**Branch-and-Bound Algorithm Applied to Uncertainty Quantification
Of A Boiling Water Reactor Station Blackout**

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Abstract:

Evaluation of the impacts of uncertainty and sensitivity in modeling presents a significant set of challenges in particular to high fidelity modeling. Computational costs and validation of models creates a need for cost effective decision making with regards to experiment design. Experiments designed to validate computation models can be used to reduce uncertainty in the physical model. In some cases, large uncertainty in a particular aspect of the model may or may not have a large impact on the final results. For example, modeling of a relief valve may result in large uncertainty, however, the actual effects on final peak clad temperature in a reactor transient may be small and the large uncertainty with respect to valve modeling may be considered acceptable. Additionally, the ability to determine the adequacy of a model and the validation supporting it should be considered within a risk informed framework. Low fidelity modeling with large uncertainty may be considered adequate if the uncertainty is considered acceptable with respect to risk. In other words, models that are used to evaluate the probability of failure should be evaluated more rigorously with the intent of increasing safety margin. Probabilistic risk assessment (PRA) techniques have traditionally been used to identify accident conditions and transients. Traditional classical event tree methods utilize analysts' knowledge and experience to identify the important timing of events in coordination with thermal-hydraulic modeling. These methods lack the capability to evaluate complex dynamic systems. In these systems, time and energy scales associated with transient events may vary as a function of transition times and energies to arrive at a different physical state. Dynamic PRA (DPRA) methods provide a more rigorous analysis of complex dynamic systems. Unfortunately DPRA methods introduce issues associated with combinatorial explosion of states. This paper presents a methodology to address combinatorial explosion using a Branch-and-Bound algorithm applied to Dynamic Event Trees (DET), which utilize LENDIT (L-Length, E-Energy, N-Number, D-Distribution, I-Information, and T-Time) as well as a set theory to describe system, state, resource, and response (S2R2) sets to create bounding

functions for the DET. The optimization of the DET in identifying high probability failure branches is extended to create a Phenomenological Identification and Ranking Table (PIRT) methodology to evaluate modeling parameters important to safety of those failure branches that have a high probability of failure. The PIRT can then be used as a tool to identify and evaluate the need for experimental validation of models that have the potential to reduce risk. In order to demonstrate this methodology, a Boiling Water Reactor (BWR) Station Blackout (SBO) case study is presented.

Keywords: PRA; Dynamic PSA; Metric Scales; Sets; Branch-and-Bound

1 Introduction

Uncertainty quantification in modeling of dynamic systems with respect to risk analysis presents a series of unique challenges. Modeling of such systems can be extremely complex and modeling simplifications to improve computational efficiency can often increase uncertainty into the model. In addition, the dynamic nature of a nuclear power plant and the changes in the state space of the plant and the timing of those changes adds additional complexity. Dynamic Probabilistic Risk Assessment (DPRA) allows for modeling of complex dynamic systems as well as allowing for uncertainty quantification with respect to risk. As systems and transients become more complex, DPRA suffers from state or combinatorial explosion. The number of required simulations grows exponentially if one aims to evaluate timing of events in the transient as well as sensitivity and uncertainty analyses of the models themselves. Monte-Carlo methods and Dynamic Event Trees (DET) have historically been used for DPRA. Reference [1] documented a method for optimization of DETs. Using the Branch-and-Bound algorithm [2], the solutions for DET are achieved much more efficiently to evaluate the timing of events within the transient. The optimal solution within this framework is to find the branches with the highest probability of failure. In the case of nuclear power plants (NPP), this is typically exceeding peak clad temperature of the fuel cladding.

Identification of the highest probability of failure conditions, can lead to quantification of uncertainties that can allow for analysts to determine important aspects of modeling that have the greatest impact on risk. By evaluating these aspects of the model, modeling fidelity and validation experiments can be designed to provide for an increase in safety margin, which can potentially allow plants to operate at a higher power, resulting in an increase in revenue for utility companies.

In this paper, we extend the optimization algorithm for solutions of DETs to include sensitivity and uncertainty analysis. This methodology is applied to commercial BWR and an evaluation of parameters that have previously been evaluated with regards to a Phenomenological Identification and Ranking Table (PIRT) are evaluated [3]. The resulting analysis yields a PIRT methodology that is based on risk-informed analysis as well as expert-based judgment.

The analysis was performed by using the DET option within the RAVEN framework [4][5], with the RELAP5-3D thermal-hydraulics code [6]. The DAKOTA code [7], which was coupled with RELAP5-3D, was used to perform the Sensitivity Analysis and Uncertainty Quantification (SA/UQ) at each branching point in the DET simulation. Using this information and the Branch-and-Bound algorithm, the DETs were pruned to obtain the highest probability of failure for each credible plant state. The SA/UQ results can then be used to provide analysts with risk informed decision analysis with regards to modeling and validation.

For completeness, the Branch-and-Bound (BB) algorithm is described in the next section. The BB algorithm relies on bounding functions to constrain the unrolling of unexplored nodes in the DET. The selections of bounding functions are important as they define the physical model to be evaluated. As a consequence, LENDIT (L-Length, E-Energy, N-Number, D-Distribution, I-Information, T-Time) metrics and S2R2 (State, System, Resource, and Response) sets [8] were develop to address this need. In section 4.2 the application of the BB method, LENDIT and the S2R2 sets to the DET framework is presented for the SA/UQ parameter evaluations as described above. This approach was implemented in RAVEN [4][5], which is part of the Multi-physics Object-Oriented System Environment (MOOSE) framework [9].

2 Branch-and-Bound Algorithm

The branch-and-bound algorithm is a technique that is used to reduce the search space in combinatorial optimization problems. The algorithm utilizes various system constraints to evaluate which branching paths to follow or which branching paths to disregard (prune) [2]. In general, the problem is to minimize (maximize) a function $f(x)$ of 'state' variables $(x_1, \dots, x_n)$ over a region of feasible solutions, S :

$$\min_{x \in S} f(x) \quad (1)$$

where the function f is called the objective function. The set of feasible solutions is usually determined by general conditions on the variables (e.g., non-negativity constraints, resource capacity constraints, etc.).

The mathematical approach is to find a set of potential solutions, P contained in S , that correspondence to a bounding function $g(x)$ defined over S and P such that

$$g(x) \leq f(x) \forall x \in S \subseteq P. \quad (2)$$

The bounding function defines a subspace of the solution space with the best lower (upper) bound solution value obtainable in the subspace. In this context

$$\min_{x \in P} g(x) \leq \left\{ \begin{array}{l} \min_{x \in P} f(x) \\ \min_{x \in S} g(x) \end{array} \right\} \leq \min_{x \in S} f(x) \quad (3)$$

The difficult problem is choosing P and g so that they are computationally easy to solve and provide tight bounds. One approach is to minimize g over S and at the same time modify the objective function f to ensure that for all feasible solutions, the modified objective function is less (greater) than or equal to the original objective function. Geometrically, the optimization creates a tree of potential solution (subspaces) nodes where nodes are explored (branched to their

next solution subspace) or ignored (pruned) from further evaluation. The resulting leaf nodes contain the minimized value of the objective function.

The application of the branch-and-bound optimization technique must be tailored to the SA/UQ analysis of the BWR SBO problem. This transition is accomplished by choosing appropriate parameters (state variables) used in computing the upper and lower bounds that minimize (maximize) the value of $f(x)$. In this presentation, the selected parameters represent a collection of physical scales referred to as LENDIT scales, which are described in more detail below.

3 LENDIT Scales

The LENDIT [1][8] scales are a collection of relevant physical quantities such as time, energy, and length here applied to nuclear power plant operator-action depends. These scales reflect, for example, the length (L) of the liquid level in the core; the amount of (thermal) energy (E) in the reactor; the number (N) of NPP operators; and approximate anticipated time (T), from core damage, under insufficient cooling of the decay heat. Although all LENDIT scales likely apply to a system under scrutiny, some metrics are less relevant simply because of the systems' behavior. As an example, the *a-priori* emergency preparedness includes prompt initiation of designated 'team' response (consisting of ' N ' members), information (I) of critical instrumentation, controls and hardware needed to address the post-event, and the distribution (D) of possible component states or probabilities of being in several physical states. Taken together these quantities are represented as a tuple of scales denoted by (N, I, D) .

3.1 S2R2 – State, System, Resource and Response Sets and Relational Model

The LENDIT scales are used in conjunction with a set theory approach to system analysis. Four ‘sets’ called, ‘state, system, resource and response’ or ‘S2R2’ describe key system operations that systematically describe the descriptive use of LENDIT-scales. The LENDIT-scales in combination with S2R2 are used to characterize off-normal reactor events and plant operators' (first responders) capabilities to intervene. Steady-state and normal operations are also defined by the S2R2 formulation. The S2R2 sets follow basic set operations. For example, the response set (described below) results from Boolean operations applied to the state, system and resource sets that results in possible branching conditions for each node within the dynamic event tree. One branching condition (response) might occur if the system set consists of SBO 'AND' SAFETY RELIEF VALVE (SRV) closed 'AND' the state set consists of Pressure > 7.58 MPa, 'AND' the resource set includes SRV available. The resulting response set would be OPEN SRV.

The information contained in the S2R2 sets is extracted using a relational database model. A relational model assigns the logical truth-value relationship between distinct informational items. A relation, based on set theory and predicate logic, is a property that assigns truth-values (true or false) to n-tuples of attributes that defines systems or states. The relation describes how components of an n-tuple are logically connected, according to whether the property does or does not hold.

A brief description of each ‘set’ follows.

3.1.1 State Set

The state set describes the thermodynamic state of the reactor that includes its instrumentation and controls (I&C) sensors. In terms of the LENDIT scales, the state set is

defined by the following properties: temperature, pressure, liquid level, mass flowrate and valve position or function. There are additional state set elements that describe the number and distribution of sensors (N, D), the time response of sensors (T), and the physical location of an important sensor (D, L), which under emergency response, a responder has to physically attend to the sensor unit.

3.1.2 System Set

The system set includes elements of hardware-based subsystems and their components. Two examples are the emergency core cooling system (ECCS) and the gravity driven cooling system (GDCS). These subsystems are typically called upon to mitigate off-normal conditions activated on the basis of the state set elements (e.g. temperature, pressure etc.). For example, the GDCS is characterized by volume (L^3) and emptying time (T). The start-up time of emergency diesel generators (EDG) and coast-down time of the primary coolant pumps and steam turbine (both T) are of special importance under off-normal conditions, such as a SBO.

3.1.3 Resource Set

The resource set contains elements that pertain to existing safety systems, which include items that are engineered to be used under hypothetical emergencies. In addition, this set also contains elements that are not usually considered as part of the engineered safety system. For example an important resource set element may concern human factors; that is, operators as an available ‘resource’. The resource set may also be in terms of ‘tools’ such as flashlights and walkie-talkies. Perhaps most important is the knowledge that all personnel involved in the response have common information; a matter of information and distribution (I, D).

3.1.4 Response Set

The response set maintains the elements of the response metrics, whose values are determined by Boolean operations applied to the various combinations of the state, system, and resource sets described above. For example, the presence or the absence of a first responder team is given a Boolean 'Y or N'; if common or differing levels of first responder team understanding (I,D) is detected then one or multiple 'paths' of actions may be determined from the response set. Furthermore, if team members (and institution) should know the average availability time of the emergency generator (T) and the time at which the core degrades (T) then the response to an insufficiently cooled core can be addressed.

4 Dynamic Event Tree Approach

4.1 Implementation of Dynamic Event Trees in RAVEN

Dynamic Event Tree, as opposed to classical event trees, allows for the incorporation of dynamic plant state transitions at various points in time as dictated by the cumulative distribution functions [5][10]. At pre-determined points in time, typically defined as the inverse of the Cumulative Distribution Function (CDF), the SBO simulation is stopped and a branching condition in the DET is evaluated. The branching condition determines a possible change in the plant state or the continuation of the current plant state. The probabilities of each branch are dictated by the user-defined sampling of the CDF.

The DET approach was developed using the operator splitting technique described by Rabiti et al [4]. This technique uses computer tools such as Reactor Analysis and Virtual control Environment (RAVEN), Multiphysics Object Oriented Simulation Environment (MOOSE) and a developmental version of RELAP-7. For this case study, the RAVEN was coupled with

RELAP5-3D, which is similar to that of RELAP-7. The RAVEN coupling methodology implements an operator splitting technique as shown below:

$$\frac{\partial \bar{x}}{\partial t} = \bar{F}(\bar{x}, \bar{v}_{i-1}, t), \quad (4)$$

$$\bar{C} = \bar{G}(\bar{x}, t) \quad \text{for} \quad (5)$$

$$t_{i-1} \leq t \leq t_i = t_{i-1} + \Delta t,$$

and

$$\frac{\partial \bar{v}}{\partial t} = \bar{V}(\bar{C}, \bar{v}_{i-1}, t). \quad (6)$$

where $\bar{x}$ is the process or physical variables at time t . These variables in a thermal-hydraulic system represent pressure, energy, density etc. $\bar{C}$ represents a monitored vector of physical variables and $\bar{v}$ represents the controlled variables. The vector $\bar{v}$ is regulated by RAVEN.

The operator splitting technique is preferred for several reasons. First, there is an inherent time delay after which control systems react. Second, the control system reaction can shift the system between two distinct discrete states. In this case, there will be first order numerical errors, especially if the discontinuity has not been handled appropriately. RELAP5-3D is primarily responsible for handling Equations (4, 5) to compute $\bar{C}_i$. In Equation (6) $\bar{C}_i$ is used to find $\bar{v}_i$, which in turn is iterated upon again by Equations (4,5).

With respect to $\bar{C}_i$, RELAP5-3D provides control features within RELAP5-3D. RAVEN allows for additional control with regards to this variable, and in particular adjusts the timing of events.

RELAP5-3D is responsible for solving the thermal-hydraulic properties of the plant system including the user defined control system within the simulation. RAVEN controls the RELAP5-3D execution via the RELAP5-3D input file. This is accomplished by using an outer wrapper with user defined “trip” functions in the RELAP5-3D input/output files. The goal is to force the evaluation of the monitored quantity $\bar{C}_i$ that is of interest in this risk simulation. The outer wrapper in the DET simulation instructs RAVEN to compute probabilities and branching conditions to restart the simulation when a component state is challenged (e.g., Diesel Generator (DG) recovery, Battery Depletion, etc.). RAVEN solves Equation (6) and obtains a set of control parameters for the next time step $\bar{v}_{it}$.

Rabiti et al., proposed a more refined operator splitting technique, by introducing auxiliary variables to the set of Equations (4,5,6) [4]. The main purpose of the auxiliary variables is to provide an effective parametric means to support the control system. The auxiliary variables can also contain information that is in addition to the capacity of $\bar{x}$ and $\bar{v}$. Below is the new set of equations:

$$\frac{\partial \bar{x}}{\partial t} = \bar{F}(\bar{x}, \bar{v}_{t_{i-1}}, t), \quad (7)$$

$$\bar{C} = \bar{G}(\bar{x}, t) \quad \text{for} \quad t_{i-1} \leq t \leq t_i = t_{i-1} + \Delta t, \quad (8)$$

$$\frac{\partial \bar{a}}{\partial t} = \bar{A}(\bar{x}, \bar{C}, \bar{a}_{t_{i-1}}, \bar{v}_{t_{i-1}}, t), \quad (9)$$

and

$$\frac{\partial \bar{v}}{\partial t} = \bar{V}(\bar{x}, \bar{v}_{t-1}, \bar{a}, t). \quad (10)$$

Equation (9), which is the new differential equation, contains the auxiliary variable $\bar{a}$.

As seen in Equations (9,10), the auxiliary variable $\bar{a}$ is time dependent and requires time step iterations similar to the monitored variable $\bar{v}$. The auxiliary variable can also be used to track various parameters that are used. The DET analysis is assumed to be a Markovian process where the branching conditions in the DET are based on the current state of the plant and not on the history or evolution of the transient to reach the current state. The auxiliary variables are used to monitor parameters that affect the branching conditions based on the history previous to the current state. An example would be the cycling history of the SRVs where the probability of failure increases with each cycling of the valve. A flow chart of the coupling of RAVEN and RELAP5-3D is shown in Figure 1.

<Figure 1>

4.2 Branch-and-Bound Applied to DET

The branch-and-bound (BB) algorithm is applied to the DET to assess the SBO scenario for a BWR. The objective function applied to the BB technique is chosen to determine the highest probability of cladding failure (e.g., peak clad temperature exceeding 1478 K). The BB-DET formulation is designed to explore sets of branches that could possibly result in cladding failure, while ignoring all other branches. In this formulation, the branching conditions are based on probabilistic changes to system states in a nuclear power plant. The objective function is defined as follows with the variable x represented as a relationship function:

$$\max f(x) = f(\text{Clad Temperature})$$

such that

$$\text{Clad Temperature} > 1478 \text{ K}$$

Where f is the probability density function. A diagram of the objective function is presented in Figure 2.

<Figure 2>

Similarly, the bounding functional relationships are derived to support the BB algorithm. The bounding functional relationships are defined as follows:

$$g(x)=g(\text{Temperature, Pressure, Decay Heat, Entropy, Liquid Mass})$$

Using the bounding functional relationships described, the problem can be constrained by the following constraint conditions:

$$[f(x)>g(x)] \text{ 'OR' } [\text{Pressure}>g(\text{Pressure}) \text{ 'AND' } \text{Decay Heat} > g(\text{Decay Heat}) \text{ 'AND' } \text{Liquid Mass} <g(\text{Liquid Mass}) \text{ 'AND' } \text{Entropy}>g(\text{Entropy})]$$

Since the DET for a nuclear power plant evaluates a complex dynamic system the initial bounding functional relationships are well defined, however, the values associated with those functional relationships are undetermined at the beginning of the simulation. In order to formulate these functional relationships, success (i.e., no cladding failure) branches and failure branches must be identified. It is not until a leaf has been determined to end in success, that the parameters of the bounding functional relationship can be defined. Even then, the parameters can only be defined for a given system set. Once a leaf has been determined to end in success, the success information is passed to the parent node. If all children of a parent have resulted in success, the information is passed to the parent of the parent node. This results in defining the parameters for the bounding functional relationships for each particular system set. A simplified diagram of this process is shown in Figure 3. If the parameters of a functional relationship exist

for a particular system state/set, an updated bounding functions (surface) is added for that particular state. A particular system state may end with a family of bounding functions (surfaces).

As the nodes of the DET are explored, the RAVEN code evaluates whether the bounding parameters exist for that particular system condition. If the parameters for the bounding functional relationship exists, the parameters for the node are compared to the functional relationship parameters to determine if they are within the bounds of the objective function. If they are within the objective function, the branches are pruned and no further exploration is performed. If the parameters at the branch are outside of the bounding functions, exploration of the branches continues. The intent is to limit the number of simulations that will end in high probability of success and explore those states that can end in failure. A success and failure are defined in Figure 3 as “OK”, and “CD”, respectively. A description of the bounding functions for the BWR SBO transient is presented in Section 6.

<Figure 3>

5 Uncertainty Quantification Methods

Evaluating the uncertainty within this DET framework was performed by randomly sampling parameters that may have a large impact on success or failure. The intent is to determine which parameters greatly influence the final results with respect to risk. For accident sequences determined in the DET to result in clad failure, a formal PIRT process is used to determine the affects of the parameters that may have large impacts on the simulation.

The PIRT consisted of a method described in References [3]. The method involves randomly sampling various parameters and then using the correlation coefficients between variables to establish a PIRT ranking. The Spearman Rank Correlation has been chosen as it “is a nonparametric measure of statistical dependence between two variables [3].” The Spearman

Rank correlation applies to both linear and non-linear problems as opposed to the Pearson correlation coefficient, which provides a measure of the linear correlation between two variables. The Spearman Rank correlation coefficient is defined as:

$$\rho = 1 - \frac{6 \sum (x_i - y_i)^2}{n(n^2 - 1)}$$

A correlation coefficient between two variables is a value between -1 and 1. A coefficient value that approaches 1 indicates a stronger positive correlation between variables. A value approaching -1 indicates a strong negative correlation between variables. As the value of the coefficient approaches 0, correlation between variables weakens. By using this methodology, the correlation coefficients are used to develop the PIRT ranking for the various parameters of interest. A discussion of the parameters evaluated in the BWR SBO is provided in Section 10. The following criteria for the PIRT ranking was used to evaluate the specific parameters against the peak clad temperature in the model:

- $|\zeta| \in [0.01, 0.29] \Rightarrow \text{Weak relationship (PIRT=Low)}$
- $|\zeta| \in [0.29, 0.69] \Rightarrow \text{Moderate relationship (PIRT=Med)}$
- $|\zeta| \in [0.69, 1.00] \Rightarrow \text{Strong relationship (PIRT=High)}$

By identifying parameters that result in higher correlations to the parameters such as peak clad temperature, their impacts can be better understood such that reconsideration of modeling techniques and experimental validation provides a possibility for improvement in the safety margin of an operating reactor.

Implementation of this methodology was performed using the DAKOTA [7] code coupled with RELAP5-3D [6]. As discussed above, RAVEN is used to guide the DET simulation, while at each node, a DAKOTA simulation is performed to determine the effect of the various parameters at this branching condition. The DAKOTA Latin Hypercube Sampling

(LHS) method was used to more efficiently evaluate uncertainties in the models as opposed to an analogue Monte Carlo sampling. Sensitivity and Uncertainty Quantification was implemented in RAVEN by evaluating each by evaluating each branch individually as the simulation progresses. The timing of branching conditions in the DET was based on the best-estimate or nominal conditions. Therefore, the PIRT for each end-state or leaf of the tree may be different dependent upon the timing of events and changes in plant operating state. This allows for a PIRT to be evaluated based on the probability of timing of events.

A flow chart of the optimization algorithm with sensitivity and uncertainty analysis is shown in Figure 4.

<Figure 4>

6 BWR Description

In order to demonstrate the capabilities of the Branch-and-Bound algorithm for addressing highly complex DET simulations, a BWR SBO model is developed and analyzed. The BWR SBO was modeled using the RELAP5-3D code and is representative of the General Electric Mark I BWR, similar to the Fukushima Nuclear Power Plant. The BWR model consists of 3440 MWt 1152 MWe BWR reactor. The reactor containment vessel consists of a Mark I pressure suppression pool design with drywells light bulb shaped. The torus and drywell are constructed of stainless steel [11]. A diagram of the BWR model is shown in Figure 5. The RELAP nodalization diagram is presented in Figure 6.

The GE BWR has several primary features related to the SBO and are listed here. The drywell contains the reactor pressure vessel and the circulation pumps. The Pressure Suppression Pool (PSP) is also known as the wetwell. The PSP contains approximately 1 million gallons (3.7 million liters) of fresh water and is used in specific situations as ultimate heat sink. The reactor circulation pumps provide cooling capabilities to the reactor.

The reactor pressure vessel (RPV) level control system provides manual and automatic water level control. The reactor core isolation cooling system (RCIC) provides high-pressure injection of cooling water from the condensate storage tank (CST) to the RPV. A turbine driven pump takes steam from the main steam line and discharges the steam to the suppression pool. Water can also be taken from the PSP rather than the CST. The RCIC has a 600 gpm (45.5 l/s) capacity.

The High Pressure Core Injection (HPCI) is similar to RCIC with a much greater capacity. The HPCI has a capacity of approximately 5000 gpm (378.8 l/s) of flow, however, requires much lower pressures in the RPV for injection.

The pressure within the RPV is controlled by the SRVs. The SRVs can be controlled either manually or automatically. The valves maintain pressure between 1100 psig (7584 kPa) and 900 psig (6205 kPa). The actuation of the SRVs require the availability of battery power for activation. In addition to the SRVs, the Automatic Depressurization System (ADS) provides a means to depressurize or blow down the RPV.

Cooling water is typically provided by the CST. The CST consists of a 375,000 gallon (1,419,000 l) tank containing fresh water. In addition to the CST, there are two additional 500 K gallon (1892 kl) tanks available. Re-alignment of the RCIC/HPCI requires manual operator action and cannot be done automatically. The PSP can provide an additional 1 million gallons of fresh water and provides a heat sink if Alternating Current (AC) power is lost in the case of a SBO. The HPCI and RCIC can draw water from the PSP as well. Due to the large amount of fresh water and limited simulation time for this analysis, the depletion of the CST or PSP is not considered.

The BWR electrical power systems, even though not explicitly modeled in the risk analysis consists of 2 separate power grids to supply off-site power. The loss of both power grids results in loss of operability of all safety systems with the exception of ADS, SRV, RCIC, and

HPCI, which require DC battery power for operation. The diesel generators are available to provide up to 24 hours of AC power to emergency systems. In the event of a Loss-of-Off-Site-Power (LOSP) and loss of diesel generators (SBO), battery systems can provide at a minimum of 30 minutes of power to system and can be reconfigured to provide up to 4 to 6 hours of emergency power.

In the event battery power is lost and the reactor is depressurized through the ADS, emergency injection into the RPV can be provided by the firewater injection system. The firewater injection system can inject water at a flow rate of 2500 gpm (189.4 l/s) at a discharge pressure of 120 psi (827.4 kPa). This action would be performed as a matter of last resort as it would be injecting raw water into the RPV, which would result in the loss of the reactor, however, it may be necessary to maintain cooling.

<Figure 5>

<Figure 6>

A summary of the important characteristic for the BWR RELAP5-3D model are presented in Table 1. The values were obtained from Reference [11].

Table 1. Modeling characteristics for the RELAP5-3D model of the BWR SBO scenario.

Characteristic	Value
Thermal Power	3293 MW _{th}
Recirculation System	20 Jet Pumps 45200 gpm/pump
Residual Heat Removal System	4 Pumps – 10000 gpm/pump @ 20 psid
RCIC	616 gpm @ 1120 psid
Safety Relief Valve Set Points	13 Valves (Safety Set Point 1080 – 1100 psig) (Relief Set Point 1030 -1100 psig)
Safety Relief Valve Capacity	850 klbs/hour
Emergency High Pressure Injection (HPCI)	5000 gpm @ 1120 psid
Low Pressure Injection	4 pumps - 10000 gpm/pump @ 20 psid
Drywell free volume	159,000 ft ³
Drywell design temperature	281 °F
Wet Well free volume	119,900 ft ³
Wet well minimum water volume	85,000 ft ³
Wet well design pressure	58 psig

7 BWR Station Blackout Event Tree

The BWR SBO involves the following sequence of events:

- LOSSP occurs as the result of loss of both switch yard supplies – t=0 seconds
- LOSSP results in the following actions
  - Operators successfully scram the reactor

- Main steam isolation valves automatically close and provide primary containment
- Emergency diesel generators start to provide emergency AC power
- Decay heat from the reactor is provided by the Residual Heat Removal system
- Batteries are assumed to be initially functional
- Pressure is maintained by a regulating SRV
- DG fail causing a SBO
- Component Cooling Water and Service Water systems are lost challenging the RCP seals. A loss of the seals may occur resulting in a small LOCA.

The classical event trees for the SBO are presented in Figure 7 through Figure 13. Within a classical event tree, the sequence of events is fixed. As an example, the transient begins with LOSP, followed by reactor scram, emergency power from the EDG and whether SRVs close. If the emergency power fails, the plant enters a SBO and if SRVs close, the seal integrity is questioned, as well as actions to extend battery power. The HPCI and RCIC is not questioned if the seals remain in tact. Additionally, the time the SRVs open and close multiple times during the transient, at any moment the valves may stick open. In addition, the classical event tree assumes that the EDG fail immediately, which may not be the case. The time the EDG failed, allowed for a significant amount of energy to be removed from the core. In addition, once the EDG begins operation, the HPCI and RCIC system may be required to provide cooling water to the core. These systems do have the potential to fail prior to loss of EDGs. These types of sequences demonstrate the ability for DETs to evaluate the transients.

<Figure 7>

<Figure 8>

<Figure 9>

<Figure 10>

<Figure 11>

<Figure 12>

<Figure 13>

8 Dynamic Event Tree Analysis of BWR SBO

In order to optimize the DETs, the LENDIT metrics and S2R2 sets are combined to initialize the constraining functional parameters for the branch-and-bound algorithm. The initial LENDIT metrics as described above begins with establishing a sequence of possible scenarios and transitions. The DET Response sets or branching conditions are presented in Table 2. Figure 14 provides a diagram of the LENDIT metrics for the BWR SBO. The metrics are established from operating procedures as well as the classical event trees used to quantify risk. The series of events begins with the question of whether or not SBO has occurred. After SBO has occurred, the SRVs may cycle several time reducing pressure in the reactor and maintaining the pressure between 900 psig (6205 kPa) and 1100 psig (7584 kPa). During this process, one or two SRVs may fail to close resulting in what would be equivalent to a small break loss of coolant accident (LOCA). Additionally, the cooling of the RCP seals may be fatigued during a SBO. This could result in seal failure also resulting in a small break LOCA.

Coolant makeup is provided by both the HPCI and RCIC pumps. Failure of both pumps can result in loss of cooling capability to the core. If HPCI and RCIC are lost, the reactor can be depressurized and coolant inventory can be made up using the low pressure injection system

(LPIS) or the less desirable firewater system. The probability of losing HPCI and RCIC are relatively low, but are still possible.

During the SBO, it is assumed that efforts will be made to extend the battery life beyond the 30 minute design, while response crews attempt to recover either the EDGs or off-site power. Extension of the battery life can be attempted by reducing the number of components requiring battery power for operation that are not needed for transient response. Typically, the battery life can be extended to 4 to 6 hours if operational staff handles the transient response properly. The extension of the battery power life is considered in the CDFs for battery power failure.

Once the battery power has been drained, the HPCI, RCIC, and SRVs are no longer operable. Operators at this point will be required to make decisions with regards to the probability of recovery AC power. If AC power is not recoverable, firewater may be the last available means to cool the core. The use of firewater may have severe impact on the economic viability of the reactor resulting in an economical loss, however, the alternative of melting the core would be more catastrophic.

The development of the DET S2R2 sets were developed from the LENDIT metrics and the potential response for the SBO classical event tree transient. The DET sampling for the CDF for the timing of SBO, manual ADS, and failure of battery power, firewater activation, and AC power recovery are presented in Figure 15 through Figure 19 and summarized in Table 3. The DET simulation assumed a plant mission time of 24 hours as a demonstration of the process described here. It is assumed in this study that if cladding failure has not occurred in 24 hours, then the additional actions would provide AC power to the plant such as via transport of off-site DG (to the plant). The DET model simulations were ran using the RAVEN framework coupled with RELAP5-3D. Initially, a simulation was undertaken using more of “brute-force” method with no optimization to create a baseline model to determine the effects of the optimization. The

second model was tested using the Branch-and-Bound optimization technique to demonstrate the capabilities of reducing the number of branches created and thus reduction in simulation time.

The optimization of the trees involves developing bounding functions as the simulation progresses. The bounding functions are developed using a “surface” plot approach where parameters, which have been determined to be important to the simulation are evaluated. The parameters for this study were determined with respect to those important for calculating peak clad temperature. The parameters are associated with the equations for convective heat transfer and the ability to remove energy from the reactor fuel. The parameters important for heat removal are internal energy or enthalpy, core level, reactor decay heat, and reactor pressure. Essentially the parameters that define the bounding functions are directly related to the increase in energy to the system and the removal or ability to remove energy from the system. By evaluating these parameters, and looking at each node, we examine previous nodes that have been in the same system “state” and ask whether or not the previous simulations have ended in success or failure. If a node with same states has been shown to result in success, then we evaluate the above parameters to determine the energy removal capability from the core. A condition of being in the same state, implies that all reactor components (e.g diesel generators, batteries, valves etc.) are in the same configuration. We have assumed that the system is represented by a Markovian or semi-Markovian process. The parameters described above provide the amount of decay heat production that has to be removed so that the reactor reaches a safe, cooldown state. If the all of the above parameters fall within the surface of a previous existing surface, then the simulation will optimistically assume that the state will not end in a failure.

<Figure 14>

Table 2. Dynamic Event Tree Response Sets.

State	System	Resource	Response
Pressure < 6.21 Mpa	SRV Open	SRV	Branch 1 SRVs Close
	SBO	Battery Power	Branch 2 1 SRV Fail to Close
			Branch 3 2 SRV Fail to Close
	SBO	Battery	Branch 1 Battery power available
			Branch 2 Battery power depleted
ADS Trip Conditions	SBO	Battery Power	Branch 1 No ADS
			Branch 2 ADS
Pressure < 1.03 Mpa	SBO and Battery Power	Firewater Injection	Branch 1 Firewater failure
			Branch 2 Firewater initiation
	SBO	HPCI	Branch 1 HPCI Operational
			Branch 2 HPCI Failure
	SBO	RCIC	Branch 1 RCIC Operational
			Branch 2 RCIC Failure
	SBO	RCP Seals	Branch 1 RCP Seals Intact
			Branch 2 RCP Seals Failed
	SBO	Diesel Generators	Branch 1 Diesel generator failure
			Branch 2 Diesel generator recovery

Table 3. Probability distributions used in BWR SBO DET analysis.

Event	Distribution Type	Distribution Parameters	CDF Sampled Points
Failure of DG	Exponential	$\lambda=0.0033/\text{second}$	0.01, 0.1, 0.3, 0.5, 0.7
Failure of Batteries	Uniform	Hi=6 hours Low=30 minutes	0.01, 0.1, 0.3, 0.5, 0.7
Failure of HPCI/RCIC	Exponential	$\lambda=1.22503\text{E-}6/\text{second}$	0.01, 0.1, 0.3, 0.5
Pump Seal LOCA	Uniform	Hi=12 hours Low=0 hours	0.01, 0.1, 0.3, 0.5, 0.7
Recovery DG	Weibull	$\lambda=368.4$ minutes $k=0.745$	0.01, 0.3, 0.5
Activation of Firewater	LogNormal	$\mu=42$ minutes $\sigma=15$ minutes	0.01, 0.3, 0.5, 0.7
Manual ADS	LogNormal	$\mu=8.4$ minutes $\sigma=15$ minutes	0.01, 0.3, 0.5, 0.7

<Figure 15>

<Figure 16>

<Figure 17>

<Figure 18>

<Figure 19>

9 Results of Dynamic Event Tree with Optimization

Evaluating the DET for a BWR SBO, the highest probability of failure occurred with a failure of the DGs at 3.5 minutes, followed by battery failure at 63 minutes. Without recovery, cladding damage is reached at 235 minutes, or 182 minutes after battery failure. The second case resulting in failure, has DG failure at 11.4 minutes with battery failure at 195 minutes. Cladding failure occurs in this simulation at 359 minutes or 164 minutes after battery failure. The time-scale for recovery in both conditions is approximately 2 to 3 hours after battery failure regardless of the time of battery failure. This is expected as the decay heat from the fuel has reached a semi-steady state condition within this time frame.

Another risk contributing branch examines the failure of a SRV failing to close resulting in a LOCA. In this simulation, the DG fails at 3.5 minute with the SRV failing to close at 46 minutes. The battery fails at 359 minutes. The firewater injection system is challenged (i.e., conditions met to activate firewater injection) at 240 minutes, if firewater injection fails, cladding failure occurs at 417 minutes or approximately 59 minutes following battery failure. This case demonstrates the changing time-scales associated with the

complexity of a dynamic system. With a failure of the SRV, the time-scale to failure is reduced by a factor of 2.

The BWR simulation consisted of 20752 nodes with 8457 end states prior to optimization. Of the 8457 end states, 1857 end states ended in failure and 6600 end states ended in success. Using the Branch-and-Bound algorithm, the DET was pruned to approximately 5868 nodes and yielded 373 of the highest probability of failure conditions. Approximately 3141 nodes ended in success. The solution to the DET without optimization is shown in Figure 20. The solution for a DET with optimization is shown in Figure 21. It should be noted that when comparing the case with optimization to the case without optimization, the high probability failure cases (i.e., core damage) are present. The lower probability cases have been pruned. A representative diagram of several of the “surface” plots used to define the bounding functions for the BWR SBO is shown in Figure 22 through Figure 24 and summarized in Table 4 through 6. In each figure, a set of curves or surfaces is presented. Each surface represents the State set at the branching condition, where the branch has led to a success. The parameters were chosen to represent the ability to remove energy from the reactor. A success surface as shown forms the bounding functional relationships used in the branch-and-bound algorithm. Each surface is treated as an independent relationship function, which allows for handling the nonlinear thermal-hydraulic functions.

Figure 15 shows the surface plots for BWR SBO conditions, which include RCP LOCA, HPCI failure, and the failure of 1 SRV to close with battery failure. If a SBO transient is reached within these conditions, the ability to recover in the allotted mission time of 24 hours can be met. As can be seen comparing to the other figures, the reactor decay heat is lower, as would be expected since HPCI is not available. All branching conditions following

will result in success. Therefore, these surfaces provide a reasonable bounding function for the BWR SBO cases.

Figure 16 provides the bounding functions for a condition of SBO with battery failure and RCP LOCA with HPCI failure. The function provides a much broader range of failure parameter than those presented in Figure 15. The number of bounding surface is fewer than those in Figure 15, however, the difference between the previous plot and those in Figure 16, is due to the cooling capability. The state sets for those represented by Figure 16, demonstrate that the amount of stored energy in the reactor must be lower to allow for a success or the ability to reach cold safe-shutdown. The cases for Figure 16 allows for lower liquid level, higher decay heat and internal energy.

Figure 17 provides the bounding functions for cases where battery failure has not occurred. In this event, the bounding function shows a higher energy, pressure, and decay heat. However, success is highly dependent upon the liquid level in the reactor vessel. The higher level allows for a much larger heat capacity for the reactor coolant as a whole, and can withstand a significant amount of heat prior to core melt. In these conditions, if recovery occurs within 24 hours of the beginning of the simulation, core damage can be averted.

The total computation savings directly involved in the Branch-and-Bound algorithm as implemented here resulted in a computational cost savings of approximately 60%. The actual computing costs can vary depending upon the number of parallel CPUs available to compute the simulation. The BWR simulation was performed using 512 processors, with a simulation time of approximately 5.5 days. Using the Branch-and-Bound algorithm, the simulation time was reduced to 2 days.

Table 4. Summary of surface plot data for BWR SBO with battery failure 1 SRV fails to close, failure of HPCI, and RCP LOCA.

Series	Power (MW)	Pressure (MPa)	Liquid Level (m)	Internal Energy (J)
Series 1	48.50	2.54	11.08	2.17E+11
Series 2	35.74	1.52	8.02	1.33E+11
Series 3	48.50	2.84	11.00	2.19E+11
Series 4	48.50	3.27	10.92	2.22E+11
Series 5	35.68	1.47	14.59	2.74E+11
Series 6	35.50	1.49	8.79	1.53E+11
Series 7	35.28	1.41	7.03	1.16E+11
Series 8	35.61	1.51	8.28	1.41E+11
Series 9	35.61	1.51	7.98	1.31E+11

Table 5. Summary of surface plot data for BWR SBO with RCP LOCA, battery failure and stuck SRV.

Series	Power (MW)	Pressure (MPa)	Liquid Level (m)	Internal Energy (J)
Series 1	48.50	2.54	11.08	2.17E+11
Series 2	35.74	1.52	8.02	1.33E+11
Series 3	48.50	2.84	11.00	2.19E+11

Series 4	48.50	3.27	10.92	2.22E+11

Table 6. Summary of surface plot data for BWR SBO with no battery failure, RCP
LOCA and stuck open SRV.

Series	Power (MW)	Pressure (MPa)	Liquid Level (m)	Internal Energy (J)
Series 1	48.50	1.47	14.61	3.00E+11
Series 2	48.50	2.21	38.59	2.24E+11
Series 3	47.92	3.70	46.14	3.05E+11
Series 4	47.92	4.41	46.58	3.30E+11

<Figure 20>

<Figure 21>

<Figure 22>

<Figure 23>

<Figure 24>

Optimization of the DETs allows for a risk informed analysis of modelling sensitivity and uncertainty. The intent of this sensitivity and uncertainty analysis is to identify those modelling parameters that have the large impact on reactor safety or in other words, peak clad temperature (PCT). The DET optimization approach allows us to identify the timing of events with the highest probability of failure and allows for a sensitivity and uncertainty quantification analysis of those parameters based on a risk informed analysis rather than strictly on the judgement of an analyst as would be performed in a traditional PIRT analysis. Expert based knowledge is an invaluable resource in complex dynamic systems. However, timing of events and potential deviations from plant conditions may even lead to incorrect analysis from plant experts. By utilizing the approach described above with optimization, plant damage states can be identified and the optimization to find the largest contributors to risk can be evaluated based on the actual timing of those events leading to failure. Traditional classical event tree analysis has the potential to miss accident sequences that are contributors to risk. The methods described above are not intended to replace expert knowledge, but rather to utilize the knowledge of experts in creating an optimization algorithm to identify potential scenarios more effectively.

10 Sensitivity and Uncertainty Analysis

With an optimized DET established and performing a sensitivity and uncertainty analysis on parameters that can be modelled or adjusted in a RELAP5-3D input file, a PIRT analysis can be conducted. In reviewing Reference [12], a small subset of the parameters is chosen. Using the DAKOTA simulation tool described above, a correlation matrix can be obtained to determine various phenomenological effects associated with the actual model relative to the results of interest (i.e., peak clad temperature). In reviewing the PIRT analysis, which is based on expert knowledge, a small subset of parameters were chosen to determine the effects on the core. These parameters were chosen as those that have a impact on ability to remove thermal energy from the system. These parameters included initial reactor power, SRV capacity, HPCI capacity, RCIC capacity, and firewater capacity. Additional parameters may be chosen, however, for demonstration, the above parameters were identified as important to energy removal. For each of the end nodes, the effects of the parameters discussed above on the peak clad temperature and liquid level, was examined. The intent was to determine which of the components as modelled had the largest impact on core damage. There are a significant number of parameters that can be evaluated. However, the study here was limited to the parameters that could be of interest. In the PIRT analysis, there were 1047 different phenomenological effects identified. Most of those parameters were a direct response function that may or may not be calculated in RELAP. The intent of this study was to identify the parameters that have a direct input into RELAP and to evaluate the effects of those parameters directly on core damage.

Table 5 provides a listing of the subset of parameters used in this analysis. In order to evaluate the uncertainty of the parameters, a Latin Hypercube Sampling is performed on each parameter for each branch condition. Each parameter was assumed to have a uniform distribution with an uncertainty of +/-10% for this demonstration case. The branch resulting in the highest probability of failure is used in this discussion to demonstrate the method. As discussed previously, approximately 373 leaves ended in failure and 3141 leaves ended in success in an optimized DET. Without the use of the optimization, the number of simulations would have increased by a factor of 3 or an increase of 66%, resulting in much greater simulation time and time needed for data reduction. The parameters were sampled using 100 samples varying the parameters. The importance of each parameter is determined from the correlation matrix, that provides an indication of that particular parameter and the results on the objective function.

Table 5 . Sensitivity parameters evaluated in SBO study.

Parameter	Nominal Value
SRV flow area	0.085 ft ² (0.0079 m ²)
HPCI capacity	5600 gpm (424.3 l/s)
RCIC capacity	5600 gpm (424.3 l/s)
Firewater capacity	2500 gpm (189.4 l/s)
Power	3293 MW
Zr heat conduction	0.0021 BTU/s/ft/F (13.03 W/mK)

Zr heat capacity	35.18 BTU/ft ³ /F (2.35×10^6 J/m ³ K)
U heat Conduction	3.906×10^{-4} BTU/ft ³ F (2.43 W/m K)
U heat capacity	56.55 BTU/ft ³ /F (3.82×10^6 J/m ³ K)

The above results were used to evaluate their potential effects on liquid level and peak clad temperature. The results are presented in Table 6 with a summary of the ranking for each parameter. Evaluation of the parameters with regards to the peak clad temperature, the initial reactor power has the greatest impact on the PIRT ranking followed by firewater injection. The failure condition did not include firewater injection activation in the model and thus is not applicable. The next important component for modeling in the failure case is the thermal conductivity of Zr cladding, however, it is still ranked low. During SBO with loss of battery power, there is no capability to remove heat from the core and peak clad temperature would be related to the rate at which water is boiling off in the reactor. As this simulation has ended in failure, the mean peak clad temperature for the simulation using Latin Hypercube Sampling was 2165 K with an standard deviation of 119 K. The cladding would have failed before these temperatures were met, however, the methods for this type of analysis are still valid for performing a sensitivity analysis. A data associated with the sampling of reactor power, SRV capacity and HPCI capacity, and Zr thermal conductivity are shown in Table 6 and Figure 25 through Figure 28. The data demonstrates that the initial reactor power has the largest impact

on PCT, and a reduction in the initial reactor power directly results in a decrease in PCT during the transient.

The case involving successful firewater injection was modeled to evaluate the effects on the ability of operators to inject firewater into the reactor. The plots of initial reactor power, SRV capacity, and firewater injection are shown in Figure 29 through Figure 31. The initial reactor power and firewater injection capacity for this case, demonstrates that the modeling or initial conditions are important to the simulation. As the initial reactor power is decreased, the PCT in this simulation decreases as would be expected. Firewater injection capacity is also demonstrated as being important (PIRT=High). The capacity of the SRVs to remove energy from the system are negligible in this case, as the SRV became stuck during the transient at 46 minutes. Battery failure occurred at 185 minutes leaving a significant time for the reactor to blow down and thus the impacts on the conditions important to recovery (i.e., reactor pressure) are not significant. Firewater injection began after 230 minutes leaving approximately 3 hours for the reactor to blow down and still recover.

Table 6. PIRT ranking for failure condition in SBO.

Parameter	Failure condition		Firewater injection recovery	
	Correlation Coefficient (PCT)	PIRT Ranking	Correlation Coefficient (PCT)	PIRT Ranking
SRV flow area	0.083	Low	-0.194	Low
HPCI capacity	0.068	Low	0.051	Low
RCIC capacity	0.189	Low	0.203	Low

Firewater capacity	N/A	N/A	-0.930	Hihg
Power	0.785	High	0.817	High
Zr thermal conductivity	0.225	Low	0.063	Low
Zr heat capacity	0.133	Low	-0.026	Low

<Figure 25>

<Figure 26>

<Figure 27>

<Figure 28>

<Figure 29>

<Figure 30>

<Figure 31>

11 Conclusions

This paper provides a methodology for optimizing DET and applies that methodology to create a risk informed PIRT based on the dynamics of the system. The PIRT developed using a dynamic event tree method can be used to design experiments and modeling parameters that are important to safety. The capabilities to evaluate the uncertainty in models with regards to a risk informed decision analysis allows for modeling decisions and validation experiments to be designed to address needs to increase the margin of safety of a nuclear power plant. The difficulties in identifying scenarios driving risk, particularly within a dynamic PRA framework, requires expensive computational tools or the use of optimization algorithms. We have proposed a methodology using the Branch-and-Bound algorithm to evaluate branches and prune branches that most likely will yield little results. The Branch-and-Bound algorithm as proposed in this research has reduced the computational costs by 60%, with additional cost saving that may be identified in later evaluations. The bounding functions that are identified in a particular sequence, may provide a seed function for later evaluations of similar transients. The use of the LENDIT metrics with S2R2

set theory allows for the decision process of parameters that are important to energy removal to be evaluated to establish a bounding surface such that nodes can be pruned if they are deemed to fit within the surface. Nodes outside of the surface are evaluated further, while pruning nodes that have resulted in significantly less probability than the highest probability of failure identified in the model.

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Figure 1. RAVEN/RELAP5-3D Framework.

Figure 2. Objective function of the Dynamic Event Tree.

Figure 3. Example of an updated DET using the Branch-and-Bound algorithm.

Figure 4. Flow chart of the Branch-and-Bound Algorithm Applied to Dynamic Event Trees.

Figure 5. Diagram of the GE Mark I BWR.

Figure 6. Nodalization diagram of the BWR RELAP5-3D Model.

Figure 7. Classical ET for Loss of Off-Site Power.

Figure 8 LOOP-1A Event Tree (transfer from LOOP ET or SBO-OP Tree).

Figure 9. SBO Event Tree (transfer from LOOP Event Tree).

Figure 10 SBO-OP Event Tree (transfer from SBO Tree).

Figure 11 SBO-T1 Event Tree (transfer from SBO Tree)

Figure 12 SBO-1 Event Tree (transfer from SBO Tree). Note: SBO-2 event tree is identical to SBO-1.

Figure 13 SBO-1 Event Tree.

Figure 14. LENDIT Metrics for the BWR SBO Transient.

Figure 15. CDF for DG Failure used in SBO Analysis ($CDF = 1 - e^{-0.0033t(sec)}$).

Figure 16. CDF for Battery Failure used in SBO Analysis ($CDF = \frac{t(hours)-0.5}{6.0-0.5}$).

Figure 17. CDF for DG Recovery used in SBO Analysis ($CDF = 1 - e^{-\left(\frac{t(min)}{368.4}\right)^{0.745}}$).

Figure 18. CDF for Manual Activation of ADS used in SBO Analysis ($CDF = 1/2 + 1/2 \operatorname{erf} \left[\frac{\ln(t(min))-42}{\sqrt{2}(15)} \right]$).

Figure 19. CDF for Firewater Injection used in SBO Analysis ($CDF = 1/2 + 1/2 \operatorname{erf} \left[\frac{\ln(t(min))-8.4}{\sqrt{2}(15)} \right]$).

Figure 20. Dynamic Event Tree Solution Without Branch-and-Bound Optimization

Figure 21. Dynamic Event Tree Solution Without Branch-and-Bound Optimization

Figure 22. Surface plots for BWR SBO with battery failure 1 SRV fails to close, failure of HPCI, and RCP LOCA.

Figure 23. Surface plot for BWR SBO with RCP LOCA, battery failure and stuck SRV.

Figure 24. Surface plots for BWR SBO with no battery failure, RCP LOCA and stuck open SRV.

Figure 25. Sensitivity analysis associated with initial reactor power for Case 1

Figure 26. Sensitivity analysis associated with SRV capacity for Case 1.

Figure 27. Sensitivity analysis associated with HPCI capacity for Case 1.

Figure 28. Sensitivity analysis associated with Zr thermal conductivity for Case 1.

Figure 29. Sensitivity analysis associated with initial reactor power for Case 2.

Figure 30. Sensitivity analysis associated with SRV capacity for Case 2.

Figure 31. Sensitivity analysis associated with firewater injection for Case 2.

RELAP5-3D

$$\frac{\partial x}{\partial t} = F(x, v_{t-1}, t)$$

$$C = G(x, t)$$

$$\frac{\partial \bar{a}}{\partial t} = \bar{V}(x, v_{t-1}, \bar{a}, t)$$

C_I $\bar{a}$

v_I

RAVEN (Control Logic)

$$\frac{dv}{dt} = V(C, v_{t-1}, \bar{a}, t)$$

BWR-SBO	DG-FTR	BATTERY-1	SRV-FTC-1	BATTERY-2	DG-REC-1	SRV-FTC-2	BATTERY-3	DG-REC-2	#	END-S
									1	OK
									2	OK
								[PASS, PASS]	3	OK
									4	OK
								[PASS, PASS]	5	OK
									6	OK
								[PASS, PASS]	7	OK
									8	OK
								[PASS, NONE]	9	OK
									10	OK
								[PASS, FAIL]	11	CD
									12	OK
								[PASS, PASS]	13	OK
								[PASS, NONE]	14	OK
								[PASS, FAIL]	15	OK
								[NONE, NONE]	16	CD
								[PASS, FAIL]	17	OK
								[NONE, NONE]	18	CD
								[PASS, FAIL]	19	OK
									20	CD
								[PASS, NONE]	21	OK
								[PASS, FAIL]	22	OK
									23	CD
								[PASS, FAIL]	24	OK
								[FAIL, FAIL]	25	CD
									26	CD

Loss of Offsite Power Initiator (Grid-Related)	Reactor Shutdown	Emergency Power	SRV's Close	Higher Pressure Injection (RCIC or HPCI)	Suppression Pool Cooling	#	End State (Phase - PH1)
IE-LOOPGR	RPS	EPS	SRV	HPI	SPC		

Transfer from LOOP or SBO-OP	Manual Reactor Depress	Low Pressure Injection (CS or LPCI)	Alternate Low Pressure Injection	Loss of Residual Heat Removal System	Containment Venting	Late Injection	#	End State (Phase - PH1)
TRANSFER	DEP	LPI	VA	RHR	CVS	LI		

Emergency Power	SRV's Close	Recirc Pump Seal Integrity	RCIC	HPCI	Actions to Extend ECCS Operation	#	End State (Phase - PH1)
EPS	SRV	RPSL	RCI	HPI	EXT		

Offsite Power Recovery	High Pressure Injection (HPCI or RCIC)	Suppression Pool Cooling	#	End State (Phase - PH1)
OPR	HP	SPC		

Actions to Extend ECCS Operation	Offsite Power Recovery	Diesel Generator Recovery	Containment Venting	Late Injection	#	End State (Phase - PH1)
EXT	OPR	DGR	CVS	LI		

Actions to Extend ECCS Operation	Manual Reactor Depress	Firewater Injection	Offsite Power Recovery	Diesel Generator Recovery	Containment Venting	Late Injection	#	End State (Phase - PH1)
EXT	DEP	FI	OPR	DGR	CVS	LI		

ONE STUCK OPEN SRV	RCIC	HPCI	OFFSITE POWER RECOVERY	DIESEL GENERATOR RECOVERY	#	End State (Phase - CD)
P1	RCI	HCI	OPR	DGR		