Numerical and experimental investigation of nonlinear focused waves-current interaction with a submerged plate

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ABSTRACT

This paper concerns the interaction of focused waves with uniform current and a submerged horizontal plate in the context of the two-dimensional nonlinear potential theory. A fully nonlinear NWT (numerical wave tank) based on a time domain HOBEM (higher-order boundary element method) is applied to the numerical solutions. In the model, the Goda's method is modified to discriminate incident and reflected waves, whereas the acceleration potential approach is introduced to determine the hydrodynamic forces on the structure. The corresponding experiments are also conducted in a two-dimensional glass-walled wave tank at Jiangsu University of Science and technology. The developed model is verified with published numerical results and validated with experimental test. Through a systematic parametric study, it is found that the maximum reflection or minimum transmission for different currents occur at the same plate width because of the compensating Doppler effects of current. The spectral analysis reveals that the energy dissipation occurs primarily in the fundamental waves when focused waves propagating over the plate. The current has a stronger influence on the wave forces than on wave surface elevation. Furthermore, the 'secondary loading cycle' phenomenon is also observed in the time series of the wave forces for a following current.

Keywords: Focused waves, HOBEM, Fully nonlinear, Submerged plate, Wave-current interaction, Experiment

1. Introduction

Stem-supported submerged-horizontal-plate structures in coastal and offshore zones can be applied as an eco-friendly breakwater for reducing transmitted and reflected wave height (Graw 1992; Rahman 2006; Peng et al. 2013; Lin et al. 2014). As compared with the traditional breakwaters such as rubble mound breakwaters, caisson breakwaters and vertical wall breakwaters, the attractive merits of such breakwater include less dependent on seabed condition, waters exchanging freely and most importantly inexpensive in construction. When the incoming waves propagate over the immersed horizontal plate breakwater, the strong pulsating flow opposite to the wave propagation direction below the plate is induced due to the impermeable boundary condition of the plate. This interesting phenomenon motivated the conception of a submerged plate wave energy converter (WEC) (Carter et al. 2006; Ozer and Ozdamar, 2007). Therefore, the submerged horizontal plate can be not only used as a breakwater for protecting the coastline and moored

vessel, but also can be employed as a wave energy conversion device for capturing the wave's oscillating motions.

Extensive theoretical and experimental researching works have been performed on wave scattering problems by a submerged horizontal plate. [Dick \(1968\)](#) was one of the first researchers to measure the characteristics of regular wave scattering by a thin and solid submerged plate. He confirmed the application of Dean's theory for the propagation of waves over a thin submerged plate at deep water depth and developed an empirical and theoretical relationship for reflection coefficients across the wave numbers. [Siew and Hurley \(1977\)](#) discussed the hydrodynamic issues of long waves interaction with a submerged plate using matched asymptotic expansions in the context of linear potential theory. Along the same lines, [Patarapanich \(1984a\)](#) analyzed systematically the dependence of the reflection coefficient on the ratio of the plate length and the wavelength. [Patarapanich \(1984b\)](#) further presented the numerical solutions of wave scattering by a thin horizontal plate with covering the whole range of water depth and placed location by means of the finite element method (FEM). In addition, [Cheong et al. \(1996\)](#) compared these numerical solutions of FEM with independently developed analytical results of eigen-function expansion method. [Carter \(2005\)](#) studied the effects of the flow generated over a submerged plate on the interaction of waves with plate using the boundary element method (BEM).

However, these mentioned studies are limited to the linear potential theory, and they are not appropriate when the nonlinear scattering by the submerged plate is considered. Recognizing the importance of this problem, some researchers have devoted significant efforts to the development of nonlinear scattering analysis models. [Brossard and Chagdali \(2001\)](#) conducted experiments to study higher harmonic waves generated by regular waves over a submerged horizontal plate, and they found that the proportion of higher harmonic wave components in the total waves becomes greater with the decrease of submerged depth. Later, [Brossard et al. \(2009\)](#) further investigated the resonant behaviour of waves above the submerged plate, and showed that the first-order and second-order resonant behaviours are linked to the fundamental wavelength and the bound harmonic wavelength over the plate, respectively. [Liu et al. \(2009\)](#) applied the fully nonlinear numerical wave tank (NWT) to re-examine the nonlinear wave scattering by a submerged thin plate as described in [Brossard et al. \(2009\)](#). They also conducted relevant model tests and validated the numerical solutions obtained by using the desingularized boundary integral equation method.

All above studies of the wave interaction with a submerged horizontal plate were still based on regular wave tests, which will not represent some extreme wave events. The interaction of extreme waves and a submerged horizontal plate, however, were not investigated in the foregoing works. In fact, these submerged-horizontal-plate structures are constantly exposed to a harsh ocean environment. The extreme wave which appears in a natural sea state, can excite great wave forces on structures and then devastate them. Thus, some efforts to characterize the generation of extreme waves and interaction of extreme waves with the structure have intrigued a number of researches. [Baldock et al. \(1996\)](#) investigated free surface evolution of the crest and trough focused wave groups and their experimental results reveal that the wave-wave interactions can lead to wave crest at the focus position becomes much higher and narrower, while the focused wave trough becomes much lower and broader as compared with the first-order and second-order analytical solutions. [Ning et al. \(2008, 2009\)](#) employed higher-order boundary element method (HOBEM) to simulate uni-directional steep wave groups in an infinite water depth and finite water

depth respectively. [Sriram et al. \(2015\)](#) adopted second-order wave maker theory to generate focused waves in intermediate water depths, and the results show that the spurious, free and bound wave components can be accurately simulated in main wave regions when using second-order paddle displacement. [Westphalen et al. \(2013\)](#) numerically and experimentally studied the nonlinear interactions between focused waves and fixed horizontal/vertical cylinders. Their numerical results, predicted by the computational fluid dynamics (CFD), correlate reasonably with the measured data. [Li et al. \(2012, 2014\)](#) measured and investigated the run-up of multi-directional focused waves and wave forces on a vertical circular cylinder, and quoted that the influences of the wave parameters on the surface spatial profile of extreme waves are significant. All of these studies are concerned with the case of focused waves without accounting the current effects.

In many coastal regions, extreme waves and currents coexist simultaneously, and focused waves can be induced by currents. Moreover, extreme waves can lead to high-frequency vibrations of structure which is called 'ring' phenomenon, and currents may weaken or enhance the ratio of the energy in higher harmonic waves to that in the fundamental waves. However, most investigations on the wave-current interaction are generally restricted to the regular waves or random waves and currents. For example, [Thomas et al. \(1981, 1990\)](#) numerically and experimentally investigated the linear and nonlinear interactions between regular waves and currents; [Ren \(1997\)](#) used Boussinesq model to simulate the effects of currents on regular wave propagation; [Rey et al. \(2002\)](#) measured the wave-current interaction with a submerged horizontal plate, and shown that currents have slight contribution to maximum and minimum reflection coefficient of plate; [Rey and Touboul \(2011\)](#) further investigated the regular and irregular wave scattering by a submerged plate in the presence of a uniform current, and their improved potential flow model is good agreement with the experimental results; [Li and Lin \(2015\)](#) adopted Reynolds Averaged Navier-Stokes (RANS) equations to calculate wave-current impacts on a surface-piercing structure for regular and irregular waves; [Lin and Ning \(2014\)](#) used the time-domain HOBEM to develop a fully nonlinear NWT based on two-dimensional potential flow theory, and they focused on current effects on the fundamental wave and the second free wave by changing wave amplitude, wave frequency and current velocity.

While several studies of the problem of wave-current interaction with structure have been performed, almost all of these studies are for regular waves or irregular wave conditions, whereas it could be computationally more efficient, and physically realistic in certain conditions ([Kharif and pelinovsky et al., 2003](#)), to model problems for extreme waves and current. The question about the extreme waves-current interaction with a submerged horizontal plate has not been studied. It is the motivation of the present work to address this particular problem.

To this end, this paper aims to report the interaction of nonlinear focused waves with uniform current and a submerged horizontal plate from both numerical simulations and small-scale experiments. For the analysis, a two dimensional fully nonlinear NWT based on potential flow theory is established by an efficient HOBEM. A spatial-temporal focusing method is used to generate extreme waves by specifying the second-order analytical velocity at the inflow boundary, such that the incident waves can be reflected from the submerged obstacle and then be absorbed in a damping layer front. The current effect can be realized by introducing the current speed to the instantaneous free surface, which is updated by using the fourth-order Runge-Kutta time-marching scheme with mixed Eulerian-Lagrangian approach at each time step. Physical experiments on the interactions of waves, current and submerged plate are also conducted in the wave flume of the

Ocean Engineering Laboratory at the Jiangsu University of Science and Technology in order to validate the numerical model and results. The paper is proceeded as follows: Mathematical Formulations about the fully nonlinear model are expressed in section 2. In Section 3, the experimental set-up, submerged plate, wave and current parameters are provided in detail. The comparisons among the present numerical results, existing numerical results and model measured data are given in Section 4. In the presence of current, the effects of design parameters, such as incident wave amplitude, current velocity, immersion, plate width and frequency bandwidth on the reflection coefficients, transmission coefficients, power density and wave loads are investigated. Lastly, the conclusions are given in Section 5.

2. Numerical model

When the wave steepness is increased, the nonlinearities of the focused waves can increase rapidly. Due to the complexity of the wave breaking process and interaction of waves, current and structure, a powerful and robust numerical model is required to simulate these focused waves correctly and efficiently. A fully nonlinear numerical model based on a time-domain HOBEM is applied in this paper.

Fig. 1. Schematic diagram of a two dimensional numerical wave-current tank

2.1 Mathematical formulation

The influence of focused waves and a uniform current on a submerged horizontal plate in a two dimensional fluid domain is shown in Fig. 1. The fluid is assumed to be incompressible, inviscid with irrotational flow and such that a velocity potential can be used to describe the fluid motion. A Cartesian coordinate system is defined with the origin on the still water level $z=0$, with the z -axis positive upwards. As shown in Fig.1, h denotes the static water depth, h_s is the submerged depth of plate, B is the plate width, and W is the plate thickness. The whole computational domain is defined by Ω , while the symbols S_F , S_B , S_I and S_O , represent the free surface, the plate and bottom surfaces, the input boundary and the output boundary, respectively. Considering the effects of current, the total velocity potential can be expressed as $\Phi(x, z, t) = U(x)x + \phi(x, z, t)$, where $U(x)$ is the steady uniform current velocity whose strength varies with the local flume depth $h(x)$ as required by the mass conservation, and $\phi(x, z, t)$ is the perturbation potential caused by waves. Both velocity potentials $\Phi(x, z, t)$ and $\phi(x, z, t)$ satisfy the Laplace equation:

$$\nabla^2 \Phi = \nabla^2 \phi = 0 \quad \text{in } \Omega \quad (1)$$

On the instantaneous free surface, both fully non-linear kinematic and dynamic boundary conditions are satisfied and the mixed Eulerian-lagrangian method is adopted to track the time-dependent free surface nodes with water particles motion in both horizontal and vertical directions. Two damping layers designed at the two ends of the NWT are expected to suppress

outgoing wave reflection at the output boundary and re-reflection at the input boundary. Then, the resulting equations on the free surface and the fluid-structure interface can be modified as follows in the Lagrangian form:

$$\begin{cases} \frac{d\mathbf{X}(x, z)}{dt} = \nabla\varphi + U\mathbf{i} - \mu_1(x)(\eta - \eta^*) - \mu_2(x)\eta \\ \frac{d\varphi}{dt} = -g\eta + \frac{1}{2}|\nabla\varphi|^2 - \mu_1(x)(\varphi - \varphi^*) - \mu_2(x)\varphi \end{cases} \quad \text{on } S_F \quad (2)$$

where $\mathbf{X}$ denotes the position vector of a fluid particle, η is the free surface elevation above the still water level, d/dt is the material derivative defined by $d/dt = \partial/\partial t + \nabla\varphi \cdot \nabla$, φ^* and η^* are the analytic solutions (incident waves) in the absence of the obstacle in the same computational domain. Damping terms μ_1 and μ_2 are defined as follows:

$$\mu_{1(2)}(x) = \begin{cases} \omega \left(\frac{x - x_{1(2)}}{L_b} \right)^2 & (x_1 - L_b < x < x_1 \text{ or } x_2 + L_b > x > x_2) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where x_1 and x_2 are the starting positions of damping zones 1 and 2, respectively, L_b is the length of the damping zone and determined to be 1.5 maximum wavelength in incident wave components for the present model.

The boundaries at the tank bottom, the plate surface and the backward wall are considered to be impermeable and no flow condition at these surfaces S_b and S_o is imposed.

For the inlet boundary S_i , a spatial-temporal focusing method proposed by [Wu and Yao \(2004\)](#) is adopted to generate the focused waves by imposing a theoretical particle velocity profile along the vertical input boundary. In this paper, the second-order Stokes irregular wave theory given by [Schäffer \(1996\)](#) and [Ning et al. \(2009\)](#) is used, i.e.

$$\frac{\partial\varphi}{\partial n} = v^{(1)} + v^{(2)} \quad \text{on } S_i \quad (4)$$

and the corresponding surface profiles are given by

$$\eta = \eta^{(1)} + \eta^{(2)} \quad (5)$$

The linear velocity $v^{(1)}$, linear wave elevation $\eta^{(1)}$, second-order velocity $v^{(2)}$ and second-order elevation $\eta^{(2)}$ are then given by

$$v^{(1)} = \sum_{i=1}^{N_f} \frac{ga_i k_i \cosh k_i(z+h)}{\omega_i \cosh k_i h} \cos[(k_i(x-x_0) - \omega_i(t-t_0) + \varepsilon_i)] \quad (6)$$

$$\eta^{(1)} = \sum_{i=1}^{N_f} a_i \left(1 - \frac{k_i U}{\omega_i}\right) \cos[(k_i(x-x_0) - \omega_i(t-t_0) + \varepsilon_i)] \quad (7)$$

$$v^{(2)} = \sum_{i=1}^{N_f} \sum_{j>i} \left\{ \begin{aligned} & (k_i + k_j) a_i a_j \frac{G^+ \cosh[(k_i + k_j)(z + h)]}{D^+ \cosh[(k_i + k_j)h]} \\ & \cos[(k_i + k_j)(x - x_0) - (\omega_i + \omega_j)(t - t_0) + (\varepsilon_i + \varepsilon_j)] + \\ & (k_i - k_j) a_i a_j \frac{G^- \cosh[(k_i - k_j)(z + h)]}{D^- \cosh[(k_i - k_j)h]} \\ & \cos[(k_i - k_j)(x - x_0) - (\omega_i - \omega_j)(t - t_0) + (\varepsilon_i - \varepsilon_j)] \end{aligned} \right\} +$$

$$\sum_{i=1}^{N_f} k_i a_i^2 \frac{G^+ \cosh[2k_i(z + h)]}{D^+ \cosh[2k_i h]} \cos[2k_i(x - x_0) - 2\omega_i(t - t_0) + 2\varepsilon_i]$$

$$\eta^{(2)} = \sum_{i=1}^{N_f} \sum_{j>i} \left\{ \begin{aligned} & a_i a_j H^+ \left(1 - \frac{k_i + k_j}{\omega_i + \omega_j} U\right) \cos[(k_i + k_j)(x - x_0) - \\ & (\omega_i + \omega_j)(t - t_0) + (\varepsilon_i + \varepsilon_j)] + a_i a_j H^- \left(1 - \frac{k_i - k_j}{\omega_i - \omega_j} U\right) \\ & \cos[(k_i - k_j)(x - x_0) - (\omega_i - \omega_j)(t - t_0) + (\varepsilon_i - \varepsilon_j)] \end{aligned} \right\} +$$

$$\sum_{i=1}^{N_f} a_i^2 H^+ \left(1 - \frac{k_i}{\omega_i} U\right) \cos[2k_i(x - x_0) - 2\omega_i(t - t_0) + 2\varepsilon_i]$$

where ω_i and k_i are wave frequency and wave number of each component wave respectively. N_f is the number of the wave frequency. ε_i is the initial phase, which is given by uniformly distribution random numbers, and

$$D^\pm = g(k_i \pm k_j) \tanh[(k_i \pm k_j)h] - (\omega_i \pm \omega_j)^2 \quad (10)$$

$$G^\pm = -g^2 \left[\begin{aligned} & \frac{k_i k_j}{\omega_i \omega_j} (\omega_i \pm \omega_j) [1 \mp \tanh(k_i h) \tanh(k_j h)] + \\ & \left[\frac{k_i^2}{2\omega_i \cosh^2(k_i h)} \pm \frac{k_j^2}{2\omega_j \cosh^2(k_j h)} \right] \end{aligned} \right] \quad (11)$$

$$H^\pm = \frac{(\omega_i \pm \omega_j)}{g} \frac{G^\pm}{D^\pm} + F^\pm \quad (12)$$

$$F^\pm = -\frac{g}{2} \frac{k_i k_j}{\omega_i \omega_j} \frac{\cosh^2[(k_i \mp k_j)h]}{\cosh(k_i h) \cosh(k_j h)} + \frac{1}{2} [k_i \tanh(k_i h) + k_j \tanh(k_j h)] \quad (13)$$

Considering current effects on wave propagating along the wave flume, the following modified dispersion relation is used to solve the wave number k_i :

$$(\omega_i - k_i U)^2 = g k_i \tanh(k_i h) \quad (14)$$

As the above boundary value problem is simulated in the time domain, the initial water surface conditions should be considered and the calm conditions are required as follows:

$$\phi|_{t=0} = \eta|_{t=0} = 0 \quad (15)$$

2.2 Higher-order boundary element method

By using the Green's second identity, the foregoing boundary value problem can be transformed the following boundary integral equation:

$$\begin{aligned} \alpha(p)\phi(p) - \int_{S_{B+I+O}} \phi(p) \frac{\partial G(p,q)}{\partial n} ds + \int_{S_F} G(p,q) \frac{\partial \phi(p)}{\partial n} ds = - \int_{S_{B+I+O}} G(p,q) \frac{\partial \phi(p)}{\partial n} ds \\ + \int_{S_F} \phi(p) \frac{\partial G(p,q)}{\partial n} ds \end{aligned} \quad (16)$$

where p and q are the source (x_0, z_0) point and field point (x, z) , respectively; α is the solid angle coefficient given Ning et al. (2009); G is the image Green function and can be written as

$$G = \frac{1}{2\pi} (\ln r_1 + \ln r_2), \text{ where } r_1 = \sqrt{(x-x_0)^2 + (z-z_0)^2} \text{ and } r_2 = \sqrt{(x-x_0)^2 + (z+z_0+2h)^2}.$$

Then these boundary surfaces in Eq. (16) are discretized with three-node line elements based on quadratic shape functions which are defined by Ning et al. (2015). These elements with collocation points are isoparametric, i.e. the physical variables are interpolated by the same shape functions. Thus, an arbitrary physical variable $M(\xi)$ within each element can be expressed by a combination of corresponding nodal values and quadratic shape functions:

$$M(\xi) = \sum_{n=1}^N H_n(\xi) M_n \quad (17)$$

where $M(\xi)$ denotes $x/z/\phi/\frac{\partial \phi}{\partial n}$, local intrinsic coordinate ξ lies in the interval $(-1, 1)$, H_n is the shape functions corresponding to the n -th node in the local system, and N is the number of nodes of each element.

On the other hand, numerical integration is performed using Gauss-Legendre quadrature with 4 integration points within each element. Then Eq. (16) can be the following discretized integral equation:

$$\begin{aligned} \alpha(p)\phi(p) - \sum_{i=1}^{Ne_2} \sum_{j=1}^4 \sum_{n=1}^3 H_j^n \phi_n \frac{\partial G(p,q(\xi_j))}{\partial n} \Big| J_j \Big| w_j + \sum_{i=1}^{Ne_1} \sum_{j=1}^4 \sum_{n=1}^3 H_j^n G(p,q(\xi_j)) \frac{\partial \phi_n}{\partial n} \Big| J_j \Big| w_j = \\ \sum_{i=1}^{Ne_1} \sum_{j=1}^4 \sum_{n=1}^3 H_j^n \phi_n \frac{\partial G(p,q(\xi_j))}{\partial n} \Big| J_j \Big| w_j - \sum_{i=1}^{Ne_2} \sum_{j=1}^4 \sum_{n=1}^3 H_j^n G(p,q(\xi_j)) \frac{\partial \phi_n}{\partial n} \Big| J_j \Big| w_j \end{aligned} \quad (18)$$

where J_j represents the Jacobian matrix relating the physical coordinates to the local intrinsic coordinates within an element, Ne_1 and Ne_2 are the numbers of the discretized elements on the free surface and solid surfaces (input, plate and outlet boundaries), respectively. The discretized integral Eq. (18) is transformed into a linear algebraic system and then solved by using Gauss elimination scheme.

After solving the boundary integral equation (18), a Runge-Kutta 4th-order (RK4) method is applied to advance the fully nonlinear free and plate surface conditions Eq. (2) in time. Once the new variables $(\phi, x$ and $\eta)$ are known at next time step, it is necessary to remesh the computational domain and the following criterion is used to find which old element the new node belongs to:

$$\left| L_0 - \sum_{i=1}^{M_1} L_i \right| \rightarrow 0.0 \quad (19)$$

where L_0 and L_i are the length of the old element and the length of a sub-element consisting of adjacent two nodes, respectively, M_1 is the number of sub-elements surrounding the considered new node. As the three-node line elements are used to approximate the entire curved boundary, $M_1=2$ is chosen for the number of all sub-elements.

2.3 Hydrodynamic forces

The hydrodynamic forces $\mathbf{F}=\{F_1, F_2\}$ and moment $\mathbf{M}=\{F_3\}$ on the submerged horizontal plate can be obtained by integrating the pressure over the wetted body surface

$$F_i = \int_{S_p} P n_i ds = -\rho \int_{S_p} \left(\frac{\partial \varphi}{\partial t} + \frac{1}{2} |\nabla \varphi|^2 + U \frac{\partial \varphi}{\partial x} \right) n_i ds \quad i=1, 2, 3 \quad (20)$$

where ρ is the density of the fluid. $\mathbf{n}=(n_1, n_2)$ is the normal unit vector (the positive direction points out of the fluid domain). $n_3=n_1(z-z_0)-n_2(x-x_0)$, wherein (x, z) is the spatial coordinate on the plate, and (x_0, z_0) denotes the rotation center which coincides with the mass center.

The accurate calculation of the time derivative φ_t of velocity potential is very crucial in solving correct hydrodynamic forces on the body surface in Eq. (20). Compared with backward finite difference method, the acceleration potential method proposed by Tanizawa (1995) can avoid numerical instability and inaccurate results induced by nodes moving with the time for full nonlinear problems. In this method, the acceleration potential satisfies the Laplace equation in the fluid domain, and the flowing boundary conditions

$$\frac{d\varphi}{dt} = -g\eta + \frac{1}{2} |\nabla \varphi|^2 \quad \text{on } S_F \quad (21)$$

$$\frac{\partial \varphi_t}{\partial n} = 0 \quad \text{on } S_B, S_O \quad (22)$$

$$\frac{\partial \varphi_t}{\partial n} = \frac{\partial^2 \varphi_t}{\partial n \partial t} \quad \text{on } S_I \quad (23)$$

Thus, the boundary integral equation similar to Eq. (16) can be obtained to determine the acceleration potential. On the other hand, partial derivatives of the velocity potential on body surface in Eq. (20) can be obtained from follows:

$$\begin{bmatrix} \frac{\partial \varphi}{\partial x} \\ \frac{\partial \varphi}{\partial z} \end{bmatrix}^{(t)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ n_x & n_z \end{bmatrix}^{-1(t)} \begin{bmatrix} \frac{\partial \varphi}{\partial \xi} \\ \frac{\partial \varphi}{\partial n} \end{bmatrix}^{(t)} \quad (24)$$

where symbol (t) denotes to the current time step.

Finally, the hydrodynamic forces and moment can be obtained by calculating the Eq. (20). Noted that the hydrodynamic forces in the present study is the total fluid force minus the mean hydrostatic force $-\rho g z n_i$.

2.4 Solution for reflection and transmission coefficients

As long time simulations are required in case of the submerged horizontal plate, the focused waves in front of the plate are composed of incident waves and reflected waves from plate. The Goda's method (Goda and Suzuki, 1976) can be easily extended to consider focused waves for

discriminating the incident and reflected wave amplitudes. For given free-surface elevation at two points x_1 and $x_1 + \Delta l$ at front of the submerged plate, the incident wave amplitude $a_I(m)$ and reflected wave amplitude $a_R(m)$ of each component can be expressed respectively as follows

$$a_I(m) = \frac{|\hat{\eta}_m(x_1 + \Delta l) - \hat{\eta}_m(x_1)e^{-ik_m^I \Delta l}|}{2|\sin(k_m^I \Delta l)|} \quad (25)$$

$$a_R(m) = \frac{|\hat{\eta}_m(x_1 + \Delta l) - \hat{\eta}_m(x_1)e^{ik_m^R \Delta l}|}{2|\sin(k_m^R \Delta l)|} \quad (26)$$

where the subscript m denotes the m -th wave component, and the number of wave components is the half of collected data. The terms $\hat{\eta}_m(x_1)$ and $\hat{\eta}_m(x_1 + \Delta l)$ can be obtained by applying the fast Fourier transform of the time series of free-surface elevation at the position x_1 and $x_1 + \Delta l$ respectively. The wave numbers of incident wave K_m^I and reflected wave K_m^R for each component can be determined by the following modified dispersion equation:

$$\left(\frac{2\pi m}{T_t} - k_m^I U\right)^2 = gk_m^I \tanh(k_m^I h) \quad m=1,2,\dots \quad (27)$$

$$\left(\frac{2\pi m}{T_t} + k_m^R U\right)^2 = gk_m^R \tanh(k_m^R h) \quad m=1,2,\dots \quad (28)$$

where T_t is total sampling time. The transmitted wave height a_T can be directly obtained by analyzing the numerical results behind the submerged plate. Thus, the reflection coefficient K_R and transmission coefficient K_T can be calculated by

$$K_R = \frac{\sqrt{\sum_{m=1}^M a_R^2(m)}}{\sqrt{\sum_{m=1}^M a_I^2(m)}}, \quad K_T = \frac{\sqrt{\sum_{m=1}^M a_T^2(m)}}{\sqrt{\sum_{m=1}^M a_I^2(m)}} \quad (29)$$

The spectra of incident, reflected and transmitted focused waves, $S_I(f)$, $S_R(f)$ and $S_T(f)$ can be determined by

$$S_I(f_m) = \sum_{j=-p}^p 0.5H(j)a_I^2(m-j)/\Delta f \quad (30)$$

$$S_R(f_m) = \sum_{j=-p}^p 0.5H(j)a_R^2(m-j)/\Delta f \quad (31)$$

$$S_T(f_m) = \sum_{j=-p}^p 0.5H(j)a_T^2(m-j)/\Delta f \quad (32)$$

where $H(j)$ is the smooth function, and the rectangular window function is selected as smooth function in this paper. Δf is the size of the step between two frequencies.

3. Experiments

In order to validate the numerical solutions developed in the preceding sections, we conducted corresponding physical experiments in the two dimensional wave flume (40-m long, 0.8-m wide, and 1.2 m deep) located at Jiangsu University of Science and Technology. The glass-walled wave flume was equipped with a piston type wave maker capable of generating incident waves, and the downstream end of the wave flume was installed with a 4-m length region of foam acting as a wave absorber. Before the experiment, the flow quality in the flume was examined and current velocity was measured using an acoustic Doppler velocimeter (ADV). Fig. 2 shows the current velocity variations along both the x direction and the z direction. The current velocity fluctuates around the mean value and the bias error is within 5% of the mean velocity. Thus, the velocity measured by the ADV is the x -component of the current velocity, and it is steady and uniform. In order to measure accurately the evolution of focused waves, 7 capacitance wave gauges having an accuracy of 0.5 mm were arranged at the centerline of the flume. The crest/trough focused point was at the position of wave gauge #4 (10.0 m away from the wave-maker) by modulating the initial wave phases in the experiment. Fig. 3 shows the test set-up, and Table 1 lists the locations of wave gauges, relative to the focused position, i.e. wave gauge #4. The submerged horizontal plate was made of acrylic plate, and its center was located at below the focused point. The submergence depths of the plate were $h_s=0.1$ m, 0.15 m and 0.2 m and the water depth was $h=0.4$ m. The length, width, and thickness of the plate were 0.8 m, 0.8 m and 0.01 m, respectively. The plate was mounted at the desired submergence depth by four vertical steel frames clamped to the flume bottom, and eighteen force gauges were evenly attached to the plate surface to measure the vertical wave loads on the plate, as indicated in Fig. 3. In the tests, all the pressure data from these gauges were simultaneously recorded by a computer control system for 82 s at a time interval of 0.02s (total 4096 data). The time histories of the wave forces were then obtained by integrating the pressure distributions along the model.

Fig. 2. Current velocity distribution on a vertical cut through the center of the flume for incoming velocity of 0.2 m/s: (a) current velocity variations 4 and 70 mm below the free surface along the x direction and (b) current velocity variations along the z direction

Table 1 Distances of all other gauges from #4 ($d_{i4}=x_i-x_4$)

d_{14}	d_{24}	d_{34}	d_{54}	d_{64}	d_{74}
-3.1 m	-2.9 m	-0.4 m	0.4 m	2.9 m	3.1 m

Fig. 3. Experimental set-up

Table 2 Input focused waves characteristics

Case	Frequency band f (Hz)	Input amplitude A_I (m)	Current velocity U (m/s)
1	$0.75 \leq f \leq 1.25$	0.04	0.0
2	$0.75 \leq f \leq 1.25$	0.04	0.1
3	$0.75 \leq f \leq 1.25$	0.04	-0.1
4	$0.75 \leq f \leq 1.25$	0.04	0.2
5	$0.75 \leq f \leq 1.25$	0.04	-0.2
6	$0.65 \leq f \leq 1.35$	0.04	0.0
7	$0.65 \leq f \leq 1.35$	0.04	0.1
8	$0.65 \leq f \leq 1.35$	0.04	-0.1
9	$0.65 \leq f \leq 1.35$	0.04	0.2
10	$0.65 \leq f \leq 1.35$	0.04	-0.2
11	$0.55 \leq f \leq 1.45$	0.04	0.0
12	$0.55 \leq f \leq 1.45$	0.04	0.1
13	$0.55 \leq f \leq 1.45$	0.04	-0.1
14	$0.55 \leq f \leq 1.45$	0.04	0.2
15	$0.55 \leq f \leq 1.45$	0.04	-0.2
16	$0.75 \leq f \leq 1.25$	0.03	0.0
17	$0.75 \leq f \leq 1.25$	0.03	0.1
18	$0.75 \leq f \leq 1.25$	0.03	-0.1
19	$0.75 \leq f \leq 1.25$	0.05	0.0
20	$0.75 \leq f \leq 1.25$	0.05	0.1
21	$0.75 \leq f \leq 1.25$	0.05	-0.1
22	$0.75 \leq f \leq 1.25$	0.06	0.0
23	$0.75 \leq f \leq 1.25$	0.06	0.1
24	$0.75 \leq f \leq 1.25$	0.06	-0.1

25	$0.75 \leq f \leq 1.25$	0.07	0.0
26	$0.75 \leq f \leq 1.25$	0.07	0.1
27	$0.75 \leq f \leq 1.25$	0.07	-0.1

Varying the input amplitude, frequency bandwidth and current velocity produced a total of 50 experimental pairs of crest/trough focused waves. Twenty-seven pairs spanning the wave and current parameter range were chosen for detailed analysis in the present study, and their details were listed in Table. 2. Owing to nonlinearity during the wave propagation, the wave focused time t_0 and focused position x_0 were unknown, and the focused amplitude was larger than input assumed amplitude. In order to simplify the experimental procedure, the input assumed amplitude and focused position were adjusted to make the maximum crest/trough of the focused waves occur at the focused position indicated by wave gauge #4 and equal to the desired amplitude. The component wave amplitude a_i can be determined by a specified constant-steepness spectrum, which can be expressed as

$$a_i = \frac{A_I}{k_i^0 \sum_{j=1}^{N_f} 1/k_j^0} \quad (33)$$

where A_I denotes the total input wave amplitude. k_i^0 is the component wave number without accounting the current effects. Fig. 4 shows the amplitude distribution of each wave component for Case1, Case 6, Case 11, Case 16, Case 19 and Case 25 indicated in Table 2. It can be seen from these figures that the characteristics of a specified wave with focused position x_0 and focused time t_0 are mainly determined by the input amplitude A_I and the frequency bandwidth Δf . In each focused wave test run, 4096 data points were acquired at a sample rate of 50 Hz for determining reflection and transmission coefficients. The reflection coefficient and transmission coefficient can also be obtained by the relevant methods introduced in Section 2.

Fig. 4. Constant-steepness input wave amplitude spectrum for Case1, Case 6, Case 11, Case 16, Case 19 and Case 25.

4. Numerical and experimental results

4.1 Comparison with published results

First, the validity and accuracy of the present model for simulating nonlinear focused waves-current interaction are established by comparing our numerical results against the existing numerical and experimental results obtained by Wu and Yao (2004). The input conditions are: the current velocity $U=0, \pm 0.1$ m/s, the water depth $h=0.3$ m, and the number of wave components $N=32$ spaced uniformly over the frequency band [0.69 Hz, 1.47 Hz]. The component wave amplitude a_i is determined by a linear steepness wave spectrum, which can be expressed as:

$$a_i = \frac{k_N^0 - k_i^0}{k_i^0 (k_N^0 - k_1^0)} G \quad (34)$$

where G denotes a gain factor, achieving a linear slope ($a_i k_i^n$) spectrum. Here, three different values $G=0.0145$, 0.017 and 0.0125 are selected for without current ($U=0.0$ m/s), following current ($U=0.1$ m/s) and opposing current ($U=-0.1$ m/s), respectively. In the numerical simulations, the focus position is chosen at $x_0=4.8$ m, and the focus time $t_0=11.35$ s. The corresponding computational domain is $8\lambda_{max}$ long (where λ_{max} represents maximum wavelength in incident wave components), and two damping layers of $1.5\lambda_{max}$ are defined at the ends of NWT. Based on the convergent numerical test, the time step is taken as $T_{min}/50$, the mesh size defined as $\Delta x=\lambda_{min}/15$ in x direction and $\Delta z=h/10$ in z direction, wherein T_{min} and λ_{min} denote the minimum period and wavelength of wave components. Fig. 5 depicts the predicted time series of surface displacement at different positions in zero, following and opposing currents. The good agreement between our numerical solutions with the experimental data obtained by [Wu and Yao et al. \(2004\)](#) validates and confirms the capability of the present formulation and numerical method in resolving the nonlinear focused waves-current interaction. In addition, it can be seen from these figures that the actual focal location and time occurred around $x_0=5.8$ m and $t_0=11.94$ s, downstream of the theoretical focusing point. This is associated with the nonlinearity of wave propagation.

Fig. 5. Comparisons of calculated free-surface wave elevations predicted by the present model (solid lines) and experimental data of Wu and Yao (marks) at different points. (a) without current ($U=0.0$ m/s), (b) following current ($U=0.1$ m/s) and (c) opposing current ($U=-0.1$ m/s).

To further validate the present model, the developed numerical wave tank (NWT) is then applied to simulate regular wave propagating over a submerged plate in the absence of current. In this case, the incident wave amplitude is $A_f=0.01$ m, the still water depth is $h=0.3$ m, the current velocity is $U=0.0$ m/s, the plate length is $B=0.6$ m, the plate thickness is $W=0.01$ m, and the plate is placed at the depth of $h_s=0.15$ m. The length of NWT is defined as 16 times the wave length (16λ), time step defined as $T/70$, meshed with 240×20 cells on free-surface and ends of NWT, and

the optimal mesh size on the plate surface as $\Delta x=B/50$ and $\Delta z=W/2$ after numerical convergent tests. Fig. 6 gives the time series of simulated free surface elevation at the upstream point (11.58, 0) and downstream point (18.73, 0) of the plate. The comparison with the published experimental results from Liu et al. (2009) is also presented. The good agreements between the numerical simulations and experimental data suggest that the present numerical model can correctly capture the main features of wave scattering by the submerged plate, especially for the reflections which comprise the reflections from both front part and the rear edge of the plate.

Fig. 6. Comparisons of calculated free-surface wave elevations predicted by the present model (solid lines) and experimental data of Liu et al. (marks) at $T=1.1$ s, $h=0.3$ m, $h_s=0.15$ m and $A_f=0.01$ m. (a) a probe position of 3.35 m from the front of the plate, (b) a probe position of 3.2 m from the rear of plate

4.2 Validation with experimental results

In this section, the simulated focused waves-current-submerged plate interaction are compared with those obtained from our experiments. The length of NWT is defined as 20 m, of which $1.5\lambda_{max}$ is used as damping layers at the two ends of NWT, and the major details of waves, current and plate are described in section 3. The optimal grid size on plate surfaces is determined as $\Delta x=B/80$ and $\Delta z=W/2$, while the free-surface, the input boundary and the output boundary are meshed with $\Delta x=\lambda_{min}/15$ and $\Delta z=h/10$. Fig. 7a-b gives the surface time history series at fore-end of the plate (wave gauge #3), focused position (wave gauge #4), back-end of the plate (wave gauge #5) resulting from different current velocities (Case 1, Case 2 and Case 3), respectively with submergence depth $h_s=0.1$ m. In order to make all wave components spatially and temporally focus at the specified position x_0 and time t_0 , it should be to satisfy the relationship $x_0/t_0 \geq \lambda_{min}/2T_{min}$. The x-axis is the difference between time t and focused time t_0 for comparative purposes. In all the cases, there is a reasonable agreement between the numerical and experimental results. The slight discrepancy between the predicted and measured wave crest elevation, especially for the position of wave gauge #4 and wave gauge #5, can be attributed to the possibly un-captured physics (such as flow separation at the edges of the plate) and inherent viscous effects. Both in the prediction and measurement, it can be seen that the surface elevations at the weather side of the plate are still smooth and periodic, but they are quite distorted at the mid-position and the lee side. This means that a number of higher-harmonic waves are introduced into the flow when focused waves propagate along the submerged plate. In addition, the largest crests at mid-position and back-end of the plate are very near to breaking because of the significant nonlinear wave features in the experiment. Thus, the present numerical method can be applied to accurately simulate the evolution processes of focused waves. It is noted that the largest wave crest elevation for an opposing current is apparently smaller than that for zero current, while the peak value for a following current is slimly larger than that for zero current. This interesting

phenomenon can be explained from the energy transformation between focused waves and current. when incident waves are against a current, some higher-order wave components are truncated to result in a reduced the wave crest elevation, and vice versa for a following current. The peak values for different currents also correspond to the different times. That is, owing to waves-current interactions, the waves at the front edge of plate (the point upstream from focused position) in the presence of a following current run slower than that for zero current, and vice versa for an opposing current. However, the opposite is true at the trailing edge of the plate (the point downstream from focused position). This is consistent with the numerical results given by Wu and Yao et al. (2004).

Fig. 8 shows snapshots of the wave profiles at $t=12.64$ s, 16.26 s and 19.88 s for Case 1, Case2 and Case 3. The submergence of the plate is $h_s=0.1$ m. It can be found from these figures that the numerically simulated results correlate well with the measured values, and the outgoing wave energy is completely absorbed in the damping layer at the downstream end of the NWT, indicating the effectiveness of the damping zone. When the incoming focused waves propagate over the plate, besides wave diffraction, higher-harmonic waves are generated and prominent at the back-end of the plate, whose wavelengths are much shorter than that of incident waves. Furthermore, the highest crest increases in the presence of a following and decreases in the presence of an opposing current, similar to the characteristics shown in Fig. 7.

Fig. 7. Comparisons of numerical and experimental surface history series at different position for the following, opposing, and zero currents with $A_f=0.04$ m, $h_s=0.1$ m and $f=0.5$ Hz. (a) wave gauge #3, (b) wave gauge #4 and (c) wave gauge #5

Fig. 8. Comparisons of numerical and experimental temporal profile at different times for the following, opposing, and zero currents with $A_I=0.04$ m, $h_s=0.1$ m and $f=0.5$ Hz. (a) time $t=12.66$ s (b) time $t=16.26$ s and (c) time $t=19.88$ s

4.3 Reflection and transmission coefficients

We simulated and measured the surface elevations at #1, #2 and #6 and analyzed this data in terms of the effects of the design parameters, such as current velocity U , incident wave amplitude A_I , frequency bandwidth Δf , submerged depth h_s and the plate width B on reflection/transmission coefficients. The relationship between the reflection/transmission coefficients and various parameters is shown in Figs. 9-12. In these figures, the reflection/transmission coefficients are calculated using the Eqs. (26)-(29).

Fig. 9 presents the variation of the reflection and transmission coefficients with current velocity U for different values of Δf at $A_I=0.04$ m, $B=0.8$ m, $h_s=0.1$ m. The figure reveals that the reflection coefficients monotonically increase as the current velocity increases. The reason may be that the a following current can absorb the energy of focused waves to reduce the wave amplitude, and the reflected waves will be increased by the same current, resulting in an increased reflection coefficient. However, the transmission coefficient reaches its minimum around $U=-0.1$ m/s and then increases to a larger value with increasing current velocity. This implies that the wave-blocking performance of the submerged plate can be enhanced with decreasing current velocity but an strong opposing current can worsen the effectiveness in reducing wave height.

Comparing the reflection and transmission coefficients with varying frequency bandwidths Δf , it can be found both the predicted and measured values that the wave group with the larger frequency bandwidths has a larger reflection coefficient and a smaller transmission coefficient. The reflection coefficient given by the presented numerical model correlates well with the measured values, and the transmission coefficient is slightly over-predicted. This because the viscous effects at the back-end of the plate cannot be reproduced by the present nonlinear potential

theory. Thus, the irrotational analysis only leads the overestimation of the transmission coefficient but has little effects on the flow at upstream from the plate compared with the measured values, similar to that of the regular waves obtained by Lin et al. (2014). Moreover, the variation trend of both reflection and transmission coefficients with current velocity can be captured well by the fully nonlinear NWT.

Fig. 9. Comparisons of the (a) reflection and (b) transmission coefficients of the focused waves predicted by the present model with experimental data for varying values of Δf , with $A_I=0.04$ m, $B=0.8$ m, $h_s=0.1$ m

For extreme waves, the wave amplitude has a definite nonlinear effect on the reflected and transmitted wave characteristics. Fig. 10 shows the variations of the reflection and transmission coefficients with the incident wave amplitude A_I for varying values of current velocity U when $\Delta f=0.5$ Hz, $B=0.8$ and $h_s=0.1$ m. In this particular case, the reflection and transmission coefficients increase in the presence of a following current and decrease in the presence of an opposing current, which is similar to the conclusion of Fig. 9. Overall, the reflected waves for different currents and the transmitted waves for zero current are not very sensitive to the change of incident wave amplitude. However, the transmission coefficients decrease approximately linearly in the presence of a uniform current as the wave amplitude increases, which may be because the waves-current-plate nonlinear interaction become strong and the higher-harmonic components become significant by the large wave amplitude. These higher-harmonic components have shorter wavelength, and thus the wave-blocking performance of the plate is more efficient. In these figures, the reflection coefficient can be predicted well by the numerical model, and the transmission coefficient is slightly overestimated by the numerical model for the same reasons stated previously.

Fig. 10. Variations of the (a) reflection and (b) transmission coefficients of the focused waves versus the incident wave amplitude A_I for varying values of current velocity U , with $\Delta f=0.5$ Hz, $B=0.8$ m, $h_s=0.1$ m

Fig. 11. Comparisons of the (a) reflection and (b) transmission coefficients of the focused waves predicted by the present model with experimental data for varying values of h_s , with $A_f=0.04$ m, $\Delta f=0.5$ Hz, $B=0.8$ m

All the curves of Figs.9-10 are given for a fixed immersion depth of plate. To further validate the numerical model and examine the effects of the position of the submerged plate on the reflection and transmission coefficients of focused waves in the presence of a uniform current, deeper submergence depth experiments of $h_s=0.15$ m and $h_s=0.20$ m are also conducted and compared with the counterpart of immersion depth $h_s=0.1$ m. For this experiment, $A_f=0.04$ m, $\Delta f=0.5$ Hz and $B=0.8$ m are selected for given parameters. Both numerical solutions and experimental results are shown in Fig. 11 and they are generally in good agreement. It is seen both in theory and experiment that the reflection coefficient decreases with increasing submerged depth, and the opposite is true for transmission coefficient. Therefore, to be an effective breakwater device, the horizontal plate should be located closed to the free surface. On the other hand, the total reflection and transmission coefficients increase in the presence of a following current and decrease in the presence of an opposing current, which is due to energy transfer between the current and focused waves.

The effects of the plate width, B , on the reflection coefficient R and the transmission coefficient T are calculated and shown in Fig. 12 for the following fixed parameters: $A_f=0.04$ m, $\Delta f=0.5$ Hz and $h_s=0.1$ m. In the figure, the reflection coefficient R (the transmission coefficient T) reaches its maximum (minimum) around $B=0.6$ m and then decreases (increases) monotonically with the increase of B . This is because the total reflection commonly consists of the flow generated from the fore-end of the plate, the back-end of the plate, and below the plate. When the reflections from the two ends of the plate are in phase (i.e. the phase difference between two ends is 0°), the focused waves are at resonance and the interaction is significant. Thus, the reflections (transmissions) achieve its maximum (minimum) at the resonant point. In addition, the reflection and transmission for different currents exhibit the same trend, but the corresponding magnitude is larger for a following current than that for zero current and vice versa for an opposing current. The position of the maximum and minimum is almost same with currents. This indicates that the current has a weak influence on the occurring condition of the peak or trough value but change the magnitude of the reflection and transmission. Taking a following current as an example, the incident wavelength is increased according to the modified dispersion relation (Eq. (14)), while the reflected wavelength is reduced by the Doppler effect of current. Accordingly, the phase differences at the front edge are identical among the cases of an following current, zero current and an opposing current.

Fig. 12. Computed (a) reflection and (b) transmission coefficients of the focused waves predicted versus the plate width B for varying values of current velocity U , with $A_f=0.04$ m, $\Delta f=0.5$ Hz, $h_s=0.1$ m

4.4 Spectra

Further features of the non-linearity of the transmitted waves are now discussed. Fig. 13 depicts the comparisons of experimental and numerical wave frequency spectra, which are given by using Fourier transforms (FFTs) of the wave-surface elevation time histories at wave gauge # 6 with $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz, 0.7 Hz and 0.9 Hz. It can be seen that the contributions of representing different harmonic nonlinear wave-surface displacements include the low frequency term (2nd-order difference term), fundamental frequency term (wave frequency term), second-harmonic term (double frequency 2nd order sum term) and small super-harmonic term. In addition, the measured fundamental frequency components are smaller than that of numerical, while second-harmonic terms are almost identical between the experimental and numerical tests although the potential theory neglects viscous effects. This suggests that energy dissipation occurs primarily in the fundamental waves when focused waves propagate along a submerged plate. These figures also reveal that the presence of a following current can worsen the wave-blocking performance compared with the case of zero current and vice versa for an opposing current. The total wave-surface displacement decreases with the increase of frequency bandwidth, and proportion of the second-harmonic term becomes greater. The reason may be that the wave group has more higher harmonic components when the frequency bandwidths become larger. That will reduce the transmitted wave height owing to reduction of effective wavelength in the range of wide frequency bandwidth.

Fig. 13. Comparisons of experimental and numerical wave-surface elevation spectra at #6 with different current velocity for $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and (a) $\Delta f=0.5$ Hz, (b) $\Delta f=0.7$ Hz, (c) $\Delta f=0.9$ Hz

Figs. 14-17 present the spatial spectral evolution process of focused waves propagating over a submerged horizontal plate at current velocity $U=0$, -0.1 m/s, 0.1 m/s and -0.2 m/s, respectively. The following fixed parameters: $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz, are selected. Both experimental data and numerical solutions show that the energy is gradually transferred from the first harmonic mode to the higher harmonic modes owing to the wave non-linearity and the nonlinear interaction between waves and plate. Furthermore, a following current can enhance this energy exchange and vice versa for an opposing current. During the focusing and interacting process, the wave energy is broken up to the higher and lower harmonic modes. However, the third and small fourth harmonic components disappear afterwards. This can be explained by the change of water depth at front edge and trailing edge of the plate. When the focused waves arrive at the front edge of the plate, the water depth abruptly decreases, and the wave non-linearity increases, resulting much higher harmonic waves. With the waves propagating along the plate, the wave energy is redistributed due to the interaction between the fundamental waves and the higher harmonic waves. After the waves propagating over the plate, the water depth suddenly increases, and locked waves can be released to the corresponding free waves to a large extent. It is also interesting that the presence of a strong opposing current as shown in Fig. 17 further reduces the wave energy distribution, and leads to smaller energy exchange between the fundamental wave and higher harmonics. This example typically illustrates that the energy exchange in the presence of a strong opposing current seems to be reversible, and the wave non-linearity decreases after waves propagating over the plate.

Fig. 14. Spatial spectral evolution at different gauges on current velocity $U=0.0$ m/s for $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz

Fig. 15. Spatial spectral evolution at different gauges on current velocity $U=-0.1$ m/s for $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz

Fig. 16. Spatial spectral evolution at different gauges on current velocity $U=0.1$ m/s for $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz

Fig. 17. Spatial spectral evolution at different gauges on current velocity $U=-0.2$ m/s for $A_f=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz

4.5 Wave and current Loads on the plate

The hydrodynamic forces of the focused waves on the submerged plate in the presence of a uniform current are now examined. Fig. 18 numerically depicts, for cases 1-4, the time history series of the total non-dimensional vertical hydrodynamic force $F_z/(\rho g A_0 B)$ and bending moment $M_y/(\rho g A_0 B^2)$ on the plate. Two important and valuable conclusions can be drawn through force analysis. First, the wave forces including the vertical force and bending moment in the presence of a following current are larger than that in the absence of a current and vice versa for an opposing current, similar to the feature shown in Fig. 13. Second, the position of the peak value almost is identical for different currents. However, the number of higher-order variation observed in the time history series of the wave forces increase with a following current and decrease with an opposing current. Thus, a following current sometimes leads the violent motion of the wave surfaces and may lead high-frequency vibrations of submerged structures. These issues can significantly

influence the design with the submerged plate and specially concern with the 'ringing' or 'secondary loading cycle' phenomenon.

Fig. 18. The total force elevations on current velocity $U=0$ and ± 0.1 m/s for $A_I=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz: (a) vertical wave force (b) bending moment

Fig. 19. The envelopes of pressure of (a) lower and (b) upper surfaces of the plate on current velocity $U=0$ and ± 0.1 m/s for $A_I=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz

Fig. 19 shows the non-dimensional pressure crest distribution ($P/(\rho g A_0)$) of lower and upper surfaces along the plate length $B=0.8$ m, with $A_I=0.04$ m, $B=0.8$ m, $h_s=0.1$ m and $\Delta f=0.5$ Hz. In each case, the fully nonlinear numerical solutions are compared with the experimental data. It can be seen that the experimental data can be reproduced well by the developed theory for various current velocity. As is confirmed in our above study, the pressure distribution increases in the presence of a following current and reduces in the presence of an opposing current. In addition, the pressure amplitude of upper surface of the plate is larger than that of lower surface of the plate. This reason may be that the nonlinear standing waves are induced by the reflections from front and rear edge of upper surface of the plate, and thus the wave motion obviously increases. However, the strong pulsating flow opposite to the wave propagation direction at the lower surface of the plate is generated due to the impermeable boundary condition of the plate surface, which causes the pressure value decreases.

Fig. 20 shows the changes in the total calculated vertical hydrodynamic force and bending moment versus current velocity for different frequency bandwidths when $A_I=0.04$ m, $B=0.8$ m and $h_s=0.1$ m. The force and bending moment are dealt with non-dimensional method removing the effect of incident wave amplitude A_I . It can be seen that the vertical wave force reaches its maximum and then decreases to a smaller value with the increase of current velocity U , and the position of its peak value varies with bandwidth. The amplitude of bending moment increase as frequency bandwidth decreases. Moreover, the current has a stronger impact on the bending moment for smaller bandwidth. Actually, the effect of the current on wave forces is mainly caused by the following three mechanisms: First, the wave surface elevation is changed by the current.

Second, the wave frequency is also influenced from the modified dispersion Eq. (14). Finally, the contribution of the term $U \frac{\partial \phi}{\partial x}$ in Eq. (20) should be considered. The bending moment is interestingly increased by further reducing the current velocity to -0.2m/s and increasing the frequency bandwidth to 0.9 Hz. This indicates that the current effects on the wave forces are more significant than on the wave surface elevation. In addition, the maximum wave force/bending moment is larger at a smaller frequency bandwidth, which may be due to the stronger shoaling effect.

Fig. 20. Variations of (a) vertical wave force and (b) bending moment on the plate versus current velocity for different frequency bandwidths with $A_f=0.04$ m, $B=0.8$ m and $h_s=0.1$ m

5. Conclusions

The present paper describes an investigation of the nonlinear transformation and scattering of focused waves over a submerged horizontal plate in the presence of a uniform current. A time-domain fully nonlinear NWT is established using HOBEM and devoted to numerical solutions in the context of two-dimensional potential theory and mixed Eulerian-Lagrangian technique. The focused waves are generated through the superposition of different wave frequency components at a specific time and position, and the current effects are realized by introducing the current velocity to the free surface boundary conditions. A robust and stable acceleration potential approach is applied to obtain the hydrodynamic forces on the plate, and the reflection/transmission coefficients are determined using the modified Goda's method. The numerical results obtained by the present nonlinear model are found to agree well with the published numerical results and our experimental tests, which have been conducted in a two-dimensional glass-walled wave tank. In particular, the numerical model provides a good description of the higher-harmonic wave components induced by the strong shoaling effect.

The results indicate that the reflection coefficient increases as the current velocity and frequency bandwidth increase, whereas the transmission coefficient is enhanced by a strong opposing current and reduced with increasing the frequency bandwidth. The reflection coefficient for different currents has no significant change as increasing the incident wave amplitude. The larger wave amplitude leads to smaller transmission coefficient in the presence of a uniform current, which occurs because, for a fixed submergence depth of the plate, the larger incident wave amplitude in the current increases the energy exchange from the fundamental waves to the higher harmonics. The shallower horizontal plate more effectively interact with focused waves to lessen wave transmission. The resonance can be obtained by varying the plate width for different currents, when the reflections from the two ends of the plate are in phase. The occurring condition of the maximum reflection or the minimum transmission is not sensitive to the presence of current,

because the compensating Doppler effects of current results in the same phase difference among zero current, following current and opposing current.

Through the spectral analysis of numerical and experimental data, the viscous effects induced by the submerged plate are the major contributor to the fundamental waves, and thus the present numerical tool based on potential theory can predict accurately the higher harmonic wave components. In addition, the proportion of second harmonics in total wave energy increases with the frequency bandwidth. The energy exchange can be intensified by a following current, which is the opposite to that for an opposing current. The focused and de-focused process of extreme waves propagating along the plate are reversible for a strong opposing current owing to the truncation of higher harmonics.

The 'ringing' phenomenon characterized by a secondary loading cycle can be observed in the following current case, and significantly affects the plate design and fatigue issue. Generally, the pressure value of upper surface of the plate is larger than that of lower surface of the plate. This reason is that the nonlinear standing waves can be excited on the upper surface, and the strong pulsating flow opposite to the wave propagation direction can be generated on the lower surface. Furthermore, current effects on wave forces are stronger than on the wave surface elevation, and the maximum wave force decreases with the frequency bandwidth.

The current effects on higher harmonic wave forces on the plate are complicated, and the mechanism of the secondary loading cycle with different frequency bandwidth remains unclear. The total vertical wave force and bending moment is correlated with the plate width and the frequency bandwidth, and more work will be done to provide insights into the secondary loading cycle.

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