“High runs” of a ship in multi-chromatic seas

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Abstract

When a ship operates in steep irregular waves, it can attain, in intermittent time intervals, abnormally high speed, due to waves' effect. Such events may occur in following seas and they will be called hereafter “high runs”. Investigations have been carried out for this peculiar type of ship motion, in three directions: firstly, the statistics of high runs are calculated by numerical simulation, exploring, in particular, their dependence on the wave spectrum and the sea state. Secondly, a rather neglected up to now analytical method, proposed in the '60s by Grim, for the quantification of the probability of high run occurrence, is assessed, against a direct numerical approach. Finally, the actual velocity of a ship in high run incidents is investigated. Suitable metrics are applied on the difference between characteristic velocity values associated with wave's form, and surge velocity, during high runs.

Keywords: *ship, nonlinear, motion, surging, surf-riding, high run*

1. Introduction

In an irregular sea, a realisation of surf-riding is perceived as a prolonged high speed run of the ship, occurring in a time interval when the waves, in the vicinity of the ship, maintain sufficient steepness to enable the phenomenon. A statistical approach for calculating the probability of surf-riding of a ship through numerical modelling and simulation, in reference to an environment of extreme irregular waves, could be then based on the direct counting of those special time intervals in which her speed is maintained consistently above the normally expected range of speed fluctuation in waves. Any individual realisation of such behaviour will be called hereafter “a high run”. It can be considered as the generalisation of surf-riding, for a multi-frequency wave environment, including also transitory and unclassified phenomena which, from a dynamics perspective, may not be exactly surf-riding. Whilst the mechanism of inception of such phenomena requires deep consideration of system’s phase-space, empirically, it can be recognised by the up-crossing of an appropriate surge velocity threshold, such as *the instantaneous wave celerity*. The concept of instantaneous wave celerity, and its phenomenological connection with high run events in irregular seas, has been discussed in Spyrou et al 2014a.

A high-run’s temporary end may be, similarly, defined by the down-crossing of a suitable threshold. For this however, as will be discussed later, there is no clear consensus on its definition.

Literature on the topic is scarce. However, in a pioneering (but rather oversighted) work, Grim had attempted to determine how a ship could be accelerated by waves and then maintain a speed higher than her mean speed, for extended time intervals, in irregular seas (Grim 1963). He had called such phenomena “long-runs”. By a string of eloquent, and yet quite plain, analytical approximations, he had produced statistical estimates of their existence and duration, based on up-crossings of a speed level that he had considered as critical.

The term “high-run” is often used in oceanography to indicate the realisation of a sequence of nearly regular high waves (Longuet-Higgins 1984). Use of such a concept has been made in the past for assessing ship stability in irregular stern quartering seas (e.g. Takaishi et al 2000). In the present context, however, the meaning of the term is very different, referring to a ship’s unintended high speed motion and not to wave phenomena. In fact, use of the term “long-run” could be ambiguous here, because sometimes, the addressed extreme motions are not characterised by a velocity plateau as was Grim’s assumption.

The aim of our study was the examination of the probabilistic properties of high run events. It is well-known that, the longer a ship maintains a speed higher than normal, the more likely it is to experience the broaching-to instability (Spyrou 1995). The importance of the topic is thus prevalent. The adopted approach is developed as follows: firstly, a campaign of numerical simulations with direct recording of the high run intervals was performed. Targeted quantities were: a) the mean duration of high run; and b) the mean time between successive high runs. This has produced the statistical dependence of the phenomenon on the characteristics of the wave field. Then, the key elements of Grim’s method were implemented, taking advantage however of the modern numerical calculation capability. Thus, alternative probability figures were derived which were contrasted against those obtained by direct counting. Our final goal was to figure out which characteristic wave velocity definition conforms best with surge velocity during high run incidents. We selected as initial candidates: a) the instantaneous wave celerity at the position of the ship on the wave (this is a quantity that is varied in time-space, hence it should be calculated at every step); and b) the celerity that corresponds to the peak frequency of the spectrum (receiving, thus, a constant value during simulation runs deriving from a certain spectrum). The distance of these celerities from surge velocity during high runs was systematically assessed, on the basis of suitable metrics.

2. High run statistics

2.1 Mathematical model

The mathematical model of surge motion in following seas was written for an earth-fixed observer, as follows:

$$(m - X_{\dot{\xi}})\ddot{\xi} - (\tilde{\tau}_2 \dot{\xi}^2 + \tilde{\tau}_1 n \dot{\xi} + \tilde{\tau}_0 n^2) + (r_1 \dot{\xi} + r_2 \dot{\xi}^2 + r_3 \dot{\xi}^3) - \sum_{i=1}^N F_{xi} \sin(k_i \xi - \omega_i t + \varepsilon_i + \varepsilon_{fi}) = 0 \quad (1)$$

where ξ is the longitudinal position of the ship and m , $X_{\dot{\xi}}$ are her mass and “surge added mass” respectively. In the summation term denoting wave force, k_i , ω_i and ε_i stand respectively for the i harmonic’s wave number, frequency and random phase. F_{xi} denotes the amplitude, and ε_{fi} the phase, of the i harmonic wave force component. Also, n is the propeller rate and r_i , $\tilde{\tau}_i$ are polynomial coefficients appearing in the resistance and thrust force expressions, respectively.

2.2 “High run” definition

An apparent choice of a velocity threshold whose up-crossing would signal a high run, is the instantaneous wave celerity. Yet, it is known that attraction towards surf-riding is very likely to have started from a slightly earlier time (and thus from a lower velocity). If this early stage is neglected, a small underestimation of the probability should be expected. As down-crossing threshold was set, as first step, the nominal speed. This lower threshold is not very likely to be crossed by speed fluctuations occurring during a high-run. The nominal speed is a safe choice from this point of view, although a conservative one, possibly contributing to a slight overestimation of probability. This may be statistically cancelled out, at least partly, with the underestimation linked with the beginning of the high run. As an extra condition, we request, the surge velocity to be always higher than the nominal speed, in order to exclude, for relatively short wave lengths and mild wave height conditions, cases that, qualitatively, should not be counted as high runs. In Figure 1 are shown time segments of high run in accordance to the presented definition. It is desired to obtain the statistics of high runs’ duration, as well as of the time interval between successive

occurrences. The mean duration is obtained by summing up all individual durations; and then dividing by the number of occurrences:

$$\bar{t}_{high\ run} = \frac{\sum_{i=1}^N t_{high\ run}^{(i)}}{N} \quad (2)$$

A similar formula is applied for the mean time between high runs.

2.3 Simulation settings

The ship selected for applying the calculation schemes is an ONR “tumblehome topside”, well-known from several previous studies (for example, Spyrou et al 2014a). A JONSWAP spectrum is considered, discretized by applying a fixed frequency increment $\delta\omega = 2\pi/t_{sim}$ where $t_{sim} = 300$ s is the so-called “basis simulation time”. The total simulation time was set to a selected multiple of the basis time (up to $40 \times t_{sim}$). Four ranges (from narrow to broad) around spectrum’s peak were separately examined, containing the wave frequencies participating in the simulations. In Figure 2 are indicated the wave amplitudes obtained from the spectrum, considering frequency ranges $0.2\omega_p$ and $0.4\omega_p$. A different choice would have been to modify the wave amplitude so that the variance remains constant. In that case the wave amplitudes obtained would be considerably higher (see again Figure 2). In the current study wave realizations were produced according to the first method, meaning that, the increase of the frequency range increased also the energy.

Lastly, in Table 1 appear the values of the remaining simulation parameters. Sensitivity studies, in relation to sea state, narrowness of spectrum and simulation time, were carried out. We run 100 wave realizations per parameters’ setting. The nominal and the initial speed of the ship, in each scenario, were not changed (for extra explanations see Spyrou et al 2014b).

2.4 Results

In Figure 3 appear characteristic high run durations, predicted through simulation. Vast differences are noticed, some high runs lasting just a few seconds and others reaching 1000 seconds! The probability density function (pdf) of mean duration, based on 100 simulations, is shown in Figure 4. The effect of sea condition on mean duration, as also on certain percentiles, appears in Figures 5 and 6 for a very narrow spectrum. Statistical convergence, with respect to the simulation time, is confirmed from Figure 7.

2.4.1 Effect of considered wave frequency range on mean duration of high run

When spectrum's width is narrow, mean times are higher, becoming more pronounced around low peak periods (Figure 8). Furthermore, as spectrum's width is broadened, so does the curve of mean high run duration around its peak value. The trend depends on the assumed significant wave height and it is more pronounced at higher significant wave heights.

2.4.2 Effect of wave frequency range on mean time between high runs

The mean time between successive high runs is increased with the peak period (Figure 9). However, it is reduced as the significant wave height is increased (i.e. they become more frequent), with a very significant drop from a narrow spectrum to a wide one (Figure 10). The broader the spectrum's width, the more frequent the high run occurrence. There seems to be a sharp increase of the mean time between high runs, as soon as a certain value of peak period is exceeded. On the other hand, the significant wave height seems more influential when the frequency range is narrow.

3. Analytical approach and comparison

3.1 Key points

Grim had targeted the probabilistic quantification of the occurrence and duration of high runs (in his words "long runs"), taking into account the strongly nonlinear character of surge motion when the phenomenon occurs, in a following irregular sea (Grim 1963). The lack of computer power for the demanding numerical calculations, together with the lack of a theory for surf-riding, at that time,

had led him to incorporate several simplifying assumptions whose influence was unknown. He had focused on the condition generating unusually high surge acceleration; and on the duration of the ensuing high speed run, which he assumed represented by a velocity plateau. Thus a simple, trapezoidal structure of ship speed was assumed during such incidents. Next is presented a summary of Grim's method.

3.2 Mathematical model set-up

The surge equation is written with respect to an inertial system that moves with the ship's constant nominal speed V . The method assumes that time is paused at the instant t when a high run's acceleration begins. Later time is measured through a new time variable τ :

$$\ddot{x}_0 + N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3) / (m - X_{\dot{u}}) = \int_0^\infty \cos \left[\frac{\omega^2}{g} x_0 - \omega \left(1 - \frac{\omega \cdot V}{g} \right) (t + \tau) + \varepsilon \right] \sqrt{f_x^2(\omega) S(\omega) d\omega} \quad (3)$$

The distance variable x_0 determines ship's position. Function $N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3)$ refers to the resistance and thrust forces, f_x is the RAO of the Froude-Krylov surge wave force [divided by the mass (including added mass) of the ship] and S is the wave spectrum. To maintain consistently with the model of section 2, the term $N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3)$ should take the following form

$$N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3) = [3r_3 V^2 + 2(r_2 - \tilde{\tau}_2)V + r_1 - \tilde{\tau}_1 n] \dot{x}_0 + (3r_3 V + r_2 - \tilde{\tau}_2) \dot{x}_0^2 + r_3 \dot{x}_0^3 + R(V) - T(V; n) \quad (4)$$

where resistance and thrust, at the nominal speed are, respectively:

$$\begin{aligned} R(V) &= r_1 V + r_2 V^2 + r_3 V^3 \\ T(V; n) &= \tilde{\tau}_2 V^2 + \tilde{\tau}_1 V n + \tilde{\tau}_0 n^2 \end{aligned} \quad (5)$$

At the beginning of a high run, the ship is assumed in constant acceleration for a time duration τ_1 .

When the critical velocity V_{crit} (beginning of velocity plateau) is reached, the ship maintains it for a time interval $\tau_2 - \tau_1$ (see eq. 6, below).

$$\dot{x}_0 = \begin{cases} b \cdot \tau, & 0 \leq \tau \leq \tau_1 \\ b \cdot \tau_1, & \tau_1 \leq \tau \leq \tau_2 \end{cases} \quad (6)$$

Thus, the required acceleration for realising the high run should be:

$$b = (V_{crit} - V) / \tau_1. \quad (7)$$

Whilst, for the regular sea, Grim identified celerity as the critical speed, he gave no similar indication for the choice of critical speed in an irregular sea.

The wave force in (3) is considered through its integral for a finite duration τ_1 (“impulse function”).

This leads to the key idea of producing an impulse spectrum. Integration of (3), in time, yields an equation based on momentum:

$$\int_0^{\tau_1} \left[\ddot{x}_0 + \frac{N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3)}{(m - X_u)} \right] d\tau = \int_0^{\tau_1} \int_0^{\infty} \cos \left[\frac{\omega^2}{g} x_0 - \omega \left(1 - \frac{\omega \cdot V}{g} \right) (t + \tau) + \varepsilon \right] \sqrt{f_x^2(\omega) S(\omega)} d\omega d\tau \quad (8)$$

Calculating partly the force integral, leads to the following expression of the impulse (for the detailed formulation see Grim 1963):

$$\int_0^{\infty} \cos \left[\omega \left(1 - \frac{\omega V}{g} \right) t + \varepsilon' \right] \sqrt{f_x^2(\omega) T^2(I^2 + Y^2) S(\omega)} d\omega \quad (9)$$

where:

$$T^2(I^2 + U^2) = \left\| \int_0^{\tau_1} e^{i \left[-\omega \left(1 - \frac{\omega V}{g} \right) \tau + \frac{\omega^2 b}{g} \frac{\tau^2}{2} \right]} d\tau \right\|^2 \quad (10)$$

The term $f_x^2(\omega) T^2(I^2 + Y^2) S(\omega)$ is the sought impulse spectrum, while ε' is another, still random, phase. Since the maximum value of the impulse is of interest, the cosine term is set to 1. The impulse is a random function and Grim assumed that its amplitude follows the Rayleigh distribution. In analogy to the mean wave amplitude, the mean impulse amplitude (or some other percentile) is obtained from the square root of the area under the impulse spectrum, for ω from 0 to ∞ .

$$\bar{I}^{(1/n)} = \alpha_{1/n} \sqrt{\int_0^\infty f_x^2(\omega) T^2(I^2 + Y^2) S(\omega) d\omega} \quad (11)$$

where the coefficient $\alpha_{1/n}$ obtains specific values depending on the average of the impulse highest $1/n$ amplitudes. For example, $\alpha_{1/10} = 1.8$ corresponds to the average of the 1/10 highest amplitudes. Additionally, the probability to exceed this average value can be obtained from the Rayleigh density function (3.92%). So, eq. 7 is transformed to the next equation which is solved for $\alpha_{1/n}$ in order to obtain the probability to reach a critical velocity within time τ_1 :

$$b \cdot \tau_1 + \int_0^{\tau_1} \left[N(\dot{x}_0, \dot{x}_0^2, \dot{x}_0^3) / (m - X_u) \right] d\tau = \bar{I}^{(1/n)} \quad (12)$$

By integrating eq. 3, up to $\tau = \tau_2$, and repeating the above procedure, a statistical estimate of the time duration of high run is produced.

3.3 Application and results

Grim's method has been applied through the next steps:

- The critical velocity is set equal to the celerity of spectrum's peak frequency.
- The probability to exceed the targeted velocity in a given time τ_1 is calculated.
- Assuming that the critical velocity has been reached in time τ_1 , we calculate the probability the high run duration to exceed certain values $\tau_2 - \tau_1$.
- The procedure is repeated by selecting various critical velocity levels, deriving from the nominal speed.

The nominal speed is 12 m/s and the spectrum is JONSWAP with its full frequency range.

3.3.1 Accelerated motion

In Figures 11 and 12 are shown plots of the calculated probability the ship speed to exceed the defined velocity threshold (as such was selected the wave celerity corresponding to the peak frequency). One could regard the time τ_1 as a fraction of the apparent wave period, i.e. it is

comparable to the time, during an encounter wave cycle, when the ship is pushed forward by the wave. Given the selected nominal speed and range of spectrum peak periods, τ_1 value should take values in the range 18-26 s. To consider the dependence of probability value on the height of the velocity threshold, in Figure 13 several alternative velocity thresholds have been set. It is remarked that, the threshold $1.3V_{nom}$ corresponds to the wave celerity of $T_p = 10$ s.

3.3.2 Duration of high run

The statistics of high run duration depends on the time τ_1 (see Figure 14). It appears that, the sooner the threshold is reached the longer the high run will last. However, according to Figure 11, the probability of a velocity-threshold-crossing becomes significant once $\tau_1 > 20$ s. We examine, further, the effect of peak period and significant wave height on high run duration, with the condition that the threshold has been reached (figures 15 and 16 were, hence, drawn for $\tau_1 = 20$ s). By increasing the peak period, the high run occurs less frequently. One notes in Figure 16 the substantial decrease of the duration, for lower significant wave heights. Also, from Figure 17 it is recovered that, setting a higher velocity threshold yields a significant reduction of probability.

3.4 Comparison with direct counting

We contrasted the statistics of high run duration, obtained with the method described in section 2, against similar results based on Grim's approach described in the present section (Figure 18). Grim's method yields higher probability values for the longer high runs, in comparison to the result obtained by direct counting. This could be verified further by the fact that, the mean duration derived from Grim's method (150 s) is approximately equal with the mean obtained for the 80th percentile of the simulation data. A qualitatively similar tendency was noticed also in other sea states.

In interpreting any discrepancies between the results of the two methods, one should take into account their main differences: firstly, Grim's method assumes a constant (and equal to the targeted

threshold) surge velocity during a, as he had called it, “long run” (possibly inspired by the regular wave case when surf-riding occurs). Nonetheless, we have observed fluctuations (sometimes strong) in high runs. Furthermore, the velocity thresholds that bound the high run in the two methods, are different. In the direct counting method, it is specified by the instantaneous wave celerity and the nominal speed while on Grim’s approach the limit threshold is constant and equal to the celerity corresponding to the peak wave frequency.

4. Correlation of celerity and mean speed in high runs

4.1 Objectives of the study and selection of metrics

The aim of this final study is to examine whether one can objectively establish that, the instantaneous wave celerity truly dictates the mean surge velocity during high run events. Two methods of calculation of the instantaneous wave celerity $c(t)$ in irregular waves were discussed earlier (see Spyrou et al. 2014c). The one used here accrues from the concept of instantaneous frequency. The mean speed $\bar{U}(t)$ is based on several speed values sampled between successive speed maxima and minima. Details of the calculation procedure are found in Spyrou and Themelis (2013). To evaluate the difference between the two time-varying processes of interest (the instantaneous celerity and the mean speed), two error metrics commonly employed in studies addressing discrepancies of time histories are used (Sarin et. al. 2010). The first metric is the well-known Euclidean vector norm:

$$L_2 = \|\mathbf{c} - \bar{\mathbf{U}}\| = \left(\sum_{i=1}^N |c_i - \bar{U}_i|^2 \right)^{1/2} \quad (13)$$

Where $\mathbf{c}, \bar{\mathbf{U}}$ are the discretised time histories (vectors of equal dimension N) of instantaneous celerity and mean speed, respectively. It should be noted that the Euclidean vector norm cannot distinguish an error due to phase difference from an error due to magnitude.

The second error metric has been proposed by Sprague and Geers (2004). It combines the error M due to magnitude differences (eq. 14) and the error P due to those of phase (eq.15):

$$M = \sqrt{\frac{\psi_{AA}}{\psi_{BB}}} - 1 \quad (14)$$

$$P = \frac{1}{\pi} \cos^{-1} \left(\frac{\psi_{AB}}{\sqrt{\psi_{AA} \cdot \psi_{BB}}} \right) \quad (15)$$

where

$$\psi_{AA} = \frac{\sum_{i=1}^N c_i^2}{N}, \quad \psi_{BB} = \frac{\sum_{i=1}^N \bar{U}_i^2}{N} \quad \text{and} \quad \psi_{AB} = \frac{\sum_{i=1}^N c_i \bar{U}_i}{N} \quad (16)$$

The combined error is:

$$C_{er} = \sqrt{M^2 + P^2} \quad (17)$$

These two error metrics are applied not only for the instantaneous celerity versus the mean speed, but also for the celerity corresponding to the peak frequency versus the mean speed, because the latter is also a strong candidate for representative velocity of surf-riding.

4.2 Simulation settings

The “tumblehome” vessel is assumed operating at nominal speed 14 m/s. Ranges of wave frequency with gradually increasing width are tested (based on a JONSWAP spectrum), in order to study the effect of a transition from a “narrow” to a “broad-band” spectrum.

Per frequency range, 10 realisations are generated. The significant wave height and the peak period are 6 m and 10 s, respectively. The total simulation time is 5000 s; however the first 2000 s of each run are excluded from further processing. The time step is 1 s.

4.3 Results

Error mean values according to the two applied metrics are obtained. To ensure that the comparison addresses, exclusively, time segments of true high run occurrence, we introduced a velocity

condition entailing, in order to be activated the error calculation, the mean surge velocity to be greater than the nominal speed ($\bar{U}(t) > U_{nom}$) (we call this as “1st velocity condition”). We tested also a slightly modified version of this condition: $\bar{U}(t) > 1.1 U_{nom}$ (“2nd velocity condition”). Since the instantaneous celerity has, often, a fluctuating character, we calculate also the error between the mean of the surge velocity and the mean of the instantaneous celerity (calculated also, from averaging of several values between maxima).

Typical time histories on which the two metrics are applied are shown in Figure 19. Errors according to the Euclidean metric are seen in Figures 20 and 21, for the first and the second velocity condition, respectively. Moreover, Figures 22 and 23 indicate the error values according to the Sprague and Speers metric. The results based on the Euclidean metric suggest that, the discrepancy of mean speed from the celerity of peak frequency is consistently less than that of mean speed from instantaneous celerity. The same conclusion is drawn when the Sprague-Geers metric is used, if the “1st velocity condition”, introduced at the beginning of this sub-section, is applied. This trend is prevalent also if a higher velocity threshold (identified earlier as the “2nd velocity condition”, $\bar{U}(t) > 1.1 U_{nom}$) is imposed, in order to cut out time intervals of ordinary ship motion which should not be considered in the correlation study (see Figures 21 and 23).

However, for the second velocity condition the situation is reversed and the correlation of instantaneous celerity with mean speed is superior, for frequency ranges up to $45\% \omega_p$. Even better correlation is achieved when the mean of the instantaneous celerity (per apparent wave cycle) is taken, in place of the instantaneous celerity itself. In general, the error increases as the frequency range of the spectrum is broadened.

5. Concluding remarks

We have addressed, in a rather semi-empirical manner, phenomena of abnormal high speed motions which can be exhibited by ships operating in steep following irregular waves. We have introduced the term “high run” for characterizing those time intervals when the speed is maintained above the normally expected range of speed fluctuation in waves. The high run is basically the generalisation of surf-riding, for a multi-frequency wave environment.

The statistics of high runs in irregular seas were investigated from two different perspectives: a) by a simulation-based scheme based on the direct counting of high run occurrences; and, b) by an approximate semi-analytical method that was proposed by Grim over 50 years ago. In comparison to the direct counting method, the analytical one seems to overestimate the probability of high runs with longer duration.

Several other conclusions have accrued from the “direct counting” investigation: for a narrower spectrum the high runs last longer but they are sparser. As spectrum’s width is broadened the curve of mean high run duration tends to acquire a less pronounced peak. High runs become more frequent as the significant wave height is increased. On the other hand, there seems to exist a threshold spectrum peak period beyond which high runs become considerably rarer.

The topic remains open since the dynamical origins of these events, for a multi-frequency wave environment, are not completely understood yet.

Another topic addressed in the paper was the actual velocity of a ship when performing a high run. Three characteristic celerity definitions were picked, for further investigation: the instantaneous celerity; the mean instantaneous celerity per apparent wave cycle; and the wave celerity corresponding to spectrum’s peak. For each one of these cases, the error between ship velocity and celerity was evaluated by two different error metrics: a Euclidian one and a metric accounting for both amplitude and phase errors. Our investigation indicated that, during high runs, ship velocity shows good correlation with the mean instantaneous celerity, when an error metric of the second

type is employed. However, an issue adding uncertainty to such an investigation is, which should be the ship velocity threshold below which values should be filtered out as being, by definition, irrelevant to phenomena like surf-riding. In our view, this low cut-off velocity threshold should be set higher than the nominal speed and yet, not too close to the time-varying wave celerity curve.

The investigation of the statistical properties of high run events is of particular importance due to their possible connection with ship tendency for broaching-to behavior. Future research should target systematically the statistical correlation of these two phenomena, for various multi-frequency wave environments.

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Figure 1 Schematic definition of high run.

Figure 2 Wave amplitudes for 2 frequency ranges and their modified values when the variance is kept constant. $H_S=6$ m and $T_P=10$ s.

Figure 3 Recorded durations of high run incidents in different simulations.

Figure 4 *pdf* of mean high run duration [$H_s = 6 \text{ m}, T_p = 9.5 \text{ s}, 40 \times t_{sim}$, frequencies in $10\% \omega_p$ (one side)].

Figure 5 Mean value and characteristic percentiles (10th and 90th) referring to high run's duration, as T_p is varied [$H_s = 6 \text{ m}$, spectrum width $0.1\omega_p$ (one-sided) and simulation time $40 \times t_{sim}$]. Standard deviations are also shown.

Figure 6 As in Figure 5, for varied significant wave height and fixed peak period.

Figure 7 Demonstration of the convergence of statistics.

Figure 8 Mean duration of high run for a gradually broader spectrum, as the peak period is varied.

Figure 9 Mean time between successive high runs as the peak period is varied.

Figure 10 Mean time between successive high runs as the significant wave height is varied.

Figure 11 Probability to exceed the celerity value corresponding to the peak frequency of the spectrum, within a certain time, for $H_s = 6 \text{ m}$.

Figure 12 Probability to exceed the celerity value corresponding to the peak frequency of the spectrum, within a certain time, for $T_p = 10 \text{ s}$.

Figure 13 Probability to exceed various values of critical velocity (defined by a constant times the nominal speed) as a function of τ_1 ($H_s = 6 \text{ m}$ and $T_p = 10 \text{ s}$).

Figure 14 *pdf* of high-run duration for certain wave parameters (3 cases of τ_1).

Figure 15 Probability of exceeding a duration value, for several peak periods ($\tau_1 = 20 \text{ s}$).

Figure 16 Probability of exceeding a duration value, when the threshold speed is the celerity that corresponds to the peak frequency of the spectrum ($\tau_1 = 20 \text{ s}$).

Figure 17 Probability of high run duration for various velocity thresholds ($\tau_1 = 20 \text{ s}$).

Figure 18 Comparison of probabilities of high run duration between simulation statistics and Grim's method.

Figure 19 Time histories of surge velocity (upper diagram), instantaneous wave celerity contrasted with surge velocity (middle) and mean surge velocity (low). Continuous and dashed straight lines show the wave celerity of the peak frequency and the nominal speed, respectively. The simulations were based on a frequency range $20\% \omega_p$ (both sides).

Figure 20 Calculated error according to Euclidean metric (1st velocity condition)

Figure 21 Calculated error according to Euclidean metric (2nd velocity condition).

Figure 22 Error according to the Sprague and Geers metric, when satisfying the 1st velocity condition.

Figure 23 Error according to the Sprague and Geers metric when satisfying the 2nd velocity condition

Table 1: Values of simulation parameters

Parameter	Value	Units
L_{OA}	154	(m)
V_{nom}	12	(m/s)
Fr	0.308	
$V_0 = \dot{\xi}(0)$	10	(m/s)
Number of wave realisations per simulation scenario	100	
H_S	(3-6)	(m)
T_P	(8.5-13)	(s)
% ω_p (one side)	(5-30)	
Total simulation time	$t_{sim} - 40 \times t_{sim}$	(s)

Graphical Abstract

Comparison of probabilities of high run duration between simulation statistics and Grim's method.