Application of principal component analysis (PCA) and improved joint probability distributions to the inverse first-order reliability method (I-FORM) for predicting extreme sea states

Aubrey Eckert-Gallup¹, Cédric Sallaberry, Ann Dallman, Vincent Neary

Sandia National Laboratories, Albuquerque, NM 87185, USA

Abstract

Environmental contours describing extreme sea states are generated as the input for numerical or physical model simulations as a part of the standard current practice for designing marine structures to survive extreme sea states. Such environmental contours are characterized by combinations of significant wave height (H_s) and energy period (T_e) or peak period (T_p) values calculated for a given recurrence interval using a set of data based on hindcast simulations or buoy observations over a sufficient period of record. The use of the inverse first-order reliability method (I-FORM) is standard design practice for generating environmental contours. This paper develops enhanced methodologies for data analysis prior to the application of the I-FORM, including the use of principal component analysis (PCA) to create an uncorrelated representation of the variables under consideration as well as new distribution and parameter fitting techniques. These modifications better represent the measured data and, therefore, should contribute to the development of more realistic representations of environmental contours of extreme sea states for use in survivability analysis for marine structures.

¹Corresponding author. Tel.: 505-844-4508; Email: acecker@sandia.gov

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1. Introduction

The current practice for designing marine structures to survive extreme sea states is to apply nonlinear time domain numerical simulations to predict structural response to a short-term extreme wave or wave group. Extreme wave design generally includes the following steps as outlined in Coe et al. (2014): (1) consideration of hindcast simulations or buoy observations of sufficient duration and appropriate location; (2) application of extreme value theory and models used for extrapolation to events more extreme than those observed in a shorter period of record; (3) generation of environmental contours consisting of pairs of significant wave height (H_s) and either peak period (T_p) or energy period (T_e) that elicit extreme structural responses for a given return period; (4) identification of one or more extreme sea states, which can be used with a wave spectrum (often Pierson-Moscowitz or JONSWAP) to reconstruct time series data generated with a random-phase/amplitude model as input for numerical or physical model simulation (Berg, 2011). The energy period is widely used in wave energy applications (Lenée-Bluhm et al., 2011) and has been found to be more robust than the peak period due to a high sensitivity to spectral peak.

Vanem and Bitner-Gregersen (2014) summarize methods for generating environmental contours. These methods include the traditional inverse first-order reliability method (I-FORM) utilizing the Rosenblatt transformation (Rosenblatt, 1952), described in detail by Winterstein et al. (1993), and newer methods that avoid the Rosenblatt transformation by employing Monte Carlo simulations of a joint probability model (Huseby et al., 2013). Silva-González et al. (2013) use the Nataf transformation to capture the correlation between inputs and create environmental contours using the marginal distributions

24 and correlation coefficients of the environmental variables in a particular problem of inter-
25 est. The I-FORM continues to be standard design practice for generating environmental
26 contours used for estimating extreme sea states of a given recurrence interval or return pe-
27 riod (DNV, 2014). Environmental loads associated with these extreme sea states are used
28 to design various marine structures, including ships (DNV, 2002), dynamic risers (DNV,
29 2001), position moorings (DNV, 2010a), offshore floating platforms (DNV, 2010b), and
30 wave energy converters (WEC) (DNV, 2008).

31 The use of the traditional I-FORM is clearly developed in Haver (2009) and has been
32 applied by Berg (2011) in accordance with the recommendations in the DNV standard on
33 position mooring (DNV, 2010a). The first step of this basic application of the I-FORM
34 described in these works includes fitting distributions and parameter models to the signifi-
35 cant wave height (H_s) and energy period (T_e) observations such that their interdependency
36 is captured. The treatment of this interdependency in Haver (2009) includes a joint prob-
37 ability distribution using parameters developed by Nygaard and Johannessen (2000). The
38 second step of the traditional I-FORM application uses these fitted distributions to esti-
39 mate extreme sea states by expressing the probability level for the return period of interest
40 (e.g., 100 years) as the radius of a circular isoline in the standard normal space. This circle
41 is defined by equiprobable points at discrete angular intervals determined by the combi-
42 nation of two variables with individually varying probabilities. The probabilities of each
43 of these variables are used to evaluate the inverse of each of the previously defined proba-
44 bility distributions for the variables of the original input space in order to transform each
45 point into the original space of interest, creating a final environmental contour.

46 Examples of this application can be found in Vanem and Bitner-Gregersen (2014).
47 In some cases, environmental contours for a 100-year return period calculated using this
48 traditional application fail to cover the observations taken over a relatively short (8 to 10-
49 year) period of record. This problem was addressed by Berg (2011) by inflating the contour

50 resulting from the I-FORM using an arbitrary percentage value. The shortcomings of the
51 environmental contours generated using the traditional methodology along with attempts
52 to correct these problems as a post-processing step clearly show that this methodology
53 requires further exploration and modification in order to generate more realistic contours.

54 Others have addressed the need to modify the treatment of input variables prior to the
55 use of the I-FORM approach. For example, Forristall and Cooper (1997) use empirical
56 orthogonal functions (EOFs) in order to create more appropriate design water current pro-
57 files using the I-FORM. In their paper, EOFs are used to simplify the time series data for
58 ocean currents into a series of simpler modes, reducing the original vertical profile. These
59 simplified input variables are then evaluated using the I-FORM to generate design current
60 profiles. The use of EOFs by Forristall and Cooper (1997) shows that it is important to en-
61 hance methods for addressing the complexity of input data before applying the probability
62 distribution fittings required by the I-FORM. The importance of capturing the correlations
63 between input variables is also addressed by Silva-González et al. (2013). In the current
64 work, input data complexity will be reduced using principal component analysis (PCA).

65 This paper describes techniques for improving the traditional I-FORM through princi-
66 pal component analysis and enhanced extreme value models for the subsequent uncorre-
67 lated random variables. Following the use of PCA to reduce the correlation between the
68 input variables, the I-FORM is applied with modifications to the traditional distribution
69 and parameter fitting models for the subsequent uncorrelated variables. These modifica-
70 tions are shown to contribute to the development of more accurate and reasonable repre-
71 sentations of environmental contours for extreme sea states. Significant wave height and
72 energy period data collected at four buoys is used in this analysis: National Data Buoy
73 Center (NDBC) 46212 offshore of Northern California in 40 m depth with 76608 hourly
74 observations taken from January 22, 2004 to December 31, 2012, NDBC 46022, also off-
75 shore of Northern California in 675 m depth with 122467 hourly observations taken from

76 January 1, 1996 to December 31, 2012, NDBC 51202 offshore of Oahu in 82 m depth
77 with 105192 hourly observations taken from January 1, 2001 to December 31, 2012, and
78 NDBC 46050 offshore of Oregon in 128 m depth with 126614 hourly observations taken
79 from January 1, 1996 to December 31, 2012. The measured spectral data from these buoys
80 was used to calculate significant wave height and energy period at each site.

81 This paper is organized as follows. Section 2 describes the use of PCA to create a
82 set of uncorrelated variables that will be used to generate better-fitting environmental con-
83 tours. Section 3 discusses an improved methodology for creating distribution and param-
84 eter models for the variables of interest so that the I-FORM can be applied. Section 4
85 details the application of the I-FORM to the problem under consideration and presents the
86 results of the implementation of this methodology. Section 5 provides concluding remarks
87 and describes recommendations for future work.

88 **2. Principal component analysis**

89 The concept of principal components was initially introduced by Pearson (1901) and
90 more formally developed by Hotelling (1933). PCA provides a powerful transformation
91 that works to reduce the dimensionality of a problem by considering that higher order
92 components have a low impact on the variance of the data. The underlying goal of PCA is
93 to develop a new orthogonal basis in which the variables will be (1) uncorrelated and (2)
94 sorted such that the first variable represents the direction in which the data has the largest
95 variance and each subsequent variable leads to the next largest variance. Traditionally,
96 the variables in the new basis are called principal components while the values associated
97 to each variable in this new basis for each point are called z-scores. The mathematical
98 tools used to generate this new basis are based on classical linear (matrix) algebra applied
99 to the covariance matrix, taking advantage of its structure as a square, symmetrical, and
100 non-singular matrix (Jackson, 1991). The application of PCA to the current problem is

101 mathematically analogous to the use of the empirical orthogonal functions utilized in For-
102 ristall and Cooper (1997) for the purpose of generating water current profiles.

103 *2.1. Motivation*

104 The wave data across four sites of interest was studied and compared as a first step
105 in the process of improving upon the traditional application of the I-FORM. This study
106 included the creation of a representation of data density in order to understand the under-
107 lying patterns and trends masked by a traditional scatterplot representation of the data. The
108 density around each point was calculated based on the number of observations included
109 within a fixed circular neighborhood with a radius ϵ defined by $\epsilon = 2000/N$ where N is
110 the number of points in each dataset. Examples of the data density calculated for each
111 of the four sites of interest are shown in Figure 1. These density representations help to
112 characterize the developmental patterns present in the data by showing the differences in
113 frequencies across the entire data set. The results of this density study motivated the use
114 of PCA to capture the relation between the sea state variables of interest.

115 *2.2. Application of PCA to the current problem*

116 Principal component analysis was used to remove the correlation between energy pe-
117 riod and significant wave height for each dataset. The application of PCA generates two
118 new variables that will be called component one (C_1) and component two (C_2), where C_1
119 represents the component with the largest variance. The application of PCA to the orig-
120 inal H_s and T_e data yields a coefficient matrix defining a linear combination that allows
121 for rotation into the principal component space. The rotational axes defined by this linear
122 combination are shown in Figure 2. The general form of this coefficient matrix is shown
123 below:

$$\mathbf{V} = \begin{bmatrix} v_{1,1} & v_{1,2} \\ v_{2,1} & v_{2,2} \end{bmatrix} \quad (1)$$

124 where $v_{2,2} = -v_{1,1}$ and $v_{1,2} = v_{2,1}$. For $i = 1, \dots, n$ where n is the number of observations in
 125 the data set under consideration, the equations for each component based on the applica-
 126 tion of the coefficient matrix above for each point $p_i = (H_{s_i}, T_{e_i})$ are shown in Equations 2
 127 and 3:

$$\mathbf{U} = \begin{bmatrix} H_{s_1} & T_{e_1} \\ \vdots & \vdots \\ H_{s_n} & T_{e_n} \end{bmatrix} \quad (2)$$

$$\mathbf{C} = \mathbf{U}\mathbf{V} = \begin{bmatrix} C_{1_1} = H_{s_1}v_{1,1} + T_{e_1}v_{2,1} & C_{2_1} = H_{s_1}v_{1,2} + T_{e_1}v_{2,2} \\ \vdots & \vdots \\ C_{1_n} = H_{s_n}v_{1,1} + T_{e_n}v_{2,1} & C_{2_n} = H_{s_n}v_{1,2} + T_{e_n}v_{2,2} \end{bmatrix} \quad (3)$$

129 In order to fulfill the requirements for subsequent elements of the extreme event analysis
 130 (i.e., fitting probability distributions to the data), the rotated components must also be
 131 shifted upwards along the y axis to ensure that they are greater than zero. This is achieved
 132 by applying a shift s where $s = |\min(C_2)| + 0.1$. The final equations for the components
 133 defined by a single original data point $p_i = (H_{s_i}, T_{e_i})$ are then given by:

$$C_{1_i} = H_{s_i}v_{1,1} + T_{e_i}v_{2,1} \quad (4)$$

$$C_{2_i} = H_{s_i}v_{1,2} + T_{e_i}v_{2,2} + s \quad (5)$$

134 The components defined by these equations are used throughout the remainder of the ex-
 135 treme event analysis until they are transformed back into the original space defined by the
 136 sea state variables H_s and T_e . PCA is a bijective transformation, meaning that there is only
 137 one way to transform back into the original input space and that the inverse transformation
 138 is also a simple linear combination. Given a point $p_i = (C_{1_i}, C_{2_i})$ in the principal compo-
 139 nent space, the transformation back to the corresponding point in the original input space

140 defined by H_s and T_e is defined by:

$$H_{s_i} = \frac{C_{1_i}v_{1,1} + (C_{2_i} - s)v_{1,2}}{v_{1,1}^2 + v_{1,2}^2} \quad (6)$$

$$T_{e_i} = \frac{C_{1_i}v_{1,2} - (C_{2_i} - s)v_{1,1}}{v_{1,1}^2 + v_{1,2}^2} \quad (7)$$

141 These equations are used to transform the calculated values for the extreme event contour
142 into the input space following the application of the I-FORM described in the remainder
143 of this paper.

144 2.3. Discussion

145 There is a definite correlation between energy period and significant wave height, as
146 shown in Figures 1 and 2. Dissociating these two variables and treating them indepen-
147 dently, as is done in the traditional approach to extreme sea state characterization, under-
148 estimates the inherent dependency that these variables share. The principal components in
149 the new basis will be uncorrelated and, thus, the application of PCA will capture some of
150 the dependency between the initial variables H_s and T_e . Correlation is, however, only one
151 aspect of the dependency between these two variables. Therefore, some dependency that
152 is not captured by PCA remains. This dependency is expressed in the nonlinear develop-
153 ment of data density over time, as shown in Figure 1. This remaining dependency will be
154 taken into account in the distribution fitting portion of the I-FORM application, and will
155 be discussed in Section 3.2.

156 3. Distribution and parameter model fitting

157 Following the rotation of the input variables into the principal component space using
158 PCA, distribution and parameter models must be developed for the subsequent component
159 variables so that the I-FORM can be applied to develop environmental contours describing

extreme sea states. These distribution and parameter models are developed such that they capture some of the remaining dependency between C_1 and C_2 that was not taken into account through the application of PCA.

3.1. Distribution fitting for Component 1

Following the rotation of the dataset into the principal component space, the cumulative distribution function (CDF) of C_1 is fitted with an inverse Gaussian distribution using a maximum likelihood estimation technique applied in Matlab. This component was chosen for the initial fitting because it has the largest variance, as can be seen in Figure 2. The inverse Gaussian distribution was used both because it provides a good fit of the CDF shape of C_1 observed in the datasets under study and because of the simplicity of its defining parameters in terms of interpretation. The equation for the CDF of the inverse Gaussian distribution is given by:

$$F(x) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} + 1\right)\right) \quad (8)$$

where Φ represents the CDF of the standard normal distribution and μ and λ can be roughly interpreted as representing the location and scale of the inverse Gaussian distribution (Folks and Chhikara, 1978). The results of the inverse Gaussian fitting of the CDF of C_1 for all four study sites are shown in Figure 3. This figure also shows the distribution fitting at each site at the highest quantiles. This area is the most important in terms of goodness of fit for the application of the I-FORM to follow because this area will be evaluated during the extrapolation of these distributions as the final environmental contour is constructed. While the inverse Gaussian distribution fits the CDF of C_1 for three of the study sites well, the fitting is not as good as expected for NDBC 51202, especially at the highest quantiles. This fitting issue predicts a smaller value for C_1 than is observed at a given quantile for this site, resulting in an underestimation of extreme events by the final

183 environmental contour, as seen in Figure 11(c). The selection of a more generic distribu-
 184 tion or a mixed distribution to fit the variation in behavior for C_1 over the selection of study
 185 sites might contribute to the creation of more accurate extreme sea state contours across a
 186 wider variety of study sites and remains as an area of possible future improvement.

187 3.2. Binning Component 2 according to Component 1

188 The values of C_2 are sorted according to their corresponding C_1 values and split into
 189 bins containing 250 observations up to the last group, which contains all remaining points.
 190 This number was chosen based on the original sample sizes of the study sites under con-
 191 sideration because it is large enough to generate a good fit and small enough so that the
 192 influence of C_1 on C_2 can be captured with a fine enough discretization. The sensitivity
 193 of the root mean square error of the distribution parameter fitting models, described in
 194 the following section, and the final environmental contour to the size of the bins of C_2 is
 195 discussed in detail in Section 3.4. This binning scheme and the distribution and parameter
 196 fittings that follow help to capture some of the remaining dependency between C_1 and C_2 .

197 The CDFs for all bins of C_2 are shown for all study sites in Figure 4. The CDF of C_2
 198 for each bin must be fitted with a probability distribution in order for the I-FORM to be
 199 applied. The parameters of the C_2 distributions are fit as functions of the representative
 200 value of C_1 for each bin in order to represent the dependency between C_1 and C_2 that
 201 has not yet been captured. Based on the shape of the CDFs for each bin of C_2 , a normal
 202 distribution was chosen to fit each CDF of C_2 . This fit seems appropriate considering
 203 the symmetry of the data, shown in Figure 4. The parameters that define the normal
 204 distribution are the mean, μ , and standard deviation, σ . The equation for the CDF of the
 205 normal distribution is given by:

$$\Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) \right] \quad (9)$$

where erf represents the error function (Johnson et al., 1995). The CDFs for selected bins of C_2 along with their corresponding normal distribution fits for all study sites are shown in Figure 5. These normal distribution fits were generated using a maximum likelihood estimation technique for μ and the square root of the unbiased estimate of the variance for σ .

3.3. Fitting functions for Component 2 distribution parameters μ and σ

The sets of μ and σ values that are created following the fitting of a normal distribution to the CDFs of C_2 are shown as functions of the mean value for C_1 for each bin at each study site in Figure 6. Based on the trends observed in the data, it was determined that a simple linear approximation could be used to fit μ as a function of C_1 and a constrained quadratic approximation could be used to fit σ . The fitting functions for μ and σ are as follows:

$$f_{\mu}(C_1) = m_1 C_1 + m_2 \quad (10)$$

$$f_{\sigma}(C_1) = s_1 C_1^2 + s_2 C_1 + s_3 \quad (11)$$

In order to ensure that the quadratic fit for σ remains positive for small values of the mean of C_1 , the least squares fit for this parameter is subjected to the following constraints:

$$s_3 \geq 0 \quad (12)$$

$$s_3 - \frac{s_2^2}{4s_1} \geq 0 \quad (13)$$

These constraints ensure that both the y-intercept and the minimum of this quadratic model remain greater than or equal to zero. The fitting function for μ was fit to the data using a basic linear least squares technique. The σ function was fit using a penalty function that applies the constraints given above to a least squares objective function evaluated using a direct search, derivative-free method applied in Matlab. The final models for μ and σ at

each study site for the binning scheme described above are shown in Figure 6 along with the RMSE for each fit.

Although these models do not perfectly represent the variations present in the data, they allow for smooth extrapolations that create more practically applicable environmental contours. This is especially true when the data appears to be unstructured and may contain multimodal distributions, as is seen in Figure 1(c).

3.4. Bin size sensitivity study

A sensitivity study was conducted to determine the impact of the size of the C_2 bins on both the root mean square error (RMSE) of the fitting models for μ and σ and the final environmental contour at one site (NDBC 46212). Bin sizes were tested at intervals of 25 data points from a bin size of 50 points up to 10000 points. For each of these bins, the RMSEs for the fitting models of μ and σ are shown in Figures 7 and 8, respectively.

In order to stabilize the response of the parameter fitting functions with respect to the size of each bin, the values for μ and σ for the last bin were not included in the calculation of each fitting model. This ensures that the trends found in the majority of each parameter dataset, rather than possible extreme values created by a small number of data points in the last bin, are used to generate the coefficients for each parameter model. The responses of the RMSE for the models of μ and σ when the parameters of the last bin of C_2 are included in the fitting calculation are shown in blue in Figures 7 and 8 and are compared to the same responses when the last bin is not included, which are shown in green. The number of points in the final bin for each fixed bin size is also shown in these figures on a secondary y-axis. This shows that peaks in the RMSE for both μ and σ when the final bin is included in the fitting calculation often correspond to points at which there are very few points in the final bin. When these points are no longer included, the RMSE stabilizes for the μ fitting function and reaches a value between 0.025 and 0.05 (Figure 7). The RMSE also stabilizes

251 for the σ fitting function up to a bin size of approximately 5500 data points when the final
252 bin is not included in the fitting calculation(Figure 8). After this point, some peaks in the
253 RMSE still remain. These peaks may correspond to numerical instabilities in the fitting
254 algorithm caused by the large number of points included in each bin, which create a small
255 number of points with little continuity for the σ function to fit. This provides a justification
256 for choosing a bin size (e.g., 250) that is large enough to contain a representative number
257 of points for the normal distribution fit but small enough that the parameter values for each
258 parameter modeling function are continuous as a function of the mean value of C_1 for each
259 bin, allowing for the generation of a close-fitting parameter model.

260 The final contours for each modeled bin size up to a bin size of 5000 points are shown
261 in Figure 9. These contours show very little variation with a maximum significant wave
262 height difference at a single contour point of 0.26 meters and maximum energy period
263 difference of 0.08 seconds. This shows that the resulting contour will not be significantly
264 different than the same contour for a slightly different bin size when the fitting functions
265 for μ and σ have stable, relatively low RMSEs.

266 **4. I-FORM Application**

267 Following the creation of an inverse Gaussian distribution fit for component one and
268 the development of models fitting the parameters of the normal distributions of the bins
269 of C_2 as a function of the mean value of C_1 for each bin, the inverse first-order reliability
270 method (I-FORM) can be applied to construct an environmental contour for a given return
271 period.

272 *4.1. Contour creation*

273 In the standard FORM approach, a threshold value is considered and its likelihood is
274 estimated in the standard normal space. The inverse FORM approach, described by Haver

(2009), starts with an exceedance probability (i.e., a return period of 100 years) that defines the radius β of an isoline in the standard normal space. Equation 14 shows the calculation of the probability for a given return period (t_r), given in years, based on the measurement interval (t_s), given in hours. The subsequent calculation of β is shown in Equation 15:

$$p = \frac{1}{365 * \frac{24}{t_s} * t_r} \quad (14)$$

$$\beta = \Phi^{-1}(p) = \sqrt{2} \operatorname{erf}^{-1}(2p - 1) \quad (15)$$

where Φ^{-1} is the inverse CDF of the standard normal distribution. This isoline is then transposed into the original uncertain input space in order to evaluate the range of extreme values. Numerically, a discretization is used on the angle θ over the isoline to generate a set of points $U_j = (u_{1j}, u_{2j})$, represented as the following parametric function:

$$\begin{aligned} u_{1j} &= \beta \cos(\theta_j) \\ u_{2j} &= \beta \sin(\theta_j) \end{aligned} \quad \text{where } \theta_j = \frac{2\pi j}{k}, j = 1, \dots, k \quad (16)$$

For each value of j , the quantile position $Q_j = (q_{1j}, q_{2j})$ of the chosen probability of likelihood is calculated in both directions using the inverse of the standard normal cdf:

$$\begin{aligned} q_{1j} &= \Phi^{-1}(u_{1j}) \\ q_{2j} &= \Phi^{-1}(u_{2j}) \end{aligned} \quad (17)$$

where Φ^{-1} is the inverse of the equation for the standard normal CDF. The resulting quantile q_{1j} is then used to evaluate the inverse of the inverse Gaussian CDF to obtain a value for C_1 :

$$C_{1j} = F^{-1}(q_{1j}) \quad (18)$$

where F is the equation for the inverse Gaussian distribution with parameters that were fit to the distribution of C_2 obtained from the original input data, described in Section 3.1.

290 This value of C_1 is used to evaluate the normal parameter models in order to obtain a
 291 particular distribution of C_2 :

$$\begin{aligned}\mu_j &= m_1 C_{1_j} + m_2 \\ \sigma_j &= s_1 C_{1_j}^2 + s_2 C_{1_j} + s_3\end{aligned}\tag{19}$$

292 The quantile position q_{2_j} is used to evaluate the inverse of the particular normal CDF
 293 defined using the parameters calculated above in order to obtain a value for C_2 at this
 294 point:

$$C_{2_j} = \sigma_j \sqrt{2} \left[\text{erf}^{-1}(2q_{2_j} - 1) \right] + \mu_j\tag{20}$$

295 This process is performed for each value along the discretization of the isoline in the stan-
 296 dard normal space to create a complete environmental contour in the principal component
 297 space. The values of each point on this contour must then be transformed from the prin-
 298 cipal component space into the original sample space defined by the variables H_s and T_e
 299 using Equations 6 and 7, respectively. The extreme sea state contours resulting from this
 300 methodology at the four study sites under consideration are shown in Figure 10.

301 4.2. Results

302 The environmental contours generated using the modified I-FORM at each of the
 303 four study sites are shown with the corresponding contours created using the traditional
 304 methodology (e.g., Berg (2011)) in Figure 11. The contours that are developed under the
 305 new methodology are displayed along with a representation of the density of each data
 306 set under consideration, calculated as described in Section 2.1. This representation helps
 307 to emphasize the importance of considering the conjoint influence of energy period and
 308 significant wave height in order to develop extreme sea state contours that reflect the den-
 309 sity patterns in each dataset. The contours created using the new methodology appear to
 310 follow the shape of trends present within the data, a great improvement on the contours

311 created using the traditional methodology. This allows for coverage in the area of the input
312 space in which both energy period and significant wave height are high, a possible area of
313 importance for survivability analysis of marine structures. The new shape of these con-
314 tours is similar to the shape of environmental contours obtained by Silva-González et al.
315 (2013), which also provide a better coverage of the input space of interest than the contours
316 obtained using the traditional methodology.

317 At the most basic level, the extreme sea state contours created using the new method-
318 ology show a much better coverage of the data for the given period of record at each of
319 the study sites under consideration. It is expected that, given a period of record on the
320 order of tens of years, the extreme contour for a return period on the order of hundreds of
321 years should include all of the observed data. This is true for all of the study sites with the
322 exception of NDBC 51202. At this site, the complex relations between H_s and T_e are not
323 entirely captured by the application of both principal component analysis and the distri-
324 bution and parameter fitting. Investigation into the patterns that are visible in this dataset,
325 shown in Figure 1(c), found that the long fingers of observations stretching beyond the
326 majority of the points belong to individual storm events. These storm events are consistent
327 with the large annual winter storms in the North Pacific that have been found to have a
328 strong impact on significant wave height in general (Gulev and Grigorieva, 2006) and on
329 significant wave height in Hawaii in particular (Stopa et al., 2011). The extreme event con-
330 tour at this site is perturbed by these storms because they are relatively too infrequent to be
331 captured by the methodologies proposed in this paper. In addition, the distribution fitting
332 functions described in Section 3 do not seem as appropriate for the CDFs of Component
333 1 and Component 2 at this site when compared to the other three sites, as shown in Fig-
334 ures 3 and 5. This suggests that the distributions of these variables, and the corresponding
335 distributions of significant wave height and energy period, do not follow the same pat-
336 terns as those for the variables at the other three sites. This difference may stem from the

337 variations in oceans hydrodynamics amongst continental and island landforms. The wave
338 dynamics at NDBC 51202 are subject to phenomena that propagate wave energy around
339 the island, causing waves to approach from varying directions (Bascom, 1980). Further-
340 more, the topography of the Hawaiian Islands has been found to modify tradewind flow,
341 causing local wind acceleration that may also increase the wave energy found at this loca-
342 tion (Stopa et al., 2011). The combination of all of these effects, which are unique to the
343 Hawaiian islands, helps to explain the different patterns found in the data from this loca-
344 tion that contribute to inconsistencies in the final environmental contour generated for this
345 site. Possible improvements to the environmental contour at this site could be made by (a)
346 collecting more data, (b) using surrogate models, or (c) including data from a similar site.
347 The improvement of the calculated contour at sites with these types of sea state patterns
348 remains as an area of future work beyond the scope of the current analysis.

349 The maximum significant wave heights for a return period of 100 years predicted us-
350 ing the new methodologies proposed in this work are significantly higher than the maxi-
351 mum values obtained using the traditional methodology implemented as described by Berg
352 (2011) and shown in Figure 11. This particular implementation of the traditional approach
353 uses probability distribution models that do not fit the data as well as they should for ex-
354 treme value extrapolation. In an effort to find support for the much higher maximum H_s
355 values obtained using principal component analysis and the modified I-FORM method, the
356 traditional methodology was implemented with an updated double-Weibull distribution fit-
357 ting that provided a better fit for the distribution tails of the significant wave height data.
358 The results of this exploration are shown in Figure 12. These show that the improvement
359 of distribution fitting in the implementation of the traditional methodologies that focuses
360 on fitting distribution tails can result in much higher values for the maximum significant
361 wave height, supporting the maximum values found in this work.

362 5. Conclusion

363 Overall, this paper presents a significant improvement to the original method of cal-
364 culating an extreme contour of sea states. The proposed modifications, utilizing principal
365 components, better represent the measured data and, therefore, should contribute to the
366 development of more realistic representations of environmental contours of extreme sea
367 states. This can better prepare developers of marine structures for survivability analysis
368 and can be applied in many areas of interest. The increase in the maximum significant
369 wave height expected to occur for a given return period derived from the new contours de-
370 veloped in this work presents a significant development in the field of marine survivability
371 analysis.

372 There are several areas that represent possibilities for future enhancements to the
373 methodology developed in this paper. First, the use of principal component analysis to
374 create an orthogonal decomposition of the data such that the values are uncorrelated in
375 each direction only addresses one aspect of the complex relation between energy period
376 and significant wave height, as can be seen in Figure 11(c) and (d). A more complex
377 decomposition taking into account the varying relations among variables across the study
378 sites shown in these representations of density (e.g., curvature) could lead to a better rep-
379 resentation of the data and, therefore, a more accurate approximation of the extreme sea
380 state contour for a given return period. However, it is unclear how orthogonality could be
381 preserved under this new paradigm. Second, the selection of a more generic distribution or
382 a mixed distribution to fit the variation in behavior for C_1 over the selection of study sites
383 along with refinements in the models developed for the normal distribution parameters μ
384 and σ for C_2 might also contribute to the creation of more accurate extreme sea state con-
385 tours for all sites. Finally, the use of principal component analysis can easily be extended
386 to include multiple dimensions, with each new set of components ultimately presented as

387 a simple linear combination of the original variables. In future work, this could be used to
388 consider additional variables (e.g., wind and water current speed) related to the problem
389 of interest.

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(a)

(b)

(c)

(d)

Figure 1: Representation of data density for four study sites (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 2: Representation of the axes of the new basis developed using principal component analysis for NDBC 46212.

Figure 3: Component 1 CDF (left) and zoom into highest quantiles (right) for all data (blue) and inverse Gaussian model (red) for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

(a)

(b)

(c)

(d)

Figure 4: CDFs for all bins of Component 2 for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 5: CDFs for selected bins of Component 2 with normal distribution fits at full scale (left) and zoom into highest quantiles (right) for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 6: Estimates of the Component 2 normal distribution parameters μ (top) and σ (bottom) as a function of Component 1 for each bin for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 7: Root mean square error (RMSE) for the μ fitting function as a function of bin size shown when the last bin is included in the fitting (blue) and when it is not (green) along with the number of points in the final bin as a function of bin size (red) for NDBC 46212.

Figure 8: Root mean square error (RMSE) for the σ fitting function as a function of bin size shown when the last bin is included in the fitting (blue) and when it is not (green) along with the number of points in the final bin as a function of bin size (red) for NDBC 46212.

Figure 9: Final 100-year environmental contours for bins modeled at increments of 25 data points from a bin size of 50 points to a bin size of 5000 points for NDBC 46212.

(a)

(b)

(c)

(d)

Figure 10: 100-year extreme sea state contour for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 11: 100-year extreme sea state contours created by the traditional methodology(dashed line) (Berg, 2011) and the new methodology presented in this paper shown with data density(solid line) for (a) NDBC 46212, (b) NDBC 46022, (c) NDBC 51202, (d) NDBC 46050.

Figure 12: 100-year extreme sea state contour obtained using the traditional methodology (blue), using an updated version of the traditional methodology with better distribution fits (green) and using the new methodologies proposed in this work (red) for NDBC 46212.