Investigation of shock wave focusing in water in a logarithmic spiral duct, Part 1: Weak coupling

Chuanxi Wang^a, Shi Qiu^b, Veronica Eliasson^{b,c}

^a*Department of Physics and Astronomy, University of Southern California, Los Angeles, California 90089, USA*

^b*Department of Aerospace and Mechanical Engineering, University of Southern California, Los Angeles, California 90089, USA*

^c*Email: eliasson@usc.edu, Phone: (213) 740-7182, Fax: (213) 740-7774*

Abstract

Shock focusing in water is a phenomenon that can occur during the impact of a shock wave generated by an underwater explosion onto any type of convergent marine structures. To predict the dynamic material response of the marine structure, it is important to understand the shock wave dynamics during the focusing event. In this paper, both experimental investigations and numerical studies of two-dimensional shock focusing in water are presented. Here, a convergent geometry given by a logarithmic spiral curve is used to focus the shock waves. In the experiments, the interaction between three types of materials and the shock wave in water is explored by using high-speed photography. Distinct features of such flows are unveiled. Three scenarios have been considered in simulations; a rigid structure where only the water-filled region is taken into account, a fluid-structure interaction problem in which the surrounding material responses are included, and an axi-symmetric simulation to determine the three-dimensional effects. Results from the numerical simulations show that boundary conditions and initial conditions are important to accurately predict fluid properties during

Preprint submitted to Ocean Engineering

September 15, 2014

the shock focusing phase.

Keywords: Shock focusing, Underwater explosion, Fluid-structure interaction, Schlieren, Finite difference simulations

1. Introduction

Shock focusing is a fundamental tool to generate extreme conditions at the focal region. It is utilized in both basic research and various applications such as in the study of shock stability, implosion fusion reactors and medical treatment for kidney stones (Gardner et al., 1982; Lindl et al., 1992; Chaussy et al., 1980). Shock focusing typically occurs because of the reflection from complex boundaries surrounding the shock focusing media or inhomogeneities of the flow field (Sturtevant and Kulkarny, 1976). For example, a planar shock front becomes curved as it propagates inside a shock wave lithotripter, a reflector in a particular shape used to generate a focused shock wave to destroy kidney stones (Mulley, 1986). Many analytical, experimental and numerical studies have been conducted on shock focusing. Most of the work focused on the generation and stability of cylindrical and spherical converging shock waves in gases (Guderley, 1942; Perry and Kantrowitz, 1951; Takayama et al., 1987; Schwendeman and Whitham, 1987; Eliasson et al., 2008). In recent years, the use of shock wave focusing in biomedical applications has increased. Examples are shock wave lithotripsy (Mulley, 1986; Gerald, 1997; Weiland et al., 2007) and drug delivery by shock waves (Kato et al., 2000; Doukas and Kollias, 2004). The working medium for both of these applications is water or a water solution. In general, shock focusing in water generates much higher pressures than shock focusing in air, due to the

22 increase in density and speed of sound in water as compared to air (Ridah,
23 1988; Fleck and Deshpande, 2004).

24 An application of particular interest to the navy is shock loading from
25 underwater explosions (UNDEX). UNDEX poses a tremendous threat to
26 all kinds of naval structures. In particular, shock wave focusing from UN-
27 DEX can take place in several convergent sections on a naval vessel, such as
28 rudder-hull junction, propeller shaft and bow thruster. The main factor that
29 influences the focusing behavior of the shock wave in water is the geometry
30 of the convergent sections, as suggested by previous studies of shock focusing
31 (Schwendeman and Whitham, 1987; Coleman and Saunders, 1989; Eliasson
32 et al., 2007). Other factors, such as the shock strength and the material
33 properties of the surrounding section, also play a critical role in determining
34 the focusing effects of the shock wave.

35 In order to obtain an upper limit of the most severe conditions that UN-
36 DEX can generate, we propose to study shock focusing in geometric shapes
37 leading to a maximized energy at the focal region. The geometric shape
38 chosen for this study is given by a so-called logarithmic spiral which was
39 presented first by Milton and Archer (Milton and Archer, 1969). Since the
40 shape of the shock front far enough away from an UNDEX event can be
41 approximated to be planar, all simulations in this paper starts with a planar
42 incident shock wave.

43 Furthermore, multiphase interactions including liquid-gas interaction and
44 liquid-solid interaction are common in shock focusing applications. For ex-
45 ample, a study on superseismic loading shows that the fluid-solid interface
46 deflection induced by the shock impact can result in modified fluid dynamics

47 (Arienti and Shepherd, 2002). In general, multiphase interactions are chal-
48 lenging for both experimental and numerical investigations. In experiments,
49 ultrafast diagnostic methods are needed in order to characterize the fluid and
50 solid dynamics simultaneously (Espinosa et al., 2006; Giordano et al., 2005).
51 In simulations, strategies to take the effects of coupling between the fluid and
52 solid phase have been developed (Shin, 2004; Giordano et al., 2005; Young
53 et al., 2009) and are available in many different simulation packages.

54 Here, we present results of shock focusing in a water-filled convergent
55 section surrounded by a bulk material using both experiments and numeri-
56 cal simulations. The experiments consist of shock focusing in a water-filled
57 convergent section surrounded by different types of bulk materials; poly-
58 carbonate, aluminum and polymethylmethacrylate/methacrylate (PMMA).
59 The plastic materials are used because they are transparent and simultane-
60 ous wave propagation in the sample and inside the water-filled region can
61 be obtained. The metallic sample is closer to the materials that are used in
62 many marine applications. Three different types of simulations are presented
63 and compared to experiments; *(i)* a two-dimensional rigid confinement, *(ii)*
64 a two-dimensional multi-physics simulation that takes both the water in the
65 convergent section and the surrounding solid material into account, and *(iii)*
66 an axisymmetric rigid confinement. A two-dimensional rigid simulation is
67 chosen because it should give the ideal maximum pressure at the focal re-
68 gion, providing an upper limit on e.g. maximum density and pressure close
69 to the focal region, and it provides an understanding if the boundary can be
70 assumed to be rigid or not. The two-dimensional fluid-structure simulation
71 shows the importance of enabling energy from the incident shock wave to

72 being distributed to the core material, as is the case in the experiments. It
73 also shows how much the maximum pressure is reduced by assuming a linear
74 elastic response of the surrounding structure. The last set of simulations,
75 axi-symmetric, indicates the importance of taking three-dimensional effects
76 into account.

77 2. Experimental technique

78 Experiments are performed using an impact technique where a projec-
79 tile from a gas gun impacts onto the experimental specimen. The projectile
80 impact leads to generation of a propagating shock wave in a water-filled con-
81 vergent section in the experimental specimen. As the shock wave propagates
82 into the convergent section it will focus, and generate very high pressures at
83 the focal region. Two different schlieren visualization techniques are used to
84 capture the shock dynamics and the interaction with the surrounding mate-
85 rial during the focusing phase; single-shot photography with high resolution,
86 and high-speed photography.

87 The experimental setup consist of a gas gun, a 25.4 cm-diameter Z-folded
88 schlieren imaging system, and the experimental specimen. The experimental
89 technique and the visualization system have been explained elsewhere in de-
90 tail (Wang and Eliasson, 2012; Wang et al., 2013), and will be only briefly ex-
91 plained here. The experimental specimen is shown in Figure 1. The core ma-
92 terial with a convergent water-filled section is sandwiched between two trans-
93 parent optical quality polycarbonate windows measuring $182 \times 205 \times 12.7$ mm.
94 The shape of the convergent cavity is given by a logarithmic spiral and this
95 shape was chosen due to its ability to focus a shock wave with minimal re-

96 flections (Inoue et al., 1993, 1995). The shape is derived using Whitham's
 97 geometrical shock dynamic theory (Whitham, 1974) based on a shock Mach
 98 number $M_s = 1.1$ with a stiffened equation of state (Wang and Eliasson,
 99 2012). The length of the convergent cavity, also referred to as the charac-
 100 teristic length of the logarithmic spiral is set to $L = 114$ mm, Figure 1. A
 101 polycarbonate blocker is placed at the entrance of the convergent section to
 102 prevent the water from leaking out. The core materials are made of polycar-
 103 bonate, aluminum or PMMA, and the windows are made of polycarbonate.
 104 The edges between the windows and the core are sealed with silicone to
 105 prevent water leakage from the specimen. Special care is taken to remove
 106 any bubbles in the water-filled region before an experiment. Details of the
 107 wave propagation through the blocker and the experimental specimen along
 108 with shock Hugoniot curves for the various materials involved are carefully
 109 explained in our earlier work (Wang and Eliasson, 2012).

110 *2.1. Single-shot photography*

111 High-resolution single-shot photographs are taken with a Nikon D90 SLR
 112 camera with a AF-S Nikkor 18-105 mm zoom lens using an 18 ns spark
 113 light source (Nanolite, High-Speed Photo-Systeme). The advantage of this
 114 technique is the high resolution, but only one photograph per test can be
 115 obtained. In the experiments presented in this paper, the camera is operating
 116 at 100 mm focal length with 2848×4288 pixel picture resolution, which
 117 corresponds to a 163 mm $\times$ 245 mm region of the sample being visualized.
 118 During the experiments, the laboratory is in a dark room setting and the
 119 camera shutter will be open for two seconds waiting for a flash from the spark
 120 light source. When the projectile impacts onto the specimen, a trigger signal

Figure 1: (a) Experimental core and exploded view of the assembly. The core thickness is 25.4 mm for the aluminum and polycarbonate samples. For the PMMA samples, this thickness is 6.3 mm. (b) Exploded specimen assembly with (1) transparent window (only one is depicted for clarity), (2) core, (3) water-filled convergent section, and (4) blocker placed at the entrance of the cavity to serve as impact site and to prevent water from leaking out.

121 is sent out to a pulse/delay generator unit (BNC Model 575). The trigger
 122 signal comes from when the projectile breaks a laser beam just ahead of the
 123 blocker at the experimental specimen. After an appropriate time delay, the
 124 delay unit sends out a signal to trigger the spark source and the photograph
 125 can be recorded.

126 2.2. High-speed photography

127 A Phantom V711 camera, equipped with an AI-S Nikkor 50mm f/1.4 lens,
 128 is used for obtaining high-speed schlieren photos. All high-speed photographs
 129 have been processed in MATLAB using the Control Point Selection Tool in

130 the Imaging Processing Toolbox to remove any optical distortions. Before
 131 each experiment, a transparent sheet, with control points placed in a square
 132 grid pattern, was placed in the test section where the experimental specimen
 133 normally is located. The image of the grid with control points can then be
 134 corrected and then used to remove the optical distortions from the experi-
 135 mental photographs. The resolution of each image was 320×128 pixels and
 136 the time between subsequent frames was set to $8.38 \mu s$. The corresponding
 137 measurement size of the specimen is $118.5 \text{ mm} \times 47.4 \text{ mm}$. During the $1 \mu s$
 138 minimum exposure time that the high-speed camera has, the shock wave in
 139 water will travel approximately 1.4 mm , which can yield blurry images. To
 140 overcome this intrinsic limitation of the camera, a synchronized pulsed laser
 141 light source (SONY SLD1332V Laser diode combined with PicoLAS LDP-V
 142 03-100UF3 Driver module) is used to provide intensive lighting within tens
 143 of nanosecond time scale. A typical pulse duration for the experiments is
 144 40 ns , which means the exposure time is reduced to significantly reduce blur.
 145 During the experiments, when the event of interest takes place, a control sig-
 146 nal is sent to a pulse/delay generator (BNC Model 575). The pulse generator
 147 receives the signal and outputs a sequence of TTL pulses to both the light
 148 source and the camera to synchronize the two.

149 *2.3. Background oriented schlieren*

150 The previous two types of visualization techniques are suitable for discov-
 151 ering flow features and performing basic flow related calculations such as the
 152 estimation of shock wave speed etc. However, it is preferable if the density
 153 or pressure change of the fluid is available for each experiment. As suggested
 154 by Meier (Meier, 2002), a technique called Background Oriented Schlieren

(BOS) can be incorporated with schlieren photography to yield quantitative density measurements in transparent flows.

3. Experimental results

Results have been obtained using the previous mentioned techniques; single-frame high-resolution schlieren photography, high-speed schlieren photography and high-speed BOS photography. Three different materials have been used, and their properties are summarized in Table 1. Transverse, c_S , and longitudinal, c_L , wave speeds are measured using an Olympus 38DL Plus Ultrasonic Thickness Gage, and Young's modulus and Poisson's ratio are computed based on the wave speeds and density, ρ .

		Polycarbonate	Aluminum	PMMA
ρ	[kg/m ³]	1270 [‡]	2700 [‡]	1200 [†]
c_L	[m/s]	2100 ± 110 [‡]	6370 ± 120 [‡]	2800 ± 55 [†]
c_S	[m/s]	950 ± 50 [‡]	3150 ± 60 [‡]	1500 ± 30 [†]
E	[Pa]	3.2×10^9	64×10^9	5.6×10^9
ν	[−]	0.37	0.31	0.34

Table 1: Material properties for the three core materials used in the experiments. [†]values measured using Olympus 38DL Plus, [‡]from Eliasson et al. (2010).

164

3.1. Polycarbonate specimen

A polycarbonate core was used because it is transparent and therefore the wave propagation inside the core material can be visualized at the same time as the waves inside the water-filled region are investigated. A series of

168

high-speed schlieren images with a projectile impact speed of 50.6 ± 1.0 m/s using the polycarbonate specimen is shown in Figure 2. Figure 2(a) shows the specimen before impact with the undisturbed water-filled region (marked A in the figure) and the blocker (marked B in the figure). The shock front is marked by white arrows. Time $t = 0$ μ s is when the shock wave enters the water region. The shock speed along the center line for frames (c)-(h) was measured using the distance between the shock front positions, and the timing between successive frames, and the result is presented in Table 2. The error in shock speed, ± 44 m/s, is based on the uncertainty in shock location (1 pixel) divided by the time interval (8.38 μ s), and the Mach number error was calculated using error propagation rule for $M_s = u_s/c_{\text{water}}$ where $c_{\text{water}} = 1491 \pm 2.8$ m/s. The first two frames indicate a wave speed less than

Frames	(b)-(c)	(c)-(d)	(d)-(e)	(e)-(f)	(f)-(g)	(g)-(h)	(h)-(i)
PC: u_s [m/s]	–	1370*	1326*	1458	1547	1458	–
Al: u_s [m/s]	1724	1591	1591	1503	1503	1547	1503

Table 2: Speed of compression wave, U_s , between subsequent frames in Figure 2 for polycarbonate (PC) with a projectile impact speed of 50.6 ± 1.0 m/s, and Figure 4 for aluminum (Al) with a projectile impact speed of 50.7 ± 1.0 m/s. All wave speeds have an uncertainty of ± 44 m/s, except values marked * with ± 88 m/s.

180

Mach 1, which is likely caused by the strong interaction of the pressure and shear waves in the windows. There is a larger error attributed to determining the shock location (2 pixels) for these images than the rest since the region where the shock wave is located is smeared out and hard to precisely locate.

In Figure 2(g) the angle between the interface of the convergent section and the shear waves in the polycarbonate core was measured to be $34^\circ \pm 1^\circ$

186

187 on the upper edge and lower edge. From such an angle, one can estimate
 188 the shock wave speed in water by assuming that the shear wave remains
 189 constant in the surrounding solid (Eliasson et al., 2010). In later frames,
 190 Figure 2(h), this angle has decreased to $29^\circ \pm 1^\circ$. This decrease is due
 191 to the curvature change of the fluid-solid interface, and if the horizontal
 192 component of the shear wave is computed it is clear that the shock speed
 193 in water remains constant. Therefore, a transparent sample can be used
 194 to further help understand the instantaneous wave propagation inside the
 195 water-filled region. In the last frame, 2(j), a cylindrical wave (marked by a
 196 white arrow) has started to propagate into the core material as a result of
 197 the shock focusing event in the water-filled region.

198 A single-shot schlieren experiment is presented in Figure 3. This image
 199 is taken at a late time when there are extensive cavitation zones both inside
 200 the convergent water-filled cavity, and in the thin liquid films between the
 201 windows and the core material. The dark zone behind the shock wave is due
 202 to extensive disturbances, and all light is diverted from the schlieren optical
 203 path. The white arrows point at the curved shear wave in the polycarbonate
 204 core, and their symmetry is a sign of a planar impact. A curved shear wave
 205 is formed due to the nonlinear geometry of the interface between the fluid
 206 and the solid.

207 3.2. *Aluminum specimen*

208 An aluminum core material was chosen for its ability to provide more
 209 realistic wave interactions as seen in marine applications. A series of high-
 210 speed schlieren images with a projectile impact speed of 50.7 ± 1.0 m/s using
 211 an aluminum specimen is shown in Figure 4. Figure 4(a) shows the specimen

212 before impact with the undisturbed water-filled region (marked A in the
 213 figure) and the blocker (marked B in the figure). The dark region in the
 214 water-filled region of Figure 4(a)-(d) is caused by the interference of light from
 215 the laser diode, and it can be removed by using a shadowgraph technique
 216 instead of a schlieren technique. The shock speed along the center line for
 217 frames (b)-(i) was measured similarly as for the polycarbonate specimen, and
 218 the results are presented in Table 2.

219 Figure 5 shows a single-shot schlieren experiment with aluminum as sur-
 220 rounding core material. The projectile impact speed of this test is at 79 ± 1 m/s.
 221 The arrows point at (1) the shock wave in water, (2) the oblique precursor
 222 wave from the shear wave in the aluminum core material, and (3) the pres-
 223 sure wave in the polycarbonate windows. Cavitation bubbles occur between
 224 the precursor wave and the main shock front due to the interaction between
 225 the faster traveling stress waves inside the surrounding structure.

226 3.3. PMMA specimen

227 A PMMA core specimen, with a reduced thickness to one quarter of
 228 the previous cores, was used together with a background oriented schlieren
 229 method to measure density and pressure along the centerline of the water-
 230 filled region. In this particular setup, any pressure sensor that is physically
 231 attached to the core material is subjected to the risk of getting hit and
 232 damaged during the experiment, and this was determined unfeasible. There-
 233 fore, a non-intrusive visual technique, BOS, was chosen to gain knowledge
 234 of the density and pressure evolution in the convergent water-filled section,
 235 see details in Wang et al. (2013). The thickness was reduced to realize an
 236 improved BOS result because the pressure from BOS is calculated under the

assumption that the experimental domain is two-dimensional and thus three-dimensional effects were reduced by reducing the core thickness. Other dimensions of the PMMA samples stay the same as previous experiments. The projectile impact speed during the test is 50.0 ± 1.0 m/s. Two-dimensional pressure distribution at time $54.5 \mu\text{s}$ is reconstructed by feeding the displacement information into a solver by Wang et al. (2013). Two extremes of boundary conditions were used to determine the maximum (16 MPa) and minimum (10 MPa) pressure peaks. The resulting pressure profile along the center axis of the logarithmic spiral geometry for the boundary conditions yielding the lower peak pressure is plotted in Figure 6. There is a 4 mm spatial span of the pressure rise as seen in Figure 6, which is due to the finite window size of the image analysis. Behind the pressure peak, the pressure drops to 3.9 MPa, which corresponds to a long trailing expansion wave. Also, compared to the previously presented experimental results by Wang et al. (2013), using a similar projectile impact speed, the calculated peak pressure for the logarithmic spiral presented here is about three times higher than for their wedge-shaped cavity made of polycarbonate.

Figure 2: A series of high-speed schlieren images using a polycarbonate specimen. Projectile impact speed of 50.6 ± 1.0 m/s. The white arrow shows the location of the shock front in (b)-(i), and the pressure wave in the core material in (j).

Figure 3: A single-shot schlieren experiment using a polycarbonate specimen visualized at $t = 111.4 \mu s$ after the shock wave enters the water-filled region. Image shows extensive cavitation zones, not just in the water-filled cavity but also in the thin liquid layers between the windows and the core material. White arrows point at shear waves inside the polycarbonate core material.

Figure 4: A series of high-speed schlieren images using an aluminum specimen. Projectile impact speed of 50.7 ± 1.0 m/s. The white arrow shows the location of the shock front in (b)-(i).

(a) Before test

(b) $t = 39.4 \mu s$

Figure 5: A single-shot schlieren experiments using an aluminum specimen with projectile impact speed 79 ± 1 m/s. Arrows point at (1) shock wave in water; (2) oblique precursor waves from the shear wave in the aluminum core material, and (3) pressure wave in the polycarbonate windows.

Figure 6: The reconstructed pressure measurement along the center line.

254 4. Numerical method and experiments

255 In this section, results from three different types of simulations of converg-
 256 ing shocks in water are presented; (4.1) a two-dimensional rigid confinement,
 257 (4.2) a two-dimensional multi-physics simulation that accounts for the fluid-
 258 structure interaction, and (4.3) an axisymmetric rigid confinement.

259 The Overture package, a finite difference code for partial differential equa-
 260 tions, is utilized to perform all the simulations (Chesshire and Henshaw,
 261 1990). The inviscid and adiabatic flows under consideration are governed by
 262 the Euler equations of gas dynamics, i.e. conservation of mass, momentum
 263 and energy,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla p = 0, \quad (2)$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot (\mathbf{u}(\rho E + p)) = 0, \quad (3)$$

$$E = e + \frac{1}{2}(u^2 + v^2 + w^2). \quad (4)$$

where $\rho, \mathbf{u}, p$ are the density, velocity vector and pressure. In Equation (4),
 E and e are the total energy and the internal energy per unit mass, and
 u, v, w are the velocity components in three directions. A stiffened equation
 of state (sometimes referred to as Tammann-Tait EOS, Ridah (1988)) is used
 to close the system of equations. The stiffened EOS is given by

$$e = \frac{p + \gamma_\infty p_\infty}{\rho(\gamma_\infty - 1)}. \quad (5)$$

267 The variable e is the internal energy, p is the pressure, ρ is the density; and
 268 the constants $\gamma_\infty = 7.415$, $p_\infty = 2.962 \times 10^8$ Pa are chosen to match the speed

269 of sound of water in previous laboratory experiments reported by Eliasson
 270 (Eliasson et al., 2010). This EOS has also been used in previous works related
 271 to shock waves in water, such as underwater explosions (Cocchi et al., 1996)
 272 and the collapse of shock-induced bubbles in water (Johnsen and Colonius,
 273 2008).

The Rankine-Hugoniot jump conditions using the stiffened EOS are represented by the following equations

$$u_2 = a_1 \frac{2(M_s^2 - 1)}{(\gamma_\infty + 1)M_s}, \quad (6)$$

$$\rho_2 = \rho_1 \frac{(\gamma_\infty + 1)M_s^2}{(\gamma_\infty - 1)M_s^2 + 2}, \quad (7)$$

$$p_2 = \rho_1 a_1^2 \frac{2(M_s^2 - 1)}{\gamma_\infty + 1} + p_1. \quad (8)$$

274 Here, M_s , is the shock Mach number, u , is the particle velocity and a_1 is
 275 the speed of sound. Quantities with a subscript 1 denote variables in the
 276 undisturbed region ahead of the shock wave and quantities with a subscript
 277 2 denotes variables in the region behind the shock.

278 The conservation equations are solved numerically using a shock-capturing
 279 second-order Godunov method (Henshaw and Schwendeman, 2003, 2006).
 280 Composite grids are generated by overlapping structured grids (Henshaw
 281 and Schwendeman, 2006). The main reason for using composite grids is to
 282 save memory because the main part of all grids are rectangular Cartesian
 283 grids which are more efficient in terms of memory usage than curvilinear
 284 grids (Cheshire and Henshaw, 1990).

285 A measure of interest is the peak pressure at the tip of the logarithmic
 286 spiral, i.e. the focal region. This peak pressure is an indicator of the destruc-
 287 tive potential of the converging shock wave to the surrounding structure.

288 Transient high pressures will induce strong tensile and shear stresses inside
289 the surrounding solid. The stress field can in turn cause initiation and growth
290 of any defects already present in the structure, which eventually can lead to
291 crack formation (Xi and Zhong, 2001) with devastating results.

292 A grid sensitivity study using Richardson Extrapolation theory has been
293 performed on the two-dimensional rigid boundary case (details explained in
294 section 4.1). Four grid sizes were used, each one half the size of the previous:
295 0.25 mm, 0.125 mm, 0.0625 mm, and 0.03125 mm. Pressure history was
296 recorded at location $(x, y) = (-4, 0)$, which is close to the tip, located at $(0, 0)$,
297 of the logarithmic spiral. The non-dimensionalized peak pressure (multiply
298 with 2196 to get result in MPa) is plotted against Δx^2 in Figure 7, and a
299 nearly linear relation is obtained, which is in agreement with the result a
300 second order Godunov scheme theoretically gives. In addition, an Adaptive
301 Mesh Refinement (AMR) algorithm in Overture is also utilized to capture
302 the fine structures of the shock focusing process. The AMR grids are two
303 levels with a refinement factor of four each, which results in an eight times
304 smaller grid size near shocks, contact surfaces and other regions with sharp
305 derivatives. Therefore, a grid size of 0.1 mm was used to balance the cost of
306 the simulation and the accuracy of the result.

307 All simulations presented below are first run with an incident shock Mach
308 number of $M_s = 1.1$ because the logarithmic spiral is designed to most effi-
309 ciently converge the shock for this specific Mach number. Then, simulations
310 are repeated with a lower shock Mach number of $M_s = 1.03$, which is closer
311 to the one realized in the experiments.

Figure 7: Grid sensitivity study by investigating peak pressures for different initial grid sizes.

312 4.1. Two-dimensional rigid boundary simulation

313 This type of simulation, in which only the liquid part of the problem
314 is considered without any influence from the surrounding solid structure, is
315 informative in two aspects. First, it enables the possibility to investigate how
316 the geometry influences the shock focusing event. Second, it should give the
317 ideal maximum conditions at the focal point for two-dimensional simulations
318 since there is no energy lost to the surrounding structure.

319 To perform the simulation, a composite grid made of three different grids
320 overlapping each other is prepared, Figure 8(a). A rectangular Cartesian
321 grid is first generated as a background grid. All other grid components cut
322 or replace part of the background grid. The reflector grid giving the outer
323 boundary of the logarithmic spiral is obtained by marching from a spline
324 whose points are given by the equation for a logarithmic spiral. The reflector

325 grid cuts out the geometry and shares interpolation points with background
 grid.

Figure 8: (a) An overview of the composite grid used for two-dimensional rigid boundary simulations with (1) Cartesian background grid; (2) inlet grid; (3) logarithmic spiral boundary grid. (b) Initial conditions in region 1 ahead of the shock wave, and region 2 behind the shock wave summarized in Table 3.

326

327 Step functions of pressure, p , velocity, u , and density, ρ , are used as initial
 328 conditions to represent an initially planar incident shock wave. The condi-
 329 tions in the undisturbed region ahead of the shock wave are given by room
 330 temperature (21°C), atmospheric pressure (101,325 kPa) and zero velocity.
 331 The conditions behind the shock wave are computed using shock jump condi-
 332 tions, Equations (6)–(8), based on the chosen shock Mach number, $M_s = 1.1$,
 333 and the conditions ahead of the shock. The initial conditions are summarized
 334 in Table 3, and the shock location at time $t = 0$ is shown in Figure 8(b).
 335 At the left boundary of the inlet grid, a constant inflow condition (given
 336 by region 2 in Table 3) is maintained to sustain the shock wave strength.
 337 The boundary condition at the wall profile is set to be a slip-wall condition,
 338 which keeps the normal velocity of the fluid zero at the wall boundary. As
 339 a consequence of this slip-wall boundary condition, the shock front is always
 340 perpendicular to the wall, as shown in Figure 9.

	Region 1	Region 2
p [MPa]	0.1	109.8
u [m/s]	0	67
ρ [kg/m ³]	1000	1043

Table 3: Initial conditions in region 1 ahead of the shock wave, and region 2 behind the shock wave.

Figure 9: (a) A snapshot of pressure distribution at time $40.5 \mu\text{s}$. (b) Zoom in of the shock front from (a) showing that the shock front is normal to the wall as a consequence of applying slip-wall condition at the boundary.

Four snapshots at various time instances are shown in Figure 10. As the shock wave propagates, the Mach number of the shock front at the boundary increases due to the slope of the boundary. The observed peak pressure at the tip is 4.3 GPa. It is 39.2 times the initial pressure, p_2 , behind the shock wave. An additional simulation with initial shock Mach number $M_s = 1.03$ was conducted to match the experimental conditions, and the maximum pressure in this case is 319.8 MPa at the focal point. To compare with the

Figure 10: Schlieren plots at different time instants. The shock Mach number and pressure right behind the shock wave increases as the shock converges. The arrow points in the direction of the wave propagation.

348 BOS results, the maximum pressure was also measured 30 mm away from
 349 the tip. In the case where $M_s = 1.1$, the maximum pressure is 124.5 MPa,
 350 and for $M_s = 1.03$ it is 36 MPa.

351 4.2. Two dimensional fluid-structure interaction simulation

352 To better understand the loads and the coupling on the surrounding struc-
 353 ture, a two-dimensional fluid-structure interaction simulation is performed.
 354 This is a coupled problem because the shock generated in the water results
 355 in both longitudinal and transversal waves in the surrounding material. In
 356 turn, the longitudinal and transversal waves in the solid interact with the
 357 water.

The governing equations for the fluid part is the same as in the first case with a rigid confinement. Linear elasticity is assumed in the solid part and

the dynamics of the displacement is given by

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i, \quad (9)$$

$$\sigma_{ij} = \lambda(\epsilon_{kk})\delta_{ij} + 2\mu\epsilon_{ij}. \quad (10)$$

In Equation (9), u_i , σ_{ij} and f_i are the displacement function of material points in direction i , the Cauchy stress tensor, and the body force in direction i . And in Equation (10), ϵ_{ij} is the strain tensor, while λ and μ are the two Lamé constants. The material properties of the aluminum core are shown in Table 1. The grid size is set to $\Delta x = 1$ mm.

The composite grid for this type of simulation consists four parts including a Cartesian grid for the fluid domain, a Cartesian grid for the solid domain, and two curvilinear grids for communication between the fluid and the solid, Figure 11. The boundary condition between the two curvilinear boundary grids is a traction boundary condition, which means that the normal stress at the fluid-solid interface is matched. For initial conditions, a shock jump is set in the fluid domain as in the previous case with the rigid boundary. The solid part is left undisturbed. As the shock wave starts to propagate into the logarithmic spiral duct, it will deform the interface between the fluid and the solid region. Because the fluid will be moving together with the deforming boundary to maintain contact, the overall velocity of the fluid behind the shock wave will be moving toward the undisturbed boundary. As a consequence, the shock front has to be at an angle less than 90° to the boundary to balance this flow condition (Arienti et al., 2003), Figure 12. Energy carried by the shock wave is attenuated by the transmission of energy into solid,

Figure 11: (a) An overview of the composite grid for the FSI simulation. The solid colored blocks are finer grids at the interface. (b) A blowup of the marked region of the interface from (a). (1) solid domain; (2) boundary grid for the solid domain; (3) boundary grid for the fluid domain; (4) fluid domain. Interpolation points are marked as black dots. The boundary condition between grid (1) and grid (2) is given by a traction boundary condition.

378 Figure 12. This results in a lower peak pressure of 900 MPa at the focal
 379 region as compared to the previous result from the rigid boundary simula-
 380 tion with a peak pressure of 4.3 GPa, because energy is transmitted from the
 381 liquid region to the surrounding solid. When the shock Mach number is low-
 382 ered to $M_s = 1.03$ the peak pressure is reduced to 320 MPa. Detailed plots
 383 of pressure along the interface for the incoming and reflected shock waves
 384 are given in Figure 13. The pressure in front of the incoming shock wave
 385 is lower than zero, due to the interaction with the pre-cursor waves, which
 386 indicates the possibility of cavitation formation. This feature is not observed
 387 in the two-dimensional rigid boundary simulation, which completely lacks
 388 pre-cursor waves.

389 A comparison with the results from this type of simulation and the ex-

Figure 12: (a) A snapshot of the FSI simulation at time $t = 43.5 \mu s$. Schlieren is plotted inside the water-filled region and the divergence of the displacement field is plotted in the solid domain. A color bar is inserted to show which colors correspond to high values (pink) and low values (blue). (b) Zoom in of shock front at the interface from (a) showing that the shock front is not perpendicular to the interface. Instead, the shock angle α is 88.2° for this particular time instance.

periments performed on the aluminum specimen show similar trends for the shock wave speed during the focusing phase. The simulated shock wave speeds are calculated at the same spatial locations as the high-speed visualization results shown in Figure 4, see Table 4. The general trends observed for the shock wave front are a deceleration of shock wave speed and also a dispersion in space, which is in agreement with the experimental observation.

The present results take the fluid-solid interaction into account but not three-dimensional effects. In order to understand how three-dimensional effects influence the peak pressure at the focal region an axi-symmetric simulation has been performed because in reality this would indicate the maximum pressure possible in this type of logarithmic spiral configuration.

Figure 13: Pressure plots along the interface at time instants $t = 26.77, 40.32, 54.89, 60.65 \mu\text{s}$. The peak pressure increases as the shock propagates towards the focal region, and again after reflection.

4.3. Axisymmetric simulation

A cylindrical symmetric or so-called axisymmetric coordinate system is used to perform the simulation. The axis of rotational symmetry is along the line where the characteristic length L is defined, Figure 1. Accordingly, the governing equations are also expressed in axisymmetric coordinates. Because the solution does not depend on the rotational angle (there is no swirl in the solution, Henshaw (2003)), the equations can be discretized and solved on a two-dimensional mesh. The center line of the logarithmic spiral is set as axis of symmetry. Slip-wall boundary conditions are again applied to the

	Experiment [m/s]	Simulation [m/s]	Difference [%]
(c)-(d)	1591	1642	3.1
(d)-(e)	1591	1640	3.0
(e)-(f)	1503	1639	8.3
(f)-(g)	1503	1515	0.8
(g)-(h)	1547	1480	4.5
(h)-(i)	1503	1515	0.8

Table 4: Shock wave speed comparison between the aluminum specimen and the two-dimensional fluid-structure interaction simulation.

410 wall boundary, and constant inflow conditions are assigned to the in-flow
411 boundary.

412 The resulting peak pressure at the focal region in the axisymmetric sim-
413 ulation is 34.2 GPa for $M_s = 1.1$ and 6.2 GPa for $M_s = 1.03$, which is higher
414 than that of the both previously described simulations. The main reason for
415 this high peak pressure is that a larger amount of energy carried by the shock
416 wave is brought to the focal region as compared with the two previous cases.
417 From the discussion above, much higher peak pressure can be achieved by
418 using a three-dimensional logarithmic spiral shape, which suggests the sig-
419 nificance of including the three-dimensional effects.

420 5. Discussion

421 Three materials were used for these experiments, and the clear plastics
422 were chosen to visualize the waves inside the water-filled region and simulta-
423 neously visualize the longitudinal and shear waves inside the core material.
424 A benefit from visualizing the transverse waves is that they provide an es-

425 timate of the instantaneous shock Mach number in the water-filled region.
 426 The aluminum core was used since it closer replicates the conditions for
 427 marine applications where shock focusing can occur. The maximum pres-
 428 sures obtained in the simulations and the BOS experiment are summarized
 429 in Table 5. Clearly, if a rigid surrounding is assumed, the maximum pres-
 430 sure will be higher than if the energy can be transmitted to a surrounding
 431 structure. The results of the axi-symmetric simulation also indicates that
 432 three-dimensional effects are important, however, the experiments are per-
 433 formed in planar geometries where the shock focusing is assumed to be close
 to two-dimensional.

Simulation	M_s	p_{max}^{30mm}	p_{max}^{tip}
2D, Ridgid	1.03	36 MPa	319.8 MPa
2D, Ridgid	1.1	124.5 MPa	4.3 GPa
Axi-symmetric, Rigid	1.03	42.2 MPa	6.2 GPa
Axi-symmetric, Rigid	1.1	146.3 MPa	34.2 GPa
2D, FSI, Aluminum	1.03	34.3 MPa	149.7 MPa
2D, FSI, Aluminum	1.1	115 MPa	900 MPa
PMMA, BOS	–	13 ± 3 MPa	–

Table 5: Maximum pressure summary for simulations and BOS experiment. Simulation results are obtained in the three different types of simulations for two shock Mach numbers ($M_s = 1.1$ and $M_s = 1.03$), and at two locations: at the tip of the logarithmic spiral, p_{max}^{tip} , and 30 mm before the tip, p_{max}^{30mm} .

434

435 The maximum pressure for the BOS measurement is about three times
 436 lower than the results from the rigid two-dimensional simulation with $M_s =$
 437 1.03, and there are several reasons for this result. First, the fluid-structure

438 interaction has not been taken into account, which will reduce the simulated
439 pressure further. Second, in the BOS evaluation, a two-dimensional assumption
440 is used, which is likely not accurate since there are waves propagating
441 out-of-plane, towards and through the windows and back. Third, while evaluating
442 the BOS result, a density change of 4 kg/m^3 was measured and then
443 a pressure increase was inferred from an equation of state taken from open
444 literature Wagner and Pruß (1995). It is possible that if a more elaborate
445 EOS was used, the calculated pressure would change.

446 The FSI simulation requires longer computational time, but provides
447 more accurate results. The cavitation bubbles seen in the high-speed photographs
448 in front of the incident shock wave for the aluminum experiments
449 can be correlated to the lower pressure ahead of the front of the shock wave
450 visualized in the FSI pressure plots in Figure 13. This result is also in
451 agreement with the experimental results from Eliasson et al. (Eliasson et al.,
452 2010), which showed clear evidence of precursor waves in the water ahead of
453 the moving shock wave when a convergent aluminum confinement was used.
454 The precursor waves occurred due to faster longitudinal and transversal wave
455 propagation in the surrounding solid compared to the shock wave speed in
456 water. Also, if the surrounding structure is smaller or thinner such that
457 waves reflecting from its outer boundary come back sooner then the coupling
458 with the liquid-filled region will be stronger. The surrounding structure can
459 also deform if it is thinner or made of a softer material, and energy will then
460 dissipate quicker and the shock will not generate as high pressure at the focal
461 region.

462 6. Summary and Conclusions

463 In this paper, basic fluid dynamics features of the shock focusing process
464 in water inside logarithmic spiral structures are discovered through single-
465 shot and high-speed photography. Quantitative measurement of the pressure
466 distributions is also obtained through BOS technique. The peak pressure
467 along the center line can be compared with the numerical simulations. Com-
468 plex fluid dynamics such as cavitation occur both in front and behind the
469 shock wave in water. Cavitation behind the shock wave is due to bubble nu-
470 cleation sites in the water-filled region that get excited when the shock waves
471 passes them. More complex fluid dynamics models would be necessary in the
472 future in order to correctly model the cavitation bubble events and possible
473 pitting and interaction with the surrounding solid surfaces.

474 Computer simulations are also conducted using the Overture package.
475 The two dimensional rigid boundary simulation confirms that the shock wave
476 undergoes a focusing process, and reaches a peak pressure at the focal re-
477 gion of the logarithmic spiral shape. The fluid-structure simulation, which
478 includes the material effects from the surrounding boundary, indicates that
479 the coupling between fluid and structure changes the shock wave dynamics.
480 As a result, the peak pressure magnitude at the focal region is lower in the
481 two dimensional FSI simulation as compared to the rigid boundary simula-
482 tion. By contrast, a rigid boundary axisymmetric simulation demonstrates
483 a much higher peak pressure when compared to the two dimensional rigid
484 boundary case, indicating a three-dimensional simulation would be necessary
485 to infer the correct response due to three-dimensional effects.

486 When comparing the experiments with the simulations, the overall dy-

namics of the shock front agrees well. The experimental reconstructed peak
 pressure from BOS is an order of magnitude lower than in the two-dimensional
 rigid boundary simulation if the incident shock Mach number is set to $M_s =$
 1.1 but only about three times lower if $M_s = 1.03$. For FSI simulations
 with an aluminum material the simulation with $M_s = 1.03$ is lower than the
 rigid simulation, but still higher than the experiment. This is attributed to
 the PMMA confinement used in the experiment, which has a lower acoustic
 impedance than aluminum and thus more energy will transmit to the
 surrounding structure. It shows clearly that initial conditions and focusing
 effects have a large influence on the thermodynamic properties at the focal
 region. There is also an uncertainty associated with the results obtained using
 BOS because the spatial averaging with a fixed size interrogation window
 smears the shock wave over several pixels. In addition, there will be error
 generated during the digital image correlation and numerical solver calculation
 process. Ideally, to reduce the error from BOS, a camera with faster frame
 rates and higher resolution is needed. Another reason that accounts for the
 differences observed between the experiments and simulations is the estimation
 of shock strength by using Hugoniot data taken from literature. If exact material
 properties of each component used in the experiments were readily available,
 this should provide more accurate data to use for shock Hugoniot matching
 techniques.

The main conclusions are summarized below:

1. Shock focusing in water can efficiently be obtained using a logarithmic spiral geometry. Results show that the focusing process at the design Mach number ($M_s = 1.1$) results in up to 13.4 times higher peak pres-

- 512 sure than the lower Mach number ($M_s = 1.03$).
- 513 2. The surrounding material influences the shock focusing in several ways:
- 514 (a) A rigid material, with a high impedance mismatch, will allow less
- 515 waves to transmit through the surrounding structure and thus
- 516 the pressure inside the water-filled domain is higher. For our
- 517 experiments, the impedance of aluminum, PMMA, and polycar-
- 518 bonate is $17.2 \pm 0.32 \times 10^6$ Ns/m³, $2.7 \pm 0.14 \times 10^6$ Ns/m³, and
- 519 $3.4 \pm 0.07 \times 10^6$ Ns/m³ respectively.
- 520 (b) The wave speeds in the surrounding material may cause cavitation
- 521 *ahead* of the incident shock wave by putting the water in tension.
- 522 3. The simulations show that three-dimensional effects are important for
- 523 realistic applications where planar two-dimensional geometries are non-
- 524 typical. Providing accurate initial conditions is important, as the simu-
- 525 lations performed here show. For example, a 6% change in initial shock
- 526 Mach number changes the maximum pressure at the focal region with
- 527 a factor close to 10.
- 528 4. BOS is a non-intrusive technique that allows for density measurements
- 529 and then pressure can be inferred from a known equation of state.
- 530 Higher resolution photographs and a higher framing rate would lead to
- 531 more accurate BOS results.

532 In the future, the degree of coupling between the surrounding structure

533 and the water-filled region will be investigated in detail. A simulation of

534 a polycarbonate or PMMA surrounding material would provide insight in

535 how much the peak pressure decays due to change in material properties.

536 This is particularly interesting for thin structures with a high degree of fluid-

structure coupling, and therefore the role of the type of material, material properties and specific geometry has to be understood. Future work also include shock focusing in thin water-filled convergent surrounding structures made of materials with more relevance to marine-type applications, e.g. steel, aluminum and carbon fiber composite. The fluid-structure coupling effect will be compared to the bulk materials presented here, and further investigated to understand the dynamic response of the surrounding structure.

7. Acknowledgments

The authors are grateful for support from Office of Naval Research through a MURI Grant Number N00014-06-1-0730 (Dr. Y.D.S. Rajapakse, Program Manager). The authors wish to thank Prof. Bill Henshaw at Rensselaer Polytechnic Institute for help with the simulations. We also want to thank the USC Viterbi Machine Shop as well as the Caltech Aero Shop.

Arienti, M., Hung, P., Morano, E., Shepherd, J.E., 2003. A level set approach to Eulerian-Lagrangian coupling. *Journal of Computational Physics* 185, 213–251.

Arienti, M., Shepherd, J.E., 2002. Superseismic loading and shock polars: An example of fluid-solid coupling. *AIP Conference Proceedings* 620, 251–254.

Chaussy, C., Brendel, W., Schmiedt, E., 1980. Extracorporeally induced destruction of kidney stones by shock waves. *Lancet* 316, 1265–1268.

Chesshire, G., Henshaw, W.D., 1990. Composite overlapping meshes for the solution of partial differential equations. *J. Comput. Phys.* 90, 1–64.

- 559 Cocchi, J., Saurel, R., Loraud, J., 1996. Treatment of interface problems
560 with godunov-type schemes. *Shock Waves* 5, 347–357.
- 561 Coleman, A.J., Saunders, J.E., 1989. A survey of the acoustic output of com-
562 mercial extracorporeal shock wave lithotripters. *Ultrasound in Medicine*
563 & *Biology* 15, 213–227.
- 564 Doukas, A.G., Kollias, N., 2004. Transdermal drug delivery with a pressure
565 wave. *Advanced Drug Delivery Reviews* 56, 559–579.
- 566 Eliasson, V., Apazidis, N., Tillmark, N., 2007. Controlling the form of strong
567 converging shocks by means of disturbances. *Shock Waves* 17, 29–42.
- 568 Eliasson, V., Henshaw, W., Appelö, D., 2008. On cylindrically converging
569 shock waves shaped by obstacles. *Physica D: Nonlinear Phenomena* 237,
570 2203–2209.
- 571 Eliasson, V., Mello, M., Rosakis, A., Dimotakis, P., 2010. Experimental
572 investigation of converging shocks in water with various confinement ma-
573 terials. *Shock Waves* 20, 395–408.
- 574 Espinosa, H., Lee, S., Moldovan, N., 2006. A novel fluid structure interaction
575 experiment to investigate deformation of structural elements subjected to
576 impulsive loading. *Experimental Mechanics* 46, 805–824.
- 577 Fleck, N.A., Deshpande, V.S., 2004. The resistance of clamped sandwich
578 beams to shock loading. *J. Appl. Mech.* 71, 386–401.
- 579 Gardner, J.H., Book, D.L., Bernstein, I.B., 1982. Stability of imploding

shocks in the CCW approximation. *Journal of Fluid Mechanics* 114, 41–
58.

Gerald, H., 1997. Use of extracorporeal shock waves in the treatment of
pseudarthrosis, tendinopathy and other orthopedic diseases. *The Journal*
of Urology 158, 4–11.

Giordano, J., Jourdan, G., Burtschell, Y., Medale, M., Zeitoun, D.E., Houas,
L., 2005. Shock wave impacts on deforming panel, an application of fluid-
structure interaction. *Shock Waves* 14, 103–110.

Guderley, G., 1942. Starke kugelige und zylindrische Verdichtungsstöße in
der Nähe des Kugelmittelpunktes bzw. der Zylinderachse. *Luftfahrt Forsch.*
19, 302–312.

Henshaw, W.D., 2003. *OverBlown : A Fluid Flow Solver For Overlapping*
Grids, Reference Guide, Version 1.0.

Henshaw, W.D., Schwendeman, D.W., 2003. An adaptive numerical scheme
for high-speed reactive flow on overlapping grids. *Journal of Computational*
Physics 191, 420 – 447.

Henshaw, W.D., Schwendeman, D.W., 2006. Moving overlapping grids with
adaptive mesh refinement for high-speed reactive and non-reactive flow.
Journal of Computational Physics 216, 744 – 779.

Inoue, O., Imuta, S., Milton, B., Takayama, K., 1995. Computational study
of shock wave focusing in a log-spiral duct. *Shock Waves* 5, 183–188.

- 601 Inoue, O., Takahashi, N., Takayama, K., 1993. Shock wave focusing in a
602 log-spiral duct. *AIAA J* 31, 1150–1152.
- 603 Johnsen, E., Colonius, T., 2008. Shock-induced collapse of a gas bubble in
604 shockwave lithotripsy. *J. Acoust. Soc. Am.* 124, 2011–2020.
- 605 Kato, M., Ioritani, N., Suzuki, T., Kambe, M., Inaba, Y., Watanabe, R.,
606 Sasano, H., Orikasa, S., 2000. Mechanism of anti-tumor effect of combina-
607 tion of bleomycin and shock waves. *Cancer Science* 91, 1065–1072.
- 608 Lindl, J.D., McCrory, R.L., Campbell, E.M., 1992. Progress toward ignition
609 and burn propagation in inertial confinement fusion. *Phys. Today* 45, 32–
610 40.
- 611 Meier, G., 2002. Computerized background-oriented schlieren. *Experiments*
612 *in Fluids* 33, 181–187.
- 613 Milton, B., Archer, R., 1969. Generation of implosions by area change in a
614 shock tube. *AIAA J* 7, 779–780.
- 615 Mulley, A.G., 1986. Shock-wave lithotripsy. *N Engl J Med* 314, 845–847.
- 616 Perry, R.W., Kantrowitz, A., 1951. The production and stability of converg-
617 ing shock waves. *J. Appl. Phys.* 22, 878–886.
- 618 Ridah, S., 1988. Shock waves in water. *Journal of Applied Physics* 64,
619 152–158.
- 620 Schwendeman, D., Whitham, D., 1987. On converging shock waves. *Proc.*
621 *R. Soc. Lond. A* 413, 297–311.

- 622 Shin, Y.S., 2004. Ship shock modeling and simulation for far-field underwater
623 explosion. *Computers & Structures* 82, 2211 – 2219.
- 624 Sturtevant, B., Kulkarny, V.A., 1976. The focusing of weak shock waves.
625 *Journal of Fluid Mechanics* 73, 651–671.
- 626 Takayama, K., Kleine, H., Grönig, H., 1987. An experimental investigation
627 of the stability of converging cylindrical shock waves in air. *Exp. Fluids* 5,
628 315–322.
- 629 Wagner, W., Pruß, A., 1995. The IAPWS formulation 1995 for the thermo-
630 dynamic properties of ordinary water substance for general and scientific
631 use. *Journal of Physical and Chemical Reference Data* 31, 387–535.
- 632 Wang, C., Eliasson, V., 2012. Shock wave focusing in water inside convergent
633 structures. *International Journal of Multiphysics* 6, 267–281.
- 634 Wang, C., Qiu, S., Eliasson, V., 2013. Quantitative pressure measure-
635 ment of shock waves in water using a schlieren-based visualization tech-
636 nique. *Experimental Techniques*, Article first published online: DOI:
637 10.1111/ext.12068.
- 638 Weiland, D., Lee, C., Ugarte, R., Monga, M., 2007. Impact of shockwave
639 coupling on efficacy of extracorporeal shockwave lithotripsy. *Journal of*
640 *Endourology* 21, 137–140.
- 641 Whitham, G., 1974. *Linear and nonlinear waves*. Wiley-Interscience.
- 642 Xi, X., Zhong, P., 2001. Dynamic photoelastic study of the transient stress

- 643 field in solids during shock wave lithotripsy. The Journal of the Acoustical
644 Society of America 109, 1226–1239.
- 645 Young, Y.L., Liu, Z., Xie, W., 2009. Fluid-structure and shock-bubble in-
646 teraction effects during underwater explosions near composite structures.
647 Journal of Applied Mechanics 76, p. 051303.