2)\} \cup \\
& \{r_2 : (c_{i2} \rightarrow (c_{(i+1)2}, out)|_{\Psi_5}, 0 \leq i < k, 2)\} \cup \\
& \{r_3 : (+_i \rightarrow (+_{(i+1)}, out)|_{\Psi_5}, 0 \leq i < k, 2)\} \cup \\
245 \quad & \{r_4 : (-_i \rightarrow (-_{(i+1)}, out)|_{\Psi_5}, 0 \leq i < k, 2)\} \cup \\
& \{r_5 : (\Psi_5 \rightarrow \delta, 3), r_6 : ([\delta] \rightarrow \lambda, 3)\}
\end{aligned}$$

3.6. Complexity analysis

In this part, an analysis of Π_{MM} is given. Suppose applying one rule will cost one time slice denoted as t_Ω , so some rules are using concurrently will also
250 cost one time slice. Besides, the process which includes input, computing and output by calculating component Δ will cost another one time slice denoted as t_Δ .

As mentioned above, the system has (n^*m+m) membrane for n unknowns, each has one row of equations, and n calculating component, so the arithmetic
255 operation are parallel execution.

Assuming t_Ω is less than t_Δ , can be ignored, therefore the complexity of the system can be worked out. So the sum of operation capacity is $(mk + nk + 3k)$.

Compare the complexity of Π_{MM} to that of serial computing and parallel computing. The complexity of serial computing method is $O(n^3)$. The com-
260 plexity of parallel computing method is $O(n^3/p)$, the value of p is an integer among 1 to n . The complexity of Π_{MM} is $O(n^2)$.

Obviously, only when $p=n$, the complexity of parallel computing method is $O(n^2)$, which equivalent to complexity of Π_{MM} . So Π_{MM} is the best to solve the problem.

265 4. An Example For Π_{MM}

For describing how the Π_{MM} working, here we give an example to illustrate the whole solving procedure.

Given the matrix A (4*5) matrix B(5*3) are as follows:

$$A \times B = \begin{bmatrix} 3 & -2 & -1 & 2 & 1 \\ 2 & 4 & \frac{3}{2} & -3 & -\frac{3}{2} \\ 1 & -3 & 2 & 1 & 2 \\ -1 & 2 & -\frac{5}{2} & 4 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & \frac{3}{2} \\ 4 & \frac{5}{2} & -1 \\ 2 & 3 & 5 \\ -1 & 3 & 2 \\ 3 & 1 & 2 \end{bmatrix} \quad (6)$$

In the preprocessing, the values of matrix are converted into objects in mem-
brane system. Each number is denotes as three kinds of object: positive or
negative symbol; The absolute value of the number is in faction, two objects a_{i1}
for numerator, one object a_{i2} for denominator. So the matrix A are transformed
to the objects set by rows shown in Table 2. On the other side, the matrix B is
transformed to the objects set by columns in Table 3.

Table 2: The objects of matrix A

Row/Col	1 st col	2 nd col	3 rd col	4 th col	5 th col
1 st row	$a_{11}^3, a_{12}, +1$	$a_{21}^2, a_{22}, -2$	$a_{31}, a_{32}, -3$	$a_{41}^2, a_{42}, +4$	$a_{51}, a_{52}, +5$
2 nd row	$a_{11}^2, a_{12}, +1$	$a_{21}^4, a_{22}, +2$	$a_{31}^3, a_{32}^2, +3$	$a_{41}^3, a_{42}, -4$	$a_{51}^3, a_{52}^2, -5$
3 rd row	$a_{11}, a_{12}, +1$	$a_{21}^3, a_{22}, -2$	$a_{31}^2, a_{32}, +3$	$a_{41}, a_{42}, +4$	$a_{51}^2, a_{52}, +5$
4 th row	$a_{11}, a_{12}, -1$	$a_{21}^2, a_{22}, +2$	$a_{31}^5, a_{32}^2, -3$	$a_{41}^4, a_{42}, +4$	$a_{51}, a_{52}, +5$

Table 3: The objects of matrix B

Col/Row	1 st row	2 nd row	3 rd row	4 th row	5 th row
1 st col	$b_{11}^2, b_{12}, -1$	$b_{21}^4, b_{22}, +2$	$b_{31}^2, b_{32}, +3$	$b_{41}, b_{42}, -4$	$b_{51}^2, b_{52}, +5$
2 nd col	$b_{11}, b_{12}, +1$	$b_{21}^5, b_{22}^2, +2$	$b_{31}^3, b_{32}, +3$	$b_{41}^3, b_{42}, +4$	$b_{51}, b_{52}, +5$
3 rd col	$b_{11}^3, b_{12}^2, +1$	$b_{21}, b_{22}, +2$	$b_{31}^5, b_{32}, +3$	$b_{41}^2, b_{42}, +4$	$b_{51}^2, b_{52}, +5$

After the transformation, the objects of matrix A input into initial mem-
branes shown in Table 2. In this example, k=5, input μ^5 and μ_1 as catalyst.

Apply the rules (r_1, r_2) in R_1 and (r_1, r_2, r_3, r_4) in R_2 to send objects into membrane, Fig.3 shows the inputting procedure.

Figure 3: Inputting procedure

Then generate the new membranes for matrix B. Assuming each membrane has a computing component while it generating, which completes the algebraic operations.

Input θ by 3 times to generate new membranes by rules (r_3, r_4) in R_1 shown in Fig. 4.

Figure 4: Generating membranes

Input matrix B into embedded membranes by columns according to rules (

285 $r_1, r_2, r_3, r_4)$ in R_3 shown in Fig. 5.

Figure 5: Input first column of matrix B

Repeat the process until all columns are in the membranes shown in Fig. 6.

Figure 6: End of inputting

Input catalyst Ψ_3 two times to applying rules r_1 in r_4 to get the membrane structure in Fig. 7, then input catalyst $\beta_j (1 \leq j \leq m)$ to applying rules r_2 in R_4 to get the membrane structure in Fig. 8.

Figure 7: Membrane division

Figure 8: Membrane permeation

290 The objects in membrane j are send into inner membranes for preparing calculation by rules in (r_1 to r_{15}) in R_5 shown in Fig.9.

Figure 9: Preparing calculation

Apply the rule set R_6 is used to complete the multiplication shown in Fig. 10 and the rule set R_7 is used to complete the addition shown in Fig. 11.

Figure 10: Multiplication calculation

Figure 11: Addition calculation

After all the calculation rules have been applied, the results are in each marked membrane shown in Fig. 12. The solution of the product can be read out from the membrane.

Figure 12: Results of matrix multiplication

5. Conclusions

Matrix multiplication is one of important matrix arithmetic operations. In this paper, we proposed a membrane system to complete matrix multiplication based on parallel computing of membranes. In the Π_{MM} system, the multiplication process and addition process can be carried on in different membrane at the same time, the complexity of Π_{MM} system is $O(n^2)$, so the efficient of the parallel membrane system is better than single-processor and p-processors. The further researches are focused on how to optimize the rule sets to improve the efficiency of P system, and how to apply P system for other numerical computation.

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