Interfacial force study on a single bubble in laminar and turbulent flows

Jinyong Feng, Igor A. Bolotnov¹

North Carolina State University, Raleigh, 27606, North Carolina, USA

Abstract:

Two-phase flows are present in various industrial processes in engineering fields ranging from light water reactor engineering to petrochemical engineering. In this paper, we conduct the interfacial force study on a single bubble under both laminar and turbulent flow scenarios. Advanced finite-element based flow solver (PHASTA) with level-set interface tracking method is used to perform the studies. The interface tracking approach is verified and validated by analyzing the interfacial forces, i.e., drag and lift forces, and comparing the results with the experiment-based data and correlations. The sign change of transverse migration direction is observed at $Eu = 3.4$ which is close to the experimental observations. A set of parametric studies, including relative velocity, bubble deformability and turbulent intensity, are performed to analyze the impact of homogeneous turbulent flow on the drag force. A new drag coefficient closure model is proposed which agrees well with the DNS data considering both laminar and turbulent flow. Those studies can complement the experimental database to obtain improved closure laws for interfacial forces and are important contribution to the multiphase computational fluid dynamics (M-CFD) closure model development.

Keywords: direct numerical simulation, interface tracking method, drag force, lift force, homogeneous turbulent flow, shear turbulent flow, bubble deformability

1. Introduction

¹ Corresponding author. Tel.: +1 518 542 8939
E-mail addresses: igor_bolotnov@ncsu.edu (Igor A. Bolotnov)

As a widely encountered phenomenon in various engineering systems, two-phase flow remains an active topic of research in the nuclear engineering community due to its importance in light water reactors (LWR). Accurate prediction of bubbles' behavior in two-phase flow is important because the pressure drop and heat transfer characteristics of bubbly flow sensitively depend on the void fraction distribution, and more importantly, in boiling flows this distribution affects the on-set of departure from nucleate boiling (DNB) phenomena. While experimental studies allow limited observations of large systems under realistic conditions (e.g. measurement of pressure drop changes and other integral parameters), it is generally difficult to independently achieve full control of the separate force effects and to obtain detailed, microscale data directly usable to develop physics-based models. The advancement in high-performance computing (HPC) and direct numerical simulation (DNS) coupled with interface tracking methods (ITMs) provides reliable analysis tool for the two-phase flow high-fidelity modeling. When properly verified and validated, this approach allows to perform additional studies in larger parametric domain than experiments, and thus help better understand the physics-based trends in bubble/turbulence interactions.

The multiphase computational fluid dynamics (M-CFD) codes heavily rely on the interfacial closure laws to model the bubble distribution and dispersion in the domain. The closure laws normally developed based on experimental data (Tomiya et al., 2002; Tomiya et al., 1998) and analytic solutions for very simple conditions. For an excellent interfacial closure law, it should have the following three features: firstly, it is supposed to be based on actual physics and can really describe physical phenomena of bubbly flows under different operating scenarios; secondly, the closure law should be a formula as simple and general as possible, and its profile should be continuous and should not show abnormal changes; thirdly, factors influencing the closure law should be taken into account as comprehensively as possible. Although the experiments provide valuable database for the development and validation of numerical models in the nuclear industry, the rapid advancement of computer power has made the DNS approach feasible in studying complex fluid dynamics problems. Compared with most simulation approaches, DNS solves the Navier-Stokes equations directly without any turbulence closure models. Since it is first-principle based, DNS is regarded as a reliable tool to complement the experiments to develop two-phase closure models.

Among the numerous interfacial forces, drag and lift forces are of special importance, which have direct influence on stream-wise mean velocity and lateral distribution of bubbles in two-phase flows. It has been observed in the experiments that the lateral migration of bubbles strongly depends on the bubble deformability, which typically depends on the bubble size and it can be described by dimensionless number (e.g. Eötvös number). Small, spherical bubbles in upflow condition tend to migrate toward the pipe wall which causes a wall-peaked bubble distribution, whereas large, deformable bubbles tend to migrate towards the pipe center which results in a core-peak bubble distribution (Liu J., 1993; Hibiki et al., 2001). Lu (Lu and Tryggvason, 2008) revealed that this phenomenon is caused by the bubble deformability, not the size of the bubbles by simulating the bubble behavior in turbulent bubbly flow. The migration of bubbles can be explained by the shear-induced lift force model. In this paper, we analyze the lift forces acting on a single bubble in low shear laminar flow (3.8 s^{-1}) and our results are consistent with the experimental observations (Dijkhuizen et al., 2010; Tomiyama et al., 2002).

While the development of closure models for two-phase laminar flow has reached a certain level of confidence, more studies and investigations are required for turbulent flows. In addition to the laminar flow study, we perform the dimensionless study to analyze the impact of homogeneous turbulent flow on the interfacial forces. A set of parametric studies has been performed to quantify the relationship between drag force and typical two-phase turbulent flow parameters, i.e., turbulent intensity, bubble deformability and relative velocity. The ultimate goal of this research is to gain insight on the interfacial forces and improve the existing closure laws. Once the expression for interfacial forces and turbulence are correctly developed, the physics behind the phenomena occurring in bubbly flows can be fully captured by the multiphase CFD approach.

2. Review of interfacial force closure laws

The past several decades witnessed the development of numerical methods based on the continuum (Euler/Euler) and the discrete particle (Euler/Lagrange) approaches to simulate the multiphase flow. In both approaches, the momentum exchange between the two phases is accounted by the source term which can be expressed as a superposition of terms representing different physical mechanism specifically,

$$M_k = M_k^D + M_k^L + M_k^{VM} + M_k^W + M_k^{TD} \quad (1)$$

where the individual terms on the right hand side are: drag force, lift force, virtual mass force, wall force and turbulent dispersion force, respectively. A correct description of the closure laws for the drag, lift and virtual mass forces is of great importance in numerical simulation of bubbly flows.

2.1. Drag force

In bubbly flow, the drag force plays a more important role on the flow pattern than lift, virtual mass and turbulent dispersion forces. It determines the gas phase residence time and terminal velocity of the bubbles. The drag force is expressed as (Anglart et al., 1997)

$$M_l^D = -M_g^D = \frac{1}{8} C_D A''' \rho_l |u_g - u_l| (u_g - u_l) \quad (2)$$

where C_D and A''' are the drag force coefficient and the interfacial area density, respectively. The interfacial area density can be determined for large range of the gas volume fraction using the Ishii and Mishima's correlation (Ishii and Mishima, 1980).

$$A''' = \frac{4.5}{D} \frac{\alpha - \alpha_{gs}}{1 - \alpha_{gs}} + \frac{6\alpha_{gs}}{d_b} \frac{1 - \alpha}{1 - \alpha_{gs}} \quad (3)$$

where α_{gs} is the void fraction in the small bubble region, and $\alpha_{gs} = \alpha$ for spherical bubbles.

For a single bubble, the drag force experienced by the bubble is expressed as

$$F_D = \frac{1}{2} C_D A v_r^2 \rho_l \quad (4)$$

where A is the bubble cross-sectional area.

The origin of the drag force is due to the resistance experienced by a bubble moving in the liquid. Various drag closure laws have been proposed in the literatures in the last few decades. Well-known closure laws include those proposed through the analytical solutions of Hadamard and Rybczynski (Hadamard, 1911) for creeping flow past a spherical gas bubble,

$$C_D = \frac{16}{Re_b} \quad (5)$$

which is valid only for low Re_b , i.e., $Re_b \leq 1$. Re_b is the bubble Reynolds number and it's defined as $Re_b = \frac{\rho v_r D}{\mu}$. It assumes that the interface is completely free from surface-active contaminants. Surface-active contaminants may cause marked changes in internal circulation and drag for a bubble or drop, but the effect on shape is negligible at low Reynolds number.

Moore (Moore, 1963) proposed a correlation for the spherical bubble rising at high Reynolds number ($Re_b \gg 1$) based on analytical derivation,

$$C_D = \frac{48}{Re_b} \left(1 - \frac{2.21}{\sqrt{Re_b}} \right) \quad (6)$$

Various correlations have since been proposed to bridge the gap between these two drag closure laws. One of the most widely accepted correlation is proposed by Tomiyama (Tomiyama et al., 1998). This correlation is based on spherical bubbles in pure systems

$$C_D = \min \left[\frac{16}{Re_b} (1 + 0.15 Re_b^{0.687}), \frac{48}{Re_b} \right] \quad (7)$$

Tomiyama et al. (Tomiyama et al., 1998) also proposed a formula for the drag coefficient, C_D , in a slightly contaminated system as:

$$C_D = \min \left[\frac{24}{Re_b} (1 + 0.15 Re_b^{0.687}), \frac{72}{Re_b} \right] \quad (8)$$

Legendre and Magnaudet (Legendre and Magnaudet, 1998) pointed out that the presence of liquid velocity gradient increased the drag force acting on a bubble and proposed the following drag multiplier ϕ_{Sr} :

$$\phi_{Sr} = 1 + 0.55 Sr^2 \quad (9)$$

where Sr is the non-dimensional shear rate defined by

$$Sr = \frac{d\omega}{v_r} \quad (10)$$

Here ω is the magnitude of the liquid velocity gradient and v_r is the relative velocity between the bubble and the liquid. While this correlation is based on the linear shear flow with a wide range of Reynolds number ($0.1 \leq Re \leq 500$) and shear rate ($0 \leq Sr \leq 1$), it's useful to investigate the impact of turbulent shear flow on the correlation.

Bagchi and Balachandar (Bagchi and Balachandar, 2003) studied the effect of freestream isotropic turbulent flow on the drag and lift forces on a spherical particle. Within the range they discussed where the particle Reynolds number is about 60-600 and the freestream turbulence intensity is about 10-25%, they observed that the turbulence does not have a substantial and systematic effect on the time-average mean drag and the standard drag correlation results in a reasonably accurate prediction of the mean drag obtained from the DNS.

When we developed the proportional-integral-derivative (PID) based controller to evaluate lift and drag forces in laminar flows, we demonstrated excellent agreement with the drag correlation

developed by Tomiyama (Tomiyama et al., 1998) over a wide range of bubble Reynolds numbers. While the development of closure model for two-phase laminar flow has reached a certain level, more studies and investigations are required for the turbulent flows. In this paper, we want to numerically analyze the influence of homogeneous turbulent flow on the drag force experienced by a single bubble and compare its behavior with the laminar flow based on Tomiyama's correlation. A set of parametric studies will be performed to justify the relationship between typical two-phase turbulent flow parameters and drag force.

2.2. Lift force

The lift force on the bubble comes from the velocity gradient of the continuous phase and relative velocity between the bubble and the liquid phase. It has a significant effect on the void fraction distribution in bubbly flows. Bubbles traveling through a fluid in shearing motion will experience lift force transverse to the direction of motion. In simple shear flow, the lift force exerted on a single bubble is expressed as

$$F_L = -C_L |v_r| \frac{\partial u}{\partial y} \quad (11)$$

Drew and Lahey (Drew and Lahey, 1979) derived the following expression for the ensemble-averaged interfacial lift force:

$$M_l^L = -M_g^L = C_L \rho_l \alpha_g (u_g - u_l) (\nabla \times u_l) \quad (12)$$

where C_L is the lift force coefficient.

Lift force governs the transverse migration direction of bubbles in continuous liquid field. The magnitude and direction of the lift force is related to bubble parameters, such as size and deformability. The key factors influencing the lift coefficient are the bubble Reynolds number, shear rate and the bubble deformability. Earlier experimental research in two-phase upflow (Tomiyama et al., 2002) made observations that small bubbles tends to migrate toward the pipe wall which causes a wall-peak bubble distribution, whereas large bubbles tend to migrate toward the pipe center which results in a core-peak bubble distribution. For a spherical bubble in upflow, the shear lift coefficient C_L is positive so that the lift force acts in the direction of decreasing liquid velocity, i.e. in the direction towards the pipe wall.

Legendre and Magnaudet (Legendre and Magnaudet, 1998) investigated the lift force experienced by the spherical bubble over a wide range of Reynolds number ($0.1 \leq Re \leq 500$) and shear rate ($0 \leq Sr \leq 1$) and proposed the following lift coefficient model

$$C_L = \sqrt{\left(\frac{6}{\pi^2} \frac{2.255}{(ReSr)^{0.5} \left(1 + \frac{0.2Re}{Sr}\right)^{1.5}} \right)^2 + \left(\frac{1}{2} \frac{1 + \frac{16}{Re}}{1 + \frac{29}{Re}} \right)^2} \quad (13)$$

They observed that when the shear rate increases, i.e. $Sr = O(1)$, a small but consistent decrease of the lift coefficient occurs while a very significant increase of the drag coefficient, essentially produced by the modifications of the pressure distribution, is observed.

While Legendre and Magnaudet's closure model is valid over the range they discussed, they didn't consider the impact of bubble deformation on the lift coefficient which is more important for the practical applications. One of the noticeable experimental research efforts on the lift force evaluation has been carried out by Tomiyama (Tomiyama et al., 2002) who used a linear shear velocity field in viscous liquids. The following dimensionless numbers were used to describe the bubble characteristics in the experiment: Morton number Mo characterizing the shape of bubbles in a surrounding fluid or continuous phase; Eotvos number Eo defining the ratio of buoyancy to surface tension forces, and the velocity gradient ω of a simple shear flow. These are defined as

$$Mo = \frac{g(\rho_L - \rho_G)\mu_L^4}{\rho_L^2 \sigma^3} \quad (14)$$

$$Eo = \frac{g(\rho_L - \rho_G)D^2}{\sigma} \quad (15)$$

$$\omega = \left| \frac{dv_L}{dy} \right| \quad (16)$$

The experiments (Tomiyama et al., 2002) varied these parameters within the ranges of $-5.5 \leq \log_{10} Mo \leq -2.8$, $1.39 \leq Eo \leq 5.74$, and $0 \leq \omega \leq 8.3 \text{ s}^{-1}$. The closure law for the lift coefficient proposed by Tomiyama is based on the modified Eotvos number given as:

$$C_L = \begin{cases} \min(0.288 \tanh(0.121Re), f(Eo_H)), & Eo_H < 4 \\ f(Eo_H) = 0.00105Eo_H^3 - 0.0159Eo_H^2 - 0.0204Eo_H + 0.474, & 4 \leq Eo_H < 10.7 \end{cases} \quad (17)$$

$$Eo_H = \frac{g(\rho_L - \rho_G)D_H^2}{\sigma} \quad (18)$$

Dijkhuizen et al. (Dijkhuizen et al., 2010) did both numerical and experimental investigation on the lift force acting on the single bubble and found that for the spherical bubble, the numerical simulation using front tracking method agreed well with the correlation presented by Legendre and Magnaudet (Legendre and Magnaudet, 1998). For large deformed bubbles, a good agreement with the correlation proposed by Tomiyama et al. (Tomiyama et al., 2002) was found over a wide range of liquid viscosities. However, the experimental results were quite different from Tomiyama's results. The author attributed those difference to the impact of contamination in the liquid.

2.3. Virtual mass force

The third term in Eq. (1) represents the virtual mass force, which comes into play when one phase is accelerating relative to the other one. In the case of a bubble accelerating in a continuous liquid phase, this force can be described by the following expression,

$$M_l^{VM} = -M_g^{VM} = C_{VM} \alpha_g \rho_l \left[\frac{D_l u_l}{Dt} - \frac{D_g u_g}{Dt} \right] \quad (19)$$

where C_{VM} is the virtual mass coefficient, which for a spherical particle is equal to 0.5 (Drew and Lahey, 1993).

2.4. Wall force

The term M_K^W represents the wall force. When a rising bubble comes in close proximity to a wall, the normally uniform drainage of the fluid around the bubble changes dramatically. The non-slip condition at the wall slows the drainage rate between the bubble and the wall which in turn increases the drainage rate on the opposite side. The net effect of this asymmetry is to create a hydrodynamic force which acts to drive the bubble away from the wall. The wall force is expressed as

$$M_l^W = -M_g^W = C_W \alpha_g \rho_l \frac{|u_g - u_l|^2}{D} \mathbf{n}_w \quad (20)$$

where the coefficient C_W has been used as given by (Antal et al., 1991) and $\mathbf{n}_w$ is the unit outward normal vector on the surface of the wall.

The modeling work of Jiao and Podowski (Jiao and Podowski, 2012) revealed that the bubble motion is not constrained by the wall force when the distance from the walls is larger than the

bubble diameter. Shaver (Shaver and Podowski, 2015) proposed a modification to the lift coefficient based on the distance from the wall given in Eq. (21)and suggested using modified lift force near the wall instead of using wall force.

$$C_L^d = \begin{cases} 0, & \text{for } \frac{y}{D} < \frac{1}{2} \\ C_L \left(3 \left(\frac{2y}{D} - 1 \right)^2 - 2 \left(\frac{2y}{D} - 1 \right)^3 \right), & \text{for } \frac{1}{2} \leq \frac{y}{D} \leq 1 \\ C_L, & \text{for } 1 < \frac{y}{D} \end{cases} \quad (21)$$

where C_L^d is the modified lift coefficient, C_L is the nominal lift coefficient and is determined in the usual manner, y is the bubble distance from the wall and D is the bubble diameter.

2.5. Turbulent dispersion force

The last term in the Eq. (1) is the turbulent dispersion force. The turbulent dispersion force is defined when the liquid velocity continuously and randomly fluctuates and acts on the gas void fraction. The turbulent dispersion force takes into account the effect of the turbulent eddies in the liquid on the bubbles. The expression of the turbulent dispersion force is proposed by Lahey et al. (Lahey et al., 1993),

$$M_l^{TD} = -M_g^{TD} = C_{TD} \rho_l k_l \nabla \alpha_g \quad (22)$$

where k_l is the kinetic energy of the turbulence of the continuous phase. The turbulent dispersion force coefficient, C_{TD} , for bubbly two-phase flow was 0.1 (Lahey et al., 1993). The turbulent dispersion coefficient varies from 0.0 to 0.5 depending on the operational conditions (Pourtoysi et al., 2014).

3. Numerical method

3.1. Governing equations

The direct numerical simulations are performed with our research CFD-solver, PHASTA (Jansen, 1999). PHASTA is a Parallel, Hierarchic, higher-order, Adaptive, Stabilized (finite element method-based (FEM)) Transient Analysis flow solver for both incompressible and compressible flows. The flow solver has been proven to be an effective tool for a multitude of different types of simulations including, Reynolds-Averaged Navier-Stokes (RANS), Large-Eddy Simulation (LES), Detached-Eddy Simulation (DES), and DNS. Various benchmarks (Bolotnov et al., 2011; Jansen, 1993; Behafarid et al., 2013) have verified and validated the PHASTA code

for the simulation of turbulent flows. PHASTA and its advanced features, such as anisotropic adaptive algorithms (Sahni et al., 2006) and the LES/DES models (Tejada-Martínez and Jansen, 2006) have been coupled with an interface tracking method known as the level set method (Nagrath et al., 2006; Sethian, 1999; Sussman et al., 1999).

The spatial and temporal discretization for the Incompressible Navier-Stokes (INS) equations used in PHASTA have been described in (Whiting and Jansen, 2001). The strong form of the INS equations is given by:

$$\text{Continuity} \quad u_{i,i} = 0 \quad (23)$$

$$\text{Momentum} \quad \rho u_{i,t} + \rho u_j u_{i,j} = -p_{,i} + \tau_{ij,j} + f_i \quad (24)$$

where ρ is density, u_i is the i^{th} component of velocity, p is the static pressure, τ_{ij} is the viscous stress tensor, and f_i is the body force along the i^{th} coordinate, including surface tension and gravitational forces. The viscous stress tensor for an incompressible flow of a Newtonian fluid is related to the fluid's dynamic viscosity, μ , and the strain rate tensor, S_{ij} , as:

$$\tau_{ij} = 2\mu S_{ij} = \mu(u_{i,j} + u_{j,i}) \quad (25)$$

We utilize the Continuum Surface Tension (CST) model (Brackbill et al., 1992) which is formulated to numerically model the surface tension effects and it's included in f_i .

3.2. Level-set method

To distinguish between liquid phase and gas phase, the level set method (Sussman et al., 1999; Sussman and Fatemi, 1999; Sussman et al., 1998; Sethian, 1999) is utilized in our research for interface tracking. This involves modeling the interface as the zero-level set of a smooth function, φ , where φ is often called the first scalar and it represents the signed distance from the interface. Thus the interface is defined by $\varphi = 0$. In case of no phase change in the flow the scalar, φ , is advected with a moving fluid according to

$$\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = 0 \quad (26)$$

where $\mathbf{u}$ is the flow velocity vector. The liquid phase is indicated by a positive level set. Since evaluating the jump in physical properties using a step change across the interface is challenging numerically, the properties near the interface are defined using a smoothed Heaviside kernel function, H_ϵ

$$H_\varepsilon(\varphi) = \begin{cases} 0, & \varphi < -\varepsilon \\ \frac{1}{2} \left[1 + \frac{\varphi}{\varepsilon} + \frac{1}{\pi} \sin\left(\frac{\pi\varphi}{\varepsilon}\right) \right], & |\varphi| < \varepsilon \\ 1, & \varphi > \varepsilon \end{cases} \quad (27)$$

where ε is the interface half-thickness. The fluid properties are then defined as

$$\rho(\varphi) = \rho_1 H_\varepsilon(\varphi) + \rho_2 (1 - H_\varepsilon(\varphi)) \quad (28)$$

$$\mu(\varphi) = \mu_1 H_\varepsilon(\varphi) + \mu_2 (1 - H_\varepsilon(\varphi)) \quad (29)$$

Although the solution may be reasonably good in the immediate vicinity of the interface, the distance field may not be correct throughout the domain since the turbulent flow field distort the level set contours. Thus, the level set has to be corrected with a re-distancing operation by solving the following partial differential equation (PDE) (Thomas et al., 2014; Sussman et al., 1999; Sussman and Fatemi, 1999).

$$\frac{\partial d}{\partial \tau} = S(\varphi)[1 - |\Delta d|] \quad (30)$$

where d is a scalar that represents the corrected distance field and τ is the pseudo time over which the PDE is solved to steady-state.

3.3. PID controller

The proportional-integral-derivative (PID) bubble controller is implemented in the PHASTA code to apply variable body force on the single bubble in order to balance the interfacial forces, namely, drag and lift force (Thomas et al., 2015). The averaged control forces can be extracted from the simulation and then used to compute lift and drag using Newton's third law. The PID controller consists of three error feedback mechanisms: the proportional part which represents the current error; the integral part which represents the accumulation of past errors; the derivative part which represents the prediction of future error. The error output for the bubble control algorithm is treated as the distance between the bubble's current position and its initial position. For each simulated time step, the bubble controller assesses the bubble's location and velocity and uses those variables as inputs for control loop feedback. The mathematical form of the PID-based bubble controller is given below (Thomas et al., 2014)

$$CF_i^{(n+1)} = c_1 \overline{CF_i^{(n)}} + c_2 \left[CF_i^{(n)} + c_3 dx_i^{(n)} + c_4 dx_i^{2(n)} + c_5 v_i^{(n)} \right] \quad (31)$$

where $CF_i^{(n)}$ is the i^{th} component of the control force at time n , $\overline{CF_i^{(n)}}$ is the historical average of the i^{th} component of control force at time n , $dx_i^{(n)}$ is the i^{th} component of the bubble position difference between time n and time zero, $v_i^{(n)}$ is the i^{th} component of the bubble's average velocity at time n . The historical average, $\overline{CF_i^{(n)}}$, is analogous to the integral input. The linear and higher order location difference terms function as the proportional input. And the velocity and velocity difference terms resemble the derivative input.

The application and implementation of PID bubble controller is an extensive learning process. Through iterative test-trials, we are able to find the appropriate control coefficients to keep the bubble nearly steady. A set of control coefficients are universally applicable for all the cases discussed in the paper and the value of control coefficients are provided in Table 1.

Table 1. Control coefficients of the PID bubble controller

<u>Control coefficients</u>	c_1	c_2	c_3	c_4	c_5
<u>Value</u>	<u>0.2</u>	<u>0.8</u>	5.0×10^6	5.0×10^4	5.0×10^6

3.4. Capability validation

A set of validation tests (Thomas et al., 2015) were done for a range of relative velocity values where the bubble Reynolds number were varied. The bubble Reynolds number is defined as

$$Re_b = \frac{\rho_L v_r D}{\mu_L} \quad (32)$$

By using the PID bubble controller, we are able to balance the drag and lift forces, extract the information of control force and calculate the drag/lift coefficient based on Eq. (4). The results are summarized in Table 2.

Table 2. Drag coefficients at different bubble Reynolds numbers

Case #	1	2	3	4	5	6	7	8	9	10
v_r (m/s)	0.0125	0.0175	0.025	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Re_b	72.93	102.11	145.86	233.38	291.73	350.07	408.42	466.77	525.11	583.46
C_D	0.6805	0.4493	0.3172	0.2075	0.1722	0.1520	0.1372	0.1266	0.1198	0.1142

Figure 1. Estimated drag coefficient from PHASTA simulation (solid line with symbols) as a function of bubble Reynolds number. Dashed line shows Tomiyama's experiment-based correlation (Tomiyama et al., 1998), Eq. (7).

Figure 1 shows the estimated drag coefficients extracted from the PHASTA simulations plotted against bubble Reynolds number. The influence of the bubble Reynolds number on the drag coefficient is drastic at $Re_b < 200$, however, the influence of the bubble Reynolds number on the drag coefficient becomes smaller and smaller with continuously increasing the bubble Reynolds number. At low bubble Reynolds number, the estimated coefficients agree very well with the correlation. It's noted for higher bubble Reynolds number, the correlation begins to under-predict the estimated coefficients. Since the correlation was developed for spherical bubbles, the under prediction is expected to occur when the numerical results are obtained for a slightly deformed bubble. In the simulations the deformation occurs due to increased relative velocity and relatively larger drag force.

4. Results and discussion

The results on the lift and drag force estimation in both laminar and turbulent flow are provided in this section. The evaluation of the lift force in laminar shear velocity field is conducted to verify and validate our methodology. Set of parametric studies are conducted to

evaluate the drag force in homogeneous no-shear turbulent flow conditions. We propose a new drag closure law based on the obtained database.

4.1. Evaluation of lift force in laminar shear flow

Some preliminary studies have been reported in (Thomas et al., 2015), a PID bubble controller is introduced to control the bubble under shear flow condition. The effect of shear rate and wall distance on the interfacial forces under laminar shear flow condition were investigated and high shear turbulent flows were created and the results were compared with the laminar shear flow cases. In the presented work we significantly expand the scope and capabilities of this approach to systematically evaluate the interfacial forces acting on a bubble in various conditions. We numerically analyze the lift force acting on a single bubble in shear flow with the exactly same parameters as Tomiyama's experiment did (Tomiyama et al., 2002), including bubble diameter, shear rate and surface tension to validate our approach against the experimental data. By using the appropriate relative velocity, we are able to maintain earth gravity and the same extended bubble diameter as shown in Figure 7. The estimation of relative velocity is based on the force balance between buoyancy force and drag force.

$$F_D = F_b \quad (33)$$

$$\frac{1}{2} \rho_l A v_r^2 C_D = V (\rho_l - \rho_g) g \quad (34)$$

$$g = \frac{3}{4} \frac{\rho_l}{\rho_l - \rho_g} \frac{C_D v_r^2 D_H^2}{D^3} \quad (35)$$

with C_D is the drag coefficient, v_r is the relative velocity, D is the spherical bubble diameter, D_H is the extended bubble diameter. If we assume the ratio of D_H to D to be a constant, then Eq. (35) can be simplified as

$$g \sim \frac{C_D V_r^2}{D} \quad (36)$$

The validation of this assumption depends on the flow conditions. For spherical bubble, the ratio of D_H to D remains constant, close to 1, while for deformable bubble, the value of the ratio can be slightly larger than 1. Our simulation results show that the range of $\frac{D_H}{D}$ is within the range of 1.04 ~ 1.32 for different deformable bubble shapes.

Tomiyama's drag coefficient correlation is based on the bubble Reynolds number.

$$C_D = \min \left\{ \frac{16}{Re} (1 + 0.15 Re^{0.687}), \frac{48}{Re} \right\} \quad (37)$$

This way gravity can be evaluated as:

$$g \sim \min \left\{ \frac{16 V_r \mu}{D^2 \rho} \left(1 + 0.15 \left(\frac{\rho V_r D}{\mu} \right)^{0.687} \right), \frac{48 \mu V_r}{\rho D^2} \right\} \quad (38)$$

Since the fluid properties are constant in this case, the value of the gravity can be expressed as $g = g(D, V_r)$. Based on the expression for the gravity, we can observe that as the bubble diameter increases, the relative velocity is expected to increase to get the corresponding Earth gravity.

The proposed velocity profile is $(3.8y + v_r)$ m/s and the shear rate is identical to the physical experiment performed in (Tomiyama et al., 2002). The relative velocity, v_r , is case dependent and based on the force balance as discussed above. We apply uniform shear inflow velocity condition on the streamwise direction, constant velocities on the top and bottom surface in lateral direction and natural pressure on the outflow surface. The surface tension is $0.061 \frac{N}{m}$. The air properties are based on 25 °C and 1 atm which are identical to the experiment setup as well.

In order to capture the curvature behavior of a very deformable bubble, we propose to use the unstructured meshing instead of structured Cartesian mesh approach. Another advantage to use the unstructured meshing capabilities (Jansen, 1993) is that we could define a specific refinement region which helps us increase the mesh resolution in the region containing bubble while maintaining the total mesh size under the computational budget. The selected bubble diameters are 2.84 mm, 3.52 mm, 4.16 mm, 4.85 mm and 5.54 mm, which are identical to the experiment.

4.1.1. Computational mesh study

The mesh refinement study was completed to ensure that the bubble control algorithm was built and implemented properly. For this study, the bubble diameter of 5.54 mm case is simulated with five different resolutions to verify the lift and drag results. The mesh configuration is shown in the Figure 2. The number of elements across bubble diameter are 25, 35, 45, 55 and 65, respectively. It should be mentioned that in order to balance the mesh cost while maintaining consistent study, the finest mesh region is compressed in the stream-wise direction for the 65 elements case. However, as shown in the picture (6) of Figure 2, the entire bubble is kept in the finest mesh region when the bubble reaches a steady state.

Figure 2. Mesh configuration of the mesh study cases. The images from (1) to (4) represent the case with 25, 35, 45 and 55 elements across bubble diameter. Both (5) and (6) represent the case with 65 elements across bubble diameter.

The comparison of the control forces is given in the Figure 3 and Figure 4. It demonstrates that the control force evolution for 25 elements case is slightly different from the other cases. As the resolution increases, the cases reach convergent interfacial force results.

Figure 3. Comparison of control force in streamwise (x) direction for mesh study. Mesh type 1 to 5 represent the 25, 35, 45, 55 and 65 elements across bubble diameter, respectively.

Figure 4. Comparison of lateral control force in lateral (y) direction for mesh study. Mesh type 1 to 5 represent the 25, 35, 45, 55 and 65 elements across bubble diameter, respectively.

The control force is averaged from 0.3 s to 0.5 s, where the bubble reaches a quasi-steady state. The drag and lift coefficients are calculated as well and provided in the Table 3. For 45, 55 and 65 elements resolution cases, the drag coefficient difference between them is very small and can be ignored. The convergence of the results is evaluated using standard grid convergence index (Roache, 1998). This is an approach to examine the spatial convergence of a numerical

simulation. The order of grid convergence involves the behavior of the solution error defined as the difference between the discrete solution and the exact solution:

$$E = f(h) - f_{exact} = Ch^p + H.O.T. \quad (39)$$

where C is a constant, h is some measure of grid spacing, p is the order of convergence and $H.O.T.$ represents the higher order terms of truncation error. A direct evolution of p can be obtained from three solutions using a constant grid refinement ratio r

$$p = \ln \left(\frac{f_3 - f_2}{f_2 - f_1} \right) / \ln(r) \quad (40)$$

where f_i represents the calculated drag coefficient in the presented research and r represents the ratio of the number of elements across bubble diameter between two adjoint mesh setups.

Roache (Roache, 1998) defines a grid convergence index (GCI) to provide consistent manner in reporting the results of grid convergence studies and it is based upon a grid refinement error estimator derived from the theory of generalized Richardson Extrapolation (Richardson, 1911; Richardson and Gaunt, 1927). The GCI on the fine grid is defined as

$$GCI_{fine} = \frac{F_s |\varepsilon|}{r^p - 1} \quad (41)$$

where F_s is a factor of safety. The factor of safety is recommended to be 3.0 for comparisons of two grids and 1.25 for comparisons over three or more grids.

The GCI for the coarser grid is defined as

$$GCI_{coarse} = \frac{F_s |\varepsilon| r^p}{r^p - 1} \quad (42)$$

Based on the Eq. (40) to (42), we calculate the GCI of drag coefficient for different mesh setup and it's shown in Figure 5. As the mesh resolution increases, the results approach the true solution. Based on the mesh study, 45 elements across bubble diameter is enough to ensure the control force is converged and maintain reasonable mesh cost. All the mesh study cases were carried out using 64 processing cores and the computational cost is provided in Table 3.

Figure 5. GCI of drag coefficient versus various mesh setup.

Table 3. Summary of the mesh study results

Number of elements across bubble diameter	25	35	45	55	65
Total mesh size (million)	0.3	1.1	2.3	3.1	3.8
Computational cost (wall-clock hours)	7.0	24.5	64.8	126.7	158.1
Number of elements across interface	1.98	2.48	3.20	3.92	4.62
Drag force (N)	1.057E-3	1.030E-3	1.026E-3	1.024E-3	1.023E-3
Lift force (N)	-6.309E-5	-4.881E-5	-4.494E-5	-4.282E-5	-4.400E-5
Drag coefficient	0.788	0.769	0.765	0.763	0.763
Lift coefficient	-0.598	-0.463	-0.426	-0.406	-0.417

4.1.2. Lift evaluation results

The mesh study demonstrates that 45 nodes across bubble diameter is sufficient for obtaining consistent results with interface tracking approach. Due to the various sizes of the bubbles, the mesh and geometrical configuration is case dependent while we keep 45 nodes across the bubble diameter to gain consistence. The mesh configuration of the unstructured mesh model case with diameter of 2.84 mm is shown in Figure 6. The detailed information of the case setup is given in Table 4. The initial spherical bubble diameter and shear velocity field are created to be exactly

the same as in experimental conditions. This way, we could obtain a direct validation of DNS results with the experimental results.

Table 4. Case setup of lift force evaluation study.

Case #	1	2	3	4	5
Bubble diameter (m)	0.00284	0.00352	0.00416	0.00485	0.00554
Simulation domain (m ³)	(0.016, 0.015, 0.015)	(0.018, 0.018, 0.018)	(0.020, 0.020, 0.020)	(0.027, 0.025, 0.025)	(0.027, 0.025, 0.025)
Number of elements across bubble diameter	45	45	45	45	45
Total mesh size (million)	2.19	1.84	2.59	2.19	2.33

Figure 6. Mesh configuration of the unstructured model case with $D = 2.84 \text{ mm}$.

The PID controller is applied to balance the drag force and lift force. After getting the instantaneous control force history from the solver, we average the control force and calculate the corresponding coefficients. After the initial transition, the control forces reach a statistically steady state where we extract and average the control forces over a certain amount of simulation time. To discuss the magnitude of bubble deformability, a dimensionless number, i.e., Eotvos number, is calculated:

$$Eo = \frac{\Delta\rho g D^2}{\sigma} \quad (43)$$

where $\Delta\rho$ is the density difference between the two phases, g is the gravitational acceleration, D is the characteristic length which is the spherical bubble diameter in our cases. At the same time, some researchers use modified Eo number for deformable bubble and replace spherical bubble diameter with extended bubble diameter, D_H , in their approach. The definition of Eo_H is provided in Eq. (18). Figure 7 shows a schematic illustrating the spherical and deformable bubble migration direction in laminar shear flow.

Figure 7. Illustration of the migration direction for spherical and deformable bubble. $V_{lateral}$ represents the lateral migration velocity of the bubble. V_g and V_l represent the streamwise velocity of the bubble and liquid, respectively.

The results from the simulation are compared and validated with the experiment measurement and it is given in Table 5. The extended bubble diameter and Eo number obtained from the simulation are quite close to the experiment and the relative errors of the measured extended bubble diameter and Eo number between simulation and experiment are within 1.4% and 9.3%, respectively.

Table 5. Comparison of the simulation and experiment results.

Case number	PHASTA Simulation							Tomiyaama's Experiment		
	V_r	D	D_H	Eo	gravity	C_D	C_L	D	D_H	Eo
1	0.160	0.00284	0.00296	1.54	9.86	1.131	0.317	0.00284	0.00298	1.54
2	0.198	0.00352	0.00397	2.58	10.74	0.852	0.145	0.00352	0.00396	2.36
3	0.200	0.00416	0.00487	3.24	9.65	0.879	0.028	0.00416	0.00488	3.30
4	0.219	0.00485	0.00610	4.45	9.73	0.722	-0.173	0.00485	0.00607	4.48
5	0.220	0.00554	0.00730	5.96	9.99	0.765	-0.426	0.00554	0.00720	5.85

The evolution of the control force is given in the Figure 8 and Figure 9. Since the drag force is proportional to the bubble cross-sectional area and velocity square which is shown in Eq. (4), the drag force will increase as the bubble size and relative velocity increases. Figure 9 also demonstrates that as the bubble diameter increases which leads to larger Eo value, the bubble becomes more deformable and the migration direction is flipped. This is consistent with the validated experimental data (Dijkhuizen et al., 2010; Tomiyama et al., 2002).

Figure 8. Evolution of control force in streamwise (x) direction for the lift force evaluation study.

Figure 9. Evolution of control force in spanwise (y) direction for the lift force evaluation study.

Table 6. Comparison of bubble and vortex shape between experiment and simulation.

Tomiyama's experiment	PHASTA Simulation	Streamlines around the bubble	Lift coefficient
$C_L = 0.317$
$C_L = 0.145$
$C_L = 0.028$
$C_L = -0.173$
$C_L = -0.426$

As the shear flow interacts with the bubble, the streamlines will form vortex in the wake region which serves as an indicator for the direction of the lift force. We used tracer filter in Paraview (Squillacote, 2007) to show the shape of vortex behind the bubble.

Table 6 shows that for small bubble diameters, i.e., 2.84 mm, 3.52 mm and 4.16 mm, the streamline behind the bubble is almost symmetric along the central line, which indicates negligible lift force in the y direction. However, for large bubble diameter, i.e., higher EO number, the bubble experiences lift force sign change and tends to migrate in $+y$ direction. It has been shown here that the migration direction change of the bubble in shear field is related to the presence of a slanted wake behind the deformable bubble which is caused by the interaction between the wake and shear field. Comparing the cases with diameters of 4.85 mm and 5.54 mm, the strength of vortex is stronger and the magnitude of lift force is increasing. There is no symmetric structure forming behind the deformable bubble. Those observations are consistent with the plot of lift force shown in Figure 9. The lift coefficients for different bubble sizes are provided as well. The bubble shape from simulation is compared with images from experiment in Table 6. The results demonstrate very good agreement. The difference of extended bubble diameter between simulation and experiment is within 1.5% for any of the five cases.

Figure 10. Lift coefficient evaluation. (a) shows the C_L versus Eo_H for different experiment data and presented research; (b) shows the C_L versus Eo from presented research.

The lift coefficient from our DNS results is compared with Dijkhuizen's data (Dijkhuizen et al., 2010) and Tomiyama's data (Tomiyama et al., 2002). Figure 10 shows that the general trend of lift coefficient versus Eo_H follows Tomiyama, however, our results has a slightly lower value of lift coefficient. Tomiyama predicted the transition happens at $Eo_H = 6$, while in our cases it's around 5 with Eo_H . For spherical bubble, our results fit Dijkhuizen data and overpredicts the lift coefficient for more deformable bubble. In Dabiri's simulation (Dabiri et al., 2013), the authors used Eo number to discuss bubble deformation and the lift sign change was observed at $Eo =$

2.5. In our case, the lift sign change happens at $Eo = 3.4$ shown in Figure 10(b). To summarize, the transverse migration of bubble behavior is observed and validated in our DNS simulation. The trend follows Tomiyama's data and the estimated lift coefficient is slightly smaller compared to Tomiyama's.

4.2. Evaluation of the drag coefficient in homogeneous turbulent flow

The lift force study provides insight on the transverse migration of bubble in laminar shear flow. Our results are very close to the experimental observations (Tomiyama et al., 2002; Dijkhuizen et al., 2010). We now expand our interfacial force study to the turbulent flows, which is more common flow condition in most engineering applications. We start with the single bubble drag force estimation in homogeneous turbulent flow. Note that in those conditions there is no mean velocity shear and the only interfacial force acting on the bubble is the drag force. To generate the transient inflow boundary conditions representing fully resolved homogeneous turbulent flow we use the approach described in (Feng and Bolotnov, 2016). To briefly summarize it here, the following steps are performed. First, a single-phase unstructured mesh domain is created with a fully resolved grid of spheres to produce turbulent wakes. We apply a pressure gradient to drive the flow through the grid of spheres to maintain a certain velocity, similar to the typical experimental setup for generating uniform turbulence (Rose, 1966; Lance and Bataille, 1991). Periodic boundary conditions are applied on the inflow and outflow boundaries. After the liquid turbulence level reach a statistically steady state conditions we use a set of virtual probes to record the history of the instantaneous two-dimensional velocity profile. The probes are located at the outflow boundary and the recorded instantaneous field is used as the transient inflow boundary condition for the two-phase single-bubble production simulations. Figure 11 shows the examples of both turbulence-generating (left) and production (right) runs.

Figure 11. Left image shows the generation of homogeneous turbulent flow. The dashed rectangle indicates the probe plane. The simulation on the right utilizes the inflow recorded on the left.

Uniform distribution of velocity and turbulent kinetic energy (TKE) are observed and shown in Figure 12. The averaged velocity is 0.297 m/s and the averaged turbulent kinetic energy is $6.74 \times 10^{-5} \text{ m}^2/\text{s}^2$. The generation of homogeneous turbulent field were carried out on Mira supercomputer at Argonne Leadership Computing Facilities (ALCF) using 8,192 processing cores over about 100 wall-clock hours. Utilizing the instantaneous velocity profile as inflow condition for two-phase simulation, we are able to analyze the effect of turbulent intensity, bubble deformation and relative velocity on the drag force in turbulent flow conditions.

Figure 12. Velocity (dash line) and turbulent kinetic energy (solid line) profiles versus spanwise direction (y) for the single-phase homogeneous turbulent flow.

Figure 13. Simulation domain of drag force evaluation case.

For the two-phase flow simulation, a single bubble with a diameter D is placed in the domain with size of $(7.5D, 5D, 3D)$ as shown in Figure 13. The bubble is located at the center of the lateral plane while the streamwise position is case dependent. For the parametric studies, we keep the mesh configuration identical and only change the flow parameters we want to investigate. Figure 13 shows the simulation domain of the drag force evaluation case together with the mesh configuration in the whole domain.

The domain length is carefully chosen to capture enough flow information upstream and downstream of the bubble. The fluid properties of the simulation cases correspond to water and air at normal temperature and pressure conditions (Table 7).

Table 7. Fluid properties summary.

	Liquid	Gas
Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	996.5	1.161
Viscosity $\left(\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)$	8.544×10^{-4}	1.845×10^{-5}

PID approach used in our simulation effectively varies the gravity in the domain to maintain the bubble position via buoyancy force. While simulated bubble size is 5 mm , this does not correspond to the realistic bubble size in Earth gravity. To obtain the corresponding bubble size in real physical experiment, we compare the values of the dimensionless number, Eo in both simulated and Earth gravity conditions as shown in the following equations:

$$Eo_r = Eo_s \quad (44)$$

$$\frac{\Delta\rho_r g_r D_r^2}{\sigma_r} = \frac{\Delta\rho_s g_s D_s^2}{\sigma_s} \quad (45)$$

$$D_r = D_s \sqrt{\frac{g_s \sigma_r}{g_r \sigma_s}} \quad (46)$$

where subscript r represents the real experiment and s is the numerical simulation. Based on the given fluid properties, bubble size and relative velocity, the corresponding bubble diameter in the physical experiment has a range of 0.45~1.66 mm shown in Table 8, which is close to the range of the bubble sizes expected in the light water reactor cores.

Based on our previous research experience (Thomas et al., 2015), the interface tracking method will lead to consistent results with at least 20 elements across the bubble diameter. In this paper, since we deal with a deformable bubble (1.66 mm), we decide to use 25 elements across the bubble diameter to better capture the curvature behavior. The total mesh size is 1.73 million for all the homogeneous two-phase turbulent flow cases.

A PID bubble controller was previously implemented (Thomas et al., 2015) into the PHASTA code to control the bubble at fixed position (with respect to all three axis) and extract the interfacial forces in the 3 directions. Typically when the bubbles are experimentally observed rising in a standing fluid, they develop wake instabilities and the trajectories of those bubbles are not vertical, but include a transverse motion as well (Hibiki and Ishii, 1999). In order to quantify if the transverse motion of the bubble has an effect on the drag force, a separate simulation has been performed in the presented research by removing the lateral PID control of the bubble. This allowed to the bubble to freely move in the lateral direction and compare the resultant drag force with the bubble controlled in all 3 directions. Figure 14, Figure 15, and Figure 16 compare the evolution of bubble position with lateral control force on and off. The drag force results are very close in both cases shown in Figure 14. Based on the mean control force, the drag coefficients are consistent and the relative error is within 2%.

Figure 14. Comparison of the bubble position evolution in streamwise (x) direction with lateral control force (solid black line) and without lateral control force (dash red line) for the validation of PID bubble controller.

Figure 15. Comparison of the bubble position evolution in y direction with lateral control force (solid black line) and without lateral control force (dash red line) for the validation of PID bubble controller.

Figure 16. Comparison of the bubble position evolution in z direction with lateral control force (solid black line) and without lateral control force (dash red line) for the validation of PID bubble controller.

In the following sections we will present the results on the effects of turbulent intensity, bubble deformability and relative velocity on the drag force experienced by the bubble.

4.2.1. Turbulent intensity effect

In the single-phase turbulent flow, the turbulence energy spectrum is considered to follow the Richardson's (Richardson, 1922) description. Large scale eddies are generated in the regions of high velocity gradients where the mean kinetic energy is converted into turbulent kinetic energy. These larger scale eddies are not stable and undergo continuous breakage process till the Kolmogorov length scale is achieved where the viscous dissipation of these smallest turbulent eddies occurs. The decay of turbulent flow generally follows the power law (Comte-Bellot and Corrsin, 1966) and it can be written as

$$\frac{k}{U_o^2} = A \left(\frac{x - x_o}{M} \right)^{-n} \quad (47)$$

where x_o is the virtual origin, M is the spacing between the two grids. Values of the decay exponent n between 1.15 and 1.45 are reported in the literature (Comte-Bellot and Corrsin, 1966; Mohamed and LaRue, 1990; Kolmogorov, 1941; Kang et al., 2003). The value of A varies

widely depending on the geometry of the grid and the Reynolds number. In our case, the decay exponent is found to be 1.5, which is close to the experiment-based range.

Since the homogeneous turbulent flow follows the power decay law as shown in Figure 17 (Feng and Bolotnov, 2016), we can separately put single bubble in different streamwise position to get different turbulence intensities at the bubble location. We reduce the mean inflow velocity from 0.3 m/s to 0.1 m/s while keeping the same Reynolds stress in the two-phase simulation to achieve the desired relative velocity between the liquid and the bubble. And we use fixed bubble Weber number value of 0.68. Weber number reflects the relative importance of the fluid's inertia compared to its surface tension. The Weber number is defined as:

$$We = \frac{\rho v_r^2 D}{\sigma} \quad (48)$$

where ρ is the liquid density, v_r is the relative velocity, D is the bubble diameter and σ is the surface tension. The corresponding bubble Reynolds number is 583. The turbulent intensity is defined as

$$I = \frac{u'}{U} \quad (49)$$

where u' is the root-mean-square of the turbulent velocity fluctuations and U is the mean velocity. In this case, the mean velocity is 0.1 m/s. The u' can be computed as

$$u' = \sqrt{\frac{1}{3}(u_x'^2 + u_y'^2 + u_z'^2)} = \sqrt{\frac{3}{2}k} \quad (50)$$

Figure 18 shows the separate bubble positions and the turbulent eddies surrounding the bubble. The eddies are visualized by taking a fixed-value contours of the so called Q-criterion, which is defined as

$$Q = \frac{1}{2}[|\Omega|^2 - |S|^2] \quad (51)$$

where $\Omega = \frac{1}{2}[\nabla v - (\nabla v)^T]$ is the vorticity tensor and $S = \frac{1}{2}[\nabla v + (\nabla v)^T]$ is the rate of strain tensor. In Figure 18 we have used the contour values of ± 20 .

Figure 17. Decay of turbulent kinetic energy as a function of x/M . Dashed line represents the power decay curve with decay exponent of 1.5 and solid line represents the results from DNS.

Figure 18. Separate bubbles' position for the turbulent intensity effect study. Bubble local turbulent intensities from left to right are, 3.44%, 2.86% and 2.34%, respectively.

In the presented study, the local turbulent intensities at the bubble location vary from 2.34% to 3.44% (Figure 18). Higher turbulent intensities would lead to stronger interactions between the bubble and liquid turbulent eddies. Higher intensities create additional challenges to steadily control the bubble using the PID approach. The control force fluctuation is defined and calculated as follows

$$\Delta CF = \frac{\sum_{i=1}^N |CF_i - \overline{CF}|}{N} \quad (52)$$

where CF_i is the instantaneous control force at timestep i and $\overline{CF}$ is the time-averaged control force.

The magnitude of control force fluctuation are 2.46×10^{-5} , 5.83×10^{-5} and 7.04×10^{-5} , and it increases with turbulent intensity. However, the averaged control forces for different turbulent intensities are quite close and the drag coefficients are 0.0965, 0.0970, and 0.0930, respectively. It supports the hypothesis that the drag coefficient doesn't strongly depend on the turbulent intensity at least over the range we investigate in this research. Figure 19 shows the plot of the drag coefficient versus turbulent intensity. The relative difference is within 2.6%. Based on Tomiyama's correlation (Tomiyama et al., 1998), the drag coefficient is estimated at 0.0823 for laminar conditions.

Figure 19. Comparison of the drag coefficient from PHASTA simulation (square symbols), the drag coefficient from Tomiyama's correlation (dash-dot line) and the control force (circles) as a function of turbulent intensity.

4.2.2. The effect of bubble deformability

In order to study the bubble deformability effect we only modify the surface tension coefficient in our simulations while keeping the other parameters fixed. We perform simulations with five different Weber numbers (0.34, 0.68, 1.35, 2.03 and 2.71) to represent different bubble deformation levels. The corresponding bubble size in physical experiment (at Earth gravity conditions) is given in Table 8. After the flow field around bubbles reaches a quasi-steady state, the deformable bubble has a stable bubble shape as shown in Figure 20.

Table 8. Summary of the bubble deformability study.

Case #	1	2	3	4	5
We	0.34	0.68	1.35	2.03	2.71
Spherical bubble diameter (mm)	0.45	0.64	0.91	1.16	1.66
Extended bubble diameter (mm)	0.45	0.67	0.95	1.30	2.00

Figure 20. Velocity field around different deformable bubbles.

Figure 21. Comparison of the drag force (solid line), the drag coefficient for homogeneous turbulent flow condition (dash line) and the drag coefficient for laminar flow condition (dash-dot line) versus different bubble deformation level

Since the bubble has the wide range of deformation level, it would be a mistake to use the spherical bubble diameter to calculate frontal area of the bubble to estimate the drag coefficient

based on the drag force obtained in the simulations. Considering the bubble deformation, the frontal area is calculated based on the extended bubble diameter (D_H) as shown in Figure 7. Figure 21 shows the drag force and drag coefficient versus different bubble deformation levels. The bubble cross-sectional area increases with the bubble deformability from $0.25\pi D^2$ to $0.30\pi D^2$. Since the drag force is proportional to the cross-sectional area shown in Eq. (4), the drag force increases with bubble deformability. For the drag coefficient, it decreases initially which is similar to the trend for laminar flow. However, the drag coefficient increases sharply and reaches the maximum for the most deformable bubble. It may be due to the effect of bubble-induced turbulence which causes the violent convective and temporal acceleration between the liquid and gas. Legendre and Magnaudet (Legendre and Magnaudet, 1998) pointed out that the presence of liquid velocity gradient increases the drag force acting on the bubble. Since the liquid velocity gradient around the deformable bubble is higher compared to a spherical bubble, the observed increase in drag force is consistent with the experiments.

4.2.3. The effect of relative velocity

In addition to the bubble deformability and local turbulent intensity studies, the relative velocity effect is also investigated in our research. Four values of relative velocities, 0.08 m/s, 0.10 m/s, 0.12 m/s and 0.15 m/s, are chosen. The bubble in each case is placed at the position where the turbulent intensity is expected to be the same in each case. We also keep the same bubble deformation level ($We = 0.68, D = 0.64 \text{ mm}$), by changing the surface tension coefficient. The summary of the cases setup is given in Table 9. Figure 22 confirms that the bubble shape is very close when we maintain the same Weber number.

Table 9. Summary of the relative velocity study.

Case #	1	2	3	4
We	0.68	0.68	0.68	0.68
Relative velocity (<i>m/s</i>)	0.08	0.10	0.12	0.15
Surface tension (<i>N/m</i>)	0.0471	0.0735	0.1059	0.1654

Figure 22. Comparison of bubble shapes for different relative velocities.

Figure 23 provides the comparison of the drag coefficient as a function of bubble Reynolds number. It shows that the drag coefficient calculated from PHASTA simulation are quite close to the experiment-based correlation under the turbulent flow condition. However, the difference increases with higher bubble Reynolds number. A modified closure model is proposed to predict the drag coefficient at this Reynolds number range. As shown in the Figure 23, the proposed model fits well with the drag coefficient data point from both laminar and turbulent flow conditions. We expect it to be valid within the Re_b of 900.

$$C_D = \min \left[\frac{16}{Re_b} (1 + 0.15 Re_b^{0.687}), \frac{48}{Re_b} (1 + 3 \times 10^{-10} Re_b^{3.3189}) \right] \quad (53)$$

Figure 23. Drag coefficient comparison between Tomiyama's correlation and DNS-based correlation. Data point from laminar flow simulation has been reported in (Thomas et al., 2015).

5. Conclusions

The results presented in this paper evaluated the lift force and drag force in laminar shear flow and turbulent flow using DNS coupled with interface tracking approach. For laminar shear flow, we numerically simulate the two-phase shear flow with the exactly same setup of experiment and obtain fully consistent results. We observe the sign change of lift force and compare it with available experimental data. We present the study on the effect of turbulent flow on the drag force experienced by a single bubble covering a parametric space which includes bubble deformability, relative velocity and turbulent intensity. A new correlation is proposed to predict the drag coefficient over the range we discussed and expected to be valid for bubble Reynolds number up to 900. Future work will include the interfacial force study under high shear flow conditions, as well as the investigation of the wall effect on the interfacial forces. The results and methods presented herein have provided an outlook for developing state-of-the-art lift and drag coefficients closure model in the future and it can be used to complement the existing analytical, numerical, and experimental interfacial force correlation database.

Acknowledgement

This work was supported by the National Science Foundation (award 13339933 under CBET-Fluid Dynamics Program). Computational resources were supported by U.S. Nuclear Regulatory Commission's Faculty Development Program. An award of computer time was provided by the ASCR Leadership Computing Challenge (ALCC) program. This research used resources of the Argonne Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC02-06CH11357. The solution presented herein made use of the Acusim linear algebra solution library provided by Altair Engineering Inc. and meshing and geometric modeling libraries by Simmetrix Inc.

Nomenclature

Notation

A	Bubble cross-sectional area (m^2)
A'''	Interfacial area density (m^{-1})
C_D	Drag coefficient
C_L	Lift coefficient
C_{TD}	Turbulent dispersion coefficient
C_{VM}	Virtual mass coefficient

C_W	Wall coefficient
$C F_i^{(n)}$	i^{th} component of the control force at time n
d	Scalar in re-distancing equation (m)
D	Spherical bubble diameter (m)
D_H	Extended bubble diameter (m)
E_o	Spherical bubble-based Eotvos number
E_{oH}	Extended bubble-based Eotvos number
f	Body force
F_D	Drag force (N)
F_L	Lift force (N)
F_s	Safety factor for GCI calculation
F_{TD}	Turbulent dispersion force (N)
F_{VM}	Virtual mass force (N)
F_W	Wall force (N)
g	Gravity (m/s^2)
h	Grid spacing
H_ε	Heaviside kernel function
I	Turbulent intensity
k_l	Liquid turbulent kinetic energy (m^2/s^2)
M	Spacing between two grids (m)
M_k	Momentum exchange term of phase k ($kg/(m^2 \cdot s^2)$)
M_k^D	Momentum exchange term of phase k caused by drag force ($kg/(m^2 \cdot s^2)$)
M_k^L	Momentum exchange term of phase k caused by lift force ($kg/(m^2 \cdot s^2)$)
M_k^{VM}	Momentum exchange term of phase k caused by virtual mass force ($kg/(m^2 \cdot s^2)$)
M_k^{TD}	Momentum exchange term of phase k caused by turbulence dispersion force ($kg/(m^2 \cdot s^2)$)
M_k^W	Momentum exchange term of phase k caused by wall force ($kg/(m^2 \cdot s^2)$)
Mo	Morton number
n	Power decay exponent
n_w	Unit normal vector
p	Pressure (N/m^2) or order of convergence for GCI
Q	q-criterion (s^{-2})
r	Grid refinement ratio
Re	Reynolds number
Re_b	Bubble Reynolds number
Re_τ	Friction Reynolds number
S	Strain rate tensor (m/s)
Sr	Non-dimensional shear rate (s^{-1})
t	Time (s)
v_r	Relative velocity between liquid and gas (m/s)
$\vec{w}$	Pseudo velocity in re-distancing equation (m/s)
We	Weber number
u'	Root-mean-square of the turbulent velocity fluctuations (m/s)
u_g	Gas velocity (m/s)
u_l	Liquid velocity (m/s)

U_o	Mean streamwise velocity in wind tunnel experiment (m/s)
U	Mean velocity (m/s)
x	x direction coordinate (m)
x_o	Virtual origin (m)

Greek letters

α	Void fraction
α_{gs}	Void fraction in the small bubble region for
δ	Channel width (m)
ε	Interface half-thickness in level-set equation (m)
ε_d	Interface half-thickness in re-distancing equation (m)
φ	Scalar in advection equation (m)
ϕ_{sr}	Dimensionless drag multiplier
μ_L	Liquid dynamic viscosity (N·s/m ²)
ν_l	Liquid kinematic viscosity (m ² /s)
ω	Shear rate (s ⁻¹)
Ω	Vorticity tensor (s ⁻¹)
σ	Surface tension (N/m)
τ	Pseudo time in re-distancing equation (s)
τ_{ij}	Viscous stress tensor (N/m ²)
ρ_l	Liquid density (kg/m ³)

References

- Anglart, H., Nylund, O., Kurul, N., Podowski, M., 1997. CFD prediction of flow and phase distribution in fuel assemblies with spacers. Nucl. Eng. Des. 177, 215-228.
- Antal, S., Lahey, R., Flaherty, J., 1991. Analysis of phase distribution in fully developed laminar bubbly two-phase flow. Int. J. Multiphase Flow 17, 635-652.
- Bagchi, P., Balachandar, S., 2003. Effect of turbulence on the drag and lift of a particle. Physics of Fluids (1994-present) 15, 3496-3513.
- Behafarid, F., Shaver, D., Bolotnov, I., Antal, S., Jansen, K., Podowski, M., 2013. Coupled DNS/RANS Simulation of Fission Gas Discharge During Loss-of-Flow Accident in Generation IV Sodium Fast Reactor. Nucl Technol 181, 44-55.
- Bolotnov, I.A., Jansen, K.E., Drew, D.A., Oberai, A.A., Lahey, J.T., R., 2011. Detached Direct Numerical Simulation of Turbulent Two-phase Bubbly Channel Flow. Int. J. Multiphase Flow 37, 647-659.
- Brackbill, J.U., Kothe, D.B., Zemach, C., 1992. A continuum method for modeling surface tension. Journal of computational physics 100, 335-354.

- Comte-Bellot, G., Corrsin, S., 1966. The use of a contraction to improve the isotropy of grid-generated turbulence. *J. Fluid Mech.* 25, 657-682.
- Dabiri, S., Lu, J., Tryggvason, G., 2013. Transition between regimes of a vertical channel bubbly upflow due to bubble deformability. *Physics of Fluids (1994-present)* 25, 102110.
- Dijkhuizen, W., van Sint Annaland, M., Kuipers, J., 2010. Numerical and experimental investigation of the lift force on single bubbles. *Chemical engineering science* 65, 1274-1287.
- Drew, D., Lahey, R., 1993. Analytical modeling of multiphase flow. *Particulate two-phase flow* , 509-566.
- Drew, D.A., Lahey, R.T., 1979. Application of general constitutive principles to the derivation of multidimensional two-phase flow equations. *Int. J. Multiphase Flow* 5, 243-264.
- Feng, J., Bolotnov, I.A., 2016. Evaluation of bubble-induced turbulence using direct numerical simulation.
- Hadamard, J.S., 1911. Mouvement permanent lent d'une sphere liquide et visqueuse dans un liquide visqueux. *C. R. Acad. Sci.* 152, 1735.
- Hibiki, T., Ishii, M., 1999. Experimental study on interfacial area transport in bubbly two-phase flows. *Int. J. Heat Mass Transfer* 42, 3019-3035.
- Hibiki, T., Ishii, M., Xiao, Z., 2001. Axial interfacial area transport of vertical bubbly flows. *Int. J. Heat Mass Transfer* 44, 1869-1888.
- Ishii, M., Mishima, K., 1980. Study of two-fluid model and interfacial area .
- Jansen, K.E., 1993. Unstructured grid large eddy simulations of wall bounded flows. *Annual Research Briefs, Center for Turbulence Research, NASA Ames/Stanford University* , 151.
- Jansen, K.E., 1999. A stabilized finite element method for computing turbulence. *Comput. Methods Appl. Mech. Eng.* 174, 299-317.
- Jiao, H., Podowski, M., 2012. An analysis of multidimensional models of gas/liquid flows. *Transactions of the American Nuclear Society* 107, 1393.
- Kang, H.S., Chester, S., Meneveau, C., 2003. Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation. *J. Fluid Mech.* 480, 129-160.
- Kolmogorov, A.N., 1941. Dissipation of energy in locally isotropic turbulence. 32, 16-18.
- Lahey, R., de Bertodano, M.L., Jones, O., 1993. Phase distribution in complex geometry conduits. *Nucl. Eng. Des.* 141, 177-201.

- Lance, M., Bataille, J., 1991. Turbulence in the liquid phase of a uniform bubbly air–water flow. *J. Fluid Mech.* 222, 95-118.
- Legendre, D., Magnaudet, J., 1998. The lift force on a spherical bubble in a viscous linear shear flow. *J. Fluid Mech.* 368, 81-126.
- Liu J., T., 1993. Bubble size and entrance length effects on void development in a vertical channel. *Int.Journal of Multiphase Flow* 19, 99-113.
- Lu, J., Tryggvason, G., 2008. Effect of bubble deformability in turbulent bubbly upflow in a vertical channel. *Physics of Fluids (1994-present)* 20, 040701.
- Mohamed, M.S., LaRue, J.C., 1990. The decay power law in grid-generated turbulence. *J. Fluid Mech.* 219, 195-214.
- Moore, D., 1963. The boundary layer on a spherical gas bubble. *J. Fluid Mech.* 16, 161-176.
- Nagrath, S., Jansen, K., Lahey Jr, R.T., Akhatov, I., 2006. Hydrodynamic simulation of air bubble implosion using a level set approach. *Journal of Computational Physics* 215, 98-132.
- Pourtousi, M., Sahu, J., Ganesan, P., 2014. Effect of interfacial forces and turbulence models on predicting flow pattern inside the bubble column. *Chemical Engineering and Processing: Process Intensification* 75, 38-47.
- Richardson, L.F., Gaunt, J.A., 1927. The deferred approach to the limit. Part I. Single lattice. Part II. Interpenetrating lattices. *Philosophical Transactions of the Royal Society of London. Series A, containing papers of a mathematical or physical character* 226, 299-361.
- Richardson, L.F., 1911. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character* 210, 307-357.
- Richardson, L., 1922. Weather prediction by numerical methods.
- Roache, P.J., 1998. *Verification and Validation in Computational Science and Engineering*. Hermosa.
- Rose, W., 1966. Results of an attempt to generate a homogeneous turbulent shear flow. *J. Fluid Mech.* 25, 97-120.
- Sahni, O., Müller, J., Jansen, K.E., Shephard, M.S., Taylor, C.A., 2006. Efficient anisotropic adaptive discretization of the cardiovascular system. *Comput. Methods Appl. Mech. Eng.* 195, 5634-5655.

- Sethian, A.J., 1999. Level Set Methods and Fast Marching Methods. Cambridge University Press.
- Shaver, D.R., Podowski, M.Z., 2015. Modeling of interfacial forces for bubbly flows in subcooled boiling conditions. 113, 1368-1371.
- Squillacote, A.H., 2007. The ParaView Guide: A Parallel Visualization Application. Kitware.
- Sussman, M., Almgren, A.S., Bell, J.B., Colella, P., Howell, L.H., Welcome, M.L., 1999. An adaptive level set approach for incompressible two-phase flows. Journal of Computational Physics 148, 81-124.
- Sussman, M., Fatemi, E., Smereka, P., Osher, S., 1998. An improved level set method for incompressible two-phase flows. Comput. Fluids 27, 663-680.
- Sussman, M., Fatemi, E., 1999. An efficient, interface-preserving level set re-distancing algorithm and its application to interfacial incompressible fluid flow. Siam J. on Scientific Computing 20, 1165-1191.
- Tejada-Martínez, A.E., Jansen, K.E., 2006. A parameter-free dynamic subgrid-scale model for large-eddy simulation. Comput. Methods Appl. Mech. Eng. 195, 2919-2938.
- Thomas, A.M., Fang, J., Bolotnov, I.A., 2014. Estimation of Shear-induced Lift Force in Laminar and Turbulent Flows.
- Thomas, A.M., Fang, J., Feng, J., Bolotnov, I.A., 2015. Estimation of shear-induced lift force in laminar and turbulent flows. Nucl Technol 190, 274-291.
- Tomiyama, A., Kataoka, I., Zun, I., Sakaguchi, T., 1998. Drag coefficients of single bubbles under normal and micro gravity conditions. JSME international journal. Series B, fluids and thermal engineering 41, 472-479.
- Tomiyama, A., Tamai, H., Zun, I., Hosokawa, S., 2002. Transverse migration of single bubbles in simple shear flows. Chemical Engineering Science 57, 1849-1858.
- Whiting, C.H., Jansen, K.E., 2001. A stabilized finite element method for the incompressible Navier-Stokes equations using a hierarchical basis. Int. J. Numer. Methods Fluids 35, 93-116.

References

- Anglart, H., Nylund, O., Kurul, N., Podowski, M., 1997. CFD prediction of flow and phase distribution in fuel assemblies with spacers. *Nucl. Eng. Des.* 177, 215-228.
- Antal, S., Lahey, R., Flaherty, J., 1991. Analysis of phase distribution in fully developed laminar bubbly two-phase flow. *Int. J. Multiphase Flow* 17, 635-652.
- Bagchi, P., Balachandar, S., 2003. Effect of turbulence on the drag and lift of a particle. *Physics of Fluids (1994-present)* 15, 3496-3513.
- Behafarid, F., Shaver, D., Bolotnov, I., Antal, S., Jansen, K., Podowski, M., 2013. Coupled DNS/RANS Simulation of Fission Gas Discharge During Loss-of-Flow Accident in Generation IV Sodium Fast Reactor. *Nucl Technol* 181, 44-55.
- Bolotnov, I.A., Jansen, K.E., Drew, D.A., Oberai, A.A., Lahey, J.T., R., 2011. Detached Direct Numerical Simulation of Turbulent Two-phase Bubbly Channel Flow. *Int. J. Multiphase Flow* 37, 647-659.
- Brackbill, J.U., Kothe, D.B., Zemach, C., 1992. A continuum method for modeling surface tension. *Journal of computational physics* 100, 335-354.
- Comte-Bellot, G., Corrsin, S., 1966. The use of a contraction to improve the isotropy of grid-generated turbulence. *J. Fluid Mech.* 25, 657-682.
- Dabiri, S., Lu, J., Tryggvason, G., 2013. Transition between regimes of a vertical channel bubbly upflow due to bubble deformability. *Physics of Fluids (1994-present)* 25, 102110.
- Dijkhuizen, W., van Sint Annaland, M., Kuipers, J., 2010. Numerical and experimental investigation of the lift force on single bubbles. *Chemical engineering science* 65, 1274-1287.
- Drew, D., Lahey, R., 1993. Analytical modeling of multiphase flow. *Particulate two-phase flow* , 509-566.
- Drew, D.A., Lahey, R.T., 1979. Application of general constitutive principles to the derivation of multidimensional two-phase flow equations. *Int. J. Multiphase Flow* 5, 243-264.
- Feng, J., Bolotnov, I.A., 2016. Evaluation of bubble-induced turbulence using direct numerical simulation.
- Hadamard, J.S., 1911. Mouvement permanent lent d'une sphere liquide et visqueuse dans un liquide visqueux. *C. R. Acad. Sci.* 152, 1735.
- Hibiki, T., Ishii, M., 1999. Experimental study on interfacial area transport in bubbly two-phase flows. *Int. J. Heat Mass Transfer* 42, 3019-3035.

- Hibiki, T., Ishii, M., Xiao, Z., 2001. Axial interfacial area transport of vertical bubbly flows. *Int. J. Heat Mass Transfer* 44, 1869-1888.
- Ishii, M., Mishima, K., 1980. Study of two-fluid model and interfacial area .
- Jansen, K.E., 1993. Unstructured grid large eddy simulations of wall bounded flows. *Annual Research Briefs*, Center for Turbulence Research, NASA Ames/Stanford University , 151.
- Jansen, K.E., 1999. A stabilized finite element method for computing turbulence. *Comput. Methods Appl. Mech. Eng.* 174, 299-317.
- Jiao, H., Podowski, M., 2012. An analysis of multidimensional models of gas/liquid flows. *Transactions of the American Nuclear Society* 107, 1393.
- Kang, H.S., Chester, S., Meneveau, C., 2003. Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation. *J. Fluid Mech.* 480, 129-160.
- Kolmogorov, A.N., 1941. Dissipation of energy in locally isotropic turbulence. 32, 16-18.
- Lahey, R., de Bertodano, M.L., Jones, O., 1993. Phase distribution in complex geometry conduits. *Nucl. Eng. Des.* 141, 177-201.
- Lance, M., Bataille, J., 1991. Turbulence in the liquid phase of a uniform bubbly air–water flow. *J. Fluid Mech.* 222, 95-118.
- Legendre, D., Magnaudet, J., 1998. The lift force on a spherical bubble in a viscous linear shear flow. *J. Fluid Mech.* 368, 81-126.
- Liu J., T., 1993. Bubble size and entrance length effects on void development in a vertical channel. *Int. Journal of Multiphase Flow* 19, 99-113.
- Lu, J., Tryggvason, G., 2008. Effect of bubble deformability in turbulent bubbly upflow in a vertical channel. *Physics of Fluids (1994-present)* 20, 040701.
- Mohamed, M.S., LaRue, J.C., 1990. The decay power law in grid-generated turbulence. *J. Fluid Mech.* 219, 195-214.
- Moore, D., 1963. The boundary layer on a spherical gas bubble. *J. Fluid Mech.* 16, 161-176.
- Nagrath, S., Jansen, K., Lahey Jr, R.T., Akhatov, I., 2006. Hydrodynamic simulation of air bubble implosion using a level set approach. *Journal of Computational Physics* 215, 98-132.
- Pourtousi, M., Sahu, J., Ganesan, P., 2014. Effect of interfacial forces and turbulence models on predicting flow pattern inside the bubble column. *Chemical Engineering and Processing: Process Intensification* 75, 38-47.

- Richardson, L.F., Gaunt, J.A., 1927. The deferred approach to the limit. Part I. Single lattice. Part II. Interpenetrating lattices. Philosophical Transactions of the Royal Society of London. Series A, containing papers of a mathematical or physical character 226, 299-361.
- Richardson, L.F., 1911. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam. Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character 210, 307-357.
- Richardson, L., 1922. Weather prediction by numerical methods.
- Roache, P.J., 1998. Verification and Validation in Computational Science and Engineering. Hermosa.
- Rose, W., 1966. Results of an attempt to generate a homogeneous turbulent shear flow. J. Fluid Mech. 25, 97-120.
- Sahni, O., Müller, J., Jansen, K.E., Shephard, M.S., Taylor, C.A., 2006. Efficient anisotropic adaptive discretization of the cardiovascular system. Comput. Methods Appl. Mech. Eng. 195, 5634-5655.
- Sethian, A.J., 1999. Level Set Methods and Fast Marching Methods. Cambridge University Press.
- Shaver, D.R., Podowski, M.Z., 2015. Modeling of interfacial forces for bubbly flows in subcooled boiling conditions. 113, 1368-1371.
- Squillacote, A.H., 2007. The ParaView Guide: A Parallel Visualization Application. Kitware.
- Sussman, M., Almgren, A.S., Bell, J.B., Colella, P., Howell, L.H., Welcome, M.L., 1999. An adaptive level set approach for incompressible two-phase flows. Journal of Computational Physics 148, 81-124.
- Sussman, M., Fatemi, E., Smereka, P., Osher, S., 1998. An improved level set method for incompressible two-phase flows. Comput. Fluids 27, 663-680.
- Sussman, M., Fatemi, E., 1999. An efficient, interface-preserving level set re-distancing algorithm and its application to interfacial incompressible fluid flow. Siam J. on Scientific Computing 20, 1165-1191.
- Tejada-Martínez, A.E., Jansen, K.E., 2006. A parameter-free dynamic subgrid-scale model for large-eddy simulation. Comput. Methods Appl. Mech. Eng. 195, 2919-2938.
- Thomas, A.M., Fang, J., Bolotnov, I.A., 2014. Estimation of Shear-induced Lift Force in Laminar and Turbulent Flows.

- Thomas, A.M., Fang, J., Feng, J., Bolotnov, I.A., 2015. Estimation of shear-induced lift force in laminar and turbulent flows. *Nucl Technol* 190, 274-291.
- Tomiyama, A., Kataoka, I., Zun, I., Sakaguchi, T., 1998. Drag coefficients of single bubbles under normal and micro gravity conditions. *JSME international journal. Series B, fluids and thermal engineering* 41, 472-479.
- Tomiyama, A., Tamai, H., Zun, I., Hosokawa, S., 2002. Transverse migration of single bubbles in simple shear flows. *Chemical Engineering Science* 57, 1849-1858.
- Whiting, C.H., Jansen, K.E., 2001. A stabilized finite element method for the incompressible Navier-Stokes equations using a hierarchical basis. *Int. J. Numer. Methods Fluids* 35, 93-116.

Table 1. Control coefficients of the PID bubble controller

Control coefficients	c_1	c_2	c_3	c_4	c_5
Value	0.2	0.8	5.0×10^6	5.0×10^4	5.0×10^6

Table 2. Drag coefficients at different bubble Reynolds numbers

Case #	1	2	3	4	5	6	7	8	9	10
v_r (m/s)	0.0125	0.0175	0.025	0.04	0.05	0.06	0.07	0.08	0.09	0.10
Re_b	72.93	102.11	145.86	233.38	291.73	350.07	408.42	466.77	525.11	583.46
C_D	0.6805	0.4493	0.3172	0.2075	0.1722	0.1520	0.1372	0.1266	0.1198	0.1142

Table 3. Summary of the mesh study results

Number of elements across bubble diameter	25	35	45	55	65
Total mesh size (million)	0.3	1.1	2.3	3.1	3.8
Computational cost (wall-clock hours)	7.0	24.5	64.8	126.7	158.1
Number of elements across interface	1.98	2.48	3.20	3.92	4.62
Drag force (N)	1.057E-3	1.030E-3	1.026E-3	1.024E-3	1.023E-3
Lift force (N)	-6.309E-5	-4.881E-5	-4.494E-5	-4.282E-5	-4.400E-5
Drag coefficient	0.788	0.769	0.765	0.763	0.763
Lift coefficient	-0.598	-0.463	-0.426	-0.406	-0.417

Table 4. Case setup of lift force evaluation study.

Case #	1	2	3	4	5
Bubble diameter (m)	0.00284	0.00352	0.00416	0.00485	0.00554
Simulation domain (m ³)	(0.016, 0.015, 0.015)	(0.018, 0.018, 0.018)	(0.020, 0.020, 0.020)	(0.027, 0.025, 0.025)	(0.027, 0.025, 0.025)
Number of elements across bubble diameter	45	45	45	45	45
Total mesh size (million)	2.19	1.84	2.59	2.19	2.33

Table5

Table 5. Comparison of the simulation and experiment results.

Case number	PHASTA Simulation							Tomiyaama's Experiment		
	V_r	D	D_H	Eo	gravity	C_D	C_L	D	D_H	Eo
1	0.160	0.00284	0.00296	1.54	9.86	1.131	0.317	0.00284	0.00298	1.54
2	0.198	0.00352	0.00397	2.58	10.74	0.852	0.145	0.00352	0.00396	2.36
3	0.200	0.00416	0.00487	3.24	9.65	0.879	0.028	0.00416	0.00488	3.30
4	0.219	0.00485	0.00610	4.45	9.73	0.722	-0.173	0.00485	0.00607	4.48
5	0.220	0.00554	0.00730	5.96	9.99	0.765	-0.426	0.00554	0.00720	5.85

Table 6. Comparison of bubble and vortex shape between experiment and simulation.

Tomiyama's experiment	PHASTA Simulation	Streamlines around the bubble	Lift coefficient
$C_L = 0.317$
$C_L = 0.145$
$C_L = 0.028$
$C_L = -0.173$
$C_L = -0.426$

Table 7. Fluid properties summary.

	Liquid	Gas
Density($\frac{\text{kg}}{\text{m}^3}$)	996.5	1.161
Viscosity ($\frac{\text{kg}}{\text{m}\cdot\text{s}}$)	8.544×10^{-4}	1.845×10^{-5}

Table 8. Summary of the bubble deformability study.

Case #	1	2	3	4	5
We	0.34	0.68	1.35	2.03	2.71
Spherical bubble diameter (mm)	0.45	0.64	0.91	1.16	1.66
Extended bubble diameter (mm)	0.45	0.67	0.95	1.30	2.00

Table 9. Summary of the relative velocity study.

Case #	1	2	3	4
We	0.68	0.68	0.68	0.68
Relative velocity (<i>m/s</i>)	0.08	0.10	0.12	0.15
Surface tension (<i>N/m</i>)	0.0471	0.0735	0.1059	0.1654

Figure1

[Click here to download high resolution image](#)

Figure2_1
[Click here to download high resolution image](#)

Figure2_2
[Click here to download high resolution image](#)

Figure2_3
[Click here to download high resolution image](#)

Figure2_4
[Click here to download high resolution image](#)

Figure2_5
[Click here to download high resolution image](#)

Figure2_6
[Click here to download high resolution image](#)

Figure3

[Click here to download high resolution image](#)

Figure4

[Click here to download high resolution image](#)

Figure5

[Click here to download high resolution image](#)

Figure6

[Click here to download high resolution image](#)

Figure7
[Click here to download high resolution image](#)

Figure8

[Click here to download high resolution image](#)

Figure9

[Click here to download high resolution image](#)

Figure10_a

[Click here to download high resolution image](#)

Figure10_b
[Click here to download high resolution image](#)

Figure11

[Click here to download high resolution image](#)

Figure12

[Click here to download high resolution image](#)

Figure13_a
[Click here to download high resolution image](#)

Figure13_b
[Click here to download high resolution image](#)

Figure14

[Click here to download high resolution image](#)

Figure15

[Click here to download high resolution image](#)

Figure16

[Click here to download high resolution image](#)

Figure17

[Click here to download high resolution image](#)

Figure18_a
[Click here to download high resolution image](#)

Figure18_b
[Click here to download high resolution image](#)

Figure18_c
[Click here to download high resolution image](#)

Figure19

[Click here to download high resolution image](#)

Figure20_1
[Click here to download high resolution image](#)

Figure20_2
[Click here to download high resolution image](#)

Figure20_3
[Click here to download high resolution image](#)

Figure20_4
[Click here to download high resolution image](#)

Figure20_5
[Click here to download high resolution image](#)

Figure21

[Click here to download high resolution image](#)

Figure22_a

[Click here to download high resolution image](#)

Figure22_b
[Click here to download high resolution image](#)

Figure22_c
[Click here to download high resolution image](#)

Figure22_d
[Click here to download high resolution image](#)

Figure23

[Click here to download high resolution image](#)