Eigendecomposition Model of Resistance Temperature Detector with Applications to S-CO₂ Cycle Sensing

Alexander Heifetz* and Richard Vilim

Nuclear Engineering Division, Argonne National Laboratory

9700 South Cass Ave., Argonne, IL 60439

*Corresponding Author Email: aheifetz@anl.gov, Fax: 630-252-3250, Tel. 630-252-4429

ABSTRACT

Super-critical carbon dioxide (S-CO₂) is a promising thermodynamic cycle for advanced nuclear reactors and solar energy conversion applications. Dynamic control of the proposed recompression S-CO₂ cycle is accomplished with input from resistance temperature detector (RTD) measurements of the process fluid. One of the challenges in practical implementation of S-CO₂ cycle is high corrosion rate of component and sensor materials. In this paper, we develop a mathematical model of RTD sensing using eigendecomposition model of radial heat transfer in a layered long cylinder. We show that the value of RTD time constant primarily depends on the rate of heat transfer from the fluid to the outer wall of RTD. We also show that for typical material properties, RTD time constant can be calculated as the sum of reciprocal eigenvalues of the heat transfer matrix. Using the computational model and a set of RTD and CO₂ fluid thermo-physical parameter values, we calculate the value of time constant of thermowell-mounted RTD sensor at the hot side of the pre-cooler in the S-CO₂ cycle. The eigendecomposition model of RTD will be used in future studies to model sensor degradation and its impact on control of S-CO₂.

Keywords: eigendecomposition, thermal hydraulics, resistance temperature detector, super-critical carbon dioxide

I. INTRODUCTION

Super-critical carbon dioxide (S-CO₂) is a promising thermodynamic cycle for advanced nuclear reactors and solar energy conversion applications [1,2]. Before S-CO₂ cycles can be implemented in large -scale applications, a number of studies are currently conducted on various aspects of the cycle, including efficient strategies for S-CO₂ control. We have previously investigated the performance of proportional-integral (PI) controller of S-CO₂ cycle, designed to respond to change in the demand of electrical power generation [3]. In the PI controller in our studies, set point was taken as the temperature measurement on the hot side of the pre-cooler. Our studies were performed under idealized conditions of instant measurements. In reality, temperature sensors have time constants, which depend on heat transfer properties of the process fluid and sensor materials. In nuclear power applications, temperature measurements which require higher accuracy are usually performed using resistance temperature detectors (RTD) [4-7]. One of the challenges in practical implementation of S-CO₂ cycle is high corrosion rate of components and sensor material due to toxic nature of the working fluid. In this paper, we develop a computational model of resistance temperature detector (RTD) sensing, which is based on eigendecomposition computation of radial heat transfer in a long cylinder. This model allows for estimating RTD time constant for given geometrical and thermo-physical parameter values of the sensor and CO₂ fluid. The advantage of this RTD model is that it allows for modeling changes in sensor response time due to material degradation in a computationally efficient manner. The RTD model we have developed will be integrated into custom plant simulation software to allow real-time computation of sensor response time for given material and fluid parameters. Modeling the changes of RTD performance due to material degradation, and the impact of these changes on S-CO₂ cycle control will be pursued in our future work.

This paper is organized as follows. In Section II, we describe the basis properties of RTD and develop sensing model using eigendecomposition model of heat transfer. In Section III, we develop the model of heat transfer coefficient from the process fluid to the RTD inserted into stream. In Section IV, we obtain numerical estimates for the time constant of thermowell-mounted RTD measuring CO₂ temperature in the regime close to the fluid critical point. Conclusions of the study in this paper are presented in Section V.

II RESISTANCE TEMPERATURE DETECTOR (RTD) MODEL

II.ARTD Basic Properties

A typical RTD is a long and narrow cylindrical metallic structure with a diameter 0.6cm to 1cm, which is inserted into a plant cooling system pipe to an immersion depth of 5cm to 10cm in the process fluid [4]. Heat sensing element is the bottom 1cm to 3cm long section of the RTD. Figure 1 depicts temperature sensing with RTD inserted into a process fluid. Typical dimensions are presented in Table 1. Nomenclature for thermo-physical material properties of RTD and process fluid is listed in Table 2.

The heat sensing element is a cylindrical layered structure, at the core of which is a thin platinum wire wound on a grooved cylinder called mandrell. The basis of temperature measurements is that resistivity of platinum wire changes with temperature. Change in resistivity can be detected in real time with external circuit. The mandrell, which is made of an insulation material, such as Alumina (Al₂O₃), is enclosed by a concentric cylinder of insulation material of the same type. The insulation, in turn, is enclosed by a concentric cylinder of stainless steel called sheath. Depending on the installation mode, RTD's are either direct immersion (wet-type) or thermowell mounted (well-type). A wet-type RTD is installed into a transverse hole in the

cooling system pipe, so that the sheath is in direct contact with the process fluid. A well-type RTD is installed into a thermowell, which had been previously mounted in the cooling system pipe. A thermowell is a hollow stainless steel cylinder with an average outer diameter of 1cm to 2cm, which is in direct contact with the process fluid. This paper considers the performance of well-type RTD. From the point of view of heat transfer in the sensor, thermowell is considered as an integral component of RTD. It is assumed that the sheath is in direct contact with the thermowell, i.e. there are no air gaps. A schematic drawing of transverse cross-section of well-type RTD is shown in Figure 2. Note that mandrell is an inner part of the insulation, and therefore no material boundary is drawn around mandrell. Radial dimensions and material regions of the RTD in this report are presented in Table 3. Radius of mandrell indicates the location of the sensing wire. Although surfaces of RTD are slightly slanted, in this paper we make straight cylinder approximation of the RTD shape.

II.B. Model of Heat Conduction in RTD

In general, heat conduction in RTD is a 3-D problem, described by [8]

$$\nabla^2 T + \frac{W}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (1)$$

Where T is the temperature, W is the heat generation per unit volume and time, k is thermal conductivity, and $\alpha = k/\rho c$ is thermal diffusivity (ρ is density and c is specific heat capacity). In cylindrical coordinates (r, θ, z) , Eq. (1) becomes

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{W}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (2)$$

Since transverse dimensions of well-type RTD ($\sim 1\text{cm}$) are smaller than the axial length of the sensing element ($\sim 3\text{cm}$), we can make a long cylinder approximation. That is, we assume there is

no heat flow in the z -direction, so that the z -derivative can be set to zero. In addition, we make an assumption of radial-only heat flow because of azimuthal symmetry, so that the θ -derivative can be set to zero as well. In order to develop 1-D radial model of heat conduction in RTD, we need an expression for heat flux. For steady-state radial heat flow through the wall of a hollow cylinder, Eq. (2) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = 0 \quad (3)$$

subject to the boundary conditions $T(r_1) = T_1$ and $T(r_2) = T_2$ at the inner and outer radii, respectively. Solution to Eq. (3) is

$$T(r) = \frac{T_1 - T_2}{\ln(r_2/r_1)} \ln\left(\frac{r}{r_1}\right) + T_1 \quad (4)$$

Heat flux through the normal area $2\pi rL$, where L is the axial length of the cylinder, is given as

$$Q = -k(2\pi rL) \frac{\partial T}{\partial r} \quad (5)$$

Using Equation (4), the heat flux is

$$Q = \frac{2\pi kL}{\ln(r_2/r_1)} (T_1 - T_2) \quad (6)$$

Next, we develop a general formalism for calculating transient response of a cylindrical RTD. In our model of heat transfer, the RTD is subdivided into concentric cylindrical nodes numbered 1 to n . Material boundaries coincide with node boundaries, but there may be more than one node inside the same material. A schematic drawing of five-node RTD with fluid boundary condition is shown in Figure 3. In this figure, insulation layer (which includes mandrell) has been subdivided into three nodes, while sheath and thermowell have each been each designated as

single nodes. The inner and outer boundaries of the node i are the radii r_{i-1} and r_i , respectively, where $r_0 = 0$. The central radius of each node i is

$$r_{i-1/2} = (r_i + r_{i-1}) / 2 \quad (7)$$

For each node i , temperatures at the left boundary, the center, and the right boundary of the node are T_{i-1} , $T_{i-1/2}$, and T_i , respectively. Material parameters of each node i are the axial length L , density ρ_i , specific heat c_i , heat conductivity k_i , and volume

$$V_i = \pi (r_i^2 - r_{i-1}^2) L \quad (8)$$

Radial dimensions and material parameters of five-node RTD model are presented in Table 4. A schematic drawing of node radii and temperatures for the five-node RTD model is shown in Figure 3. Note that the location of the sensing wire is approximately $r_{3/2}$, so that the temperature measured by RTD is given by $T_{3/2}$. Rate of change of temperature $T_{i-1/2}$ in the center of each node i is given by the heat flow balance equation

$$\rho_i V_i c_i \frac{dT_{i-1/2}}{dt} = Q_{i-1,i-1/2} - Q_{i-1/2,i} \quad (9)$$

Where $Q_{i-1,i-1/2}$ is the heat flux flowing from the inner radius r_{i-1} to the center radius $r_{i-1/2}$

$$Q_{i-1,i-1/2} = \frac{2\pi k_i L}{\ln(r_{i-1/2}/r_{i-1})} (T_{i-1} - T_{i-1/2}) \quad (10)$$

And $Q_{i-1/2,i}$ is the heat flux flowing from the center radius $r_{i-1/2}$ to the outer radius r_i

$$Q_{i-1/2,i} = \frac{2\pi k_i L}{\ln(r_i/r_{i-1/2})} (T_{i-1/2} - T_i) \quad (11)$$

Both quantities $Q_{i-1,i-1/2}$ and $Q_{i-1/2,i}$ are defined to be positive in the radially outward direction.

Temperatures in the nodes i and $i+1$ are linked by the heat flux continuity at the nodal boundary

r_i

$$Q_{i-1/2,i} = Q_{i,i+1/2} \quad (12)$$

Where $Q_{i,i+1/2}$ is the heat flux flowing from the inner radius r_i to the center radius $r_{i+1/2}$ of the node

$i+1$

$$Q_{i,i+1/2} = \frac{2\pi k_{i+1} L}{\ln(r_{i+1/2}/r_i)} (T_i - T_{i+1/2}) \quad (13)$$

At the outer boundary of the n^{th} node r_n , which is in contact with the fluid, heat flux is given as

$$Q_{n,n+1/2} = h_f A_n (T_n - T_f) \quad (14)$$

Where h_f is the heat transfer coefficient, T_f is the temperature of the fluid, and A_n is the surface area

$$A_n = 2\pi r_n L \quad (15)$$

Performing algebraic substitutions and simplifications, we arrive at the final solution consisting of a system of n ordinary differential equations which give rates of change of temperatures at the centers of each node (for notational convenience we are replacing half-integer subscripts with integer values):

$$\frac{dT_i}{dt} = \alpha_i \beta_{i-1} T_{i-1} - \alpha_i (\beta_{i-1} + \beta_i) T_i + \alpha_i \beta_i T_{i+1}, \quad 1 \leq i \leq n \quad (16)$$

Where for consistency we define

$$T_0 = 0 \quad (17a)$$

$$T_{n+1} = T_f \quad (17b)$$

The constants are defined as

$$\alpha_i = \frac{2}{\rho_i c_i (r_i^2 - r_{i-1}^2)}, \quad 1 \leq i \leq n \quad (18)$$

And

$$\beta_0 = 0 \quad (19a)$$

$$\beta_i = \frac{k_i k_{i+1}}{k_i \ln \left(\frac{r_{i+1} + r_i}{2r_i} \right) + k_{i+1} \ln \left(\frac{2r_i}{r_i + r_{i-1}} \right)}, \quad 1 \leq i \leq n \quad (19b)$$

$$\beta_n = \frac{k_n h_f r_n}{k_n + h_f r_n \ln \left(\frac{2r_n}{r_n + r_{n-1}} \right)} \quad (19c)$$

In addition, we define

$$\gamma = \gamma_n = \alpha_n \beta_n \quad (20a)$$

$$\gamma_i = 0, \quad i \neq n \quad (20b)$$

Next we introduce the vector of temperatures and its time derivative

$$\mathbf{T} = (T_1 \quad \dots \quad T_n)^T \quad (21)$$

$$\frac{d\mathbf{T}}{dt} = \left(\frac{dT_1}{dt} \quad \dots \quad \frac{dT_n}{dt} \right)^T \quad (22)$$

We define the forcing function vector as $T_f \mathbf{F}$, where

$$\mathbf{F} = \gamma (0 \quad 0 \quad \dots \quad 1)^T \quad (23)$$

The system of differential equations can be written in the matrix form as

$$\frac{d\mathbf{T}(t)}{dt} = -\mathbf{A}(t) \cdot \mathbf{T}(t) + T_f(t) \mathbf{F}(t) \quad (24)$$

Where $\mathbf{A}$ is a tri-diagonal matrix

$$\mathbf{A} = \begin{pmatrix} \alpha_1(\beta_0 + \beta_1) & -\alpha_1\beta_1 & 0 & \cdots & 0 \\ -\alpha_2\beta_1 & \alpha_2(\beta_1 + \beta_2) & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -\alpha_{n-1}\beta_{n-1} \\ 0 & 0 & \cdots & -\alpha_n\beta_{n-1} & \alpha_n(\beta_{n-1} + \beta_n) \end{pmatrix} \quad (25)$$

Physical units of entries of the matrix $\mathbf{A}$ and vector $\mathbf{F}$ are s^{-1} . Solution of the matrix Eq. (24) is subject to the initial condition

$$\mathbf{T}(0) = (T_1(0) \quad \cdots \quad T_n(0))^T \quad (26)$$

Using the 1-D model of heat transfer in RTD, we can determine the response time by applying a step change forcing function to RTD, which is initially in steady state. The time constant is calculated as the time required for the RTD internal temperature to approach within e^{-1} of the new steady-state value. In order to perform calculations of RTD response, we need the auxiliary routines for calculating the values of entries in the matrix $\mathbf{A}$ and the vector $\mathbf{F}$. List of symbols of thermophysical material parameters of the RTD materials and the process fluid is given in Table 2. In general, all of the parameters are temperature-dependent, so that the system of ordinary differential equations (ODE) in Equation (24) is non-linear.

II.C.Derivation of RTD Time Constant Using Eigendecomposition Method

In general, RTD response can be calculated by numerically solving the nonlinear system of ODE's in Equation (24). From the correlations listed in Table 5 [9], RTD material and process fluid parameters are weakly dependent on temperature. To observe that, note that all correlations in Table 5 have a form $x(T) = a_0 + a_1T + \dots + a_nT^n$, where $n \leq 4$ and $a_n/a_0 < 10^{-n}$. If the change in temperature is small, i.e., $\Delta T \ll T$, the change in a variable Δx can be calculated from the Taylor series expansion of the terms $a_n(T + \Delta T)^n \approx a_nT^n(1 + n\Delta T/T)$. The change in a variable can be estimated as $\Delta x = (\Delta T/T) \sum n a_n T^n$. From Table 7, typical value of interest is $T \approx 30^\circ\text{C}$. For $n \geq 2$,

$na_n T^n < a_0$, so that $\Delta x \ll x$. If the temperature does not change appreciably over the observation time, the values of the matrix $\mathbf{A}$ and vector $\mathbf{F}$ can be approximated as time-independent constants, and calculated using temperature values at the start of the transient. Under this approximation, the system of ODE's in Equation (24) is linear. Typical power plant transients involve temperature changes of several percent, so that conditions for linearity are satisfied in most cases in practice. Then, we can develop eigendecomposition solution, which allows computing the RTD time constant in terms of the eigenvalues of the heat transfer matrix $\mathbf{A}$. The eigendecomposition method has been successfully implemented in the past to solve problems such as the photon transfer [10] and the neutron transport [11] equations. The tri-diagonal matrix $\mathbf{A}$, by construction, has non-zero entries on the main diagonal and the first diagonals above and below the main diagonal. Therefore, row vectors and column vectors are linearly independent, respectively. The matrix is invertible and has non-zero eigenvalue. The case of repeated eigenvalues can be addressed by several approaches. For example, because repeated eigenvalues lead to Jordan canonical form instead of diagonal form, one could use applicable techniques for the Jordan canonical form. As another approach, one could use an adaptive mesh to generate symmetric tri-diagonal matrix, which is guaranteed to have distinct eigenvalues. Detailed discussion of the special case of repeated eigenvalues is outside the scope of the present paper.

The solution to the matrix ODE Eq. (24) is obtained using the integrating factor technique [12]

$$\mathbf{P}(t) = e^{\int \mathbf{A} dt} = e^{\mathbf{A}t} \quad (27)$$

So that the solution becomes

$$\mathbf{T}(t) = e^{-\mathbf{A}t} \mathbf{T}(0) + e^{-\mathbf{A}t} \int_0^t e^{\mathbf{A}\xi} \mathbf{F} T_f(\xi) d\xi \quad (28)$$

Next, using the spectral decomposition

$$e^{-\Lambda t} = \mathbf{S} e^{-\Lambda t} \mathbf{S}^{-1} \quad (29)$$

Where $\mathbf{S}$ the matrix of eigenvectors of $\mathbf{A}$,

$$\mathbf{S} = (\mathbf{x}_1 \quad \cdots \quad \mathbf{x}_n)^T \quad (30)$$

And Λ is the diagonal matrix of eigenvalues of $\mathbf{A}$, so that

$$e^{\Lambda t} = \begin{pmatrix} e^{\lambda_1 t} & & \\ & \ddots & \\ & & e^{\lambda_n t} \end{pmatrix} \quad (31)$$

Note that λ 's have physical units of s^{-1} , and the elements of the matrix $\mathbf{S}$ are dimensionless.

Then, Equation (28) can be written as

$$\mathbf{T}(t) = \mathbf{S} e^{-\Lambda t} \mathbf{S}^{-1} \mathbf{T}(0) + \mathbf{S} e^{-\Lambda t} \mathbf{S}^{-1} \int_0^t \mathbf{S} e^{\Lambda \xi} \mathbf{S}^{-1} \mathbf{F} T_f(\xi) d\xi. \quad (32)$$

Since the matrices $\mathbf{S}$ and $\mathbf{S}^{-1}$, and the vector $\mathbf{F}$ consist of temperature-independent constant entries, the matrices can be taken outside of the integral, so Eq. (8) simplifies to

$$\mathbf{T}(t) = \mathbf{S} e^{-\Lambda t} \mathbf{S}^{-1} \mathbf{T}(0) + \mathbf{S} e^{-\Lambda t} \left(\int_0^t e^{\Lambda \xi} T_f(\xi) d\xi \right) \mathbf{S}^{-1} \mathbf{F} \quad (33)$$

Where

$$\int_0^t e^{\Lambda \xi} T_f(\xi) d\xi = \begin{pmatrix} \int_0^t e^{\lambda_1 \xi} T_f(\xi) d\xi & & \\ & \ddots & \\ & & \int_0^t e^{\lambda_n \xi} T_f(\xi) d\xi \end{pmatrix} \quad (34)$$

Note that the first term in Equation (32) is the homogeneous solution $\mathbf{T}_h(t)$, and the second term in Equation (32) is the particular solution $\mathbf{T}_p(t)$. The homogeneous solution has the form

$$\mathbf{T}_h(t) = c_1 \mathbf{x}_1 e^{-\lambda_1 t} + \dots + c_n \mathbf{x}_n e^{-\lambda_n t} \quad (35)$$

Where we define

$$\mathbf{C} = (c_1 \quad \cdots \quad c_n)^T \quad (36)$$

Such that

$$\mathbf{S} \cdot \mathbf{C} = \mathbf{T}(0) \quad (37)$$

From Eq. (31), we expect that the matrix $\mathbf{A}$ is positive definite, i.e., all eigenvalues of $\mathbf{A}$ are positive. This is because any negative eigenvalue would lead to unphysical exponential growth of temperature in the absence of the forcing function. The particular solution has the form

$$\mathbf{T}_p(t) = d_1 \mathbf{x}_1 e^{-\lambda_1 t} \int_0^t e^{\lambda_1 \xi} T_f(\xi) d\xi + \dots + d_n \mathbf{x}_n e^{-\lambda_n t} \int_0^t e^{\lambda_n \xi} T_f(\xi) d\xi \quad (38)$$

Where we define

$$\mathbf{D} = (d_1 \quad \cdots \quad d_n)^T \quad (39)$$

Such that

$$\mathbf{S} \cdot \mathbf{D} = \mathbf{F} \quad (40)$$

Response time of the RTD can be estimated from the relaxation dynamics after the RTD is excited with a sudden step change in temperature. We consider two types of excitations: step up and step down.

III.C.1. Sudden Temperature Decrease

Let us assume that RTD is initially in steady state $\mathbf{T}(0) = T_{f0} (1 \quad \cdots \quad 1)^T$, and let the temperature forcing function be described as

$$T_f(t) = \begin{cases} T_{f0}, & t \leq 0 \\ 0, & t > 0 \end{cases} \quad (41)$$

where T_{f0} is some positive constant. Then all the integrals in the particular solution are zeros, so that $\mathbf{T}_p(t) = 0$, and

$$\mathbf{T}(t) = \mathbf{T}_h(t) = c_1 \mathbf{x}_1 e^{-\lambda_1 t} + \dots + c_n \mathbf{x}_n e^{-\lambda_n t} \quad (42)$$

The final steady-state RTD temperature of every node is zero.

III.C.2. Sudden Temperature Increase

Let us assume that RTD is initially in steady state with $\mathbf{T}(0) = 0$, and let the temperature forcing function be described as

$$T_f(t) = \begin{cases} 0, & t \leq 0 \\ T_{f0}, & t > 0 \end{cases}, \quad (43)$$

where T_{f0} is again some positive constant. From Equation (32), $\mathbf{T}_h(0) = 0$, and the integrals in the particular solution can be evaluated to obtain

$$\mathbf{T}(t) = \mathbf{T}_p(t) = b_1 \mathbf{x}_1 T_{f0} (1 - e^{-\lambda_1 t}) + \dots + b_n \mathbf{x}_n T_{f0} (1 - e^{-\lambda_n t}) \quad (44)$$

Where for each i

$$b_i = d_i / \lambda_i \quad (45)$$

The final steady-state RTD temperature of every node is T_{f0} . In either Case 1 or Case 2, the time constant for each node is

$$\tau_i = 1 / \lambda_i \quad (46)$$

RTD is in steady state when all of its nodes individually reach the steady state. Since we are solving a linear diffusion-type heat transfer problem, which excludes the possibility of temperature oscillations, a given node can reach the steady state only after all the outer nodes have attained the steady state. Hence the RTD time constant is equal to the sum of the individual node time constants.

$$\tau = \sum_{i=1}^n \tau_i, \quad (47)$$

which allows to compute RTD time constant as a sum of the inverses of the eigenvalues of the heat transfer matrix.

$$\tau = \sum_{i=1}^n 1/\lambda_i \quad (48)$$

III. CALCULATION OF HEAT TRANSFER COEFFICIENT

The temperature of the fluid stream is sensed by the tip of the RTD as depicted in Figure 1. The heat transfer at the tip will be dependent on the location of the tip within the fluid stream as indicated by the depth parameter L . The following correlation [13] can be used to calculate heat transfer coefficient h in forced convective heat transfer for fluid flow normal to circular cylinder with diameter d :

$$h = \frac{k * Nu}{d} \quad (49)$$

$$Nu = 0.43 + C * Re^m * Pr^{0.31} \quad (50)$$

$$Re = \frac{\rho * v * d}{\mu} \quad (51)$$

$$Pr = \frac{c_p * \mu}{k} \quad (52)$$

Where ρ is fluid density, μ is viscosity, c_p is specific heat, k is thermal conductivity, v is velocity, and Nu is the Nusselt number. Table 6 lists the values of constants C and m for the ranges of the Reynolds number for Equation (50). For cross-validation of the correlation in Equation (50), we also considered the correlation [14]

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5} \quad (53)$$

Valid for all values of Re and $Pr > 0.2$. Since

$$W = \rho v_m A \quad (54)$$

where the cross-sectional area of the pipe is $A = \pi D^2 / 4$ and v_m is the mean fluid velocity.

We can calculate the Reynolds number as

$$Re = \frac{Wd}{A\mu} \quad (55)$$

Next, we show that the flow at the tip is in the turbulent regime ($Re \gg 4 \times 10^3$). Turbulent flow in a circular pipe can be characterized by several layers: viscous, buffer, overlap and turbulent [8].

Velocity of the fluid depends on the layer. The layers within the pipe are determined by the value ranges of non-dimensional variable

$$y^+ = \frac{yu_*}{\nu} \quad (56)$$

Where y is the distance from the wall,

$$u_* = \sqrt{\tau_w / \rho} \quad (57)$$

Is friction velocity and

$$\nu = \mu / \rho \quad (58)$$

Is kinematic viscosity, and τ_w is the wall shear stress. The former can be obtained from the equation for the Fanning friction factor

$$f = \frac{2\tau_w}{\rho v^2} \quad (59)$$

For turbulent flow in a smooth pipe, we have [8]

$$1 / \sqrt{f} = 4.0 * \log_{10} \left(Re \sqrt{f} \right) - 0.4 \quad (60)$$

which can be asymptotically approximated as

$$f = \frac{0.046}{\text{Re}^{0.2}}, \text{ Re} \sim 10^5 \quad (61a)$$

$$f = \frac{0.0791}{\text{Re}^{0.25}}, \text{ Re} \sim 10^4 \quad (61b)$$

In Section IV.C, using fluid properties and RTD geometric parameters, it is shown that the range for y^+ is $10^4 < y^+ < 10^5$, which corresponds to the turbulent layer. Hence, the velocity of the fluid stream in contact with RTD is very closely approximated by the mean fluid velocity.

IV NUMERICAL EXPERIMENTS

IV.A. Numerical Validation of 1-D Heat Transfer Model

To validate the 1-D model of RTD developed in this paper, we calculate transient response of the RTD to a step-change forcing function using five-node 1-D model and 3-D COMSOL finite element model of heat transfer. Details of COMSOL computations are contained in [6,7]. We estimate RTD time constants from the graphs of both transient responses. In these simulations, the fluid forcing function $T_f(t)$ consists of a step change in temperature from 20°C to 70°C, where the time step in the simulations was set to be $\Delta t = 0.1$ s. Note that the temperatures selected for the step change are meant to allow for a comparison of the two responses. The values should not be taken as necessarily representative of a particular physical scenario. The ambient fluid is water. Heat transfer coefficient is calculated using the following parameter values:

$\mu = 2.2 \times 10^{-5} \text{ kg/m} \cdot \text{s}$, $D = 1 \text{ m}$, and $v = 2 \text{ m/s}$. With these values, we obtain

$h_f = 2.7 \times 10^4 \text{ W/m}^2 \cdot \text{K}$. Figure 4 displays transients of the fluid forcing function $T_f(t)$,

temperature response of RTD calculated using 3-D COMSOL model $T_{\text{COMSOL}}(t)$, and temperature

response of 1-D five-node model $T_{RTD}(t)$. Numerical solution to the 1-D five-node model was obtained by numerical integration of Equation (24) using MATLAB “ode45” ODE solver.

$T_{RTD}(t)$ is the temperature in the middle of the second node. Figure 4 shows that RTD transient responses to step change calculated using 1-D and 3-D models of heat transfer are in very good agreement. The response time was estimated as the time it takes for temperature to rise to $(1-e^{-1}) \approx 0.63$ point of the change in temperature of between the initial value and the steady-state value. In reference to Figure 4, the response time was estimated as the transient time required to reach the temperature of 51.6°C. Estimating the response time τ of the RTD from the graphs in Figure 4, both the COMSOL simulation and 1-D five-node model yield $\tau \approx 2s$.

IV.B. Numerical Investigation of RTD Time Constant Obtained from Eigendecomposition Method

The result in Equation (48) is independent of the number of nodes n in the nodal model, as long as the minimum number of nodes is chosen to account for differences in material thermal properties. In this problem, our engineering judgment suggests that the minimum number of nodes is five. Independence of the result in Equation (48) on the number of nodes is verified numerically by plotting RTD time constant τ as a function of heat transfer coefficient h_f for $n=5$, $n=10$, $n=15$, and $n=20$ total nodes at $T=50^\circ\text{C}$. The plot is shown in Figure 5.

Note that, strictly speaking, the time constant of RTD is the time it takes for the node containing the RTD sensing element to approach within $1/e$ of the steady-state temperature. When we consider the numerical values of the eigenvalues in Section III.D, we show that the last node n has the largest time constant which dominates the sum in Equation (48). The lower-numbered nodes have negligible contribution to the sum in Equation (48). Thus, RTD time

constant can be written in the form of Equation (48), which is independent of the exact numbering of the node assigned to the RTD sensing element.

From the spectral analysis developed in this section, we obtain for the five-node model $\tau = \sum_{i=1}^5 1/\lambda_i \approx 2s$. This matches the value of time constant estimated from the transient response of the 1-D five-node model obtained by numerical integration of nonlinear system of ODE's. For comparison, experimentally measured values of RTD time constants in water quoted in [4] are 3.6s, 4.5s, and 5.4s. While the value of τ calculated with eigendecomposition model in this paper is slightly smaller than the values quoted in [4], the fact that the calculated value is comparable in magnitude to literature values is encouraging. Precise calibration of the model would require measurements and simulations of an RTD with well characterized parameters.

IV.C. Estimation of time constant of RTD used in S-CO₂ cycle controller

The time constant of RTD is of interest in the recompression S-CO₂ cycle from a cycle-stability standpoint. Our previous work has shown, in particular, that cycle stability is sensitive to the temperature sensor time constant at the precoolers hot side outlet [3]. In this section, we use our model of RTD to calculate the sensor time constant at this location in the S-CO₂ cycle. The values of fluid properties and state variables at the output of the precoolers and the resulting value of the time constant of an RTD at that location were calculated for nominal transient operation near the critical point. Calculation of the S-CO₂ power transients using General Plant Analyzer and Simulator Software (GPASS) software [15,16] is described in details in [3]. Carbon dioxide fluid properties are supplied to GPASS by the NIST RefProp program [17] using the Span and Wagner Helmholtz model [18]. The transient involved a reduction in generator electric power by 10% over 10s. In the simulation, the value of target generator power was held equal to the initial

generator power $Q(0)$ for the first 5s, and then Q_{target} was reduced by 10% over the next 10s. Turbine bypass (TB) controller [1] was activated at $t = 0$. From $t = 40\text{s}$ to $t = 50\text{s}$, the TB controller was gradually turned off, while the mixed mode(MM) controller [1] was gradually turned on. For $t > 50\text{s}$, the cycle was controlled by MM controller. Precooler controller with no time delay in measurements was operating at all times. Graphs of GPASS-computed state variables of CO_2 at the HOT side of the Precooler during the first 100s are shown in Figure 6.

Table 7 lists values of CO_2 fluid properties calculated at a few state points. Notice that values of heat capacity C_p are large because the fluid is near its critical point. Clearly, the flow is in the turbulent regime ($Re \gg 4 \times 10^3$). Using fluid properties calculated by GPASS, and RTD geometric parameters, we determined that the range for y^+ is $10^4 < y^+ < 10^5$, which corresponds to the turbulent layer. Hence, the velocity of the fluid stream in contact with RTD is very closely approximated by the mean fluid velocity.

The RTD fluid-to-thermowell heat transfer coefficient for the fluid thermos-physical parameter values in Table 7 has value of $h \approx 2 \times 10^4 \text{ W/m}^2\text{K}$, as calculated from both correlations in Equations (50) and (53). Thus, we calculate that the value of RTD time constant is $\tau \approx 2\text{s}$.

V. CONCLUSIONS

We developed an eigendecomposition model of heat transfer in resistance temperature detector (RTD), which enables efficient computing time constant using the information on RTD material and ambient fluid properties. The estimates of the RTD time constant obtained from our model are comparable in magnitude to typical values quoted in literature. Motivation for the RTD model development was our prior work on super-critical carbon dioxide (S-CO_2) control, which has shown that cycle stability depends on RTD time constant. RTD model developed in this

paper can be integrated into plant simulation software to serve as a platform for developing and testing algorithms for cycle control under realistic conditions. Because of high toxicity and corrosive effects of CO₂ working fluid on materials of cycle components and sensors, the value of RTD time constant could change with time as the sensor degrades during service. Future work will investigate RTD time constant changes as a result of material degradation, and the impact of these changes on S-CO₂ cycle stability.

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Figure Captions

Figure 1. Schematic of thermo-well mounted RTD insertion into a process fluid pipe (color on-line)

Figure 2. Cross-section of RTD material layers (color on-line)

Figure 3. Eigendecomposition model of heat conduction with five nodes (color on-line)

Figure 4. Validation of five-node eigendecomposition model with COMSOL finite element simulations (color on-line)

Figure 5. Time constant dependence on heat transfer coefficient for eigendecomposition model with 5, 10, 15 and 20 nodes (color on-line)

Figure 6. CO₂ state variables at the hot side of the precooler during 10%/10s power reduction transient (color on-line)

Table Captions

Table 1. Geometrical dimensions of RTD insertion into process fluid pipe

Table 2 Nomenclature for thermos-physical RTD and process fluid properties

Table 3. Dimensions of RTD cross-sectional material layers

Table 4. Dimensions of five-node eigendecompositionmodel of RTD

Table 5. Correlations for calculating temperature-dependent thermo-physical parameters

Table 6. Values of constants for heat transfer coefficient correlation in Equation (50).

Table 7. Properties of CO₂ fluid at the hot side of the precooler during 10%/10s power reduction transient

Table 1.

Geometrical Quantity	Symbol	Value (m)
Pipe diameter	D	1
RTD outer diameter	d	0.01035
RTD immersion length	L	0.05 - 0.1
RTD sensing element length	l	0.01 - 0.03

Table 2

Thermophysical Material Properties	Description		Symbol	Units
RTD	Alumina (Aluminum Oxide)	Density	ρ_A	kg/m ³
		Specific Heat	c_A	J/kg*K
		Thermal Conductivity	k_A	W/m*K
	Stainless Steel-316	Density	ρ_S	kg/m ³
		Specific Heat	c_S	J/kg*K
		Thermal Conductivity	k_S	W/m*K
Fluid	Heat Transfer Coefficient		h_f	W/m ² *K
	Reynolds Number		Re	none
	Prandtl Number		Pr	none
	Viscosity		μ	kg/m*s
	Thermal Conductivity		k_f	W/m*K
	Density		ρ_f	kg/m ³
	Specific Heat		c_f	J/kg*K
	Free Stream Velocity		v	m/s
	Mass Flux		G	kg/m ² *s
	Pipe Diameter		D	m

Table 3.

Region	Symbol	Dimension (outer) (mm)	Material
Mandrell	r_{mand}	1.2	Alumina (Al ₂ O ₃)
Insulation	r_{ins}	2.335	Alumina (Al ₂ O ₃)
Sheath	r_{sh}	3.175	Stainless Steel-316
Thermowell	r_{well}	5.175	Stainless Steel-316

Table 4

Node	Radius	Dimension (mm)	Material
1	r_1	0.778	Alumina
2	r_2	1.557	Alumina
3	$r_3 = r_{ins}$	2.335	Alumina
4	$r_4 = r_{sh}$	3.175	Stainless Steel
5	$r_5 = r_{well}$	5.175	Stainless Steel

Table 5

Variable	Correlation
$c_A(T(K))$	$-40.92 + 4.024 * T - 5.0048 \times 10^{-3} * T^2 + 2.8852 \times 10^{-6} * T^3 - 6.2488 \times 10^{-6} * T^4$
$k_A(T(K))$	$85.868 - 0.22972 * T + 2.607 \times 10^{-4} * T^2 - 1.3607 \times 10^{-7} * T^3 + 2.7092 \times 10^{-7} * T^4$
$\rho_A(T(K))$	3900
$c_S(T(F))$	$0.1071939 + 6.034521 \times 10^{-5} * T - 4.397679 \times 10^{-8} * T^2 + 6.023013 \times 10^{-8} * T^3$
$k_S(T(F))$	$7.336869 + 5.567335 \times 10^{-3} * T - 4.511739 \times 10^{-7} * T^2 - 3.296337 \times 10^{-11} * T^3$
$\rho_S(T(F))$	$498.5886 - 0.246212 * T$

Table 6

C	m	Re
0.437	0.0895	$10^{-4} - 4 \times 10^{-3}$
0.565	0.136	$4 \times 10^{-3} - 9 \times 10^{-2}$
0.8	0.280	$9 \times 10^{-2} - 1$
0.795	0.384	$1 - 35$
0.583	0.471	$35 - 5 \times 10^3$
0.148	0.633	$5 \times 10^3 - 5 \times 10^4$
0.0208	0.814	$5 \times 10^4 - 5 \times 10^5$

Table 7.

W (kg/s)	P (MPa)	T (C)	ρ (kg/m ³)	k (W/mC)	c_p (J/kgC)	μ (kg/m*s)	Re
1692	7.533	31.88	546.44	10.172e-2	51812	3.9456e-5	5.6512e5
1721	7.548	31.96	544.57	10.158e-2	51660	3.9295e-5	5.7716e5
1833	7.547	31.91	554.48	9.6857e-2	41549	4.0177e-5	6.0122e5
1824	7.532	31.84	553.92	9.799e-2	43729	4.0123e-5	5.9908e5

Figure 1.

Figure 2.

Figure 3

Figure 4

Figure 5

Figure 6