A time-domain strip theory approach to maneuvering in a seaway

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Abstract

A time-domain body exact strip theory is developed to predict maneuvering of a vessel in a seaway. A frame following the instantaneous position of the ship, by translating and rotating in the horizontal plane, is used to set up the Boundary Value Problem (BVP) for the perturbation potentials. A boundary integral technique is used for solving the Laplace equation. Linearized free surface boundary conditions are used for stability and computational efficiency, and exact body boundary conditions are used to capture nonlinear effects. A nonlinear rigid body equation of motion solver is coupled to the hydrodynamic model to predict ship responses.

Results are presented for the turning circle maneuver of the containership S-175 in calm water and in the presence of regular waves. The results are compared with available experimental results. The simulations are able to capture general qualitative aspects and overall physics of the problem.

Keywords: S175, Maneuvering in a seaway, Turning circle test, Body-exact strip theory, Time-domain simulation

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1. Introduction

Ship maneuvering and seakeeping have traditionally been dealt with as separate sub problems. Maneuvering is predicted in calm waters and seakeeping has to do with the response of the ship in waves. These give very important information to the naval architect during the initial stages of design. But in reality, they are coupled in nature. Presence of waves are known to effect course keeping and maneuvering performance of a ship by way of wave induced drift forces. On the other hand, maintaining a given course can induce severe ship motions, increase resistance and decrease propulsive efficiency and speed.

Mathematical models have been used by several authors to study maneuvering of ships in a seaway. *Hirano et al.* (1980) used three-dimensional equations of motion in calm water to predict maneuvering performance by computing only wave drift forces. *McCreight* (1986) developed a maneuvering model in waves, in which the hydrodynamic forces were evaluated in a body-fixed coordinate system. The hydrodynamic coefficients were computed by linear strip-theory. *Ottosson and Bystrom* (1991) used a more simplified approach, where the hydrodynamic radiation coefficients were assumed to be constant based on mean encounter frequency during maneuvering motion. *Fang et al.* (2005) developed a mathematical model taking into account the frequency of encounter into the time-domain simulation.

The models mentioned above do not take into account the memory effect due to ship motions. One popular approach to do so is the use of linear convolution integrals of *Cummins* (1962). *Bailey et al.* (1997) and *Fossen* (2005) have developed unified models for maneuvering in a seaway, using this

approach. Recently *Skejic and Faltinsen* (2008) have developed a unified 4-DOF maneuvering model in which the mean 2nd order wave loads were added using a direct pressure integration scheme. A two time scale approach was used in their methodology where the high frequency seakeeping was separated from the low frequency maneuvering problem. *Lin et al.* (2006) and *Yen et al.* (2010) solved the unified problem using a three-dimensional panel method. Their study was basically an extension of the nonlinear ship motion prediction code LAMP (Large Amplitude Motion Program). Recently, *Seo and Kim* (2011) extended the time-domain motion program WISH (computer program for nonlinear Wave Induced load and SHip motion analysis) based on a B-spline Rankine panel method to couple seakeeping and maneuvering. Second order wave drift forces were computed by direct pressure integration and a modular-type maneuvering model (MMG) is integrated with the seakeeping model.

Experimental methods, although an important tool to estimate wave effects on ship maneuvering and to validate mathematical models and numerical methods, are difficult to conduct, very expensive and can suffer from scale effects. Nonetheless, several results have been presented using free-running model tests. *Hirano et al.* (1980) performed turning circle maneuvers in regular waves. *Ueno et al.* (2003) have carried out turning, zig-zag and stopping tests with a VLCC model. Recently, *Yasukawa and Nakayama* (2009) performed turning circle tests of the containership S-175 in both calm water as well as in waves for a variety of incident wave frequencies, amplitudes and headings.

There has been a lot of recent interest in CFD based methods. Although

the increase in modern computational power has made it feasible to solve the viscous flow problem in the time-domain, as is done in unsteady Reynolds-Averaged Navier-Stokes (RANS) codes, the computational effort still remains enormous.

This paper presents the development of a unified model for predicting the maneuvering of a ship in a seaway by using a time-domain, strip theory approach. A hydrodynamic frame following the instantaneous position of the ship in the horizontal plane is used to set up the BVP. This has the advantage that the path and forward speed of the ship need not be described in advance, rather all 6 degrees of freedom are determined by solving the equations of motion.

The strip theory approach allows for faster computational times and simplified body geometry definition. The use of a blended method ensures that vital nonlinearities are captured while keeping the computational time reasonable. The free surface boundary conditions are linearized. The body boundary conditions in the current formulation are as follows. The incident wave and hydrostatic loads are computed up to the intersection between the exact body position and exact incident wave surface. The radiation and diffraction components of the pressure are integrated up to the intersection of the mean free surface and the exact instantaneous body surface.

At present, the computer code, although not optimised for speed, is capable of simulation speeds of about 1.5 to 2 times of real time on a typical CPU (Intel Core i5, 2.5 GHz processor and 4 GB RAM).

2. Problem Formulation

2.1. Definition Sketch

The problem definition is as shown in Figure 1 where a freely floating rigid body such as a ship is moving either in calm water or in the presence of external waves. The objective is to predict the hydrodynamic forces acting on the ship and the resultant motions and trajectory of such a body.

Three different coordinate systems are used for solving the problem; an earth fixed inertial axis (x_e, y_e, z_e) is used to keep track of the position of the centre of gravity of the ship and the Euler angles. A hydrodynamic frame (x_h, y_h, z_h) translates in the horizontal calm water plane with translational velocities U, V and rotational yaw rate $\dot{\psi}$. It thus follows the ship such that its origin O_h is always in vertical line with the origin of the body frame, O_b . This is the frame in which the boundary value problem is formulated. A body fixed frame (x_b, y_b, z_b) rotates and translates in all 6-DOF with the body. The frame is used to compute the forces acting on the ship and to solve for the equations of motions.

The velocities U, V and $\dot{\psi}$ are the instantaneous body velocities resolved in the hydrodynamic frame.

2.2. Mathematical Formulation

The fluid flow is considered inviscid, irrotational, incompressible and unsteady. For such a flow, a velocity potential Φ , representing the perturbation potential for the absolute fluid velocity in the earth fixed frame is defined. The fluid particle velocity $\mathbf{v}$ can be written as:

$$\mathbf{v} = \nabla \Phi \tag{1}$$

Figure 1: Schematic showing the different coordinate frames used

From Equation (1), the continuity equation for the conservation of mass of the fluid reduces to the Laplace's equation

$$\nabla^2 \Phi = 0 \quad (2)$$

The linearized kinematic and dynamic free surface boundary conditions written in the moving hydrodynamic frame are,

$$\frac{\partial \eta}{\partial t} = \frac{\partial \Phi}{\partial z} + (\mathbf{U} + \dot{\Psi} \times \mathbf{r}_h) \cdot \nabla \eta \quad \text{on } z = 0 \quad (3)$$

and

$$\frac{\partial \Phi}{\partial t} = -g\eta + (\mathbf{U} + \dot{\Psi} \times \mathbf{r}_h) \cdot \nabla \Phi \quad \text{on } z = 0 \quad (4)$$

Here, η and $\mathbf{r}_h$ represent the free surface wave elevation measured from the calm water surface, and the position vector in the hydrodynamic frame respectively.

The body boundary condition ensures no normal flow through the body surface:

$$\nabla\Phi \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n} \quad \text{on } S_B(t) \quad (5)$$

where Φ is the total three-dimensional perturbation potential, $\mathbf{v}$ is the absolute velocity of a node on the body surface with respect to the earth fixed frame including velocities due to rotational effects; $\mathbf{n}$ is the unit normal vector positive out of the fluid (or into the body), and $S_B(t)$ is the exact wetted body surface.

The boundary conditions at ∞ will be discussed in detail in the following sections where the two-dimensional strip theory boundary value problem is described.

2.3. Strip Theory Formulation

The above formulations are defined for a fully three-dimensional flow field. If it is assumed that the ship is slender such that the slope of the body surface in the longitudinal direction is smaller than the slopes in the transverse direction, the gradient $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$. This forms the basis for the strip theory approximation, where the three-dimensional problem is solved as a series of individual two-dimensional problems.

2.3.1. Strip theory boundary value problem

Based on the slender-body approximation, the three-dimensional problem is reduced to a series of two-dimensional problems at each station of the ship:

$$\nabla^2 \phi(y, z, t; x) = 0 \quad (6)$$

Here $\phi = \phi(y, z, t; x)$ shall henceforth refer to the two-dimensional potential for notational convenience.

The free surface boundary conditions take the following form from Equations (3) and (4):

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} + V \frac{\partial \eta}{\partial y} + x_h \dot{\psi} \frac{\partial \eta}{\partial y} \quad \text{on } z = 0 \quad (7)$$

$$\frac{\partial \phi}{\partial t} = -g\eta + V \frac{\partial \phi}{\partial y} + x_h \dot{\psi} \frac{\partial \phi}{\partial y} \quad \text{on } z = 0 \quad (8)$$

Consistent with strip theory, the encounter frequency is assumed high such that, $\frac{\partial}{\partial t} \gg U \frac{\partial}{\partial x}$ and the downstream free surface effects are ignored.

In the far field, a radiation boundary condition is imposed such that only the incident waves are incoming. This is done numerically by incorporating an outer damping beach and modifying the free surface boundary conditions. The details for this will be explained in the sections dealing with numerical schemes (Section 3) . All simulations are carried out in deep water and so the gradient of the perturbation potential vanishes as $z \rightarrow -\infty$. This is automatically taken care of by the selection of the Green function, which will be shown in the following section.

2.3.2. Velocity Potential Decomposition and Body Boundary Conditions

The perturbation potential can be broken down into different components for proper bookkeeping:

$$\phi(y, z, t; x) = \phi_I + \phi_D + \phi_R$$

where, ϕ_I , ϕ_D , ϕ_R refer to the incident, diffracted and radiated potential, respectively.

The incident wave potential and elevation are known at all time and the linear Airy wave theory is used. The analytical expressions for the potential and wave elevation with respect to the earth fixed inertial frame are:

$$\phi_I = \frac{iga}{\omega_0} e^{-ik(x_e \cos \beta + y_e \sin \beta)} e^{i\omega_0 t} e^{k(z-\eta)} \quad (9)$$

$$\eta_I = ae^{-ik(x_e \cos \beta + y_e \sin \beta)} e^{i\omega_0 t} \quad (10)$$

Here, a , ω_0 , k , λ , and β refer to the wave amplitude, frequency, wave number, wavelength and the heading angle relative to earth fixed x-axis, respectively. The incident wave potential formulation also includes the Wheeler stretching term.

The body boundary condition (5) is re-written in terms of it's individual components in the two-dimensional frame

$$\nabla \phi_D \cdot \mathbf{N} = -\nabla \phi_I \cdot \mathbf{N} \quad \text{on } S_B(t) \quad (11)$$

$$\nabla \phi_R \cdot \mathbf{N} = \mathbf{v} \cdot \mathbf{N} \quad \text{on } S_B(t) \quad (12)$$

Here the two-dimensional strip theory unit normal $\mathbf{N}$ is used. $S_B(t)$ is the instantaneous wetted body surface. The details of the blending scheme are given in Section 3.4.

The velocity $\mathbf{v}$ used in Equation (12) is the velocity of a node on the body surface with respect to the earth fixed frame and includes all the three translational and rotational components, namely (u, v, w, p, q, r) . Since this is a strip theory formulation, care has to be taken not to include the steady forward speed component of the velocity into the body boundary condition, as this represents a steady flux of fluid into the two-dimensional domain and can cause continuity issues. In any case, solving the steady forward speed problem in a strip-theory formulation is of little use. In order to circumvent this problem, the following procedure is used. The velocity $\mathbf{v}$ can be assumed to consist of a time-varying mean component $\bar{\mathbf{v}}(t)$, and an oscillatory component $\tilde{\mathbf{v}}(t)$, such that

$$\mathbf{v} = \bar{\mathbf{v}}(t) + \tilde{\mathbf{v}}(t) \quad (13)$$

The mean component is computed at every time-step by taking the average of the body velocities over the last number of time-steps

$$\bar{\mathbf{v}}(t) = \frac{\sum_{j=i-N+1}^{i-1} \mathbf{v}(t_j)}{T_{interval}} \quad (14)$$

where i is the current time level, j is the counter variable, $T_{interval}$ is the length of the interval for averaging, and N is the number of time-steps in the interval. $T_{interval}$ is chosen as the period of the incident waves w.r.t the inertial frame. It is important to note that the averaging is done only in the x-direction, and other degrees are untouched and the velocities are fully solved for.

The modified body boundary condition is as follows

$$\nabla \phi_R \cdot \mathbf{N} = (\mathbf{v} - \mathbf{U}_{avg}) \cdot \mathbf{N} \quad \text{on } S_B(t) \quad (15)$$

where $\mathbf{U}_{avg}$ is the mean surge component that is not solved for and given by $(\bar{u}, 0, 0, 0, 0, 0)^T$.

2.4. Boundary Integral Method

To solve for the two-dimensional problem, Green's theorem is used with respect to the potential to write the fluid flow problem as a set of boundary integral equations. The source distribution method is used in this formulation and the perturbation potential is given by:

$$\phi(\mathbf{x}) = \int_C \sigma(\boldsymbol{\xi}) G(\mathbf{x}; \boldsymbol{\xi}) dC \quad (16)$$

where,

C = contour defining the perimeter of the 2-D boundary,

$\mathbf{x}$ = point anywhere in the fluid domain,

$\boldsymbol{\xi}$ = position of source points on boundary C ,

σ = unknown source strength,

G = Green function satisfying Laplace's equation

The two-dimensional Rankine source Green function used is

$$G(\mathbf{x}; \boldsymbol{\xi}) = \ln |\mathbf{x} - \boldsymbol{\xi}| \quad (17)$$

and has the following attributes

$$\nabla^2 G = 2\pi\delta(\mathbf{x} - \boldsymbol{\xi}) \quad (18)$$

$$\nabla G \rightarrow 0 \quad \text{as} \quad \mathbf{x} \rightarrow \infty \quad (19)$$

where δ is the Dirac delta function. The source strengths are determined such that they satisfy the boundary conditions. Once they are determined, the spatial derivatives of the potential can be found directly by using the derivative of Equation (16).

2.5. Forces and Moments

Applying the potential theory formulation, the Euler equations representing the momentum conservation equations reduce to the unsteady Bernoulli equation. The pressure equation derived in the translating and rotating hydrodynamic frame, takes the form:

$$\frac{p}{\rho} + \left. \frac{\partial \phi}{\partial t} \right|_h - \mathbf{U} \cdot \nabla \phi - (\dot{\mathbf{\Psi}} \times \mathbf{r}_h) \cdot \nabla \phi + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gz = c(t) \quad (20)$$

where, $\left. \frac{\partial \phi}{\partial t} \right|_h$ represents the temporal derivative of the potential taken with respect to the hydrodynamic frame.

Alternatively, the pressure can also be written with respect to the earth fixed frame

$$\frac{p}{\rho} + \left. \frac{\partial \phi}{\partial t} \right|_e + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gz = c(t) \quad (21)$$

where $\left. \frac{\partial \phi}{\partial t} \right|_e$ represents the temporal derivative of the perturbation potential with respect to the earth fixed frame.

The pressure formulation that is used in the current method is the one written in the earth-fixed frame, given by Equation (21). The pressure is integrated on the exact body surface to obtain the sectional forces and moments acting on the ship. The sectional forces and moments are then integrated along the length of the vessel to obtain the total forces. The pressure at any

point on each of the cross-sections is given by

$$\frac{p}{\rho} + \frac{\partial \phi}{\partial t} \Big|_e + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gz = 0 \quad (22)$$

and the forces and moments are given by

$$\mathbf{F} = \int_L dx \int_{C_H} p \mathbf{N} dl \quad (23)$$

$$\mathbf{M} = \int_L dx \int_{C_H} p \mathbf{r} \times \mathbf{N} dl \quad (24)$$

Similar to the potential, the pressure is segregated into different components.

To avoid numerical differencing, a separate BVP is set up for the term $\frac{\partial \phi}{\partial t}$ known as the acceleration potential formulation. The major advantage is that it uses the same influence matrix as that of the ϕ problem. The other advantage is that it is stable and not sensitive to the time-step size when computing the pressure on the body, since $\frac{\partial \phi}{\partial t}$ is solved for directly. Brief details are given Section 2.7.

2.6. Equations of Motion

Once the potentials, pressure on the hull and forces and moments acting on the body are determined, using the position and velocity of the body, the equations of motion (EOM) are set up. The formulation here basically uses the convention followed by *Fossen* (1994).

The forces and moments on the ship are described in the body frame, and the equations of motion will be solved in the body frame. The origin of the body frame O_b is set at midships and is such that at $t = 0$, it is at the calm water line. The EOM can be written in compact vector form about O_b

Figure 2: Body coordinate system showing sign conventions and the definitions of the 6-D.O.F (Illustration taken from johnclarkeonline.com)

as follows:

$$m[\dot{\mathbf{v}}_b + \dot{\boldsymbol{\omega}} \times \mathbf{r}_g + \boldsymbol{\omega} \times \mathbf{v}_b + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_g)] + \mathbf{m}_1^* \dot{\mathbf{v}}_b + \mathbf{m}_2^* \dot{\boldsymbol{\omega}} = \mathbf{F} \quad (25)$$

$$\mathbf{I}_b \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}_b \boldsymbol{\omega} + m \mathbf{r}_g \times (\dot{\mathbf{v}}_b + \boldsymbol{\omega} \times \mathbf{v}_b) + \mathbf{m}_3^* \dot{\mathbf{v}}_b + \mathbf{m}_4^* \dot{\boldsymbol{\omega}} = \mathbf{M} \quad (26)$$

where, $\mathbf{I}_b$ represents the generalised moment of inertia matrix, m is the mass of the ship, $\mathbf{v}_b$ is the translational velocity of the body, $\boldsymbol{\omega}$ is the angular velocity of the body, and $\mathbf{r}_g$ is the position vector of the CG. The forces acting on the body are referred to as $\mathbf{F}$ and the external moments taken about the body axis are given by $\mathbf{M}$.

The generalised radiation impulsive added mass matrix, $\mathbf{m}^*(t)$ represents one part of the solution of the radiation acceleration potential formulation. The acceleration potential formulation is described in Section 2.7. The generalised force vector can be described by $\mathbf{F} = (\mathbf{F}, \mathbf{M})$. This can be written out in terms of the different components:

$$\mathbf{F} = \mathbf{F}^{FK} + \mathbf{F}^{DIFF} + \mathbf{F}^{RAD} + \mathbf{F}^S + \mathbf{F}^{HS} + \mathbf{F}^{VISC} + \mathbf{F}^{PROP} + \mathbf{F}^{CALM} + \mathbf{F}^{RUDDER} \quad (27)$$

where,

F^{FK} = exact Froude-Krylov incident wave force

F^{DIF} = diffracted wave force

F^{RAD} = radiated wave memory force

F^S = force due to $\nabla\phi \cdot \nabla\phi$ term in the pressure equation

F^{HS} = exact hydrostatic force

F^{VISC} = empirical viscous forces in roll, sway and yaw DOF

F^{PROP} = propeller thrust force

F^{CALM} = calm water resistance

F^{RUDDER} = rudder force

The vessel's position and orientation are described with respect to the earth fixed coordinates by three translations and three rotations, given by ξ .

$$\xi = [\xi_1, \xi_2, \xi_3, \phi, \theta, \psi]^T \quad (28)$$

Here, $\{\xi_1, \xi_2, \xi_3\}$ represents the three translations and $\{\phi, \theta, \psi\}$ represents the three Euler angles in roll, pitch and yaw respectively. The velocities in the earth fixed frame are obtained from the body velocities by using the standard transformation matrix.

2.7. The Acceleration Potential

The pressure equation (22) requires the evaluation of the temporal derivative $\frac{\partial\phi}{\partial t}\big|_e$. In this methodology, a direct formulation is used to set up a BVP for this term, called the “acceleration potential”, φ .

This method has a number of advantages. Firstly it allows for direct solution at each time-step, instead of relying on numerical differencing. It also avoids numerical difficulties with repanelization techniques. Secondly it can be written in terms of the body accelerations $\dot{\mathbf{v}}$. It can therefore be moved to the other side of the rigid body equations of motion. Overall it improves stability.

The formulation used here is based on the work of *Bandyk* (2009). Detailed background study, derivations, validations and comparison with other similar formulations can be found in *Bandyk* (2009) and *Bandyk and Beck* (2011).

The idea is that a BVP can be set up for φ in a similar manner as is done for the velocity potential ϕ , since the acceleration potential also satisfies the Laplace equation. The deep water and far field boundary conditions are satisfied in the same manner as in the velocity potential problem.

The body boundary condition for the diffraction component is,

$$\mathbf{N} \cdot \nabla \varphi_e^D = -\mathbf{N} \cdot \nabla \varphi_e^I - (\boldsymbol{\omega} \times \mathbf{N}) \cdot (\nabla \phi^D + \nabla \phi^I) - \mathbf{N} \cdot [(\mathbf{v} \cdot \nabla)(\nabla \phi^D + \nabla \phi^I)] \quad (29)$$

and that for the radiation component is,

$$\mathbf{N} \cdot \nabla \varphi_e^R = \{\dot{\mathbf{v}}_{\mathbf{b}} \cdot \mathbf{N} + (\dot{\boldsymbol{\omega}} \times \mathbf{r}_{\mathbf{b}}) \cdot \mathbf{N}\} - (\boldsymbol{\omega} \times \mathbf{N}) \cdot \nabla \phi^R - \mathbf{N} \cdot [(\mathbf{v} \cdot \nabla)(\nabla \phi^R)] \quad (30)$$

The free surface boundary condition is straightforward and follows directly from the dynamic free surface boundary condition in Equation (8) for the ϕ problem.

$$\varphi_e|_{FS} = -g\eta \quad \text{on } z = 0 \quad (31)$$

It is to be noted that the entire formulation is set up with respect to the earth fixed frame. It could alternatively be derived in the hydrodynamic frame.

The radiation acceleration potential φ^R is split into two components, the impulsive $\varphi^{R, Imp}$ and memory $\varphi^{R, Memory}$, subject to the following boundary conditions

$$\varphi^R = \varphi^{R, Imp} + \varphi^{R, Memory} \quad (32)$$

$$\mathbf{N} \cdot \nabla \varphi^{R, Imp}_e = \dot{\mathbf{v}}_b \cdot \mathbf{N} + (\dot{\boldsymbol{\omega}} \times \mathbf{r}_b) \cdot \mathbf{N} \quad \text{on} \quad S_B(t) \quad (33)$$

$$\mathbf{N} \cdot \nabla \varphi^{R, Memory}_e = -(\boldsymbol{\omega} \times \mathbf{N}) \cdot \nabla \phi^R - \mathbf{N} \cdot [(\mathbf{v} \cdot \nabla)(\nabla \phi^R)] \quad \text{on} \quad S_B(t) \quad (34)$$

$$\varphi^{R, Imp} = 0 \quad \text{on} \quad S_{FS} \quad (35)$$

$$\varphi^{R, Memory} = \varphi^R_e|_{FS} \quad \text{on} \quad S_{FS} \quad (36)$$

The body boundary conditions are applied on the instantaneous wetted body surface, $S_B(t)$. It is observed that the impulsive component is only a function of body accelerations and is reduced into six canonical problems. For notational convenience, let χ represent $\varphi^{R, Imp}$. Then χ can be represented as follows

$$\chi = \sum_{i=1}^6 \dot{v}_i \chi_i \quad (37)$$

where $\dot{v}_i$ and χ_i are the acceleration, and the acceleration potential respectively due to a unit body acceleration in the i^{th} mode of motion, respectively. They are such that they satisfy the following boundary conditions

$$\mathbf{N} \cdot \nabla \chi_i = N_i \quad \text{for} \quad (i = 1, \dots, 6) \quad \text{on} \quad S_B(t) \quad (38)$$

$$\chi_i = 0 \quad \text{for} \quad (i = 1, \dots, 6) \quad \text{on} \quad S_{FS} \quad (39)$$

The solution of the χ_i problem produces the $\mathbf{m}^*$ impulsive added mass matrix seen in Section 2.6. The components are given by

$$m_{ij}^* = \rho \int_L dx \int_{S_B(t)} \chi_j N_i dl \quad (40)$$

These terms are then moved to the other side of equations (25) and (26), in order to stabilize the equations of motion time-stepping scheme. On the other hand, the solution of $\varphi^{R, \text{Memory}}$ goes directly into the pressure equation and is integrated over the entire body to give F^{RAD} , the radiation memory force in Equation (27).

2.8. External Forces

The external forces acting on the body constitute the last four terms in the Equation (27), namely the viscous forces in roll, sway and yaw, propeller thrust force, calm water resistance and forces from control surfaces such as rudder, fins or thrusters.

2.8.1. Viscous Forces

Potential theory does not capture the effects of viscosity. Seakeeping problems are dominated by inertial effects. However, there are scenarios where viscosity plays a vital role. This is especially true of vortical flows characterized by separation such as roll damping. Crucially, slow motion maneuvering is viscous dominated since potential theory predicts zero damping force at the zero frequency limit.

Additional roll viscous damping is used in this formulation. The model is based on the empirical corrections made by *Himeno* (1981). The amount

of damping is a function of the hull form, the section shape, forward speed and frequency.

The maneuvering coefficients used in the present work, for the maneuvering of the container ship S-175 in calm water and waves, are the PMM based derivatives given in *Son and Nomoto* (1981).

$$X_{visc} = X_{vr}vr + X_{vv}v^2 + X_{rr}r^2 \quad \text{surge damping} \quad (41)$$

$$Y_{visc} = Y_vv + Y_rr + Y_{vvv}v^3 + Y_{rrr}r^3 + Y_{vvr}v^2r + Y_{vrr}vr^2 \quad \text{sway damping} \quad (42)$$

$$N_{visc} = N_vv + N_rr + N_{vvv}v^3 + N_{rrr}r^3 + N_{vvr}v^2r + N_{vrr}vr^2 \quad \text{yaw damping} \quad (43)$$

where X_{visc} , Y_{visc} and N_{visc} are the viscous damping forces in surge, sway and yaw respectively. These terms would show up on the right hand side of a typical modular-type mathematical maneuvering model (MMG model). The forces proportional to the body accelerations, namely the added mass terms are accounted for by potential theory.

2.8.2. Propeller Thrust Force

The thrust model used in the current method is the one presented in *Son and Nomoto* (1981). The thrust force is given by

$$T_{prop} = (1 - t)\rho n^2 D^4 K_T \quad (44)$$

where T_{prop} , t , n , D and K_T are the propeller thrust force, the thrust-deduction fraction, propeller revolutions per second, propeller diameter, and thrust coefficient, respectively. The thrust coefficient, K_T is expressed as a

linear function of the advance ratio, J

$$K_T = 0.527 - 0.455J \quad (45)$$

$$J = u'_P U_{net} / nD \quad (46)$$

where u'_P represents the effective wake fraction given by

$$u'_P = \cos v' [(1 - w_p) + \tau(v' + x'_p r')^2] \quad (47)$$

Here, w_p is the wake fraction in the straight-ahead condition, v' and r' are the non-dimensional sway and yaw velocities, x'_p is the x-location of the propeller plane, and τ is a parameter to account for the lateral velocity components.

2.8.3. Calm water Resistance

The calm water resistance is estimated according to the quadratic curve fit for low Froude numbers, given in *Son and Nomoto* (1981)

$$X(u) = X'_{u|u|} 0.5 \rho L^2 u |u| \quad (48)$$

where $X'_{u|u|}$ is the total calm water resistance coefficient at model scale set at a constant value of $X'_{u|u|} = -0.0004226$ for the containership S-175. This model is deemed sufficient for the low Froude number, $F_n = 0.15$, that is used for the maneuvering simulations in the present research. Consistent with the definition of all body velocities, u represents the instantaneous velocity which includes both the oscillatory as well as the time varying mean component.

2.8.4. Rudder Forces

The hydrodynamic forces acting on the ship due to rudder action is modeled according to *Son and Nomoto* (1981) and is written as follows:

$$X_{Rudder} = c_{RX} F'_N \sin \delta' \quad (49)$$

$$Y_{Rudder} = (1 + a_H)F'_N \cos \delta' \quad (50)$$

$$K_{Rudder} = -(1 + a_H)F'_N \cos \delta' \times z'_R \quad (51)$$

$$N_{Rudder} = (1 + a_H(x'_H/x'_R))F'_N \cos \delta' \times x'_R \quad (52)$$

where X_{Rudder} , Y_{Rudder} , K_{Rudder} , and N_{Rudder} are the non-dimensional rudder forces in surge, sway, roll and yaw respectively. Variables δ' , x'_R , and z'_R represent the rudder angle, and the x- and z-directional centre of normal force acting on the rudder, respectively. Parameters c_{RX} , a_H , and x'_H represent the interaction of the hull and rudder which can be obtained experimentally or through empirical methods. The rudder normal force F'_N is expressed as follows:

$$F'_N = -\frac{6.13\Lambda}{\Lambda + 2.25} \cdot \frac{A_R}{L^2}(u_R'^2 + v_R'^2) \sin \alpha_R \quad (53)$$

$$\alpha_R = \delta + \tan^{-1}(v'_R/u'_R) \quad (54)$$

here A_R and Λ represent the area and aspect ratio of the rudder, respectively. α_R , u'_R , and v'_R represent the effective inflow angle, and the x- and y- components of the effective inflow velocity into the rudder, respectively. The effective inflow speed and angle are affected by the hull wake, the flow straightening effect of the hull, propeller and ship motions. In the presence of incident waves, the wave particle velocities could also have a significant effect on the rudder forces. To account for these effects, semi-empirical formulas are used. In the current method, the experimental data and coefficients used in *Son and Nomoto* (1981) for the calm water condition are used.

3. Numerical Techniques

3.1. Source Distribution Formulation

The mixed boundary value problem is solved for the perturbation potentials ϕ_R and ϕ_D and their derivatives. In the present work, a source distribution technique is used. Desingularised sources are placed above the free surface nodes and constant strength panels are used on the body. The potential at any point on the fluid is then given by:

$$\phi_i = \sum_{S_F} \sigma_j G(\mathbf{x}_i; \boldsymbol{\xi}_j) + \int_{S_B} \sigma_j G(\mathbf{x}_i; \boldsymbol{\xi}_j) dl \quad (55)$$

where,

ϕ_i = velocity or acceleration potential

$\mathbf{x}_i$ = field point anywhere in the domain

$\boldsymbol{\xi}_j$ = location of a source point

σ_j = source strength of source point

S_B = instantaneous hull contour

S_F = free surface contour

$G(\mathbf{x}_i; \boldsymbol{\xi}_j)$ = Rankine source Green function, given by

$$= \ln r$$

$r = |\mathbf{x}_i - \boldsymbol{\xi}_j|$ (the distance between a field point and a source point)

The first term on the RHS of Equation (55) is the contribution of the isolated desingularised free surface sources and the second term the influence from the panels on the body.

Applying the Dirichlet and Neumann boundary conditions on the free surface and body respectively, and using Equation (55), the integral is discretized to form a system of linear equations to be solved for the source strengths σ_j .

$$\sum_{S_F} \sigma_j \ln |\mathbf{x}_i - \boldsymbol{\xi}_j| + \int_{S_B(t)} \sigma_j \ln |\mathbf{x}_i - \boldsymbol{\xi}_j| dl = \phi_i \quad \mathbf{x}_i \in S_F \quad (56)$$

$$\sum_{S_F} \sigma_j \frac{\partial \ln |\mathbf{x}_i - \boldsymbol{\xi}_j|}{\partial N} + \int_{S_B(t)} \sigma_j \frac{\partial \ln |\mathbf{x}_i - \boldsymbol{\xi}_j|}{\partial N} dl = \frac{\partial \phi_i}{\partial N} \quad \mathbf{x}_i \in S_B \quad (57)$$

These equations (56) and (57) can be written in discretized matrix form, $A_{ij}\sigma_j=b_i$, where A_{ij} is the influence matrix, which is determined from the LHS of the equations, σ_j are the unknown source strengths, and b_i the RHS of the equations. The matrix is inverted using a LU decomposition technique.

Once the source strengths are determined, the potentials and their derivatives are determined on the body and free surface.

3.2. Domain Discretization

3.2.1. Body panelization

The body is modeled using flat panels as they are more capable of handling complex body geometries. The body panel size is fixed for the entire simulation, i.e the panelization is done only once, using a preprocessor. This ensures that the size of the body panels are maintained about the same as that of the free surface resolution, which is important from a numerical point of view.

At each time step, the intersection of the body with the free surface is determined to obtain the wetted panels that would be used to solve the

B.V.P. To compute the pressure on the panel that intersects the free surface, only the contribution from the wetted portion of the panel is considered.

3.2.2. Free Surface Discretization

On the free surface, desingularised sources are distributed above the free surface nodes. The desingularization distance, L_d is given by

$$L_d = l_d \Delta s^\alpha \quad (58)$$

where l_d is set to 3.0 and α to 0.5 based on the study by *Cao et al.* (1991). In the present formulation, Δs is the internodal spacing on the free surface and scaled by the body panel size.

The free surface is divided into two segments, the inner region which resolves the waves and an outer region representing the numerical beach. The length of the domain is set based on the wavelength of the incident waves. Typically, two wavelengths are used for the inner region and two wavelengths for the beach on each side of the station. To resolve the radiated waves, the number of nodes used per wavelength is usually $\frac{\lambda}{30}$.

The numerical beach is important to satisfy the far field boundary condition and ensure there are no wave reflections. It is thus a crucial element to performing stable long time simulations. The details of the beach will be discussed in Section 3.3.

3.3. Time Integration

The free surface elevation and potentials are evolved in time by integrating the free surface boundary conditions given by Equations (7) and (8). In the body exact formulation, as the body moves relative to the water surface,

the free surface nodes are also displaced horizontally. The rate of displacement is termed the “moving node velocity”, and the integrations include this contribution.

$$\frac{\delta\eta}{\delta t} = \frac{\partial\phi}{\partial z} + V\frac{\partial\eta}{\partial y} + x_h\dot{\psi}\frac{\partial\eta}{\partial y} + v_m\frac{\partial\eta}{\partial y} - \mu\eta \quad \text{on } z = 0 \quad (59)$$

$$\frac{\delta\phi}{\delta t} = -g\eta + V\frac{\partial\phi}{\partial y} + x_h\dot{\psi}\frac{\partial\phi}{\partial y} + v_m\frac{\partial\phi}{\partial y} - \mu\phi \quad \text{on } z = 0 \quad (60)$$

Here $\frac{\delta}{\delta t}$ refers to the time derivative taken by following a free surface node. v_m is the velocity of the moving node. The last terms in the equations (59) and (60) are the damping terms that are added to generate artificial damping at the beach.

The beach is based on the work by *Cointe et al.* (1990). The damping coefficient μ is given by

$$\mu(y) = \alpha\omega \left(\frac{y - y_0}{\lambda} \right)^2, \quad y_0 \leq y \leq y_0 + \beta\lambda \quad (61)$$

where y_0 is the location of the start of the beach, ω and λ are the angular frequency and wavelength of the incident or radiated waves. The parameters α and β are used to tune the strength of damping and length of the damping zone respectively. Based on the study by *Tanizawa* (1996), α is set to 1 and $\beta = 2$.

To numerically integrate the free surface equations (59) and (60) and the rigid body equations (25) and (26), an explicit fourth-order Adams-Bashforth scheme is used

$$F^{n+1} = F^n + \frac{\Delta t}{24}(55F_t^n - 59F_t^{n-1} + 37F_t^{n-2} - 9F_t^{n-3}) \quad (62)$$

where F denotes the function being numerically integrated, F_t is the time-derivative, and the superscripts indicate the time level at which they are

Figure 3: Details at a given station

evaluated. The quantities that are integrated using the scheme are $F = \phi, \eta$ on the free surface and $F = \mathbf{v}, \boldsymbol{\xi}$ for body equations of motion.

The time-step size is set based on the period of the wave. Typically the values of Δt used are in the range of $T/200$ to $T/100$. It also depends on the spatial resolution of the free surface. Numerical instabilities can occur if Δt is chosen to be too large. The general guideline used here is based on the stability criterion described in *Park and Troesch (1992)* and *Wang and Troesch (1997)*

$$S = \pi g \Delta t^2 / \Delta x \quad (63)$$

where S is the stability index dependent on the spatial discretization Δx and time-step size Δt . An acceptable value of S is in general dependent on the type of boundary condition and the integration scheme. As a conservative estimate, the value of $S \leq 1$ is used to ensure a stable free surface throughout the computations.

3.4. Blending Scheme

The present formulation uses a blended approach to ensure computational efficiency while capturing essential nonlinearities. The free surface boundary

conditions are linearized while the degree of nonlinearity accounted for in the body boundary conditions depend on the choice of the blending scheme.

In the present formulation, the incident wave and hydrostatic loads are computed up to the intersection of the body position and exact incident wave surface. To obtain the radiation and diffraction forces, the corresponding pressure components are integrated on the exact instantaneous body surface up to the intersection of the mean free surface.

The body-exact approach also has the ability to allow sections to completely emerge out of the water surface and re-enter. This can account for impact loads on ship sections.

3.5. Radial Basis Functions

The acceleration potential formulation requires the evaluation of the derivatives, $\frac{\partial \phi}{\partial x}$, $\frac{\partial^2 \phi}{\partial x \partial y}$ and $\frac{\partial^2 \phi}{\partial x \partial z}$. These terms are approximated using radial basis functions (RBF), by interpolating for the required quantities once the two-dimensional BVP is solved on each station. Detailed descriptions can be found in *Bandyk (2009)* and *Subramanian (2012)*.

4. Results and Discussion

The turning circle maneuvers of the containership S-175 are simulated in calm water and regular waves, and compared with experiments conducted by *Yasukawa and Nakayama (2009)*. The propeller thrust, the hull resistance, rudder forces, and viscous linear and nonlinear forces in sway and yaw, are modeled according to *Son and Nomoto (1981)*. The basic details are given in Section 2.8. The main particulars of the hull, rudder and propeller are given in Table 1.

LBP [m]	175.0
B [m]	25.4
T [m]	9.5
S [m^2]	5540
C_w	0.7080
C_B	0.5714
Rudder area [m^2]	33.04
Aspect ratio	1.8219
Propeller diameter [m]	6.533

Table 1: Main particulars of Containership S-175

As mentioned in Section 2.8, the models in *Son and Nomoto* (1981) for the S-175, make use of semi-empirical parameters and coefficients to account for hull wake, rectification of the flow from the propeller to the rudder, and to model the rudder lift forces. In the present computations, no attempt has been made to tune these parameters to obtain a better fit with experiments.

4.1. Calm Water Turning Circle Test

At $t = 0$, the propeller is turned on and the ship is allowed to come up to the approach speed set at 6.212 m/s ($F_n = 0.15$). The propeller rps (revolutions/sec) to achieve this speed is calculated from the thrust-resistance balance and set at 0.9893 rps. In order to keep the ship on course, an automatic 4-DOF based nonlinear controller developed by *Li et al.* (2010) is used. At $t = 1200$ s, the rudder is put over at maximum deflection of $\delta = -35^\circ$ to execute a tight starboard turn. The maximum rudder deflection rate is set to $\dot{\delta} = 5^\circ/s$. The track of the ship is shown in Figure 4. The

(a) Trajectory

(b) Velocities U and V of hydrodynamic frame

(c) Yawrate $\dot{\psi}$ of hydrodynamic frame

Figure 4: Calm water turning circle maneuver of S-175, $\delta = -35^\circ$

results from the experimental data in *Yasukawa and Nakayama* (2009) are also shown for comparison. As can be seen from the plots, the agreement is quite good. The advance, transfer and steady turning diameter are in close agreement to the experimental values. Figures 4 (b) and (c) show the time-history of the translational and rotational velocities of the hydrodynamic frame as it is maneuvering.

4.2. Turning Circle Test in Regular Waves

The turning circle tests are performed for the containership S-175 in the presence of regular waves. Wave headings of $\theta = 90^\circ$ (Beam seas) and $\theta = 180^\circ$ (Head seas) are considered and λ/L is set to 0.7, 1.0, and 1.2. In order to compare with the available experimental results from *Yasukawa and Nakayama* (2009), the wave height is set equal to 3.5 m. This would correspond to $H/L = 1/50$, which implies the wave steepness changes with wavelength of the incident waves. As with the case of the calm water maneuvering test, the ship is brought up to an approach speed of 6.212 m/s ($Fn = 0.15$) and once steady state is reached ($t = 1200$ s), the rudder is put hard over and simultaneously, the waves are turned on. A brief linear ramp of two periods is used to bring the waves to full wave height.

Figures 5 and 6 show the results of the starboard ($\delta = -35^\circ$) turning circle test for $\lambda/L = 1.0$ in beam seas. The arrow in Figure 5(a) indicates the direction of the waves and the shows the track of the ship. As can be seen, the model reasonably captures the general characteristics of the turn. The ship enters into a series of steady turning circle loops, along with a net drift due to the wave drift forces. It is interesting to note that there are drift components both along and perpendicular to the direction of the wave fronts. Although

(a) Trajectory

(b) Velocities U and V of hydrodynamic frame

(c) Yawrate $\dot{\psi}$ of hydrodynamic frame

Figure 5: Turning circle of S-175 in waves, $\delta = -35^\circ$, $\lambda/L = 1.0$, Heading = Beam seas, $H/\lambda = 1/50$

(a) Trajectory

(b) Details of roll and pitch motions

(c) Details of heave motions

Figure 6: Details of roll, pitch and heave during turning maneuver, $\delta = -35^\circ$, $\lambda/L = 1.0$, Heading = Beam seas, $H/\lambda = 1/50$

the steady turning diameter, advance, transfer and tactical diameter are being over-predicted, the overall course and drift of the ship is in the right direction. Figures 5(b) and 5(c) show interesting results for the translational and rotational velocities of the hydrodynamic frame, respectively. The model captures the increase and decrease of the velocities at different phases of the turn. As can be seen, the oscillatory component of the velocities is superimposed on the time-varying mean component. Figure 6 shows the

(a) $\lambda/L = 1.0$, Heading = Head Seas,
 $H/\lambda = 1/50$

(b) $\lambda/L = 1.2$, Heading = Head Seas,
 $H/\lambda = 1/60$

(c) $\lambda/L = 1.2$, Heading = Beam Seas,
 $H/\lambda = 1/60$

(d) $\lambda/L = 0.7$, Heading = Beam Seas,
 $H/\lambda = 1/35$

Figure 7: Turning circle maneuver of S-175 waves, $\delta = -35^\circ$ (starboard turns)

details of roll, pitch and heave motions during the turning maneuver, of inset 1 in Figure 6(a). The interesting feature here is that as the ship traverses from a following sea configuration, through beam sea and into head sea, there is a shift in the frequency of encounter. This is evident from plots 6(b) and 6(c).

Figures 7 and 8 show the turning circle results for starboard and port side turns for $\lambda/L = 1.0$, $\lambda/L = 1.2$, and $\lambda/L = 0.7$ for a variety of heading

(a) $\lambda/L = 1.0$, Heading = Beam Seas,
 $H/\lambda = 1/50$

(b) $\lambda/L = 1.0$, Heading = Head Seas,
 $H/\lambda = 1/50$

(c) $\lambda/L = 1.2$, Heading = Head Seas,
 $H/\lambda = 1/60$

(d) $\lambda/L = 1.2$, Heading = Beam Seas,
 $H/\lambda = 1/60$

(e) $\lambda/L = 0.7$, Heading = Beam Seas,
 $H/\lambda = 1/35$

(f) $\lambda/L = 0.7$, Heading = Beam Seas,
 $H/\lambda = 1/50$

Figure 8: Turning circle maneuver of S-175 waves, $\delta = 35^\circ$ (port-side turns)

conditions. For $\lambda/L = 1.0$ and $\lambda/L = 1.2$, the ship is able to enter into a series of loop to loop turns and drift in the right direction. The advance, transfer and turning diameter are being over-predicted by the present method. In general, it is observed that the rudder is encountering large opposing forces that are delaying the turning rate. This seems more apparent when the ship is in a following sea configuration. It also indicates that the forward speed may be much higher than what it should be, which results in a much larger turning diameter. The other source of over-prediction could be with the prediction of the 2nd order wave drift forces and moments. The largest deviations from the experimental results occur at shorter wavelengths. This is apparent from Figures 7(d) and 8(e), which represent the starboard and portside turning maneuver for $\lambda/L = 0.7$ in beam seas, respectively. The ship is drifting by a large amount when it is in following sea condition. From Figure 7(d), it is seen that the ship in fact begins the turn pretty much in agreement with the experiments, before it turns around and enters the following sea configuration and begins drifting a long way before the rudder can again turn the ship around.

The radiation and diffraction pressures are integrated upto the intersection of the mean free surface and exact instantaneous body surface. This approach is deemed a good balance between computational accuracy and computational efficiency. However, the accuracy of this approach could drop as the wave steepness increases. Also, the effects of the diffraction forces become prominent for shorter waves. The turning tests for $\lambda/L = 0.7$ are done in relatively steep waves of $H/\lambda = 1/35$. In order to test the above hypothesis, the wave height was reduced, such that $H/\lambda = 1/50$, and the

turn simulated. The result is shown in Figure 8(f), where the ship is drifting in the right direction, and able to comfortably execute the turn. This could indicate that including the contributions of the higher order diffraction and radiation is important as wave steepness increases.

5. Conclusions

A body-exact strip theory has been developed to simulate maneuvering of a ship in a seaway. Propeller thrust force, calm water resistance, viscous forces and rudder lift forces are modeled based on semi-empirical methods, to predict the turning circle maneuver of the containership S-175 in calm waters and in the regular waves. The general qualitative aspects of the maneuver are captured by the numerical model, in particular for longer waves.

The results establish the ability of the translating and rotating hydrodynamic axis system to perform long time simulations. The results also establish the stability and robustness of the numerical scheme and the numerical damping beach by the ability to simulate hundreds of wave periods, for a variety of input conditions.

In order to make quantitative comparisons, the methodology may need improvements. In the results presented in this article, the radiation and diffraction pressures are integrated upto the intersection of the mean free surface and exact instantaneous body surface. This approach is considered a good balance between computational accuracy and computational efficiency. However, the accuracy can drop as the wave steepness increases. A complete body-exact approach taking into account the intersection of the exact body position and incident wave, may need to be implemented in the future. This

could improve not only the first order wave responses, but also the 2nd order drift force predictions.

Improved models for predicting the rudder forces and propeller rudder interactions are needed. The models that are used in the present computations are directly from the ones used in *Son and Nomoto* (1981). No attempt has been made to tune these coefficients to improve the predictions. Since the model is known to be sensitive to these parameters, it is crucial to correctly account for these factors. It is also to be noted that, the present simulations do not consider the side force from the propeller, which can be quite a significant percentage of the forward thrust.

List of Symbols

D = Propeller diameter

$\mathbf{F}$ = Total Forces on the body

G = Rankine source Green function

H = Wave height

$\mathbf{I}_b$ = Mass moment of Inertia of the body

J = Advance ratio

K_T = Propeller thrust coefficient

L = Ship length

L_d = Desingularisation distance

$\mathbf{M}$ = Total Moments on the body

m = Mass of the vessel
 $\mathbf{m}^*$ = Impulsive added mass matrix
 $\mathbf{N}$ = Two dimensional normal vector
 n = Propeller rps
 N_{visc} = Viscous maneuvering moments in yaw direction
 p = Dynamic pressure
 $\mathbf{r}_h$ = Position vector in hydrodynamic frame
 S_B = instantaneous hull contour
 S_F = free surface contour
 t = Thrust deduction fraction
 T_{prop} = Propeller thrust force
 $\mathbf{U}$ = Translational velocity of hydrodynamic frame
 $\mathbf{U}_{avg}$ = Time average of surge velocity component
 $\mathbf{v}_b$ = Translational velocity of the body
 w_p = Wake fraction
 $X(u)$ = Calm water Resistance
 X_{visc} = Viscous maneuvering forces in X direction
 $\mathbf{x}_i$ = field point anywhere in the domain
 Y_{visc} = Viscous maneuvering forces in Y direction
 δ = Rudder angle
 η = Wave elevation
 λ = Wavelength

ξ_j = location of a source point

ρ = Density of water

σ_j = source strength of source point

Φ = Three dimensional velocity potential

ϕ = Two dimensional velocity potential

χ_i = Impulsive Acceleration potential due to unit body acceleration

$\dot{\Psi}$ = Rotational rate of hydrodynamic frame

φ = Acceleration potential

$\varphi^{R, Imp}, \chi$ = Impulsive radiation acceleration potential

$\varphi^{R, Memory}$ = Memory radiation acceleration potential

ω = Angular velocity of the body

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