A Model for TSUnami FLOW INversion from Deposits (TSUFLIND)

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Abstract

Modern tsunami deposits are employed to estimate the overland flow characteristics of tsunamis. With the help of the overland-flow characteristics, the characteristics of the causative tsunami wave can be estimated. The understanding of tsunami deposits has tremendously improved over the last decades. There are three prominent inversion models: (a) Moore's advection model [Moore et al. 2007], (b) Soulsby's model [Soulsby et al. 2007], and (c) TsuSedMod [Jaffe and Gelfenbuam 2007]. TSUFLIND incorporates all three models and adds new modules to better simulate tsunami deposit formation and calculate flow condition. TSUFLIND takes grain-size distribution, thickness, water depth and topography information as inputs. TSUFLIND computes sediment concentration, grain-size distribution of sediment source and initial flow condition to match the sediment thickness and grain size distribution from field observation. Furthermore, TSUFLIND estimates the flow speed, Froude number and representative wave amplitude. The model is tested by using field data collected at Ranganathapuram, India after the 2004 India Ocean tsunami. TSUFLIND reproduces the field measurement grain-

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size distribution with less than 5% error. Tsunami speed in this test case is about 4.7 ms^{-1} at 150 meters inland and decreases to 3.3 ms^{-1} 350 meters inland from the shoreline. The estimated wave amplitude of the largest wave for this test case is about 5 to 7 meters.

Keywords: Tsunami, Tsunami sediment, Sediment transport, Flow depth, Flow speed, Inversion model

1. Introduction

The tsunami events that occurred over the last decades have caused an increase in public awareness and resulted in more research on the tsunami wave. Tsunami deposits play an important role not only in tsunami hazard assessments, but also in interpreting tsunami hydraulics [Hutchinson et al. 1997; Moore et al. 2007; Jaffe and Gelfenbuam 2007]. To draw any useful quantitative conclusions from tsunami deposits, the information from deposits about the causative tsunami needs to be extracted either by comparing parameters from the deposits with results from forward models [see Bourgeois et al. 1988; Martin et al. 2008] or by inversion models directly [see Nott 1997; Noormets et al. 2004; Jaffe and Gelfenbuam 2007; Moore et al. 2007; Soulsby et al. 2007; Smith et al. 2007; Benner et al. 2010; Nandasena and Tanaka 2013].

Tsunami inversion models attempt to link the basic information of the tsunami deposits with the overland flow characteristics. There are three prominent inversion models: Moore’s advection model [Moore et al. 2007], Soulsby’s model [Soulsby et al. 2007] and TsuSedMod model [Jaffe and Gelfenbuam 2007]. It should be noted that all these models are based on different basic assumptions and employ different information from the deposits. For

19 example, Moore’s advection model estimates tsunami flow magnitude by de-
 20 termining the combination of flow velocity and depth to move the largest
 21 grain from the sediment source to the deposition area [Moore et al. 2007].
 22 In this paper, we present a joint inversion framework (TSUFLIND), which
 23 combines these three models. TSUFLIND not only couples all these three
 24 inversion models, but also contains a new method to calculate deposit char-
 25 acteristics [Tang and Weiss 2014]. It also uses the calculated flow depth
 26 and water volume from Soulsby’s model to estimate a representative offshore
 27 tsunami wave amplitude.

28 **2. Theoretical Background**

29 *2.1. Inversion Models Employed*

30 As mentioned above, there are three prominent tsunami deposition in-
 31 version models that will be used: Moore’s advection model, Soulsby’s model
 32 and TsuSedMod model.

33 [Figure 1 about here.]

34 *(a) Moore’s model:.* Moore et al. [2007] assumes that some grains in the
 35 sediment source do not move because the tsunami flow is not strong enough.
 36 Furthermore, it is assumed that most grains are transported in suspension.
 37 Based on these assumptions, the shear velocity is determined for the largest
 38 grain in the tsunami deposits. The law of the wall can be employed to find
 39 the shear stress, which is necessary to move the largest grain to get a flow
 40 velocity U . The following equation is used to determine deposition.

$$\frac{h}{w_s} = t = \frac{l}{U} \quad (1)$$

in which w_s is the settling velocity of the sediment grain. h is the water depth, l represents the horizontal distance a grain travels to be deposited. Because of the horizontal transport, this model is also referred as an advection model.

This model was applied to deposits formed by the 1929 Grand Banks tsunami, Newfoundland, Canada [Moore et al. 2007]. In this application, it was estimated that the average flow depth was 2.5 to 2.8 m, and the flow speed was 1.9 to 2.2 ms⁻¹, which are the minima [Moore et al. 2007].

(b) *Soulsby's model*:. Soulsby's model assumes that the water depth increases linearly between 0 and γT and decreases from γT to T for any given locations. T is the inundation time and γ is a constant related to run-up time, which is between 0 and 1. $H = H_0 + \Delta h$ is the maximum flow depth at a given location during tsunami inundation and decreases toward the inundation limit, H_0 denotes the maximum water depth at the shoreline, Δh denotes the depth increment due to tsunami:

$$\Delta h = \frac{l(R_z - H_0)}{mR_z} - \frac{l}{m} \quad (2)$$

where m is the slope and R_z represents the vertical inundation limit. The thickness of the deposit for grain size i at the shoreline:

$$\zeta_0^{(i)} = \frac{C_0^{(i)} w_s^{(i)} T_d}{(1-p)\rho_s(1+\alpha^{(i)})} (1 + \alpha^{(i)}\gamma) \quad (3)$$

where $\alpha^{(i)} = \frac{w_s^{(i)} T_d}{H_0}$, $w_s^{(i)}$ denotes the settling velocity for grain size i , $T_d = (1 - \gamma)T$ is the deposition time. $C_0^{(i)}$ is the depth averaged sediment concentration

for grain size i and p is the porosity. The sediment thickness for grain size i linearly decreases with distance from the shoreline:

$$\zeta^{(i)}(x) = \begin{cases} \zeta_0^{(i)}(1 - \frac{x}{R_s^{(i)}}) & x < R_s^{(i)} \\ 0 & x \geq R_s^{(i)} \end{cases} \quad (4)$$

where $R_s^{(i)}$ is the distance between sediment extent and the shoreline for grain size i [Soulsby et al. 2007].

(c) *TsuSedMod*:. Jaffe and Gelfenbaum [2007] developed an inversion model based on sediment deposited from suspension. There are several basic assumptions in *TsuSedMod*: (1) sediment is transported in suspension and deposited when steady and uniform tsunami flow slows down; (2) suspended sediment concentration is distributed in an equilibrium profile; (3) there is no erosion caused by return flow. The model iteratively adjusts the sediment source and the shear velocity to match the sediment grain-size distributions and thickness of suspension-grading sediment layers [Jaffe et al. 2011, 2012]. For the grain size i , the sediment thickness $\Delta\eta^{(i)}$ is given by:

$$\Delta\eta^{(i)} = \frac{1}{(1-p)} \int_0^{H(x)} C^{(i)}(z) dz \quad (5)$$

where $C^{(i)}(z)$ is the sediment concentration profile of grain size i . After determining the shear velocity, the flow speed profile is calculated by :

$$U(z) = \int_{z_0}^z \frac{u_*^2}{K(z)} dz \quad (6)$$

where z_0 is the bottom roughness from MacWilliams [2004] and $K(z)$ is the eddy viscosity profile from Gelfenbaum and Smith [1986].

76 The TsuSedMod model has been applied to four modern tsunami [Jaffe
 77 and Gelfenbuam 2007; Spiske et al. 2010; Jaffe et al. 2011, 2012] and two
 78 paleotsunami [Witter et al. 2012; Spiske et al. 2013b]. For the 2009 tsunami
 79 near Satitua, Samoa, the flow speed estimated from TsuSedMod at three
 80 locations (100, 170 and 240 meters inland) were 3.6 to 3.8 ms^{-1} (bottom
 81 layer/earlier wave) and 4.1 to 4.4 ms^{-1} (top layer/later wave). These results
 82 are consistent with the 3 to 8 ms^{-1} flow speed from the boulder transport
 83 inverse model [Jaffe et al. 2011]. For more details about these three models,
 84 we refer to Jaffe and Gelfenbuam [2007], Moore et al. [2007], Soulsby et al.
 85 [2007] and Sugawara et al. [2014]

86 *2.2. Sedimentation model*

87 The method used to calculate the sediment concentration of the sediment
 88 source in TSUFLIND is similar to the one presented in Madsen et al. [1993].
 89 The grain-size distribution of the sediment source is characterized by D_{50} ,
 90 the largest grain and the smallest grain size.

91 When the entire tsunami deposit at a given location is considered, re-
 92 suspension sediment flux can be neglected and Soulsby's model is applied.
 93 However, if the individual layer in the tsunami deposit is considered, intense
 94 turbulent mixing cannot be ignored. Therefore resuspension has to be taken
 95 into account. The generation of each individual portion of the tsunami sedi-
 96 ment based on flow condition is the fundamental part to reconstruct tsunami
 97 deposits. For the entire deposit, the basic process is to calculate sediment
 98 thickness $\zeta^{(i)}(x)$ for each grain size at each point along the slope by using
 99 Eq (3) and (4) from Soulsby's model. We assume that the depth averaged
 100 sediment concentration C_0 in Eq (3) is the reference sediment concentration

101 C_r here. The reference concentration is calculated for a given flow condition
 102 with Madsen et al. [1993]:

$$C_r^{(i)} = \frac{\beta_0(1-p)f^{(i)}S^{(i)}}{1 + \beta_0 S^{(i)}} \quad (7)$$

103 where β_0 is the resuspension coefficient, $f^{(i)}$ is a fraction of the sediment of
 104 grain size i . $S^{(i)}$ is the normalized excess shear stress given by

$$S^{(i)} = \begin{cases} \frac{\tau_b - \tau_{cr}^{(i)}}{\tau_{cr}^{(i)}} & \tau_b > \tau_{cr}^{(i)} \\ 0 & \tau_b \leq \tau_{cr}^{(i)} \end{cases} \quad (8)$$

105 where τ_b is the bed shear stress and $\tau_{cr}^{(i)}$ is the critical shear stress of the
 106 initial sediment movement for grain size i [Madsen et al. 1993].

107 For a given location x , the grain-size distribution for the entire tsunami
 108 deposit is given by:

$$f^{(i)} = \frac{\zeta^{(i)}(x)}{\sum_{i=0}^N \zeta^{(i)}(x)}; i = 1, 2, 3, \dots, N \quad (9)$$

109 where $f^{(i)}$ is the percentage of grain size i in the entire sediment, $\zeta^{(i)}(x)$ is
 110 sediment thickness of grain size i and $\sum_{i=0}^N \zeta^{(i)}(x)$ is total deposit thickness
 111 for all grain sizes. N is the number of grain size classes.

112 The tsunami deposit characteristics are reconstructed by matching sedi-
 113 ment thickness and grain-size distribution with field data. In order to recon-
 114 struct deposit details, the sediment concentration cannot be depth averaged
 115 and is described as a Rouse-type suspended sediment concentration profile.
 116 In this framework, we use the method from Jaffe and Gelfenbuam [2007]
 117 to calculate the suspended sediment concentration profile. It is efficient to

118 reconstruct the deposit by calculating times of deposition. The deposition
 119 time of suspended sediment is calculated by:

$$t_j^{(i)} = \frac{z_j}{w_s^{(i)}} \quad (10)$$

120 in which $t_j^{(i)}$ is the deposition time for grain size i sediment at elevation z_j .
 121 The amount of sediment settling in each grain size class for a given elevation
 122 is tracked by

$$\zeta_j^{(i)} = \frac{C_j^{(i)}}{1 - p} \quad (11)$$

123 in which $C_j^{(i)}$ is the suspended sediment profile [Jaffe and Gelfenbuam 2007].
 124 $\zeta_j^{(i)}$ is the sediment thickness increment of the same grain size i at elevation z_j
 125 and deposited at time $t_j^{(i)}$. The deposition time and corresponding sediment
 126 thickness increment are ordered from shortest to longest. If there are multiple
 127 layers in the tsunami sediment, we can compute the grain-size distribution
 128 for each layer separately based on the depositional temporal order of the
 129 sediment thickness increments by:

$$f_k^{(i)} = \frac{\sum_{j=0}^M \zeta_j^{(i)}}{\sum_{i=0}^N \left(\sum_{j=0}^M \zeta_j^{(i)} \right)}; i = 1, 2, 3, \dots, N; j = 1, 2, 3, \dots, M \quad (12)$$

130 where $f_k^{(i)}$ is the sediment fraction of grain size i in layer k . $\sum_{j=0}^M \zeta_j^{(i)}$ is total
 131 sediment thickness with the same grain size i in sediment layer k . Index
 132 j is used to mark the original location of sediment in the water column.
 133 $\sum_{i=0}^N \left(\sum_{j=0}^M \zeta_j^{(i)} \right)$ is the total thickness of this sediment layer which contains
 134 all grain size classes. In TSUFLIND, the calculation of tsunami flow condition
 135 will use the same method as TsuSedMod model [Jaffe and Gelfenbuam 2007].

136 2.3. Result Evaluation

137 We employ the second norm to quantify the error between model and
 138 observed results as a control of the iterative procedure. The second norm of
 139 error for layer k is given by:

$$L_k = \sqrt{\frac{\sum_{i=1}^N (f_m^{(i)} - f_o^{(i)})^2}{N}} \quad (13)$$

140 $f_m^{(i)}$ and $f_o^{(i)}$ are the modeled and observed percentage for each grain size
 141 class i . With the help of L_k , we compute the average second norm value for
 142 a location with:

$$\bar{L} = \frac{1}{K} \sum_{k=1}^K L_k \quad (14)$$

143 We define $\bar{L} \leq 5\%$ as a good simulation. For the tsunami sediment thickness
 144 simulation, we employ the same process. The second norm value of error for
 145 thickness between the model result and the field observation is given by:

$$L_{th} = \sqrt{\frac{\sum_{j=1}^Q \left(\frac{th_m - th_f}{th_f} \cdot 100\% \right)^2}{Q}} \quad (15)$$

146 where th_m and th_f are the modeled and observed thicknesses for each sample
 147 location, Q is the number of sample locations. As there is only a limited
 148 number of tsunami deposit samples for the test case applied here, we use
 149 10% as the threshold value.

150 2.4. Offshore Wave Characteristics and Flooding

151 In order to estimate a representative offshore tsunami amplitude, we re-
 152 late the water volume calculated from Sousby's model at maximum inun-

153 dation with the volume calculated by numerically solving the shallow water
 154 equation. We carry out a parameter study by varying the slope (m) and the
 155 offshore wave amplitude (ξ). For more details about the parameter study
 156 and employed numerical model, we refer to Appendix B. The water depth
 157 computed from Soulsby’s model is used to calculate the volume of the in-
 158 undation water. With the help of numerical simulations (Appendix B), we
 159 derived the following formulation:

$$\begin{aligned} \xi = & \lambda_1 + \lambda_2 \cdot V + \lambda_3 \cdot m + \lambda_4 \cdot V^2 + \lambda_5 \cdot m \cdot V + \lambda_6 \cdot m^2 \\ & + \lambda_7 \cdot V^3 + \lambda_8 \cdot V^2 \cdot m + \lambda_9 \cdot V \cdot m^2 + \lambda_{10} \cdot m^3 \end{aligned} \quad (16)$$

160 Where ξ is offshore wave height, V is the water volume that covers the land
 161 at maximum inundation, m is the slope of beach profile. These constants
 162 λ in Eq 16 are $\lambda_1 = 5.06$, $\lambda_2 = 2.93$, $\lambda_3 = -0.28$, $\lambda_4 = 0.51$, $\lambda_5 = -3.04$,
 163 $\lambda_6 = 0.0014$, $\lambda_7 = 0.027$, $\lambda_8 = -0.011$, $\lambda_9 = 0.051$, $\lambda_{10} = 0.053$.

164 2.5. Inversion Framework and Coupling

165 We use the information from all three models as different components in
 166 this joint inversion framework. The steady flow condition that is presented in
 167 all models, is also presented in TSUFLIND and represents the most simplify-
 168 ing assumption. The inputs to TSUFLIND are the sediment characteristics
 169 for different sampling locations along a slope. However, it should be noted
 170 that the inversion of the flow conditions is carried out for each sample loca-
 171 tion individually. TSUFLIND uses components from Moore model, Soulsby’s
 172 model and TsuSedMod model to adjust the sediment source grain-size dis-
 173 tribution, the sediment source concentration and the average flow velocity

174 to simulate tsunami sediment thickness and grain-size distribution along the
175 slope in the deposition zone. If needed, the representative offshore wave am-
176 plitude can be computed. Figure 2 depicts the flowchart outlining how the
177 joint inversion model works.

178 [Figure 2 about here.]

179 The information needed for a successful inversion includes the grain-size
180 distribution, sediment thickness as well as the information of the slope along
181 which the tsunami sediments were sampled. It should be noted that TSU-
182 FLIND can handle volume and weight based grain-size distributions that
183 are generated with various of methods. However, in general, settling tube
184 measurements are preferred. In the inversion framework, Moore’s advec-
185 tion model is employed to calculate the initial flow speed. Because Moore’s
186 model uses the actual data from measured sediment distributions, it reduces
187 the number of iterations, significantly. The reservoir of sediments in the wa-
188 ter column is calculated by following Madsen et al. [1993], and it is assumed
189 that all grain-size distributions can be described with log-normal distribu-
190 tions. The iteration begins by computing the inundation (R_w in Fig. 1) with
191 the help of Soulsby’s model, and the initial estimate of the flow condition is
192 from Moore’s advection model. The result of this step is the local flow depth
193 and the entire sediment thickness at each sample location. Our sediment for-
194 mation module calculates the characteristics of the deposited sediments. The
195 number of iterations is controlled by the error norm between the simulated
196 and observed deposits and stop after the predefined threshold is met. The
197 model outputs are flow speed and depth, the Froude number and a range of
198 offshore reference wave amplitudes.

199 **3. Application and Example**

200 *3.1. Field Observation and Data*

201 We employ the field data [Bahlburg and Weiss 2007] from the 2004 Indian
202 Ocean tsunami to demonstrate the capabilities of our framework (Fig. 3).
203 These samples come from the coastal area in the vicinity of Ranganathapu-
204 ram, India. Bahlburg and Weiss [2007] identify sediment layers formed by
205 the tsunami in this cross section and described grain-size distributions for
206 each layer. There are some grass runners on the top of the tsunami sed-
207 iment, which indicate the return flow direction and the erosion caused by
208 the return flow. Most grain-size distributions of the sediment layers in the
209 test case are unimodal (Fig. 3b). Tsunami deposits in this cross section are
210 usually well sorted, and the mean grain size is between 0.5 and 1.5 in ϕ scale,
211 which corresponds to medium and coarse sand. Furthermore, Bahlburg and
212 Weiss [2007] observe that the mean grain size is upward and landward fining
213 in this cross section. For the inversion of flow depth, speed, Froude number
214 and offshore wave amplitude in the TSUFLIND, the deposit thickness and
215 grain-size distribution along all section are needed as inputs. Flow depth in
216 this model will take full use of both the field observations and the model
217 results from Soulsby's model.

218 *3.2. Sedimentary Simulation Results*

219 TSUFLIND first simulates tsunami deposit thickness (Fig. 3c). In the
220 test case, the largest observed thickness is about 0.22 meters at 120 meters
221 inland. For the first 100 meters in this cross section, the simulated thickness
222 from TSUFLIND is larger than the field measurement. After 200 meters

inland, the simulated thicknesses decrease quickly and generally fit with the field measurement.

TSUFLIND reconstructs sediment grain-size distributions for both the entire tsunami deposit and several vertical intervals at any given sample locations. The error of the entire tsunami sediment grain-size distribution in this test case is from 0.38% to 1.54%, which can be considered good simulation results. The error is less than 1.0% from 120 meters to 160 meters inland and then increases to 1.5% after 160 meters inland. We use four sediment samples to calculate grain-size distributions (Fig. 3d *I – IV*, response to 120 m, 160 m, 177 m and 207 m from shoreline). Beyond 160 meters inland, there are fewer coarse grains and more fine grains in the simulated grain-size distribution than the field measurement (Fig. 3d, *I, III* and *IV*).

[Figure 3 about here.]

In order to study how the grain-size distribution changes in the vertical direction, we employ the new sediment formation module (See Section 2.2) to simulate tsunami deposit grading. Figure 3a shows grain-size distribution for several vertical intervals at four different study locations. The grain size for these reconstruction results ranges from 0 to 6 in ϕ scale. The number of vertical intervals decreases toward the inland extent of the deposits. The simulated deposits exhibit features such as the well-known fining inland and fining upward.

Based on the grain-size distribution for each vertical interval (Fig. 3a), mean grain size, kurtosis, skewness and sorting factor can be calculated for locations that are at least 110 m away from the shoreline. The mean grain size

247 in the bottom portion of the deposit does not significantly change (around
 248 1.2 ϕ). However, the mean grain size decreases toward the top of the deposit
 249 about 2.2 ϕ . The change in kurtosis is about 0.8 to 1.1 in this sample.
 250 Sediment simulation results in this example also show that tsunami sediment
 251 changes from moderate sorted at the bottom to well sorted at the top. The
 252 grain-size distribution is positively skewed.

253 3.3. *Hydrodynamic Inversion Results*

254 After reconstructing the grain-size distributions, TSUFLIND calculates
 255 the flow speed and Froude number at the sample locations. For the test
 256 case, Fig.4a and 4b show the flow speed and Froude number distribution
 257 along the slope. The average flow speed decreases from 4.7 ms⁻¹ at 150
 258 meter inland to 3.3 ms⁻¹ at 350 meter inland. The Froude number, which is
 259 around 0.9, does not change significantly along the slope. Furthermore, the
 260 range of possible velocities and Froude numbers decreases from 150 meters
 261 to 350 meters inland.

262 The flow speed profile is influenced by the eddy viscosity profile and shear
 263 velocity. The eddy viscosity profile parametrizes the vertical distribution of
 264 turbulent stress. TSUFLIND follows the flow eddy viscosity profile based on
 265 laboratory data from Gelfenbaum and Smith [1986]. While the flow speed
 266 has the largest value on the water surface and decreases toward the sediment
 267 bed, TSUFLIND only calculates the depth-averaged velocities as final results.

268 [Figure 4 about here.]

269 TSUFLIND computes the water surface profile to estimate the water
 270 volume when the tsunami wave reaches the maximum inundation. With the

271 help of Eq (16), the wave amplitude can be estimated based on the slope (m)
272 and the water volume (V). For the Eastern India case, the wave amplitudes
273 range from 5 to 7 meters and the wavelength is close to 50 km.

274 4. Discussion

275 4.1. Interpretation of test case results

276 With the help of the presented model, we can reproduce tsunami sedi-
277 ments as well as infer the flow condition based on observations and laboratory
278 measurements of existing tsunami deposits. Figure 3 summarizes the results
279 of our simulation for the tsunami deposits. The apparent difference of the de-
280 posit thickness between model results and observations for distances smaller
281 than 120 meters from the shoreline can be explained by strong return flow or
282 large velocities from subsequent waves with small inundation. For distances
283 from the coastline larger than 120 meters, the deposit simulated thicknesses
284 match well with the observations. However, the observations are slightly
285 larger due to the presence of topographic change that may slow down the
286 flow (Fig 3c and d). The finer grain sizes contain the largest error between
287 observation and model result . It is likely that the topographic changes are
288 the main source of the error. However, the difference could also be a part of
289 the model uncertainty.

290 The calculated mean speed decreases from 4.7 ms^{-1} to 3.3 ms^{-1} along
291 the studied section. The speed decreases continuously shown as Fig 4a, the
292 Froude number increases and then decreases (Fig. 4b). The mean Froude
293 number is around 0.9 for this test case. As the flow depth decreases toward
294 the maximum inundation with a constant slope, the calculated decrease in

295 the Froude number can only be explained by a decrease in the velocity. At
 296 first, the flow speed decreases less slowly than the water depth, so the mean
 297 Froude number increases in this area (150 meters to 300 meters). After
 298 300 meters, the flow decelerates quickly and causes the Froude number to
 299 decrease. The flow speed and Froude number results from TSUFLIND are
 300 shown as ranges of possible values with uncertainties (Fig. 4a and 4b). The
 301 ranges of the speed and the Froude number decrease from 150 m to 350 m,
 302 which indicates the uncertainties decrease towards the sample location close
 303 to the landward sediment pinch-out. It is possible that the tsunami deposits
 304 near the maximum run-up position become thin, well-sorted and fine-grained
 305 containing less information about the flow condition. Tsunami wave ampli-
 306 tudes calculated by TSUFLIND are usually larger than real amplitudes, be-
 307 cause the mathematical relationship (Eq. 16) is based on frictionless shallow
 308 water equations.

309 *4.2. Model limitation and improvement*

310 In this study, we combine three tsunami inversion models to simulate
 311 tsunami deposit and estimate tsunami flow parameters. All three models
 312 are based on model-specified basic assumptions. A significant assumption
 313 of TSUFLIND is that the sediment transport and the deposition process
 314 during a tsunami are considered uniform in space and time. Consequently,
 315 the deposit comes from both horizontal convergence and suspension settling.
 316 TSUFLIND combines Sousby's model and TsuSedMod to simulate these two
 317 processes. This combination greatly improves the grain-size distribution sim-
 318 ulation results. However, when the tsunami flow decelerates rapidly because
 319 of bathymetric or topographic changes or any other reasons, some part of the

320 deposit would be eroded again. If the flow is strong enough, a significant part
321 of tsunami deposit may be eroded, just like the result shown in Fig. 3c from
322 shoreline to 100 meters in land. As a result, the tsunami speed calculated by
323 TSUFLIND represents an underestimation.

324 Another significant assumption of TSUFLIND is that most of tsunami
325 deposits is transported by the suspension load and ignores the contribution
326 of bed load. This assumption results into an overestimation of the tsunami
327 flow speed and increase the percentage of coarse fraction in the grain-size
328 distribution. TSUFLIND is not applicable for a case in which bed load is the
329 dominant sediment transport mode. In order to reduce the effect of bed load,
330 only the suspension-grading fraction of the measured grain size distribution
331 should be considered as input for inversion with TSUFLIND. However, it
332 should be noted that suspension-grading is not easy to identify in grain-size
333 distribution. TSUFLIND is designed to handle unimodal grain-size distribu-
334 tion. However, polymodal grain-size distributions can be deconvoluted, and
335 TSUFLIND can be employed to invert tsunami flow characteristics for the
336 different deconvoluted grain-size distributions, individually.

337 TSUFLIND has three aspects that require improvement: the applicable
338 flow condition, the accuracy of final outputs including sediment simulation
339 and flow speed calculation and capabilities to deal with post-depositional
340 processes. The improvement of the applicable area can be made by em-
341 ploying other tsunami propagation models instead of Sousby's model, which
342 can deal with non-uniform and unsteady flows. At the same time, the new
343 model needs to consider both the suspension load and bed load. Also a new
344 method for combining forward and inverse modeling will hold great promise

for deciphering quantitative information from tsunami deposits and decreasing the uncertainties in tsunami sediment transport simulation and inversion results [Sugawara et al. 2014]. Furthermore, post-depositional processes may alter the grain-size distribution and the thickness of tsunami deposit as documented by Szczuciński et al. [2007], Szczuciński [2012], Spiske et al. [2013a] and Bahlburg and Spiske [2015]. More quantitative data on how the post-depositional processes affect tsunami deposits are needed to derive empirical formulae to consider such changes in inversion model.

5. Conclusion

Modeling the tsunami sediment deposition processes and estimating tsunami flow parameters will greatly improve not only the understanding of deposition from tsunami but also the hazard assessment for extreme high-energy events. The combination of different inversion models allows the computation of a wide range of tsunami wave impacts or characteristics, ranging from sediment thickness, grain size distribution to flow speed and wave amplitude. Based on comparisons between our model results and field observation from post-tsunami surveys of the 2004-Sumatra (India), TSUFLIND appears to simulate tsunami deposit thickness and grain-size distribution with small error. Furthermore, our results show that TSUFLIND gives reasonable approximations of tsunami flow parameters like flow speed and Froude number for the 2004 Indian ocean tsunami case. However, these results are restricted by the flow condition. If there were strong return flows or subsequent waves, the results from TSUFLIND will contain some uncertainties. From a general point of view, with a simple bathymetry, the tsunami and paleotsunami

as well as other extreme events can be understood with the help of TSU-
 FLIND. More research needs to be done to improve TSUFLIND to quantify
 and reduce the uncertainties in the inversion results and expand applicable
 conditions.

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Appendix A

Table 1: Symbols List

Symbol	Unit	Description
h	m	Water depth
w_s	ms^{-1}	Settling velocity of the sediment grain
l	m	Horizontal distance a grain travels to be deposited
γ	-	Run-up time constant
T	s	Inundation time
H	m	Maximum flow depth at a given location during tsunami inundation
Δh	m	Water depth increment due to tsunami
m	-	Slope
R_z	m	Vertical water inundation limit
H_0	m	Maximum water depth at the shoreline
$\zeta_0^{(i)}$	m	Thickness of the deposit for grain size i at the shoreline
T_d	s	Deposition time

Table 1 Continued: Symbols List

$C_0^{(i)}$	m^3/m^3	Depth averaged sediment concentration for grain size i
p	-	Porosity
$\zeta^{(i)}$	m	Sediment thickness for grain size i
$R_s^{(i)}$	m	Distance between sediment extend and the shoreline for grain size i
$\Delta\eta^{(i)}$	m	Sediment thickness increment for grain size i
$C^{(i)}(z)$	m^3/m^3	Sediment concentration profile for grain size i
z_o	m	Bottom roughness
$K(z)$	$\text{kgm}^{-1}\text{s}^{-1}$	Eddy viscosity profile
$C_r^{(i)}$	m^3/m^3	Reference sediment concentration
β_0	-	Resuspension coefficient
$f^{(i)}$	%	Percentage of the sediment of grain size i
$S^{(i)}$	-	Normalized excess shear stress
τ_b	$\text{kgm}^{-1}\text{s}^{-2}$	Bed shear stress
$\tau_{cr}^{(i)}$	$\text{kgm}^{-1}\text{s}^{-2}$	Critical shear stress of the initial sediment movement for grain size i
N	-	Number of grain size classes
z_j	m	Sediment original elevation
$t_j^{(i)}$	s	Deposition time for grain size i sediment at elevation z_j
L_k	-	Second norm of error for layer k 's grain-size distribution
$f_m^{(i)}$	%	Modeled percentages for grain size class i
$f_o^{(i)}$	%	Observed percentages for grain size class i
$\bar{L}$	-	Average second norm value of grain-size distribution for a location
L_{th}	-	Second Norm value of thickness between the model result and the field observation
th_m	m	Modeled thicknesses
th_f	m	Observed thicknesses
Q	-	Number of sample locations
ξ	m	Offshore wave amplitude
V	m^3	Water volume that covers the land at maximum inundation
R_w	m	Distance of water run-up to shoreline (Inundation)
$\eta(x, t)$	m	Solitary wave form

Table 1 Continued: Symbols List

C	ms^{-1}	Wave celerity
$u(x, t)$	ms^{-1}	Horizontal velocity in shallow water equations
g	ms^{-2}	Gravitational constant
$B(x)$	m	Bed topography function
d	m	Water depth of continental shelf

379 Appendix B

380 TSUFLIND implements a simplified method to estimate the representa-
381 tive offshore tsunami wave amplitude. First of all, the water volume on the
382 land due to tsunami wave is calculated by:

$$V = \int_0^{R_w} h(x) dx \quad (17)$$

383 where V is the water volume, R_w is the distance of run-up to shoreline, $h(x)$
384 is water depth distribution function on land. To simplify this problem, we
385 assume tsunami wave is a solitary wave. The solitary wave form is given as
386 a function of distance x and time t by

$$\eta(x, t) = \xi \text{sech}^2(k(x - Ct)) \quad (18)$$

387 where

$$k = \sqrt{\frac{3\xi}{4d^3}} \quad (19)$$

388 ξ is the wave amplitude and d is the water depth of continental shelf, which
389 is assumed as 500 meters. C is the wave celerity which is expressed as:

$$C = \sqrt{g(h + H)} \quad (20)$$

390 Initial velocity in shallow water equation code is set as:

$$v_0(x, t) = \sqrt{g/h_0}\eta_{(x,t=0)} \quad (21)$$

391 We calculate the water volume when tsunami wave got the maximum run-
 392 up based on water distribution function $h(x)$. Water depth function $h(x)$
 393 comes from a shallow water equations code. The shallow water equations
 394 code used here originally is designed for studying the propagation and runup
 395 of solitary wave by using a high-resolution finite volume method to solve
 396 following equations[Delis et al. 2008]:

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = 0 \quad (22)$$

$$\frac{\partial(uh)}{\partial t} + \frac{\partial}{\partial x}(hu^2 + \frac{1}{2}gh^2) = -gh\frac{dB}{dx} \quad (23)$$

397 where $h(x, t)$ is the flow depth, $u(x, t)$ is the horizontal velocity, g is the
 398 gravitational constant, $B(x)$ is the bed topography function.

399 In this code, a conservative form of the nonlinear shallow water equations
 400 with source term is solved by using a high-resolution Godunov-type explicit
 401 scheme with Roe's approximate Riemann solver [Delis et al. 2008]. In order
 402 to get the mathematical relationship between the maximum water volume
 403 (V), slope (m) and initial wave amplitude (ξ), we design a parameter study
 404 by varying slope and wave amplitude to calculate the water volume. Finally,
 405 we use curve fitting methods to get the mathematical relationship based on
 406 parameter study data set.

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Figure 1: Conceptual model of TSUFLIND with definition of the terminology used later in the paper. For more symbols used in this paper see Appendix A.

Figure 2: Flowchart for TSUFLIND's iterative scheme to simulate tsunami deposit and estimate tsunami flow condition.

Figure 3: TSUFLIND simulation results and field measurement at Ranganathapuram, India. a: Vertical grading in grain size distribution (blue line) and mean grain size (red line) for four sampled locations (120m, 160m, 177m and 207m); b: the entire tsunami deposit grain-size distributions used as inputs to TSUFLIND (red points) and model result outputs from TSUFLIND (green line); c: tsunami deposit thickness field measurements (red points) and simulation results from TSUFLIND (green line); d: topography, wave run up and sample locations for test case (*I* : 120m; *II* : 160m; *III*: 177m; *IV* : 207m).

Figure 4: The estimated flow speeds and Froude numbers from TSUFLIND. a: Tsunami flow speed estimates are indicated by the gray area with the boundaries of maximum and minimum possible speeds. The dashed line is the average value of estimated flow speeds. b: Froude number estimates are indicated by the gray area in this figure with the maximum and minimum possible values. The dashed line is the average value of Froude number.