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Validation of Boussinesq-Green-Naghdi Modelling for Surf Zone Hydrodynamics

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Abstract

2D modelling for surf zone phenomena are validated in present work using the rotational Boussinesq-Green-Naghdi model. Three benchmark test cases are simulated: tsunami wave runup a conical island; tsunami wave runup complex shelf; and rip current and wave setup over sand bars. The computed results are compared to the experimental data including the free surface deformation and depth-averaged velocities. The simulated 2D cases fundamentally validate the model's ability in predicting wave transformation, wave breaking, wave runup and the velocity field for complex hydrodynamic conditions and give the basis for moving on to more complex applications. The lack of irrotationality would strongly contribute to the depth-varying velocity profile of rotational modelling, which has been partially proven in 1D undertow test (Zhang et al., 2014a). Unfortunately, few 2D experiments with rotational vortex data measured could be found due to the difficulty of recording the vortex characteristics. Future work would be the model application to much more complex geophysical and engineering problems, where the lack of any irrotational constrain is expected to excel.

Keywords: Boussinesq; Green-Naghdi; Rotational; Wave; Surf zone; Hydrodynamics

1 Introduction

Boussinesq-type models describe nonlinear and dispersive wave propagation in arbitrary directions with good computational efficiency and range of validity. Since the classic depth-averaged Boussinesq theory was introduced by Peregrine (1967), Boussinesq-type models have experienced a booming development in accuracy, range of application and popularity (Nwogu 1993; Wei et al., 1995; Kennedy et al., 2001; Madsen and

Schäffer, 1998; Gobbi and Kirby, 1999; Gobbi et al., 2000; Lynett and Liu, 2002a; Madsen et al., 2002; Donahue, 2015). Properties such as dispersion, shoaling gradient, and higher order harmonics have been significantly improved by asymptotic rearrangement throughout the system. Introducing an eddy viscosity model and moving shorelines enables viscous Boussinesq models to describe surf zone dynamics such as wave evolution, wave setup, and large scale wave-induced currents including rip currents and longshore currents (Schäffer and Madsen, 1993; Sørensen et al., 1998; Chen et al., 2000; Kennedy et al., 2000a,b; Lynett, et al., 2002b; Musumeci et al., 2005; Nwogu and Demirbilek, 2010; Shi et al., 2012; Bonneton et al., 2011; Kim and Lynett, 2013).

Green and Naghdi (1976) introduced an alternate but related approach dealing with the rotational orbital velocities using a polynomial structure of the velocity profile. The mass and momentum equations were solved using a weighted residual with no irrotationality constraint, so that the rotational flow is expected to be modelled naturally. Zhang et al. (2012, 2013) developed Boussinesq-Green-Naghdi models that take advantage of both Boussinesq and Green-Naghdi systems. Polynomial expansions (Shields and Webster, 1988) and Boussinesq scaling were both applied. The major interest, rotational modelling, has been partially validated at both $O(\mu^2)$ and $O(\mu^4)$ approximation levels for 1D test cases which include nonbreaking and breaking waves in surf zone, undertow and wave runup (Zhang et al., 2014 b; Panda et al., 2014). All these give the basis and confidence for further 2D surf zone modelling.

For breaking techniques, two common ones are usually adopted. The first technique is the surface roller method based on the flux version of Boussinesq equations (Schäffer and Madsen, 1993) which was further developed by Madsen et al. (1997). The second approach is the ad hoc eddy viscosity formulation originally developed by Zelt (1991), and extended by Kennedy et al. (2000a) that yields extra viscous terms in momentum equations leading to wave dissipation. Both eddy viscosity and surface roller techniques are of comparable accuracy. A third, more recent, approach turns off dispersive terms in the vicinity of the breaking roller and allows the dissipative nature of shallow water bores to remove energy from waves while conserving momentum (Shi et al., 2012; Tissier et al., 2012).

In this paper, the rotational Boussinesq-Green-Naghdi modelling is extended to 2D/quasi-3D demonstrations for complex hydrodynamics, including wave transformations and interactions, wave breaking, and moving multi-wet-dry shoreline interfaces. In order to reproduce the energy dissipation under the breaking wave crest, viscous terms in the Navier Stokes equation represented by eddy viscosity are kept by Boussinesq scaling which may be further improved by introducing extra scaling. The eddy viscosity is modeled by the

depth-integrated $k-l$ turbulent-kinetic-energy equation. The $k-l$ model (Karambas and Koutitas, 1992) is a basic approach for turbulence modelling compared to more complex turbulence models such as $k-\epsilon$ model, $k-\omega$ model, and Reynolds-stress related models. The model is arguably the simplest incomplete turbulence model, and hence it has the broad range of applicability with much less computational cost. This simplicity is helpful to make the system tractable. And similar to $k-\epsilon$ model (King et al. 2012), $k-\omega$ (Neary, 2003) model, $k-l$ is a favorable numerical coupling between the flow and turbulence equations. Benchmarked experimental cases with wave breaking, wave runup and rotational velocity fields are simulated. Numerical results reach highly satisfactory level compared to collected experimental data.

Due to the single surface assumption in Boussinesq models, there is an upper limit on accuracy for the model used here to simulate the complex free surfaces under wave breaking, in the roller region and near the wave crest. Still, the present model maintains a balance between accuracy and computational efficiency, which is ideal for intermediate-scale modelling with much lower computational cost than full Navier-Stokes solvers (e.g. Ma et al., 2012; Higuera et al., 2013).

2 Boussinesq-Green-Naghdi rotational water wave system

2.1 Scaling

The system derived is dimensionless, applying Boussinesq-shallow water scaling for non-dimensional variables, which are defined as

$$\begin{aligned} (x, y) &= k_0(x^*, y^*), & z &= h_0^{-1}z^*, & t &= k_0(g_0h_0)^{\frac{1}{2}}t^*, & h &= h_0^{-1}h^*, \\ \eta &= (h_0)^{-1}\eta^*, & P &= (\rho^*g_0h_0)^{-1}P^*, & g &= g_0^{-1}g^*, & (u, v) &= (g_0h_0)^{-1/2}(u^*, v^*), \\ w &= (k_0h_0)^{-1}(g_0h_0)^{-1/2}w^*, & \nu_t &= h_0^{-1}(g_0h_0)^{-1/2}(k_0h_0)^{-1}\nu_t^*, & \tau_{xx} &= g_0^{-1}k_0^{-1}h_0^{-2}\tau_{xx}^*, & \tau_{zx} &= g_0^{-1}k_0^{-2}h_0^{-3}\tau_{zx}^* \end{aligned} \quad (2.1)$$

where the superscript $*$ indicates dimensional variables. $\mathbf{x}^* \equiv (x^*, y^*)$ are horizontal spatial coordinates and the vertical coordinate (z^*) is oriented positive upward. g^* is gravitational acceleration. Time t^* is scaled by long wave celerity $(g_0h_0)^{1/2}$ and wavenumber k_0 , while depth h^* and surface elevation η^* scales with typical water depth h_0 . Horizontal and vertical velocities (u^*, v^*, w^*) are all scaled by wave orbital velocities. The pressure P^* is hydrostatically scaled. This scaling allows strongly nonlinear waves, although of course the system also remains valid for small amplitude waves. Eddy viscosity ν_t^* is assumed to scale with depth and gravity, and turbulent stresses use both eddy viscosity scaling and Boussinesq-shallow water scaling.

2.2 Model equations

As derived by Zhang et al. (2013), the fully $O(\mu^2)$ Boussinesq-Green-Naghdi rotational water wave model assumes a polynomial expansion for the horizontal and vertical velocity,

$$\begin{aligned}\mathbf{u} &= \mathbf{u}_0 + \mu^2 \mathbf{u}_1 f_1 + \mu^2 \mathbf{u}_2 f_2 + O(\mu^4) \\ w &= -\nabla \cdot \mathbf{u}_0 (\eta + h) q - \mathbf{u}_0 \cdot \nabla h + O(\mu^2)\end{aligned}\tag{2.2}$$

where μ is a dimensionless indicator of approximation level; $\mathbf{u} = (u, v)$; $q = (z + h)/(h + \eta)$. The polynomial basis functions $f_n(q)$ used here are,

$$\begin{aligned}f_0 &= 1 \\ f_1 &= -54/125 + q \\ f_2 &= -1/5 + q^2,\end{aligned}\tag{2.3}$$

which is the optimized set at the approximation level of $O(\mu^2)$. All horizontal velocity components $\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2$ are independent, so that higher order velocity components do not depend on lower order components as in other irrotational Boussinesq theories (e.g. Peregrine, 1967).

Insertion of the velocity expressions (2.2) into the vertically integrated mass and momentum equations gives, at $O(\mu^2)$,

$$\eta_{,t} + \nabla \cdot \left(\mathbf{u}_0 (\eta + h) + \mu^2 \sum_{n=1}^2 \mathbf{u}_n (\eta + h) g_n|_{q=1} \right) = 0\tag{2.4}$$

$$\begin{aligned}
& \mathbf{u}_{0,t}(\eta + h)g_m|_{q=1} + \mathbf{u}_0 \cdot \nabla \mathbf{u}_0(\eta + h)g_m|_{q=1} + g \nabla \eta(\eta + h)g_m|_{q=1} \\
& + \mu^2 \sum_{n=1}^2 (\mathbf{u}_{n,t}(\eta + h)\phi_{mn} - \mathbf{u}_n \eta_{,t} \varepsilon_{mn})|_{q=1} \\
& - \mu^2 [\frac{1}{2} \nabla(\nabla \cdot \mathbf{u}_{0,t})(\eta + h)^3 (g_m - \nu_m) + (\nabla \cdot \mathbf{u}_{0,t})(\eta + h)^2 \nabla(\eta + h)g_m \\
& + \nabla(\mathbf{u}_{0,t} \cdot \nabla h)(\eta + h)^2 (g_m - S_m) + \mathbf{u}_{0,t} \cdot \nabla h \nabla \eta(\eta + h)g_m \\
& - (\nabla \cdot \mathbf{u}_{0,t})(\eta + h)^2 \nabla h S_m]|_{q=1} \\
& + \mu^2 \sum_{n=1}^2 [(\mathbf{u}_n \cdot \nabla \mathbf{u}_0 + \mathbf{u}_0 \cdot \nabla \mathbf{u}_n)(\eta + h)\phi_{mn} - \mathbf{u}_n \nabla \cdot (\mathbf{u}_0(\eta + h))\varepsilon_{mn}]|_{q=1} \\
& + \mu^2 (\eta + h)^2 [(\nabla \cdot \mathbf{u}_0)^2 - \mathbf{u}_0 \cdot \nabla(\nabla \cdot \mathbf{u}_0)](\nabla \eta g_m + \nabla h(g_m - S_m))|_{q=1} \\
& + \frac{\mu^2}{2} (\eta + h)^3 \nabla[(\nabla \cdot \mathbf{u}_0)^2 - \mathbf{u}_0 \cdot \nabla(\nabla \cdot \mathbf{u}_0)](g_m - \nu_m)|_{q=1} \\
& - \mu^2 (\eta + h) \nabla \eta \mathbf{u}_0 \cdot \nabla(\mathbf{u}_0 \cdot \nabla h)g_m|_{q=1} \\
& - \mu^2 (\eta + h)^2 \nabla(\mathbf{u}_0 \cdot \nabla(\mathbf{u}_0 \cdot \nabla h))(g_m - S_m)|_{q=1} \\
& = \int_{-h}^{\eta} f_m \frac{\partial \tau_{xz}}{\partial z} dz + \int_{-h}^{\eta} \mu^2 f_m \nabla \cdot \tau_{\mathbf{x}\mathbf{x}} dz \quad , \quad m = 0, 1, 2 \quad (2.5)
\end{aligned}$$

where g_m and S_m are integral functions of f_n , e.g. $g_n \equiv \int_0^q f_n(q) dq$, with many other functions defined (Appendix A).

Integrated breaking terms are

$$\int_{-h}^{\eta} f_m \frac{\partial \tau_{xz}}{\partial z} dz = \tau_{xz}(\eta) - f_m|_{q=0} C_f \mathbf{u}_b |\mathbf{u}_b| - \int_0^1 f'_m (\nu_{t2} \mathbf{u}_{,q} / (\eta + h) + \mu^2 \nu_{t2} \nabla w) dq, \quad (2.6)$$

and

$$\begin{aligned}
\int_{-h}^{\eta} \mu^2 f_m \nabla \cdot \tau_{\mathbf{x}\mathbf{x}} dz &= \mu^2 \nabla \cdot \left[(\eta + h) \int_0^1 f_m \nu_{t1} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) dq \right] \\
&- \mu^2 (\eta + h) \int_0^1 \nu_{t1} (\nabla f_m) \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) dq \\
&- \mu^2 f_m \nu_{t1} (\nabla \eta) \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)|_{q=1} \\
&- \mu^2 f_m \nu_{t1} (\nabla h) \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)|_{q=0} \quad (2.7)
\end{aligned}$$

If there is no air-water shear stress, then $\tau_{xz}(\eta) = 0$. For bottom frictional dissipation, some sort of information is required based on bottom conditions, whether from bed roughness or vegetation type. These are often placed into a drag framework such as $\tau_{xz}(-h) = C_f \mathbf{u}_b |\mathbf{u}_b|$, where $\mathbf{u}_b$ is the near-bottom velocity, and C_f is a bottom friction coefficient normally in the order of 10^{-3} , and this approach will be adopted. A bottom stress coefficient and gradient also imply a vertical profile for the eddy viscosity, $\nu_{t2}(\mathbf{x}, z) =$

$\tilde{\nu}_{t2}(\mathbf{x})f_\nu(q)$. Combining eddy viscosity and bottom friction, the depth-varying eddy viscosity will then be $\nu_{t2}(\mathbf{x}, z) = \epsilon f_\nu C_f(\eta + h)|\mathbf{u}_b|$, where ϵ is a constant applied to control the velocity difference between surface and bed, and the influence of the bed velocity (or alternatively mean velocity) on eddy viscosity arises from scaling considerations. The simplest application will be a constant vertical profile $f_\nu(q) = 1$ (e.g. Kim and Lynett, 2013), which will use $\epsilon \approx 0.4$, but other vertical profiles may also be employed. The depth-averaged eddy viscosity ν_{t1} is modelled by the $k-l$ turbulent-kinetic-energy equation (Zhang et al. 2014 b),

$$\frac{\partial \nu_{t1}}{\partial t} + \frac{\partial \nu_{t1}}{\partial x} \sum_{n=0}^N \mathbf{u}_n g_n|_0^1 - \frac{\mu}{\nu_{t1}\sigma_k} \nabla \cdot (\nu_{t1}^2 \nabla \nu_{t1}) - \mu \frac{c^2 \bar{l}_m^2}{2} \int_0^1 \mathcal{P} dq + \frac{1}{\mu} \frac{c^2}{2\bar{l}_m^2} \nu_{t1}^2 = 0. \quad (2.8)$$

where σ_k is the turbulent Prandtl number for kinetic energy, and is generally taken to be 1.0. $\bar{l}_m$ is depth-averaged mixing length defined as $\bar{l}_m = \int_0^1 l_m dq = 4/15\kappa(\eta + h)$, where $\kappa = 0.412$ (Nezu and Rodi, 1986). A value of $c = 0.55$ yields the correct behavior for shear flow in $k-l$ models, and this is used here. According to the turbulent-viscosity hypothesis introduced by Boussinesq (1877), the production of turbulent kinetic energy is then

$$\mathcal{P} = \nu_{t1} [\nabla \mathbf{u} \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + 2\mathbf{u}_{,z} \cdot \nabla w + \frac{1}{\mu^2} \mathbf{u}_{,z} \cdot \mathbf{u}_{,z} + 2w_{,z}^2]. \quad (2.9)$$

This one-equation model for the evolution of eddy viscosity coupled with the Boussinesq-Green-Naghdi rotational wave model calculates rotational inner velocities during energy dissipation under the breaking wave crest.

2.3 Moving shoreline

We adopt a thin-layer method for moving multi-shorelines. This method was proposed by Benque (1982) to place thin-layer water over the dry area so that the whole computational domain is always unchanged. There is no need to update the wet computational boundary, although a threshold is needed to switch the governing equations into 'dummy' equations for the 'dry' region. The method used here is similar with a few minor differences in the governing equations for the 'thin layer'. Like most moving shoreline methods, a threshold, δ , to determine the dividing points between wet and dry areas is needed. If the total water depth $h + \eta > \delta$, the model equations (2.5) will be applied; otherwise, a new set of equations will be adopted as following,

$$\eta = -h + \delta, \quad (2.10)$$

$$\frac{\partial u}{\partial t} = \omega(0 - u). \quad (2.11)$$

Again here, the 'dry area' is defined where $h + \eta \leq \delta$ as a thin water layer is placed, which allows the water to propagate up to the coast but holds water in the layer by setting a damping coefficient ω for the horizontal velocity, where $\omega = C(x - x(\delta))$. Slightly different from Benque's theory, we do need to calculate the threshold positions at each computational time step so that the mass is better conserved.

3 Numerical validations

In this paper, we present three numerical test cases to further validate the 2D/quasi-3D modelling of wave transformation, wave breaking, runup, orbital velocities and rip currents. All numerical tests used a fully nonlinear $O(\mu^2)$ Boussinesq-Green-Naghdi rotational model (Zhang et al., 2013) with a fourth order Runge-Kutta time differencing and a second order spatial central differencing scheme. Two-dimensional momentum equations are solved by the alternating direction implicit method. Waves are generated using second order generation (Zhang et al., 2014 a).

The benchmarked wave run-up on a conical island (Briggs et al., 1995) test gives a view how tsunamis break down, generate currents and run up on land. This case partially validates the tsunami modelling with different wave heights. Here, numerical results of surface elevation at gauges are compared to data.

A recent experiment with complex topography was conducted in a large wave basin at Oregon State University's O.H. Hinsdale Wave Research Laboratory with surface elevations and depth-averaged horizontal velocities measured at different gauge locations (Lynett et al., 2010). This experiment further examined the surf zone dynamics such as shoaling, wave refraction, wave breaking and turbulent flow. Computed results for both water elevation and velocities are compared to data.

Since the Boussinesq-type model is able to simulate both surface waves and wave-current interaction, a rip current experimental, conducted by Haller et al. (2002), is simulated. The present model provides good agreement with the comprehensive experimental data for both surface elevation setup and current velocity field.

3.1 Solitary Wave run up a conical island

The experimental setup is shown in Figure 1. Solitary waves were generated in a $0.32m$ water depth propagating from the left to the right of the domain along the x -direction. The waves passed the conical island placed in the middle of the domain with water consequently running up both the front and lee sides of the

island. The computational domain was $25m$ long and $30m$ wide, as in the experiment. A conical island with a slope of $1 : 4$ was located at $(x, y) = (13, 15)m$ with bottom, still water level, top diameters $7.2m$, $4.64m$, and $2.2m$, respectively. Gauges 6, 9, 16, 22 are located at $(x, y) = (9.4, 15)m$, $(10.4, 15)m$, $(13, 12.42)m$, $(15.6, 15)m$.

Three tests, A, B, and C, with $H/h = 0.05, 0.1, 0.2$ respectively, are simulated, where the H/h is initial solitary wave height/depth ratio. Water depth remains the same for all tests. The model used $dx = dy = 0.1m$ grid size with a $dt = 0.02s$ time step.

Figure 2 compares the computed time series of surface elevation at four gauge locations for test A. A good agreement is shown for gauges experiencing the initial water surface peak and depression. More disagreement appears at later times. Fuhrman and Madsen (2008) proposed a reason for the deviations, which is the tail of the solitary wave in the experiment could be generated with a spurious water surface which is not ideal or accurate. These deviations are similar to those found in other literature numerical results (Chen et al., 2000; Zijlema et al., 2011; Kazolea et al., 2012). Compared to the data, gauges 9 and 22 slightly over-predict the initial wave crest passing by while gauges 6 and 16 were predicted very well. However, this still remains a good prediction.

For test B, surface elevations are predicted very well compared to the data as shown in Figure 3. A much larger surface depression is observed at gauge 9 in front of the island. The crest and trough near the initial runup at all locations are described well. The results at later times follow the trend despite the variance from the oscillating data.

Test C, with $H/h = 0.2$, is the most difficult to simulate for either a Boussinesq-type model or other methods due to the large incident wave breaking on the island causing strong nonlinear effects and higher harmonic signals. Fewer satisfactory simulations for test C were found other than for tests A or B. Figure 4 shows an accurate capturing of higher harmonics and nonlinearity during and after initial runup. The agreement is good compared to most others (Titov and Synolakis, 1998; Fuhrman and Madsen, 2008; Nicolsky et al., 2011; Zijlema et al., 2011). The 3D snapshots through the wave rushing over the island are shown in Figure 5. Panels (a) and (b) describe the wave inundates and breaks on the island. Waves collide at the back in panels (c) and (d), creating a complex surface with wave diffraction and refraction in panels (e) and (f).

3.2 Solitary wave run up on a shelf with a conical island

In order to validate the surf zone modelling including wave shoaling, transformation, breaking and run up, we simulate a recent experiment with a solitary wave propagating on complex topography (Lynett et al., 2010). Complex phenomena such as wave refraction and wave-induced currents are modeled.

Figure 6 shows the experimental setup. The basin is $48.8m$ long and $26.5m$ wide with a constant water depth of $0.78m$ at the left side of the domain. A complex bathymetry consists of two 1:30 slope beaches connected to a triangular flat submerged shelf with a conical island placed on it. The solitary wave was generated at the left side and propagated to the right of the domain along the x direction. The wave height was $0.39m$. Surface elevations were measured at 9 gauge locations, while velocities were collected at 3 ADV locations. Gauges 1–9 were placed at $(x, y) = (7.5, 0.0)m, (13.0, 0.0)m, (21.0, 0.0)m, (7.5, 5.0)m, (13.0, 5.0)m, (21.0, 5.0)m, (25.0, 0.0)m, (25.0, 5.0)m$ and $(25.0, 10.0)m$. ADVs 1–3 are located at $(13.0, 0.0)m, (21.0, 0.0)m$ and $(21.0, -5.0)m$.

The computational domain was the same as in the experiment. The solitary wave is generated using the sponge-layer generation method (Zhang et al., 2014 a) within $x = 0 - 5m$ by imposing the exact target wave signals. The wave propagates the same way in and out of the generation zone. Note that the generation zone also plays a sponge layer role, which means the wave entering this area from any direction would be absorbed so that no reflected waves get back into the computational domain and affect the results.

Figure 7 shows the snapshots of the surface elevations at six different times during the shoaling process. The wave becomes steep approaching the shelf, rushes over the island, breaks down and runs up the coast. Wave refraction, diffraction and collision are observed around the cone.

Figure 8 compares the time series of computed surface elevation at 9 gauges with the experimental data. Gauges 1 and 2 predict the wave transformation and reflection in front of the cone. Gauges 3 and 7 describe the wave passing and surrounding the island and colliding behind the island. There is an over prediction at gauge 3 of the initial wave peak. Results at gauges 4, 5, 6, 8, 9 show the wave shoaling, breaking and refraction over the entire time. There appears the phase lag at gauges 7, 8, 9 and the deviations in later time. This probably would be diminished by adopting more accurate shoreline method. The results are overall comparably good.

Figure 9 gives the time series for depth-averaged velocities, U and V , in both the x and y directions, respectively. For the x -direction velocity, the strong initial peak was predicted well followed by many strong

oscillations of the flow in front of the island at ADV1. The weaker velocity variation behind the island was capture as well. At ADV3, the computed U matches the data the best with lest affected by the island among these 3 locations. On the other side, the velocity in the y direction is negligibly small at ADVs 1 and 2, while the model results are a bit larger than measured data in amplitude for ADV 3. Still, the velocity performs as expected and follows the data's trend.

3.3 Rip Current simulation

The rip current case is a good demonstration for prediction of rotational flow and wave-current interaction. Often a short wave model is coupled with a the depth-averaged circulation model to simulate wave current interaction (Hass et al., 2003; Kennedy, 2003; Hass and Warner, 2009; Kennedy et al., 2008; Kennedy and Zhang, 2008), while the phase resolving Boussinesq-type model is able to simulate wave-current by nature (Chen et al., 1999; Johnson and Pattiaratchi, 2006; Fang et al., 2013; Fang et al., 2014). The simulated topography shown in Figure 10 is interpolated from the detailed wave basin bathymetry. The still water depth in the left deep water is around $0.367m$ and varies over the area. A regular wave train is generated in deep water propagating towards the shore. Longshore submerged bars are placed offshore with two gaps where the rip current would flow through. The computational domain is $10.6m$ long and $18.2m$ wide with a uniform grid size $dx = dy = 0.05m$. Wave height and period are $4.8cm$ and $1.0s$. The two gaps are approximately $1.8m$ long, with the upper one a bit larger (Figure 11). The slope of the bed is about $1 : 30$. Waves break down strongly on the bar while weakly over the gaps. Thus, a surface elevation setup on the bar and lower surface in the gap provides consistent pressure gradient which motivates the rip currents in the gap. The simulation time is $1700s$ with time step $dt = 0.025 * 9.81^{0.5}s$. Computed results are averaged over the later $1600s$ to avoid the startup effects.

Figure 11 shows the time-averaged, depth-averaged velocity field of the rip current. In the panel (a), the vorticity contour and arrows clearly show the rip current flowing offshore at two gaps as well as the circulation patten nearshore. The flow near the upper channel is slightly stronger due to its larger width. Panels (b) and (c) depict the measured and computed depth-averaged velocities at gauge locations. The circulation pattern and rip current flow is similar.

Cross-shore mean surface elevations of four longshore sections near the bars are compared to the data in Figure 12. In general, the agreement is good for all results. Panels (a) and (c) are at the edges of the bars while panel (b) is on the bar, and panel (d) is in front of the bar. The depression of water in the channel is

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4 predicted well in panels (a), (b), and (c). The overall surface setup and setdown are captured respectively in
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6 panels (a), (b), and panels (c), (d).
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8 Figure 13 shows the cross-shore velocity at four longshore sections. The numerical results agree with the
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10 data very well. The troughs show mean offshore currents in the channels. The nearshore velocity variation
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12 is described well in panel (a). The channel velocity at the troughs in panels (b) and (c) does not match the
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14 data as well, however, the trend and variation follows the data. Panel (d) shows that the measurement data
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16 indicates much weaker, outgoing flow at the far offshore side compared to the simulation.
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18 Figure 14 compares longshore velocities computed by the model to the longshore velocity data. The
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20 modeled result is smaller than the measured data at the upper gap in panel a near shore. The longshore
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22 velocity is modeled well in panel (b) and (d). The agreement in panel c is the lowest. Overall, the longshore
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24 velocity follows the data.
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26 27 28 4 Discussion and conclusions 29

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31 The Boussinesq-Green-Naghdi rotational water wave model has demonstrated ability in modelling complex
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33 2D surf zone hydrodynamics in this paper. For these test cases, the rotational system reaches comparable
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35 accuracy to other advanced Boussinesq-type models. The completely rotational structure might not contribute
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37 significantly to the depth-averaged velocities in these cases while would demonstrate great improvement in
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39 depth-varying velocities under rotational flows. And, of course, the surface elevation is also well resolved. The
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41 new idea of a polynomial expansion for phase resolving water wave theory has given an approach for other
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43 researchers in the field to get rid of partial or full irrotationality assumptions that are violated in turbulent
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45 flow.
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47 Although the whole model system is complete and working well, still there is great improvement room.
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49 The phase lag and the deviations in later time for case 2 would probably be diminished by adopting a more
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51 complete kinetic turbulent energy model, a more accurate wet-dry algorithm, and higher order 2D systems.
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53 However, the fundamental goal of this 2D demonstration work, which is to get a balance between accuracy
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55 and efficiency, has been achieved. More rotational features in 2D would be further explored in future.
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57 Most importantly, the rotational 2D model was validated by modelling wave breaking, shoaling, runup
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59 and nearshore flows in the presence of complex hydrodynamic conditions. The model performs stably and
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61 have comparable computational efficiencies according to the numerical construction as other Boussinesq-
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type models. Besides these benchmarked experimental validations, we are moving forward to the real world engineering and geophysical problems, which greatly affect human activities (Tomiczek et al., 2014).

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Appendices

A Integral Functions

All indefinite integrals will be assumed to have integration constants defined to give values of 0 at $q = 0$.

Thus, for example, $g_n|_0^{q=1} = g_n|_{q=1}$.

Figure 1: Conical island case setup (Briggs et al., 1995). Stars indicate gauge locations.

$$\begin{aligned}
 g_n &= \int f_n dq & r_n &= \int f'_n q dq & G_n &= \int g_n dq & \psi_{mn\nu} &= \int f'_m f'_n f_\nu dq \\
 R_n &= \int r_n dq & \phi_{mn} &= \int f_m f_n dq & \gamma_{mn} &= \int f_m g_n dq & \zeta_{mn} &= \int f_m f'_n dq \\
 \rho_{mn} &= \int f_m r_n dq & \Gamma_{mn} &= \int f_m G_n dq & \Theta_{mn} &= \int f_m R_n dq & E_{mn} &= \int f'_m f'_n dq \\
 \theta_{mn} &= \int f_m g_n q dq & \nu_m &= \int q^2 f_m dq & S_m &= \int q f_m dq & K_{mn} &= \int f'_m f'_n q^2 dq \\
 \varepsilon_{mn} &= \int f_m f'_n q dq & \Psi_{mn} &= \int f_m f_n q dq & F_{mn} &= \int f_m r_n q dq & J_{mn} &= \int f'_m f'_n q dq
 \end{aligned}$$

Figure 2: Time series at gauge locations for case A ($H/h=0.05$) (- -) measured; (-) computed. (Briggs et al., 1995)

Figure 3: Time series at gauge locations for case B ($H/h=0.1$) (---) measured; (—) computed. (Briggs et al., 1995)

Figure 4: Time series at gauge locations for case C ($H/h=0.2$) (- -) measured; (—) computed. (Briggs et al., 1995)

Figure 5: Snapshots throughout the tsunami's rushing over a conical island in case C ($H/h=0.2$). (Briggs et al., 1995)

(a) experimental setup

(b) 3D bathymetry view

Figure 6: (a) Bathymetry contours for solitary wave runup on a shelf with a conical island case (Lynett et al., 2010). (circles) pressure gauges; (stars) ADVs. ; (b) 3D bathymetry.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 7: Snapshots throughout the runup process ($H/h=0.5$; Lynett et al., 2010).

Figure 8: Time series of surface elevation at gauge locations ($H/h=0.5$; Lynett et al., 2010), (---) measured; (—) computed.

Figure 9: Time series of velocities at ADV locations ($H/h=0.5$; Lynett et al., 2010), (--) measured; (—) computed.

Figure 10: Experiment topography for rip current test (Haller et al., 2002).

Figure 11: Time-averaged current field. (a): computed vorticity and velocity field, yellow: positive, Blue: negative; (b): experimental velocity data; (c): computed velocity at data locations (Haller et al., 2002). Vectors at upper left equal 0.2 m/s in (b) and (c).

Figure 12: Time-averaged surface elevations at four longshore sections (Haller et al., 2002). (*) measured; (–) computed.

Figure 13: Time-averaged cross-shore velocities at four longshore sections (Haller et al., 2002). The ADV measurements were made 3 cm from the bottom. (*) measured; (–) computed.

Figure 14: Time-averaged longshore velocities at four longshore sections (Haller et al., 2002). The ADV measurements were made 3 cm from the bottom. (*) measured; (–) computed.