A dual-frequency fringe projection three-dimensional shape measurement system using a DLP 3D projector

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Abstract: A dual-frequency fringe projection system for three-dimensional (3D) surface shape measurement is proposed in this paper. The system includes two cameras, a DLP 3D projector, and a liquid crystal (LC) shutter glasses. The phase information related to the object height is obtained from the dual-frequency temporal method with 3-step phase-shifting algorithm. By using the DLP 3D projector and LC shutter glasses, 3-step phase-shifting high-frequency and low-frequency fringe patterns are captured only 3 times by the two cameras synchronously. The technique of image registration is applied to low-frequency fringe patterns to guarantee the accuracy of low-frequency phase for high-frequency phase unwrapping. Using the equi-phase coordinate method based on two reference planes, the phase-to-height conversion and non-sinusoidal errors reduction are carried out in one go without any extra operation or measurement time. Experimental results demonstrate that the proposed method effectively improves the measuring speed, and it is valid for measuring surface shapes with multi-steps or discontinuities.

Keywords: 3D shape measurement; fringe projection; synchronization; phase unwrapping

1 Introduction

Owing to the advantages of noncontact, whole-field, and automated measurement, fringe projection profilometry (FPP) is one of the most widely used techniques in practical applications of three dimensional (3D) shape measurements, such as object detection, online quality inspection, medical diagnostics, and so on. As for the technique of FPP, it is of fundamental importance to extract phase from the deformed fringes, which is related to the object shape. Several approaches have been proposed in previous studies, such as phase-shifting technique [1–5] and Fourier transform method [6–10]. In general, phase-shifting technique employs at least three frames to evaluate the phases, therefore, it is more accurate than Fourier transform method. Both phase-shifting technique and Fourier transform method use arctangent algorithm to demodulate the phase values in the range of $-\pi$ to π . A so-called phase unwrapping process to recover the real phase distribution is inevitable [11].

Generally, it is a challenge to perform phase unwrapping when fringe is discontinuous due to the height steps or spatially isolated surfaces. Among the available methods for absolute phase

recovery, temporal phase unwrapping is a dominant technique to retrieve the absolute phase [12]. The method makes use of a sequence of at least two phase maps with varied fringe frequencies. Specifically, for the method of dual-frequency temporal phase unwrapping [13], the high-frequency phase provides the primary phase value, and the low-frequency one provides information for unwrapping. Since each of the phase map is obtained from phase-shift patterns, six is the minimal number of frames necessary. In order to improve the measurement efficiency, several strategies have been proposed. By modulating two different spatial carrier frequencies along the orthogonal direction into one fringe pattern, Takeda et al. introduced a single-shot method for measuring the 3D shape of objects with discontinuities or spatially isolated surfaces [14]. Since two or multiple fringe patterns are overlapped to be one image, the phase resolution is poor. The advent of color CCD cameras and digital projection devices, such as LCD or digital light-processing (DLP) projectors, has advanced the fringe-projection technique. Several single snapshot 3D imaging approaches have been developed by using red, green and blue channels of color image to identify the fringe order or encode the absolute position within the projected fringe pattern. For example, Su [8-9] introduced two methods for single snapshot 3D shape acquisition: one technique is based on the combination of encoded color strips and sinusoidal fringe into the same fringe pattern, the other approach is to overlap sinusoidal fringes, binary stripes, and color grids at each pixel position to be one image. However, the algorithm of color coding depends on the tested objects, which may make the processing complicated. Furthermore, color crosstalk and nonlinear color responding of imaging devices cannot be avoided, thus, the edge of color strips is difficult to identify and that will reduce the accuracy of phase extraction. Zhou [15] proposed an improved phase unwrapping method based on a stair phase coding method. Two groups of 4-step phase-shifting fringe patterns are put into different channels of RGB color fringe patterns. The method needs four projections to retrieve the absolute phase. Huang, Zhang, *et al.* [16] modified an 80Hz DLP projector to project fringe patterns with a switching speed of 240Hz. By synchronizing a high-speed monochrome camera with this modified projector properly, a series of high speed fringe patterns, rather than a static one could be captured sequentially and repeatedly. Based on the similar high-speed strategy, Zuo [17] realized a speed of 90 fps for 3-D shape acquisition by modifying a 120 Hz DLP projector and using a four-pattern method. Compared to the four-pattern method, the dual-frequency temporal phase unwrapping method is more robust. Conventionally, the use of a high-speed camera reduces the image resolution.

To reduce the time of 3D shape measurement, a simple but effective method using a DLP 3D projector is proposed in this paper. The dual-frequency temporal phase unwrapping method with 3-step phase-shifting algorithm is used for phase retrieving. By using this method, the high-frequency

and low-frequency images can be captured simultaneously by two cameras. The present method is much suitable for fast 3D shape measurement. The principle of the method is introduced detailedly in section 2, experimental work to demonstrate the validity of the method is presented in section 3, and the conclusions are summarized in section 4.

2 principle

2.1 Phase retrieval

Due to its high accuracy and insensitivity to ambient illumination, phase-shifting algorithm is known as one of the most common techniques for phase recovery. Among N-step phase-shifting algorithm, three-step phase-shifting strategy has been extensively adopted in high-speed applications because it only needs three phase-shifting images to obtain a full-field phase map. Because of its symmetric properties, and less sensitivity to the nonlinearity errors, the step of phase shift is generally chosen as $2\pi/3$ [18]. In the above-mentioned case, the intensities of fringe patterns can be expressed as

$$I_1(x, y) = I'(x, y) + I''(x, y) \cos[\phi(x, y) - 2\pi/3] \quad (1)$$

$$I_2(x, y) = I'(x, y) + I''(x, y) \cos[\phi(x, y)] \quad (2)$$

$$I_3(x, y) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + 2\pi/3] \quad (3)$$

Where, $I'(x, y)$ is related to the background, $I''(x, y)$ represents the fringe contrast and $\phi(x, y)$ is the phase to be retrieved. From Eqs. (1) - (3), one can find that

$$I'(x, y) = (I_1 + I_2 + I_3)/3 \quad (4)$$

$$\phi(x, y) = \tan^{-1} \left[\frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3} \right] \quad (5)$$

The phase map obtained from Eq. (5) is wrapped and it needs to be unwrapped before applying to 3D shape reconstruction. In this study, the dual-frequency temporal phase unwrapping method is used to eliminate the phase ambiguity on surfaces with steps or discontinuities based on Eqs. (6) and (7) [13].

$$\phi_h^u(x, y) = \phi_h^w + 2\pi * O(x, y) \quad (6)$$

$$O(x, y) = INT \left[\frac{k\phi_l^u(x, y) - \phi_h^w}{2\pi} \right] \quad (7)$$

Where, $\phi_h^u(x, y)$ is the unwrapped phase of high-frequency fringe, $\phi_l^u(x, y)$ is the unwrapped phase of low-frequency fringe, $O(x, y)$ is the fringe order, k is the ratio of high frequency to low frequency, ϕ_h^w is the wrapped phase of high-frequency fringe. $INT [.]$ denotes rounding to the nearest integer. It is obvious for 3-step phase-shifting algorithm that six fringe patterns are required to obtain a continuous phase map.

Theoretically, the unwrapped phase of low-frequency fringe should be satisfied with the following condition for the phase unwrapping of high-frequency fringe

$$\left| \frac{(k\phi_L^u - \phi_h^w)}{2\pi} - O(x, y) \right| < 0.5 \quad (8)$$

Which can be rewritten as

$$|k\phi_L^u - [\phi_h^w + 2\pi * O(x, y)]| < \pi \quad (9)$$

From Eqs. (6) and (9), it can be found

$$|k\phi_L^u - \phi_h^u| < \pi \quad (10)$$

The maximal allowable phase error of low-frequency unwrapped phase is

$$\Delta\phi_{L_{max}}^u = |\phi_L^u - \phi_h^u/k|_{max} < \pi/k \quad (11)$$

It means that the shift of the low-frequency fringe must be less than half of the high-frequency fringe period.

2.2 Fringe projection system by using a DLP 3D projector

Thanks to its capacity of projecting 3D images or videos, the DLP 3D projector is now much more popular [19]. The core of a DLP 3D projector is the DLP link technology. The projector splits one image into two images, one left and one right, or one upper and one lower. It projects the two images alternately at 120Hz by using the fast switching characteristics of DMD. A Liquid crystal shutter glasses, also called LC shutter glasses or active shutter glasses, are used in conjunction with the DLP 3D projector. Each eye's glass contains a liquid crystal layer, it will become dark when the voltage is applied and transparent otherwise. The glasses are controlled by the DLP link pulse from projector. With the projector refreshing, the glasses can darken one of the two eyes alternately. Employing the LC shutter glasses and DLP link technology, the images are displayed at a speed of 60Hz for each eye independently. The left eye always sees only the left image, and the right eye just the right one.

Fig. 1 Proposed fringe projection system.

Fig. 1 illustrates the proposed fringe projection system. A set of dual-frequency fringe patterns, including one group of high-frequency 3-step phase-shifting fringe patterns and the other low-frequency 3-step phase-shifting fringe patterns, are loaded into the DLP 3D projector. The two split images are projected onto the tested object alternately at 120Hz. The LC shutter glasses are placed in front of camera 1 and camera 2 and synchronize with the projector refreshing. Two images can be captured at one time by the two cameras. A set of 3-step fringe patterns, namely 6 frames can be captured only 3 times. As shown in Fig. 1, camera 2 captures the low-frequency fringe patterns, which are used for the phase unwrapping of the high-frequency fringe patterns' phase from camera 1.

It can be found from Fig. 1 that the captured images from cameras 1 and 2 are not aligned with each other, and that will cause error for phase unwrapping. The image registration operation can handle this problem by solving a homography matrix, which maps the target images to the reference images. For the sake of description, images from camera 1 are regarded as reference images and those from camera 2 are target ones. Due to 8 unknown parameters in the homography matrix [20], the homography matrix is resolved by the least-square method using at least 4 corresponding points in the reference images and target ones, respectively. Then target images from camera 2 can be transformed to align with the reference images from camera 1 with a tolerance error by using Eq. (12).

$$(x', y', 1)^T = H \cdot (x, y, 1)^T \quad (12)$$

Where, H is the homography matrix, $(x', y', 1)$ is the resultant homogeneous coordinate aligned with the reference image, and $(x, y, 1)$ is the corresponding coordinate in the target image.

Fig. 2 Image registration: (a) image from camera 1; (b) image from camera 2; (c) result from applying homography matrix on (b).

For image registration, a chessboard with 36 corner points is captured by cameras 1 and 2 as shown in Figs. 2(a) and 2(b), respectively. Positions of the points on the board are extracted at subpixel accuracy and employed to calculate the homography matrix subsequently. Fig. 2(c) is the result of applying the homography matrix on the image shown in Fig. 2(b). The difference between the points in the reference and target images after registration is about 1.47 pixels. In practical

measurement, the low-frequency images cannot align with the high-frequency images perfectly. According to Eq. (11), if the shift of low-frequency fringe pattern is less than half period of high-frequency fringe pattern, the dual-frequency temporal method is suitable for phase unwrapping. In this study, the registration error (1.47 pixels) is far less than half period of the high-frequency fringe pattern. Considering other factors, such as system noise, non-linear intensity response, etc, will affect the phase accuracy of the low-frequency images. The ratio of high frequency to low frequency in Eq. (7), k , should not be too large. To the best of our knowledge, it should keep in one's mind that the value of k should be less than 8 to ensure high reliability of measurements.

2.3 System calibration

In a fringe projection 3D measurement system, calibration is to transform the 2D phase information into the world coordinates. Because the measurement result depends on the high-frequency fringes, only the camera system with high-frequency fringe patterns needs to be calibrated. In this study, the two reference planes based calibration system [21-23] is used. As shown in Fig.3, there are two parallel reference planes placed at positions z_1 and z_2 , respectively. The distance h of the two planes can be set or measured precisely. The projection plane that hosts identical phase values can be called an equi-phase plane, which is projected onto the reference planes 1, 2 and the object with the intersection O , D and B , respectively. It is obvious as seen in Fig. 3 that the phase values corresponding to points O , D and B should be equal to each other. Accordingly, from view of the camera, the plane that has the same image coordinates can be called an equi-coordinate plane, which is projected onto plane 1, 2 and the object with the intersection A , C and B , respectively. From the perspectives of equi-coordinate plane and equi-phase plane, object surface height can respectively be calculated by Eqs. (13) and (14). They are called as equi-coordinate phase [22] and equi-phase coordinate methods [23], respectively.

Fig. 3 Diagram of the two-reference-plane method.

$$z_B = \frac{\phi_B - \phi_A}{\phi_C - \phi_A} \cdot h \quad (13)$$

Where ϕ_A , ϕ_B and ϕ_C are the projected fringe phase corresponding to points A , B and C , respectively. In Eq. (13), ϕ_A , ϕ_B , ϕ_C are different, but points A , B and C have the same image coordinate.

$$z_B = \frac{x_B - x_O}{x_D - x_O} \cdot h \quad (14)$$

Where x_B , x_D and x_O are the x coordinates of points B , D and O , respectively. For a point x_B of the object surface with a phase ϕ , the identical phase value coordinates x_O , x_D can be found on reference planes 1 and 2, respectively. As a matter of fact, x_O or x_D usually does not fall on an exact pixel. Linear interpolation is applied to retrieve x coordinate in sub-pixel scale.

The phase-to-height conversion is widely to be carried out by the use of equi-coordinate phase method [17, 18, 22, 25-31]. However, the error caused by the non-sinusoidal intensity distribution is unavoidable. In order to reduce such error, many strategies were adopted [5, 32, 33]. These error elimination approaches are time-consuming due to their either intensity pre-adjusting or phase post-filtering. As presented in the researches [23, 24], the equi-phase coordinate method can handle phase-to-height conversion and non-sinusoidal error in one go. It should be noted that the phase demodulated in the previous researches is based on 4-step phase-shifting method. It is necessary to verify whether the equi-phase coordinate method is validation for 3-step phase shifting algorithm.

For the quantitative evaluations, a flat plane is sequentially placed at three locations $z = 0\text{mm}$, 25.0mm , and 50.0 mm . The first and third positions were treated as reference planes, as shown in Fig. 3. The height of the plane at the second location is measured by equi-coordinate and equi-phase methods combined with 3-step and 4-step phase-shifting methods, respectively. Fig. 4(a) is an image of non-sinusoidal fringe pattern. Results from Eq. (13) by the use of 3-step and 4-step phase-shifting methods are shown in Figs. 4(b) and 4(c), respectively. It is obvious as presented in Figs. 4(b) and 4(c) that non-sinusoidal error of 3-step phase-shifting method is far more than that of 4-step phase-shifting method. Figs. 4(d) and 4(e) are the results obtained from Eq. (14) with 3-step and 4-step phase-shifting algorithms, respectively. Fig. 4(f) presents 2D height distributions of one cross-section corresponding to the results shown in Figs. 4(b), 4(c), 4(d) and 4(e). The location of the cross-section is only labelled in Fig. 4(a) as A-A'. It can be seen from Fig. 4(f) that non-sinusoidal error of the equi-phase and 3-step phase-shifting method is about one tenth of those of the equi-coordinate and 3-step phase-shifting method. It also can be found that, based on the equi-phase method, the non-sinusoidal errors of 3-step and 4-step phase-shifting algorithms are almost equal to each other.

The RMS error is defined as the root mean square of the deviation of the measurement value from the ground truth. In this study, the ground truth is assumed as an absolutely flat plane. For the case of 3-step phase-shifting algorithm, the RMS error is 0.044mm for the equi-phase coordinate method and 0.675mm for the equi-coordinate phase method. While in the case of 4-step phase-

shifting algorithm, the RMS error is 0.041mm and 0.053mm, respectively. Considering the measurement speed and accuracy, the equi-phase coordinate method is suitable for the phase-to-height conversion in the proposed dual-frequency fringe projection system.

Fig. 4 (a) A non-sinusoidal fringe pattern; (b) and (c) results from Eq. (13) with 3-step and 4-step phase-shifting; (d) and (e) results from Eq. (14) with 3-step and 4-step phase-shifting; (f) 2D height distributions of one cross-section corresponding to (b), (c), (d) and (e) [the position of the cross-section labelled in (a) as A-A']

If the camera of the system is calibrated in advance, the x and y world coordinates of a surface point can be obtained from the camera matrix, the height and the image coordinate of the point. This is supported by the governing equation, expressed in homogeneous coordinates, which maps a 3D world point to a 2D image point [23]

$$(x_i, y_i, 1)^T = M \cdot (x_w, y_w, z_w, 1)^T \quad (15)$$

Where, M is the camera matrix, T denotes matrix transpose, x_i and y_i are image coordinates, x_w , y_w and z_w are world coordinates (z_w stands for height). The matrix representation contains three equations, two of which are linearly independent; hence the two unknowns x_w and y_w can be calculated.

3 Experimental work

In experiments, a DLP 3D projector (GIMI Z3S) was used to project the dual-frequency 3-step phase-shifting fringe patterns onto the tested objects. Images were captured by two IDS USB3.0 CCD cameras with 2048×2048 pixels at frame rate of 25 fps. One pair of LC shutter glasses were used for synchronous acquisition. Fig. 5 shows the experimental system. Two sets of tested objects were shown in Fig. 6(a) and 6(b), respectively. One of them consists a multi-step and a rotational surface structures produced by 3D-printing technology, another tested object is a mask with holes. The periods of the projected dual-frequency fringe patterns were a high-frequency of 30 pixels/fringe and a low-frequency of 150 pixels/fringe, respectively.

Fig. 5 The fringe projection system.

Fig. 6 Tested objects: (a) multi-steps and rotational surface structures; (b) a mask with holes.

For objects measurement, the world coordinate of the system can be arbitrarily chosen: the X and Y axes were set to parallel the horizontal and vertical edges of a chessboard pattern as seen in Fig. 7, respectively. The Z axis was set to the shifting direction of the chessboard pattern.

The chessboard was sequentially measured at two locations $z=0$ and 50mm for system calibration. Figs. 7(a) and 7(c), captured by camera 1, were the fringe patterns of reference planes at the first and second locations. The phase maps of the two reference planes were measured just for one time and stored for object height measurement. The background images, Figs. 7(b) and 7(d), corresponding to Figs. 7(a) and 7(c), were used for camera calibration. 36 corner points of the black

squares were extracted at subpixel accuracy, based on which the camera matrix in Eq. (15) was obtained.

Fig. 7 (a) (c) Fringe patterns, and (b) (d) background images of the calibration chessboard at two reference positions.

After camera calibration and the reference planes' position determination, the tested objects were placed into the space between the two reference planes. The dual-frequency 3-step phase-shifting fringe patterns were projected onto the objects by the DLP 3D projector and captured by the two cameras. Figs. 8(a) and 8(b) were the deformed high-frequency and low-frequency fringe patterns captured by camera 1 and camera 2 respectively, and Fig. 8(c) was the low-frequency fringe pattern after registration corresponding to Fig. 8(b).

Fig. 8 The deformed fringe patterns: (a) high-frequency image; (b) low-frequency image before registration; (c) low-frequency image after registration.

Figs. 9(a) and 9(b) were the wrapped phase maps cooresponding to high-frequency and low-frequency fringe patterns respectively. The unwrapped phase map of Fig. 9(b) is shown in Fig. 9(c) obtained by the spatial method[11]. Fig. 9(d) was the unwrapped phase of high-frequency phase obtained by the proposed dual-frequency method. The binarized background image, as shown in Fig. 9(e), was used to guide the height calculation. So there were gaps or holes between the steps in the following results. Fig. 9(f) was the height map obtained from the equi-phase coordinate method (Eq. (14)). The three-dimensional point cloud of the objects surfaces was obtained by using Eq. (15) in section 2.3 and the reconstructed 3D surface was presented in Fig. 9(g). Fig. 9(h) shows the height distribution of cross-section B-B' labelled in Fig. 9(f).

Fig. 9 (a) and (b) Wrapped phases of high-frequency and low-frequency phase fringe patterns, respectively; (c) and (d) unwrapped phase of low-frequency and high-frequency phase fringe patterns, respectively; (e) binary map of the background image; (f) height map; (g) 3D reconstructed surface; (h) height distribution of cross-section B-B' labelled in Fig. 9 (f).

As shown in Figs. 9(f) and 9(h), the average heights of step 1, 2, 3, 4 and 5 were 5.09 mm, 14.95 mm, 29.91 mm, 14.95 mm and 5.07 mm, which were very close to the nominal heights 5mm, 15mm, 30mm, 15mm and 5mm. Based on the data from measurement, the fitted radius of spherical surface was 31.67mm which perfectly matched with its actual value 32mm. The discrepancy between the measurement results and the nominal values was mainly caused by several factors such as errors in camera calibration, the non-flat surface of reference plate, and so on. Investigation in methods to reduce such errors was beyond the scope of this paper.

To further verify the validation of the proposed method, another object surface shown in Fig. 6(b) was measured. The experimental results presented in Fig. 10. The area of the interested region in Fig. 10(g) was about $220 \times 200 \text{ mm}^2$, its image size is 1760×1600 pixels. The surface with discontinuities were recognized and measured using the present method without any extra processing.

Fig. 10 Results of the measured surface with discontinuities. (a) and (b) Wrapped phases of high-frequency and low-frequency phase fringe patterns, respectively; (c) and (d) unwrapped phase of low-frequency and high-frequency phase fringe patterns, respectively; (e) binary map of the background image; (f) height map; (g) 3D reconstructed surface.

It needs to be pointed out that before the 3D reconstructions, a 5×5 mean filter was applied to the height maps shown in Figs. 9(f) and 10(f) to eliminate the random noise of the image acquisition system.

4 Conclusion

In this study, a dual-frequency fringe projection system based on a DLP 3D projector is proposed to reduce the measuring time in 3D surface measurement. Different from the conventional fringe projection systems, the proposed system includes two cameras, a DLP 3D projector, and one LC shutter glasses. The phase information related to object height is obtained from the dual-frequency temporal method with 3-step phase-shifting algorithm. By using the DLP 3D projector and LC shutter glasses, 3-step phase-shifting high-frequency and low-frequency fringe patterns are captured only 3 times by the two cameras synchronously. The technique of image registration is applied to the low-frequency images to guarantee the accuracy of low-frequency phase for high-frequency phase unwrapping. In addition, by using the equi-phase coordinate method based on two reference planes, the phase-to-height conversion and non-sinusoidal error reduction are carried out in one go. Experiments have been performed to demonstrate that the proposed method effectively improves the measuring speed, and it is valid for measuring the surfaces' shape with multi-steps or discontinuities.

It is worth to note that if the 3-step phase-shifting fringe patterns are encoded into the color channels R, G, and B, the 6 images can be captured by color cameras only one time. A potential high speed system is available, and the study will be performed in the future.

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