A sensorless method for measuring the point mobility of mechanical structures

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Abstract

This paper presents a convenient and cost-effective experimental tool for measuring the mobility characteristics of a mechanical structure. The objective is to demonstrate that the point mobility measurement can be performed using only an electrodynamic inertial exciter. Unlike previous work based on voice coil actuators, no load cell or accelerometer is needed. Instead, it is theoretically shown that the mobility characteristics of the structure can be estimated from variations in the electrical input impedance of the actuator fixed onto it, provided that the electromechanical parameters of the actuator are known. The proof of concept is made experimentally using a cheap commercially available actuator on a simply supported plate, leading to a good dynamic range from 100 Hz to 1 kHz. The methodology to assess the basic parameters of the actuator is also given. Measured data are compared to a standard shaker testing and the strengths and weaknesses of the sensorless mobility measuring device are discussed. It is believed that this sensorless

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mobility measuring device can be a convenient experimental tool to determine the dynamic characteristics of a wide range of mechanical structures.

1. Introduction

The concepts of mechanical impedance and mobility are part of the design process and analysis in structural dynamics. Developed from electromechanical and electroacoustic analogies in the 1920's [1, 2, 3, 4], they define the relationship between force and velocity at given locations of a structure. The point mobility characteristics of a mechanical structure, which is based on a collocated measurement of force and velocity, has broad and practical applications in: 1) modal testing to determine the natural frequencies, modal damping and mode shape of structures [5], 2) materials testing to determine certain specific properties of materials such as the loss factor and the modulus of elasticity [6], 3) structural monitoring in order to investigate non-uniformities, structural defects or tolerance deviation that can be detected by comparing the mobility function of a standard with a recently manufactured item [7]. It is also used in system modelling since mobility measurement performed at many locations on a structure can be used as verification of mathematical models, or the response of subsystems or cascaded substructures may be measured and the properties of the assembled system be predicted before final assembly [8], similarly to the study of complex electrical circuits. In smart structures and vibration isolation, furthermore, it allows the dynamic interaction between electromechanical transducers and structure to be involved in the development process [9]; vibration isolator, proof-mass actuator and machinery mounting can then be optimized to minimize structural vibration

1 or sound transmission [11, 12].

2 The measurement of mechanical impedance and mobility in general re-
3 quires three devices : an actuator, a force sensor and a vibration sensor
4 [6, 8, 13]. In impact testing, the structure under test is excited by a sharp and
5 clean force input of small time duration, generally by using an impact hammer
6 equipped with a load cell, and the vibration response is typically sensed by
7 using an accelerometer. In shaker testing, a vibrating-rod actuator is required
8 to mechanically excite the structure according to the amplified input signal.
9 In both cases mechanical energy is supplied to the structure with a known
10 frequency content, allowing the mobility function to be calculated from the
11 force applied and the vibration response. Achieving collocated sensing is a
12 challenging task and imperfect collocation may result in significant errors in
13 the measured mobility function [14]. An alternative approach for the direct
14 measurement of the driving point mobility of a structure has recently been
15 proposed by Sands *et al.* [15] (2009). In this method, an electrodynamic iner-
16 tial actuator combined with two accelerometers is attached to the structure
17 directly so that the assembly can be used as a mobility measuring device. The
18 two sensors are actually placed strategically on the actuator frame and proof
19 mass, allowing the mass-loading effect of the device on the structure to be
20 removed. The feasibility of the method was validated in [15] on a cantilever
21 beam. Although this approach overcomes some practical limitations encoun-
22 tered in impact testing and shaker testing, such as allowing better control
23 over the frequency, amplitude and duration of the force excitation applied
24 to the structure while compensating the mass-loading effect of the actuator
25 on the structure, it requires developing a specific device. On the other hand,

1 several authors have studied how to use electromechanical transducers as
2 both sensor and actuator simultaneously, a technique which is commonly
3 referred to as *self-sensing actuation* in the literature. The idea of self-sensing
4 actuation was developed in the early nineties by Hagood *et al.* [16] and Dosch
5 *et al.* [17] concurrently, where the piezoceramic elements were used as sensors
6 and actuators simultaneously to reduce implementation, cost and complexity,
7 by achieving truly collocated control. The principle was later implemented to
8 achieve active control of noise and vibration [18, 19, 20, 21], to implement
9 self-sensing vibration damping using electrodynamic shaker actuators [22],
10 for active acoustic attenuation [23] and for biomedical applications using an
11 electromagnetic actuator [24]. More recently, the principle has been applied
12 to an electrodynamic inertial exciter for active vibration control of a simply
13 supported plate [9] and for active structural acoustic control of an aircraft
14 panel [10]. In particular, it was shown in Ref. [10] that the vibration of the
15 structure at the actuator base can be estimated from the actuator impedances
16 and the current and voltage at the transducer's terminals, paving the way for
17 the development of a *sensorless* mobility measuring device. The approach of
18 estimating the mechanical impedance and mobility of a structure from the
19 electrical impedance of an electromechanical transducer attached onto it is
20 not new. In the mid 1990s, studies on the coupling of the electrical impedance
21 of piezoceramic materials and the mechanical impedance of structure at
22 the location of the piezo patch were reported by Liang *et al.* [25]. In the
23 early 2000s, Ling *et al.* developed a technique for measuring the mechanical
24 impedance of structures using the sensing capability of a piezoceramic inertial
25 actuator [26]. Experimental results for monitoring structural integrity of a

1 clamped-clamped beam can be found in [27], showing that some structural
2 defects can be detected by examining the changes of the electrical impedance
3 of the actuator. In the footsteps of Ref. [26], Doutres *et al.* (2010) developed
4 a methodology for determining the loss factor and the Young's modulus of
5 a porous sample from the measurement of the electrical impedance of an
6 electrodynamic loudspeaker used to apply static and dynamic stress on it
7 [28]. Bois *et al.* (2007) also proposed a method for detecting and charac-
8 terizing a delamination process in a composite structure using embedded
9 piezoelectric transducers, which allows to determine the modal parameters
10 from electromechanical impedance measurements [29].

11 This paper intends to simplify the mobility measuring device proposed by
12 Sands *et al.* [15] by jointly using the self-sensing capabilities of the inertial
13 exciter, as suggested in Ref. [10]. The proposed sensorless mobility measuring
14 device (SMMD) therefore enables the dynamic properties of mechanical
15 structures to be evaluated quickly and inexpensively. The remainder of the
16 paper is organized as follows. Section 2 provides a dynamic model of the
17 electrodynamic actuator as a two-port network, including its transduction
18 functions that are used in the analytical derivation of the mobility function of
19 the structure. In Section 3, we present the methodology used to experimentally
20 evaluate the electromechanical parameters of the actuator needed to determine
21 the transduction functions of the actuator. The proof of concept is provided
22 in Section 4 on the basis of a simply supported plate and experimental results
23 are compared to standard shaker testing. The strengths and weaknesses of
24 the SMMD are discussed in Section 5, including the possible reasons for the
25 observed limitations on the current prototype.

Figure 1: Schematic of the electrodynamic inertial exciter (a) and representation as a two-port network (b), where the current i is defined positive when flowing into the electrical port of the transducer and the two collinear velocities v_a and v_s are defined positive when oriented towards the structure; u is the voltage at the input terminals and f_s the reaction force exerted by the structure at the actuator base.

2. The electrodynamic sensoriauator

This section provides the theoretical basis about the operation and characteristics of the electrodynamic inertial exciter, and explains how it can be employed for self-sensing actuation. A lumped parameter model is used in the following assuming $\exp(j\omega t)$ harmonic dependence and linear behavior. Otherwise, some of the model parameters would need to be considered as time-varying nonlinear functions of the input variables [30].

2.1. Governing equations

Figure 1 gives a schematic representation of the actuator. The equations of motion of the proof mass M_a and frame mass M_f , and the governing equation

1 of the electrical circuit can be expressed as [10, 12, 32]

$$\begin{aligned}
j\omega M_a v_a &= Z_{ms}(\omega) (v_s - v_a) - Bl i \\
j\omega M_f v_s &= -Z_{ms}(\omega) (v_s - v_a) + Bl i - f_s \\
u &= Z_e(\omega) i + Bl (v_s - v_a)
\end{aligned} \tag{1}$$

2 where $Z_{ms}(\omega) = R_a + K_a/j\omega$ is the mechanical impedance of the spring-
3 dashpot mounting system and $Z_e(\omega) = R_e + j\omega L_e$ is the blocked electrical
4 impedance of the transducer.

5 2.2. Transduction matrix

6 The electrodynamic inertial exciter is a linear power device that can be
7 modelled as a two-port network. As illustrated in Fig. 1(b), the complex
8 magnitudes of effort (voltage) and flow (current) at the input (electrical) port
9 are related to the complex magnitudes of effort (force) f_s and flow (velocity)
10 v_s at the output (mechanical) port, and the governing equations can be
11 represented in transduction matrix by [26, 31]

$$\begin{bmatrix} u \\ i \end{bmatrix} = \begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} \begin{bmatrix} f_s \\ v_s \end{bmatrix} \tag{2}$$

12 where the frequency dependent parameters $T_{11}(\omega) = \frac{u}{f_s} |_{v_s=0}$ and $T_{21}(\omega) =$
13 $\frac{i}{f_s} |_{v_s=0}$ are the transduction function of voltage to force and the transduction
14 function of current to force, respectively, when the actuator base is fixed to a
15 rigid foundation; $T_{12}(\omega) = \frac{u}{v_s} |_{f_s=0}$ and $T_{22}(\omega) = \frac{i}{v_s} |_{f_s=0}$ are the transduction
16 function of voltage to velocity and the transduction function of current to
17 velocity, respectively, when the actuator base is free to vibrate.

The transduction functions $T_{11}(\omega), T_{21}(\omega), T_{12}(\omega)$ and $T_{22}(\omega)$ given in Eq. (2) can be derived as

$$T_{11}(\omega) = \frac{u}{f_s} \Big|_{v_s=0} = \frac{1}{j\omega M_a Bl} (Z_e(\omega) Z_{ma}(\omega) + (Bl)^2) \quad (3)$$

$$T_{21}(\omega) = \frac{i}{f_s} \Big|_{v_s=0} = \frac{Z_{ma}(\omega)}{j\omega M_a Bl} \quad (4)$$

$$T_{12}(\omega) = \frac{u}{v_s} \Big|_{f_s=0} = \frac{Z_e(\omega)}{Bl} \left(\frac{M_f}{M_a} Z_{ma}(\omega) + Z_{ms}(\omega) \right) + Bl \left(1 + \frac{M_f}{M_a} \right) \quad (5)$$

$$T_{22}(\omega) = \frac{i}{v_s} \Big|_{f_s=0} = \frac{1}{Bl} \left(\frac{M_f}{M_a} Z_{ma}(\omega) + Z_{ms}(\omega) \right) \quad (6)$$

1 where $Z_{ma}(\omega) = Z_{ms}(\omega) + j\omega M_a$, and the determinant of the transduction
2 matrix yields

$$T = T_{11}(\omega)T_{22}(\omega) - T_{12}(\omega)T_{21}(\omega) = -1 \quad (7)$$

3 showing that the electrodynamic inertial actuator is an anti-reciprocal elec-
4 tromechanical transducer. Fig. 2 shows the block diagram and the analogous
5 electrical circuit of the electrodynamic inertial actuator where the point mo-
6 bility function (in $\text{m s}^{-1} \text{N}^{-1}$) of the structure to which it is attached is defined
7 as

$$Y(\omega) = \frac{v_s}{f_s} \quad (8)$$

8 2.3. The sensoriactuator transfer functions

9 When the actuator base is prevented from vibrating, i.e. attached to a
10 rigid foundation such that $v_s = 0$, the electrical motional impedance (in Ω)
11 of the actuator can be derived from Eqs. (3-4) as

$$Z_{in0}(\omega) = \frac{u}{i} \Big|_{v_s=0} = \frac{T_{11}(\omega)}{T_{21}(\omega)} \quad (9)$$

Substituting Eq. (8) into Eq. (2), the transfer function of the reaction force to the input current (in N A^{-1}) can be written as

and the electrical input impedance of the actuator when attached to the structure can be derived as

9

1 showing that the point mobility function of the structure can affect the
 2 electrical input impedance of the actuator. Similarly, the output velocity
 3 generated at the actuator base can be expressed in terms of the input
 4 current as

$$G_i(\omega) = \frac{v_s}{i} = T_{21}(\omega) \left(\frac{T_{11}(\omega) + T_{12}(\omega)Y(\omega)}{T_{21}(\omega) + T_{22}(\omega)Y(\omega)} - \frac{T_{11}(\omega)}{T_{21}(\omega)} \right) = T_{21}(\omega) (Z_{in}(\omega) - Z_{in0}(\omega)) \quad (12)$$

5 As can be seen in Eq. (12), the open loop response function (in $\text{m s}^{-1} \text{A}^{-1}$) of
 6 the actuator at the attachment point can be readily obtained from variations in
 7 the electrical input impedance of the actuator. More details on the analytical
 8 derivation of Eqs. (10-12) are given in Appendix A.

9 2.4. The sensorless mobility measuring device

10 From Eqs. (8-12), and after some further manipulations, the mobility
 11 function of the structure at the attachment point can be expressed as

$$Y(\omega) = \frac{T_{21}(\omega)G_i(\omega)}{1 - T_{22}(\omega)G_i(\omega)} \quad (13)$$

12 Therefore, the proposed method to use the electrodynamic inertial exciter
 13 as a sensorless mobility measuring device requires 1) deriving the transduction
 14 functions T_{21} and T_{22} given in Eqs. (4) and (6), respectively, by evaluating the
 15 electromechanical parameters M_a, K_a, R_a, Bl, M_f , and 2) obtaining the open
 16 loop response function of the actuator as given in Eq. (12) by measuring the
 17 electrical input impedance of the actuator when attached to a rigid foundation
 18 and to the structure under test, as indicated in Eqs. (9) and (11), respectively.

3. Experimental assessment of electromechanical parameters

This section presents the method employed to evaluate of the basic parameters of the electrodynamic actuator described in Section 2 based on a transmissibility test and electrical impedance measurement.

Figure 3: Picture (a) and schematic representation (b) of the experimental setup used to assess the velocity transmissibility of the actuator when attached to a shaker table.

3.1. Experimental setup

Figure 3 shows the experimental setup used to assess the velocity transmissibility of the actuator. As shown in Fig. 3, a K2007E01 Mini SmartShaker (5 lbf rms) with integrated power amplifier is used to mechanically drive the inertial exciter and a VibroMet 500V laser Doppler velocimeter (sensitivity 5 mV/(ms⁻¹)) is used to sense the axial velocity v_a of the proof mass M_a . A PCB model 288D01 impedance head (sensitivity 22.4 mV/N for the force

Figure 4: Picture (a) and schematic representation (b) of the experimental setup used to measure the electrical motional impedance of the actuator when attached to a rigid foundation.

1 sensor and $10.3 \text{ mV}/(\text{m s}^{-2})$ for the accelerometer) is connected between the
 2 shaker table and the mounting system to which the actuator is attached,
 3 allowing the force and acceleration transmitted by the shaker table to be
 4 monitored. The accelerometer signal is processed by a time integrator, thus
 5 determining the axial velocity v_s at the base of the inertial exciter.

6 The electrical input impedance is determined by applying a sweep signal
 7 to the current driven actuator while monitoring the voltage induced at its
 8 terminals. As shown in Fig. 4, measurement is performed when the actua-
 9 tor base is attached to a rigid foundation, allowing the electrical motional
 10 impedance to be evaluated as given in Eq. (9). Similar to the procedure for the
 11 determination of electromechanical parameters of a loudspeaker driver [35],
 12 measurement is performed at small input level around the resonance frequency
 13 of the proof mass on its suspension so that the mechanical behavior of the
 14 drive unit is effectively linear.

3.2. Measured data

Figure 5: Frequency response function of the velocity transmissibility (a) measured when the actuator is attached to the shaker table and electrical motional impedance (b) measured when the actuator is attached to a rigid foundation (A: actuator attached at its base, B: attached at its base with the added mass, C: mounted upside down on the shaker).

Figure 5 shows the measured frequency response functions (FRFs) of the velocity transmissibility and electrical motional impedance of the actuator. As can be seen in Fig. 5, the actuator resonance frequency measured at $f_n = 75$ Hz (case A) is found to be shifted at $f_m = 62$ Hz with an added mass $m = 0.0305$ kg (case B). As shown in Appendix B, this measurement is used to determine the proof mass and effective stiffness of the actuator $M_a = 0.108$ kg and $K_a \simeq 24000$ N m⁻¹, respectively. When mounted upside down on the shaker table as shown in Fig. 6(c), the actuator resonance frequency is found to be shifted at $f_f = 222$ Hz, thus giving the mass of the actuator base $M_f = 0.0134$ kg. As shown in Fig. 5(a), the peak-transmissibility

1 equals 9.56 for all configurations tested, thus giving the effective mechanical
 2 resistance of the actuator $R_a = 5.32 \text{ N s m}^{-1}$. As can be seen in Fig. 5(b),
 3 the dc resistance of the coil is $R_e = 6.65 \Omega$ and the maximum electrical
 4 resistance at the actuator resonance frequency is $R_{em} = 10.63 \Omega$, thus giving
 5 the electrodynamic transduction coefficient $Bl = 4.6 \text{ N A}^{-1}$. The measured
 6 parameters of the actuator used in this study are summarized in Tab.1. More
 7 details about the derivation of the basic parameters of the actuator can be
 8 found in Appendix B.

Figure 6: Pictures of the actuator when attached to the shaker table at its
 base without (a) and with the added mass (b), and mounted upside down on
 the shaker table (c).

9 **4. Experimental proof of concept**

10 This section aims to experimentally confirm the use of an electrodynamic
 11 inertial actuator for self-sensing actuation, allowing the determination of the

1 mobility characteristics of a structure without the need for an external sensor.
2 For the purpose of illustration, the results presented below relate specifically
3 to a simply supported plate.

4 *4.1. Experimental setup*

5 The plate used for the tests is shown in Fig. 7, and its physical parameters
6 are summarized in Table 2. More details on the dynamic model of the simply
7 supported plate can be found in [36]. First, a Brüel and Kjaer Type 4810
8 electrodynamic mini-shaker (2.25 lbf pk) is employed to produce the input
9 force, as can be seen in Fig. 7(a). A PCB model 208B04 force sensor, the
10 sensitivity of which is 1.174 mV/N, is connected between the plate and the

Table 1: Physical parameters of the VISATON EX 60 S electrodynamic exciter measured in small-signal range.

Parameter	Notation	Value	Unit
Coil resistance	R_e	6.65	Ω
Coil inductance	L_e	0.05	mH
Transduction coefficient	Bl	4.6	N A ⁻¹
Proof mass	M_a	0.108	kg
Frame mass	M_f	0.0134	kg
Mechanical resistance	R_a	5.32	N s m ⁻¹
Suspension stiffness	K_a	24×10^3	N m ⁻¹
Resonance frequency	f_n	75	Hz

Figure 7: Pictures of the simply supported plate equipped for shaker testing (a) and with the sensorless mobility measuring device (b).

1 shaker using a stinger. As illustrated in Fig. 7(b), the SMMD is then fixed
 2 near the same location in order to be compared to the measurement obtained
 3 with the shaker. Unlike the shaker, the inertial actuator will exert a reaction
 4 force on the structure which is due to acceleration of the proof mass, without
 5 the need of a grounded support. Not shown in Fig. 7, a VibroMet 500V
 6 laser Doppler velocimeter (sensitivity $5 \text{ mV}/(\text{m s}^{-1})$) is used here to sense
 7 the vibration response on the back side of the plate. Signal acquisition is
 8 done with 18 bit precision analog inputs at a sampling frequency of 20 kHz.
 9 The digital signal processing is performed using a Speedgoat® Performance
 10 Real-Time Target Machine running on Simulink®. The physical parameters
 11 of the VISATON EX 60 S electrodynamic exciter used in this study can
 12 be found in Tab. 1. A picture of the inertial actuator used to perform the
 13 experiments is given in Fig. 8(a).

Table 2: Physical parameters of the rectangular aluminum plate.

Parameter	Notation	Value	Unit
Width	L_x	0.481	m
Length	L_y	0.421	m
Thickness	h	3.19	mm
Young's modulus	E	68.5×10^9	Pa
Mass density	ρ	2740	kg m^{-3}
Poisson ratio	ν	0.33	
Loss factor	η	0.004	

1 4.2. System calibration

2 Before making a valid measurement, the electronic circuit used for both
3 controlling the actuator and detect the voltage and current at its input
4 terminals is subject to calibration. A known precision resistor is substituted for
5 the electrical load impedance to be measured and a scaling factor is calculated
6 for subsequent measurements. A picture of the homemade electronic circuit
7 used to perform the experiments is given in Fig. 8(b).

8 4.3. Electrical input impedance

9 Figure 9(a) illustrates the frequency response function of the actuator
10 electrical motional impedance when the base is prevented from vibrating, i.e.
11 for $v_s = 0$ as given in Eq. (9). As can be seen in Fig. 9(a), there is a good
12 agreement between the measured and computed data. The actuator electrical

(a)

(b)

Figure 8: Pictures of the electrodynamic inertial actuator (a) and electronic circuit used for self-sensing actuation (b).

1 impedance is mainly resistive at low and middle frequencies, except around
 2 the mechanical resonance at 75 Hz where there is also a large increase in the
 3 actuator impedance, and becomes inductive as the frequency increases due to
 4 the coil inductance.

5 When attached to the plate, the vibration of the mass M_f of the actuator
 6 base depends on the plate mobility which in turns, will affect the actuator
 7 electrical input impedance [37], as shown in Eq. (11). Figure 9(b) illustrates
 8 the frequency response function of the input impedance of the actuator
 9 when fixed near a corner of the plate (see Fig. 7(b)). As can be seen in
 10 Fig. 9(b), there is a good agreement between the measured and computed
 11 data. Compared to Fig. 9(a), it can be noted that the actuator resonance
 12 frequency, which can be seen at 75 Hz in Fig. 9(a), interferes with the first
 13 structural resonance at 77 Hz for the plate used here. The following peaks in
 14 Fig. 9(b) are the natural frequencies of the plate which affect the actuator
 15 electrical input impedance.

Figure 9: Computed and measured frequency response function of the actuator electrical input impedance when attached to a rigid foundation (a) and when mounted on the simply-supported plate (b).

4.4. Structural response from variations in the actuator input impedance

As given in Eqs. (12) and (13), the mobility function of the structure under test can be derived from the actuator electromechanical impedances. In order to investigate the validity and accuracy of the method and to define the working range of the SMMD, we will first examine the measured FRF $Z_{in}(\omega) - Z_{in0}(\omega)$ shown in Fig. 10. As can be seen in Fig. 10 in the case of the simply supported plate, there is a good agreement between simulation and measurement in the frequency range from 60 Hz to 1000 Hz. For higher frequencies, the measured phase begins to deviate and crosses -90 degrees, in contradiction with the theory. This unwanted phase shift is presumably due to the non-ideal behavior of the voice coil transducer [39, 40, 41] from which the SMMD is developed. Therefore, the frequency range of the measured data presented below will be limited to 1 kHz.

Figure 10: Computed and measured frequency response function $Z_{in}(\omega) - Z_{in0}(\omega)$ showing the dynamic response of the structure from variations in the actuator electrical input impedance.

Figure 11: Velocity-current open loop frequency response function measured using a laser Doppler velocimeter (true) and derived from Eq. (12) (estimate).

1 4.5. Velocity-current open loop response

2 Figure 11 shows the comparison between the velocity-current open loop
3 frequency response function obtained using a laser Doppler velocimeter (LDV),
4 the beam of which is focused on the back side of the plate, and that derived
5 from Eq. (12). As can be seen in Fig. 11, there is a very good agreement
6 between measurement and estimate, showing the self-sensing capability of
7 the electrodynamic actuator. The deviation that can be observed around 75
8 Hz is due to the resonance of the actuator interfering with the first natural
9 resonance of the plate and thus influences the mobility measured using the
10 SMMD.

1 4.6. *Mobility function of the plate*

2 Figure 12 illustrates the comparison between the mobility function of the
3 plate measured by shaker testing and that obtained from Eq. (13) using the
4 SMMD. As can be seen in Fig. 12, the two FRFs show a relatively good
5 agreement in the frequency range from 60 Hz to 1 kHz. The deviation that
6 can be seen above 700 Hz in Fig. 12 is most likely due to the SMMD location
7 on the plate which is not strictly the same as for shaker testing ; this effect is
8 moreover compounded by the fact that the SMMD is coupled to the structure
9 through a larger mounting base. As mentioned above, a deviation can also be
10 seen around the actuator resonance frequency which is very close to the first
11 natural resonance of the plate used here. Notwithstanding the foregoing, Fig.
12 12 provides experimental evidence that the mobility function of a mechanical
13 structure can be obtained using only an electrodynamic inertial exciter and
14 without requiring external sensor.

15 5. Discussion

16 As confirmed experimentally in Section 4, the point mobility characteristics
17 of a mechanical structure can be estimated with enough accuracy using
18 the SMMD. With our mock-up SMMD prototype based on a commercially
19 available electrodynamic inertial exciter, the measurement range already spans
20 from 100 Hz to 1kHz. Its working range is limited to lower frequencies due to
21 the resonance of the actuator which interferes with the first natural resonance
22 of the plate used here. An actuator with a lower resonance frequency should
23 therefore improve the usable range of the SMMD. The limiting factors at higher
24 frequencies are less evident, and may be related to the basic lumped-element

Figure 12: Comparison on the mobility function of the plate at the attachment point measured by shaker testing and derived from Eq. (13) (SMMD).

1 model used here. As can be seen in Eqs. (12-13), the derivation of the mobility
 2 function involves both the measured electrical impedances of the actuator and
 3 a few parameters identified experimentally for the model presented in Fig. 2.
 4 Discrepancies between these lumped models and the actual electrodynamic
 5 motors are described in the literature about loudspeakers and are likely to
 6 impact the SMMD as well. In particular, the non-ideal behavior of the voice
 7 coil transducer from which the SMMD is developed might be significant : it is
 8 recognized that eddy currents flowing in the iron pole structure may modify
 9 its electrical impedance [39, 40]. This is usually dealt with by employing lossy
 10 inductances in the model. A second possible cause is related to the suspension
 11 creep which is not covered by our simplified model [41]. An expanded model
 12 to incorporate eddy currents and suspension creep could then be considered in
 13 future work with a view to further extending the working range of a low-cost
 14 version the SMMD, or a dedicated (more expensive) device could be designed
 15 in order to reduce the above-mentioned effects.

16 The SMMD can also overcome some common issues encountered in practice
 17 in impact testing and shaker testing. Although impact testing is non-intrusive
 18 with a low mass-loading effect exerted upon the structure and therefore
 19 convenient for small lightweight structures, some practical problems related to
 20 the input force applied to the structure or the recording of the measurement
 21 signals may limit its use in some cases [37]. For example, multiple impacts,
 22 misalignment of impacts in both position and direction over the averaging
 23 period will affect the measured mobility characteristics of the structure under
 24 test. The short duration of the impact force may lead to overload the input
 25 channel of the multichannel analyzer, while supplying little energy into the

1 structure. The duration of the contact time has also an effect on the frequency
2 content of the force, a longer contact time leading to a reduced bandwidth.
3 As a result, different force transducers have to be used according to the
4 force needed [8] and impact hammers equipped with different masses may be
5 required to cover the frequency range of interest. Note that impedance heads
6 can also be used but these devices are not free of measurement error [5, 14].
7 On the other hand, shaker testing can supply more energy to the structure
8 over a longer period of time and hence provide better control of the frequency
9 content, amplitude and duration of the mechanical excitation. A grounded
10 mounting support is necessary, however, which can make it difficult to fit
11 into small spaces. In addition, shaker testing can significantly influence
12 the dynamic characteristics of the structure under test. The stinger has
13 to be axially stiff but soft in bending in order to reduce the transmission
14 of bending moments to the armature of the shaker on one hand and the
15 force transducer and structure under test on the other hand [37]. With the
16 SMMD, no direct measurement of the applied force with a force transducer is
17 necessary. As the force excitation and velocity response occur through the
18 electrodynamic transducer, there is no misalignment of sensors. The applied
19 force and resulting velocity are therefore colocated and constrained to a single
20 direction of action by the voice coil transducer, allowing highly repeatable
21 measurement. Various electrodynamic inertial exciters are commercially
22 available that can suit a wide range of dynamic structures. In addition, the
23 mass-loading effect exerted upon the structure by the actuator can also be
24 removed mathematically, allowing measurement of the unloaded mobility
25 characteristics as described in [15]. However, one drawback of such a mobility

1 measuring device based on an electrodynamic inertial exciter is that it is
2 coupled to the structure through a relatively large mounting base, which can
3 hardly be considered as a point.

4 Compared to the concept proposed by Sands *et al.* [15] which is also
5 based on a voice coil actuator, the SMMD is developed from an "off-the-shelf"
6 inertial exciter and does not require extra sensor to determine the mobility
7 function of the structure under test. Using a SMMD, strong electric signals
8 lead to a very good Signal-to-Noise Ratio and the mobility function of the
9 structure can be evaluated by a single measurement for a broad frequency
10 range, typically from 100 Hz to 1 kHz with the current SMMD prototype,
11 using a sweep or random test signal over a duration of about one minute. In
12 comparison to the experimental technique proposed by Ling *et al.* (2001) [26]
13 for detecting mechanical impedance of structures using the sensing capabilities
14 of a piezoceramic inertial actuator, no force sensor is required to measure
15 the parameters of the transduction matrix. Significant discrepancies are
16 reported in [26] at the anti-nodes of the beam structure in their frequency
17 range of interest. According to the authors, it arises from the non-ideal
18 linearity between the electrical variables and mechanical variables due to
19 the construction of the piezoelectric inertial actuator. Above the actuator
20 resonance frequency, the dynamic range of the SMMD allows the anti-nodes
21 and nodes to be well identified from the derived point mobility function.

22 The mobility characteristics of a structure obtained using a SMMD can
23 also be informative in other contexts. Since the actuator electrical input
24 impedance is a sensitive function of the mechanical impedance of the structure
25 at the attachment point, variations thereof may be used to detect changes

1 of structural properties such as stiffness changes due to mechanical failure,
2 mass changes due to wearing, or changes of supporting conditions due to
3 malfunction of a coupled mechanical element [7, 27, 28, 29]. In addition to
4 determining the point mobility of a structure, the SMMD can also be used as
5 a self-sensing actuator [10] to achieve active vibration control of the structure
6 without having to remove it. This may help predicting the maximum feedback
7 gain that can be implemented for a specific application, and hence the stability
8 of the feedback loop where the control unit is mounted [12].

9 **Conclusion**

10 This paper presents a stand-alone experimental tool for measuring the
11 mobility characteristics of a mechanical structure using only an electrodynamic
12 inertial exciter. Unlike the state of the art techniques such as impact testing
13 and shaker testing, it is shown that mobility measurement can be performed
14 with good accuracy without involving external sensors such as accelerometers,
15 load cells or impedance heads, but directly from variations in the electrical
16 input impedance of the actuator fixed onto it. The electrical input impedance
17 is available when switching on the device or can be assessed in operation,
18 allowing early detection of structural failure, defects or other changes. The
19 proof of concept is demonstrated using a commercially available actuator
20 relatively inexpensive, leading to a good dynamic range from 100 Hz to 1kHz.
21 It is believed that the formulations presented in this paper can be used to
22 investigate new designs of electrodynamic actuator in order to optimize the
23 performance properties of this type of self-sensing actuator depending on the
24 structure to be characterized.

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5 Appendix A. Derivation of the sensor/actuator transfer functions

6 From Eqs. (1), the reaction force at the base of the actuator can be
 7 written as

$$f_s = \frac{j\omega M_a}{Z_{ma}(\omega)} Bl i - \left(j\omega M_a \frac{Z_{ms}(\omega)}{Z_{ma}(\omega)} + j\omega M_f \right) v_s \quad (\text{A.1})$$

8 where $Z_{ma}(\omega) = Z_{ms}(\omega) + j\omega M_a$, and the velocity of the proof mass M_a may
 9 be expressed as

$$v_a = -\frac{Bl}{Z_{ma}(\omega)} i + \frac{Z_{ms}(\omega)}{Z_{ma}(\omega)} v_s \quad (\text{A.2})$$

10 Substituting the mobility function given by Eq. (8) into Eq. (A.1), the
 11 transfer function between the input current and the reaction force at the
 12 attachment point can therefore be written as

$$H_i(\omega) = \frac{f_s}{i} = \frac{j\omega M_a Bl}{Z_{ma}(\omega) + j\omega M_a \left(Z_{ms}(\omega) + \frac{M_f}{M_a} Z_{ma}(\omega) \right) Y(\omega)} = [T_{21}(\omega) + T_{22}(\omega) Y(\omega)]^{-1} \quad (\text{A.3})$$

13 Note that Eq. (A.3) is equivalent to Eq. (10).

14

15 Considering now that the proof mass M_a is moving as a result of an
 16 external force acting on the actuator base, i.e. for $i = 0$, the complex ratio
 17 between the output and the input velocities can be written as [33, 34]

$$T_a(\omega) = \frac{v_a}{v_s} \Big|_{i=0} = \frac{Z_{ms}(\omega)}{Z_{ms}(\omega) + j\omega M_a} \quad (\text{A.4})$$

1 which denotes the velocity transmissibility between the proof mass and the
2 actuator base.

3

4 Substituting Eq. (A.2) into Eq. (1), the output voltage when the actuator
5 is driven by a current source is

$$u = \left(Z_e(\omega) + \frac{(Bl)^2}{Z_{ma}(\omega)} \right) i + Bl \left(1 - \frac{Z_{ms}(\omega)}{Z_{ma}(\omega)} \right) v_s \quad (\text{A.5})$$

6 and after some further manipulation, the output velocity at the actuator base
7 can be rewritten in terms of the input current and the induced voltage at the
8 transducer terminals, as

$$v_s = \frac{Z_{ma}(\omega)}{j\omega M_a Bl} \left(u - \left(Z_e(\omega) + \frac{(Bl)^2}{Z_{ma}(\omega)} \right) i \right) \quad (\text{A.6})$$

9 When attached to a rigid foundation, i.e. for $v_s = 0$, the actuator electrical
10 input impedance (in Ω) can be derived from Eq. (A.5) as

$$Z_{in0}(\omega) = \frac{u}{i} \Big|_{v_s=0} = Z_e(\omega) + \frac{(Bl)^2}{Z_{ma}(\omega)} = \frac{T_{11}(\omega)}{T_{21}(\omega)} \quad (\text{A.7})$$

11 and substituting now Eq. (8) into Eqs. (1), the electrical input impedance of
12 the actuator when attached to the structure can therefore be expressed as

$$Z_{in}(\omega) = \frac{u}{i} = Z_e(\omega) + \frac{(Bl)^2}{Z_{ms}(\omega) + j\omega M_a \frac{1 + j\omega M_f Y(\omega)}{1 + j\omega(M_a + M_f)Y(\omega)}} = \frac{T_{11}(\omega) + T_{12}(\omega)Y(\omega)}{T_{21}(\omega) + T_{22}(\omega)Y(\omega)} \quad (\text{A.8})$$

13 Note that Eqs. (A.7) and (A.8) are equivalent to Eq. (9) and (11), respectively.

14

15 The transfer function between the input current and the output velocity
16 at the attachment point can be derived from Eqs. (A.6-A.8) as

$$G_i(\omega) = \frac{v_s}{i} = \frac{Z_{ma}(\omega)}{j\omega M_a Bl} (Z_{in}(\omega) - Z_{in0}(\omega)) = T_{21}(\omega) (Z_{in}(\omega) - Z_{in0}(\omega)) \quad (\text{A.9})$$

1 Note that Eq. (A.9) is equivalent to Eq. (12). From Eqs. (A.3) and (A.9),
 2 the mobility function of the structure at the attachment point can also be
 3 expressed as

$$Y(\omega) = \frac{v_s}{f_s} = \frac{G_i(\omega)}{H_i(\omega)} \quad (\text{A.10})$$

4 Substituting Eqs. (A.3) and (A.9) into Eq. (A.10) yields

$$Y(\omega) = G_i(\omega) \frac{Z_{ma}(\omega)}{j\omega M_a Bl} + \frac{G_i(\omega)}{Bl} \left(Z_{ms}(\omega) + \frac{M_f}{M_a} Z_{ma}(\omega) \right) Y(\omega) \quad (\text{A.11})$$

5 and after some further manipulations, Eq. (A.11) can be expressed as a
 6 function of Eq. (A.9) as

$$Y(\omega) = \frac{Z_{ma}(\omega)}{j\omega M_a Bl} \frac{G_i(\omega)}{1 - \frac{1}{Bl} \left(Z_{ms}(\omega) + \frac{M_f}{M_a} Z_{ma}(\omega) \right) G_i(\omega)} = \frac{T_{21}(\omega) G_i(\omega)}{1 - T_{22}(\omega) G_i(\omega)} \quad (\text{A.12})$$

7 Note that Eq. (A.12) is equivalent to Eq. (13).

8 **Appendix B. Derivation of the electromechanical parameters**

9 The proof mass M_a is determined in two steps by measuring the velocity
 10 transmissibility of the actuator when attached to the shaker table at its base
 11 (see Fig. 6(a)), and then combined with an added mass m (see Fig. 6(b)).
 12 Denoting f_m and f_n the resonance frequencies (in Hz) of the actuator with
 13 and without added mass, respectively, the proof mass M_a can be derived after

$$M_a = \frac{f_m^2}{f_n^2 - f_m^2} m \quad (\text{B.1})$$

15 and the effective stiffness K_a of the suspension can therefore be derived as

$$K_a = (2\pi f_n)^2 M_a \quad (\text{B.2})$$

1 Similarly, the mass M_f of the actuator base can be determined from the
 2 transmissibility response as

$$M_f = \frac{K_a}{(2\pi f_f)^2} \quad (\text{B.3})$$

3 where f_f is the resonance frequency when the actuator is mounted upside
 4 down on the shaker (Fig. 6(c)).

5 The effective mechanical resistance R_a can be derived from the peak-
 6 transmissibility which is the maximum amplitude of the transmitted velocity
 7 for a given amplitude of the input velocity. For a linear system the peak-
 8 transmissibility is $|T_a(\omega)|_{max} \simeq 1/(2\zeta)$, where $\zeta = R_a/(2\sqrt{K_a M_a})$ is the
 9 damping ratio (see Eq. (A.4)), and the effective mechanical resistance is
 10 therefore given by

$$R_a = 2\zeta\sqrt{M_a K_a} \quad (\text{B.4})$$

11 Once R_a is known, the electrodynamic transduction coefficient Bl can be
 12 derived from the actuator electrical motional impedance as

$$Bl = \sqrt{R_a (R_{em} - R_e)} \quad (\text{B.5})$$

13 where R_e is the dc resistance of the coil and R_{em} is the electrical motional
 14 resistance measured at the actuator resonance frequency f_n .

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