Support vector machine-based Grassmann manifold distance for health monitoring of viscoelastic sandwich structure with material ageing

Chuang Sun, Zhousuo Zhang*, Xue Luo, Ting Guo, Jinxiu Qu, Bing Li

School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an, P.R. China.

Abstract

Subspace analysis is an effective way for Structural Health Monitoring (SHM). In current research, linear algorithms for single-subspace analysis are commonly utilized. Nonlinearity of the structure and characteristics of subspace distribution are ignored. To overcome these shortcomings, characteristics of subspace set are analyzed and a nonlinear subspace distance is defined for SHM in this paper. To calculate this distance index, vibration response signals are firstly monitored and system subspaces are extracted by subspace analysis method. Then, subspace set is viewed as a Grassmann manifold, and the manifold is modeled by Grassmann kernel-based SVM classifier to describe its nonlinear characteristics. Finally, margin in SVM classifier modeled from Grassmann manifolds corresponding to structural normal state and abnormal state, respectively, is defined as a SHM index. This index indicates the degree of the abnormal state deviating from the normal state, and it is an effective index to reflect structural states. Effectiveness of the SHM index is validated by testing data of a Viscoelastic Sandwich Structure (VSS) with viscoelastic sandwich subjected to accelerated ageing in a thermal-oxygen ambient. Analysis result shows that the designed index is very effective to indicate health state in the VSS.

Keywords: Signal subspace, Grassmann manifold distance, material ageing, SHM

1. Introduction

Viscoelastic Sandwich Structure (VSS) is a composite structure consisting of elastic-solid layers and viscoelastic layers [1]. The viscoelastic material, such as rubber and silicon, in this kind of structure is with the effect of vibration absorption and noise reduction. Hence, the VSS is always used in the place with requirement of low level of vibration and noise, such as the aerospace equipment [2, 3]. In service phase of the VSS, viscoelastic material will be ageing gradually due to the influence factors from external environment, including temperature, humidity, etc. Material ageing will degrade performance of the VSS in

* Corresponding author. E-mail address: zzs@mail.xjtu.edu.cn.

vibration and noise reduction. Ageing state assessment is meaningful to identify health state of the VSS and to keep the VSS work in the status with high performance.

Ageing state assessment falls in the domain of Structural Health Monitoring (SHM), and the method used for SHM is effective to assess ageing degree in the VSS. To perform the SHM, vibration analysis is a commonly used way [4]. The fundamental assumption in vibration-based SHM is that state change caused by structural fault, such as crack, bolt looseness and material ageing, can be represented in structural vibration signals. The vibration-based SHM has been attracting more and more attentions, and merits and shortcomings of these methods are reviewed in [5, 6]. According to difference in modeling approaches, these methods are classified into two types, that is, model-based methods and data-driven methods. In model-based method, the structure is abstracted as a number model and dynamic characteristics of the structure can then be investigated by this model. In recent research, Lin developed a numerical model to study the vibration characteristic of a symmetric elastic-viscoelastic sandwich beam [7]. Bilasse designed a number method to study the linear and nonlinear vibration of viscoelastic sandwich beams and plates [8]. Gillespie proposed an analytical model to study the fatigue failure mechanism of a sandwich structure with discontinuous ceramic tile core [9]. Lima presented a numerical fatigue damage analyses method to deal with fatigue reliability analysis of a three-layer sandwich plate [10]. The results in this literatures show the effectiveness of the model-based method to characterize structural dynamic characteristics. While, analytical number model with high precision is always necessary in real word to obtain accurate analysis result. The pursuit of accuracy for specific object limits robustness and generality of the number model.

In the data-driven method, the SHM is carried out by direct modeling and analysis on structural response signals. Structural states are assessed at the data level and analytical model is not required. The data-driven SHM has been researched, for example, Liu introduced a novel damage detection method to monitor and quantify impact damage propagation in sandwich composite wing [11]. In this method, vibration response of the wing is monitored and Kernel Principal Component Analysis is utilized to analyze vibration pattern and identify damage degree. García developed a multivariate vibration data-based approach to detect delamination fault in a composite beam [12], where the Singular Spectrum Analysis is used to analyze free decay response signals of the beam. Tibaduiza designed a data-based structural damage detection method, where damage index for damage assessment in aircraft turbine blade and skin panel is constructed by performing Principal Component Analysis on vibration response signals [13]. In these literatures, structural vibration responses are monitored and used as data source for SHM methods. Pattern recognition methods

are utilized to analyze essential patterns hidden in the response signals, and damage-related patterns are extracted and defined as index for SHM. The data-driven method only using response signals provides a convenient way for SHM, and it is helpful to realize on-line structural health monitoring.

The above mentioned methods to carry out the data-driven SHM can be viewed as subspace-based process, and similar process for SHM can also be found in [14-16]. In the subspace-based SHM method, structural state matrix, that is feature matrix, Hankel matrix or covariance matrix, is firstly constructed by structural response data. The row or column subspace of the state matrix is then obtained by subspace analysis methods, i.e., singular value decomposition or principal component analysis, to describe pattern change in the response data caused by structural damage. In subspace analysis, eigenvalues of the state matrix is utilized to distinguish structural response from noise. The noise component corresponds to the eigenvalues with smaller value is discarded [17], which is beneficial to improve stability of subspace analysis for SHM. Moreover, since only linear algebra operations, i.e., the singular value decomposition or QR decomposition is needed in subspace analysis [18], computation complexity and time consumed by performing subspace analysis for SHM is reduced [19, 20]. Complexity reduction which can improve real-time performance of the SHM algorithm is an important and meaningful requirement for structural health monitoring [21]. Subspace analysis with less calculation cost holds advantage over the analytical model-based SHM in real-time performance. Considering advantage of subspace analysis in complexity reduction and noise elimination, it is utilized for health monitoring of the VSS in this paper.

Though the results in these literatures show effectiveness of subspace-based method, shortcoming in current research on subspace analysis still exists. Only linear algorithm for single-subspace analysis is considered in current research. A set of subspaces from specific structural state holds global nonlinear characteristic in subspace distribution and cluster. Considering randomness in response signals, ignoring the global characteristics in subspace-based SHM methods will reduce effectiveness of the methods. A subspace set forms a Grassmann manifold (GM), and analysis on the manifold provides an effective way for subspace modeling. In this paper, an enhanced SHM method will be researched by performing global characteristic analysis on signal manifold. To achieve this goal, Support Vector Machine (SVM) is extended into the domain of manifold analysis to learn global characteristics of signal manifold. A global distance between two manifolds is defined by the margin in SVM classification algorithm. This distance is determined by a few critical subspaces, e.g., support vectors in SVM, which will reduce effect of signal randomness to the SHM index. To perform the above process for ageing assessment in VSS, response signals of the VSS under

stochastic excitation are firstly collected, and the Grassmann manifold is constructed from the response signals. Grassmann kernel-based SVM is then utilized to model the manifold, and the ageing state index is finally defined as the distance between the GMs corresponding to structural normal state and ageing state.

This paper is organized as following. Grassmann manifold is firstly introduced in section 2. Then, Grassmann manifold distance is presented in section 3. After that, a structural ageing index is defined in section 4. Case study is shown in section 5 and the conclusion is finally summarized in section 6.

2. Grassmann manifold constructed from response signals

2.1. System subspace analysis

System subspace analysis is foundation of the proposed ageing assessment method. In order to extract subspace from structural response signals, covariance-driven subspace extraction method [22-24] is used here. Given structural response signals $\mathbf{z}$, the Hankel matrix constituted of covariance matrix of the response signals is represented as

$$\mathbf{H}_{p,q} = \begin{bmatrix} \Lambda_0 & \Lambda_1 & \cdots & \Lambda_{q-1} \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_q \\ \vdots & \vdots & & \vdots \\ \Lambda_{p-1} & \Lambda_p & \cdots & \Lambda_{p+q-2} \end{bmatrix}, \quad q \geq p \quad (1)$$

The Hanker matrix is a block matrix, and parameter p and q denote number of block in the column and row in the matrix respectively. Λ_i is the covariance matrix of the response signals, and it can be calculated by response signal as

$$\Lambda_i \approx \frac{1}{a-i} \sum_{k=1}^{a-i} \mathbf{z}_{k+i} \mathbf{z}_k^T \quad (2)$$

where a denotes the number of the response signals. $\mathbf{z}$ is a vector and its elements are signals monitored from multiply sensors.

To analysis intrinsic pattern of the Hankel matrix, Principal Component Analysis (PCA) [25] is performed to decompose the Hankel matrix. The decomposition process is described as

$$\lambda \mathbf{s} = \mathbf{C} \mathbf{s} \quad (3)$$

with $\mathbf{C}$ denoting covariance matrix of the Hankel matrix, λ and $\mathbf{s}$ denoting eigenvalue and eigenvector of the covariance matrix respectively. The eigenvector obtained by PCA is a group of orthonormal vectors which spans a linear space of the Hankel matrix as

$$S = \text{span}[s_1, s_2, \dots, s_r] \quad (4)$$

where S is the linear subspace, and $s_i, i=1, \dots, r$ is basis vector of the subspace. Parameter r is the subspace dimension and it is calculated according to the information preservation rate ξ in PCA algorithm. The parameter ξ determines how much original information of the Hankel matrix is kept in the subspace, and it is calculated as

$$\xi = \sum_{i=1}^r \lambda_i / \sum_{i=1}^l \lambda_i \quad (5)$$

where l is the number of eigenvectors obtained by PCA decomposition. When parameter ξ is pre-set before the PCA is performed, the subspace dimension is calculated according to (5).

2.2 Grassmann manifold (GM)

In the view of manifold analysis, Grassmann manifold (GM) is consisted of a set of linear subspaces, and the element in the manifold is a linear subspace. The GM, denoted by M , is defined as a set of d -dimensional linear subspace in the D -dimensional linear space with $d \leq D$ [26, 27]. By using the linear subspace, the GM is expressed as

$$M = \{S_1, S_2, \dots, S_m\} \quad (6)$$

where M denotes the GM. S_i is the linear subspace, and m is the number of the linear subspaces that constitute the manifold. The process for constructing the manifold is shown in Fig. 1.

Fig. 1. Process for constructing the Grassmann manifold from vibration signals.

3. SVM-based Grassmann manifold distance

3.1 Brief introduction on support vector machine classifier

Support vector machine (SVM) is originally proposed by Vapnik et al. to solve binary classification problem in the occasion lacking of statistical samples [28]. As a developed machine learning tool, SVM is successfully used in addressing classification problems in machine fault diagnosis [29, 30], text classification [31] and face recognition [32].

Given a set of samples $\{\mathbf{x}_i, i=1, \dots, n\}$ from two categories with category labels being denoted by $y_i \in \{1, -1\}$, objective function of SVM is to find a hyperplane that can separate these samples. The hyperplane is described as

$$\mathbf{w} \cdot \mathbf{x} + b = 0 \quad (7)$$

Where $\mathbf{w}$ and b are the two parameters that determine the hyperplane. $b/\|\mathbf{w}\|$ denotes the distance between the origin and the hyperplane, with $\|\mathbf{w}\|$ denoting Euclidean norm of $\mathbf{w}$. Suppose all the samples satisfy the following inequality

$$y(\mathbf{w} \cdot \mathbf{x} + b) - 1 \geq 0 \quad (8)$$

This constraint can be realized by choosing suitable $\mathbf{w}$ and b . The margin between the samples from the two categories is $2/\|\mathbf{w}\|$. To maximize the margin, objective function of SVM is described as

$$\begin{aligned} \min \quad & \frac{1}{2} \|\mathbf{w}\|^2 \\ \text{s.t.} \quad & y(\mathbf{w} \cdot \mathbf{x} + b) - 1 \geq 0 \end{aligned} \quad (9)$$

The above objective function is available to handle liner separable data. When dealing with the non-separable, the slack variable and penalty factor is considered in the objective function. Accordingly, the objective function becomes

$$\begin{aligned} \min \quad & \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \xi_i \\ \text{s.t.} \quad & \begin{cases} y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \geq 1 - \xi_i, & i = 1, 2, \dots, n \\ \xi_i \geq 0 \end{cases} \end{aligned} \quad (10)$$

where $\xi_i, i=1, 2, \dots, n$ is the slack variable and C is the penalty factor.

By using Lagrangian multiplier method, the objective function is transformed into a dual quadratic

153 programming problem as

$$154 \quad \begin{cases} \max Q(\alpha) = -\frac{1}{2} \sum_{i,j=1}^n \alpha_i \alpha_j y_i y_j (\mathbf{x}_i \cdot \mathbf{x}_j) + \sum_{i=1}^n \alpha_i \\ \text{s.t. } \sum_{i=1}^n \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C, \quad i = 1, 2, \dots, n \end{cases} \quad (11)$$

155 where α is Lagrangian multiplier. By solving (11), we can get the Lagrangian multiplier α and then the
156 $\mathbf{w}$ as

$$157 \quad \mathbf{w} = \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i \quad (12)$$

158 When $\mathbf{w}$ is obtained, the parameter b can be calculated by the following equality

$$159 \quad \alpha_i [y_i (\mathbf{w} \cdot \mathbf{x}_i + b) - 1] = 0 \quad (13)$$

160 By using $\mathbf{w}$ and b , a discriminant function is generated to determine category label of a new sample $\tilde{\mathbf{x}}$ as

$$161 \quad \tilde{y} = \text{sgn}(\mathbf{w} \cdot \tilde{\mathbf{x}} + b) \quad (14)$$

162 where $\text{sgn}(\cdot)$ is the sign function and $\tilde{y}$ is the category label of the new sample. In (14), category label of
163 the new samples is expressed as the inner-product of the sample and parameter $\mathbf{w}$.

164 3.2 Grassmann kernel-based SVM classifier

165 The elements analyzed in tradition SVM are vectors, and the elements in GM are subspaces that are
166 expressed as matrices. So, the challenge for performing SVM on the GM is how to calculate the
167 inner-product of the subspaces. When performing SVM analysis on the GM, the manifold is mapped to the
168 feature space by a nonlinear mapping function φ to handle the global nonlinear characteristic of the
169 manifold. In the feature space, the GM is expressed as

$$170 \quad \tilde{M} = \{\varphi(S_1), \varphi(S_2), \dots, \varphi(S_m)\} \quad (15)$$

171 with $\tilde{M}$ and $\varphi(S)$ denoting the GM and the subspace in the feature space respectively. Inner-product of
172 subspaces in feature space can be expressed by Grassmann kernel as

$$173 \quad (\varphi(S_1) \cdot \varphi(S_2)) = G_k(S_1, S_2) \quad (16)$$

174 where $G_k(\cdot, \cdot)$ is the Grassmann kernel [33, 34]. Grassmann kernel is a symmetric and positive-defined
175 function, and it is helpful to generalize the kernel-based machine learning method to manifold analysis. The
176 Grassmann kernel is related to subspace principal angles [35, 36]. To define the Grassmann kernel, the

concept on principal angles is briefly introduced. For two subspaces S_1 and S_2 in the GM, the cosine of their principal angles is defined as maximum of inner-product of the principal vectors in the two subspaces. In the numerical algorithms to calculate the principal angles, Singular Value Decomposition (SVD) [37] on product matrix of the two subspaces is mostly used. The SVD process is described as

$$S_1^T S_2 = U \kappa V \quad (17)$$

In (17), matrix κ is a diagonal matrix and its diagonal elements are the cosine of the principals. Hence, the principal angles are calculated by the following equation

$$\theta_i = \arccos(\kappa_i), \quad i = 1, \dots, r \quad (18)$$

where θ is the principal angel and κ is the diagonal element in the diagonal matrix. r is the number of the principal angles. By means of principal angels, a new Grassmann kernel, that is the Projection Gaussian kernel, is defined in [38]. The Projection Gaussian kernel is expressed as

$$\begin{aligned} G_k(S_1, S_2) &= \exp\left(-\frac{\gamma \times \|\varphi(S_1) - \varphi(S_2)\|_F^2}{2}\right) \\ &= \exp\left(-\frac{\gamma \times \|S_1 S_1^T - S_2 S_2^T\|_F^2}{2}\right) \\ &= \exp\left(-\gamma \left(\sum_i \sin^2 \theta_i\right)\right) \end{aligned} \quad (19)$$

where $\gamma > 0$ is a parameter in the Grassmann kernel. $G_k(S_1, S_2)$ is the kernel function of S_1 and S_2 , and θ is the principal angle. Grassmann kernel of the subspace is explicitly expressed by sine function of the principal angle, which provides an available way to extend SVM to the domain of manifold analysis.

Given two GMs denoted by $\tilde{M}_1 = \{S_1, S_2, \dots, S_m\}$ and $\tilde{M}_2 = \{\tilde{S}_1, \tilde{S}_2, \dots, \tilde{S}_m\}$, the purpose of performing Grassmann kernel-based SVM is to find the hyperplane that can separate the manifolds with maximal margin. Category labels of the subspaces in the two manifolds are denoted by 1 and -1 respectively. By means of Grassmann kernel, objective function in the Grassmann kernel-based SVM algorithm is written as

$$\begin{cases} \max Q(\alpha) = -\frac{1}{2} \sum_{i,j=1}^m \alpha_i \alpha_j y_i y_j G_k(S_i, S_j) + \sum_{i=1}^m \alpha_i \\ \text{s.t. } \sum_{i=1}^m \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C, \quad i = 1, 2, \dots, m \end{cases} \quad (20)$$

197 where α is Lagrangian multiplier and $y \in \{1, -1\}$ is category labels. By solving (20), parameter $\mathbf{w}$ and
 198 b in the Grassmann kernel-based SVM is obtained, then discriminant function of the Grassmann
 199 kernel-based SVM is expressed as

$$200 \quad \tilde{y} = \text{sgn} \left(\sum_{i=1}^m \alpha_i y_i G_k(\mathbf{S}_i, \tilde{\mathbf{S}}) + b \right) \quad (21)$$

201 where $\tilde{\mathbf{S}}$ is a new subspace sample and $\tilde{y}$ is its category label.

202 3.3 SVM-based Grassmann manifold distance

203 The Grassmann kernel-based SVM classifier is introduced in section 3.2, and discriminant function can be
 204 used to determine category label of a new sample. Similar classification method by combining Grassmann
 205 kernel and SVM has been used in the domain of subspace-based image and human face identification [39,
 206 40]. Furthermore, manifolds are separated by the hyperplane and the margin in the SVM denotes distance
 207 between the manifolds. Hence, this margin can be used as a metric to measure the manifold distance. By
 208 utilizing the Grassmann kernel, the SVM-based Grassmann manifold distance is expressed as

$$209 \quad dis. = \frac{2}{\|\mathbf{w}\|} = \frac{2}{\sqrt{\sum_{i=1}^m \sum_{j=1}^m \alpha_i \alpha_j y_i y_j G_k(\mathbf{S}_i, \mathbf{S}_j)}} \quad (22)$$

210 where $dis.$ denotes Grassmann manifold distance.

211 Sketch map of the SVM-based Grassmann manifold distance is shown in Fig. 2, where the manifold
 212 distance is determined by some of the subspaces, that is, the support vectors, on the manifolds.

213
 214 Fig. 2. SVM-based Grassmann manifold distance.

4. Structural ageing index

In section 3, the Grassmann manifold distance is derived by performing SVM on manifold classification. In this section, we will define ageing index by using the Grassmann manifold distance, and process for calculating the ageing index is also introduced.

4.1 The ageing index

To define ageing index, structural response signals are monitored to construct the GM. The manifold constructed by the signals from structural normal state is denoted by $\tilde{M}_1 = \{S_1, S_2, \dots, S_m\}$, and that from structural ageing state is denoted by $\tilde{M}_2 = \{\tilde{S}_1, \tilde{S}_2, \dots, \tilde{S}_m\}$. According to the process shown in section 3, the distance between manifolds from structural normal state and ageing state is calculated and treated as the aging index (AI), expressed as

$$AI = dis.(\tilde{M}_1, \tilde{M}_2) = \frac{2}{\sqrt{\alpha_i \alpha_j y_i y_j G_k(S_i, \tilde{S}_j)}} \quad (23)$$

where α is the Lagrange multipliers in SVM algorithm and y is category label of the subspace. This ageing index indicates the degree of an ageing state deviating from the normal state. The bigger the ageing index, the more serious the ageing existing in the structure.

4.2 The process for calculating the ageing index

To calculate the ageing index, the VSS is firstly monitored by online condition monitoring technique and multi-channel response signals are collected to construct the Grassmann manifold. Then, category labels of the subspaces from structural normal state and ageing state are set to be 1 and -1 respectively. Thereafter, subspaces with category labels are used as input of the Grassmann kernel-based SVM. And the margin in SVM algorithm is calculated and treated as Grassmann manifold distance. Finally, the Grassmann manifold distance between the manifolds from structural normal state and ageing index is used an ageing index to indicate structural ageing degree. The process for calculating the ageing index is summarized in Fig. 3.

For structure with multiple ageing states, the ageing index for all the states is calculated according to the process shown in Fig. 3. Trend in the index shows the growing degree of material ageing in the structure. Process for multi-states ageing assessment is shown in Fig. 4, where AI denotes the ageing index, and AS denotes the ageing state. The AS_0 denotes the normal state without material ageing, and AS_1 , AS_2 and AS_t

241 denote ageing state 1, ageing state 2 and ageing state t respectively. t is the number of the multiple
 242 ageing state.

Fig. 3. The process for calculating the ageing index.

Fig. 4. Ageing index for multiple ageing states.

248 4.3 Rule for optimal parameters selection

249 Rule for optimal parameters selection is necessary in the calculation of ageing index. Previous works on
 250 trend analysis [41] and monotonicity analysis [42] have been carried out to analysis performance of the state
 251 parameters, which are effective to guide parameter selection. In this study, there are three parameters, that is,

the information preservation rate ξ in PCA algorithm, the penalty factor C in SVM algorithm and the parameter γ in Grassmann kernel should be set. To get an index with obvious trend, these three parameters are viewed as combined parameters. An assumption that there is a trend in the ageing index increasing with the increase of ageing degree is used to choose optimal parameters. With this assumption, a trend factor and a range metric are designed to describe characteristics of the ageing index derived from different parameter combinations.

The trend factor τ is used to restrict the increasing trend in the index, and the factor is expressed as

$$\tau = \begin{cases} 1, & \text{if } AI(i+1) > AI(i), \text{ for all } i=1, \dots, t-1 \\ 0, & \text{else} \end{cases} \quad (24)$$

where AI is the ageing index, and t is number of ageing states in the structure. The range metric R is calculated to describe span of the ageing index, and the metric is expressed as

$$R = \frac{1}{t-1} \sum_{i=1}^{t-1} [AI(t+1) - AI(t)], \quad i=1, \dots, t-1 \quad (25)$$

By means of the trend factor and the range index, the rule for optimal parameters selection is described as

$$(\xi, C, \gamma)_{opt.} = \arg \max_{(\xi, C, \gamma)} (\tau \times R) \quad (26)$$

The trend factor is set to be 1 when there is a monotonic increasing trend in the ageing index. The range metric is with a bigger value if the ageing index lies in a bigger span. By this rule, the parameters combination that generates the ageing index with the most obvious increasing trend is chosen as the optimal one. To search the optimal parameters, ranges for the parameters are chosen firstly to form a parameter space. Then the optimal parameters are chosen by grid search algorithm in the given parameter space. Sketch map for optimal parameters selection is shown in Fig. 5.

Fig. 5. The sketch map for selecting optimal parameters.

(a) Combined parameters in parameter space. (b) The optimal combined parameter.

5. Case study

To validate effectiveness of the proposed ageing index, a case study regarding to ageing assessment in a VSS is conducted in this section. In the case study, ageing test of viscoelastic material is firstly performed in an ageing test chamber. Then the ageing material is embedded in a VSS and structural response signals are monitored to identify structural ageing degree.

5.1 Viscoelastic material accelerated ageing test

The VSS analyzed in this research is a multilayer metal structure with viscoelastic material embedded in. The viscoelastic layer in this VSS is made of acrylonitrile-butadiene rubber. An accelerated ageing test on viscoelastic material is conducted to research effect of ageing to material characteristics. The viscoelastic material sample is shown in Fig. 6, with the external view showing in Fig. 6(a) and the size of the sample showing in Fig. 6(b). The sample is made as a rounded object with a hole in the middle. Diameter of the sample is 178 mm, and thickness is 2 mm.

Fig. 6. External view and size of the viscoelastic material sample.

(a) The external view of the sample. (b) The size of the sample.

In the normal environment, material ageing is a time consuming process and it is difficult to finish natural ageing test in a short time. To accelerate the ageing process, the viscoelastic material sample is placed in a ageing test chamber. External view of the ageing test chamber is shown in Fig. 7(a), and the material sample is hung in the ageing test chamber, as shown in Fig. 7(b). A thermal-oxygen environment with high temperature and high wind speed is produced in the chamber. Environmental parameter in the chamber is listed in table 1, where air temperature is set to be 110 °C and the heated air circulation is forced by the fans.

295 The air humidity in the chamber is in normal level. During the test, material characteristic of the samples is
 296 tested. Changing trend of material characteristics, that is, elastic modulus and hardness, with ageing time is
 297 shown in Fig. 8. It can be seen that elastic modulus and hardness of the viscoelastic material increase
 298 obviously with the ageing time, which indicates the accelerated ageing test is very effective to change
 299 characteristics of the viscoelastic material.

Fig. 7. The ageing test chamber.

(a) External view of the chamber. (b) Placement of the sample in the chamber.

Table 1. Environmental parameters in the ageing test chamber.

Enviromental Parameter	Air	Air humidity	Air speed
Parameter setting	110	Normal	Forced wind

Fig. 8. Change trend of material characteristics with ageing time.

(a) Trend of material elasticity modulus. (b) Trend of material hardness.

307 5.2 Structure test under stochastic extraction.

308 The VSS in this research is a bolted connection structure, and structure diagram of the VSS is shown in
 309 Fig. 9. The VSS contains multiple layers of metal and four layers of acrylonitrile-butadiene rubber. The

310 acrylonitrile-butadiene rubber is used as viscoelastic sandwich to reduce vibration and noise of the VSS.
 311 Four viscoelastic material samples are embedded in the metal layers, and metal layers and rubber layers are
 312 connected by a bolt. A preload is generated by the bolt to connect the layers as a whole. As shown in section
 313 5.1, characteristic of the viscoelastic material is changed by material ageing. Material elastic modulus and
 314 hardness increase with the increasing of ageing time. Characteristics of the viscoelastic material is a major
 315 factor that affects performance of the VSS. Vibration response signals of the VSS are related to material
 316 ageing and they can be used to assess degree of ageing in the VSS. To assess ageing degree, the VSS is tested
 317 on the stochastic excitation test table, as shown in Fig. 10.

318
 319 Fig. 9. Structure diagram of the VSS.

320
 321 Fig. 10. Stochastic excitation test of the VSS.

322 The VSS is fixed on the test table and forced vibration is produced by a vibration exciter. Frequency range

of excitation signals produced by the test table is 0-2000 Hz. During the test, twelve acceleration sensors are placed on the VSS to monitor the response signals. Six of them are on the axial direction and the other six are on the radial direction. Response signals sampled by the sensors are stored by a LMS data acquisition system, and the sampling rate is set to be 10240 Hz. The acquired response signals of the VSS with viscoelastic material ageing for 3 days, 7 days, 11 days, 17 days, 23 days and 29 days are shown in Fig. 11.

Fig. 11. Response signals of the VSS under stochastic excitation.

(a)- (f) Ageing for 3 days, 7 days, 11 days, 17 days, 23 days and 29 days respectively.

5.3 Ageing degree assessment

When the stochastic extraction test is finished, the obtained response signals are used to assess the level of material ageing. Response signals from axial direction, radial direction are respectively used to calculate the ageing index. Ageing index calculated from radial signals is with the most obvious increasing trend. Hence, signals from radial direction are used to assess ageing levels in the VSS. Response signals acquired from all the structural states are utilized to form GMs according to Fig. 1. Distance between two GMs is calculated by the process shown in Fig. 3 and treated as health index for the VSS. When calculating the ageing index, the rule for optimal parameters selection described in section 4.3 is utilized to generate a set of optimal parameters. The optimal information preservation rate ξ is chosen in the range of $[60\%, 95\%]$. The optimal penalty factor C is chosen in the range of $[2^{-15}, 2^{15}]$ and the optimal parameter γ is chosen in the range of $[1, 10]$. Step length for searching optimal values of ξ , C and γ is set to be 5, 2 and 1 respectively. The optimal values for ξ , C and γ are 70%, 2^3 and 1 respectively in this case study.

When these optimal parameters are used, the obtained ageing index is shown in Fig. 12 and value of the ageing index for all the structural states is also listed in table 2. It can be seen that the ageing index equals to 0.14, which is a smaller value, when the VSS is in normal state without material ageing. The ageing index for the normal state is not equal to 0 because patterns in the two parts of signals used to calculate health index for normal state are not absolutely identical, which causes a small margin when performing SVM on the two manifolds from the normal state. Furthermore, what gets attention is the trend in the obtained index, and a bigger value in the index is generated from the VSS with ageing in more serious level. This index increasing with ageing time provides an effective metric to reflect ageing level in the VSS.

Fig. 12. The calculated ageing index.

Table 2. The ageing index for the structural state.

Structural state	AS ₀	AS ₁	AS ₂	AS ₃	AS ₄	AS ₅	AS ₆
Ageing days	0	3	7	11	17	23	29
Ageing index	0.14	0.22	0.28	0.52	0.54	0.59	0.68

6. Comparisons

In current research, subspace-based damage indices have been presented for structural health monitoring. A comparison will be carried out to further validate effectiveness of the ageing index presented in this paper. In [43], a subspace distance, denoted by DI_1 , is defined by inner-product matrix of two subspaces, and it is expressed as

$$DI_1 = \text{mean} \left\{ \left| U_{h,nul}^T U_{a,act} \right| \right\} \quad (27)$$

where $U_{h,nul}$ is used to denote the null-subspace [14, 43] from structural healthy state. $U_{a,act}$ denotes the

active subspace from structural ageing state. In [14], a subspace distance is defined by the residue matrix, and this matrix is expressed as

$$\Delta = U_{h,nul.}^T \left(U_{a,act.} S_{a,act.} V_{a,act.}^T \right) \quad (28)$$

where $S_{a,act.}$ represents the active singular-value matrix and $V_{a,act.}$ represents the right active subspace. Based on this residue matrix, a subspace distance, denoted by DI_2 , is defined as

$$DI_2 = \frac{\{\Delta\}^T \{\Delta\}}{\Sigma_{\Delta}} \quad (29)$$

where Σ_{Δ} is estimated by the reference data as

$$\Sigma_{\Delta} = \frac{1}{N_{ref}} \sum_{i=1}^{N_{ref}} \{\Delta\}_j^T \{\Delta\}_j \quad (30)$$

In Eq.(29) and (30), $\{\cdot\}$ denotes the column stacking operator, and N_{ref} means the number of reference data.

To calculate these two subspace distances, Hankel matrix shown in Eq.(1) is firstly constructed by response signal of the VSS. Then, singular value decomposition is performed on the Hankel matrix to get the subspace. The decomposition process is expressed as [14]

$$\mathbf{H}_{p,q} = USV = [U_{act.} \ U_{nul.}] \begin{bmatrix} S_{act.} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} [V_{act.} \ V_{nul.}]^T \quad (31)$$

with subscript *act.* denoting the active subspace and *nul.* denoting the column null-space. Distance between subspaces from structural healthy state and ageing state can finally be calculated to assess ageing states in the VSS. In each structural state, 20 samples are used to calculate the subspace distance DI_1 and DI_2 . Information preservation rate of the singular value in SVD is set to be 70%, which is the same with that used in PCA. The obtained subspace distance for all the structural states is shown in figure 13, and relationship between the serial number of the samples and the structural states are shown in table 3. It can be seen on one hand that, there is fluctuation in the two subspace distance. This is caused by the stochastic excitation to the structure, and randomness in the excitation leads to the fluctuation in the subspace distance. On the other hand, there is no increasing trend in the two subspace distance with the increase of degree of material ageing. To investigate effect of information preservation rate to the subspace distance, different information preservation rates are set to calculate the subspace distance. Mean value of the subspace distance in each structural state is shown in figure 14, where the information preservation rate in SVD, denoted by μ , is set to be 70%, 75%, 80%, 85% and 90% respectively. It can be seen that, though multiple parameter values are

utilized to calculate the subspace distance, there is no increasing trend shown in the distance index.

Fig. 13 Trend of the subspace distance: (a) trend of DI_1 , (b) Trend of DI_2 .

Table 3. Relationship between the serial number and structural states.

Structural state	AS ₀	AS ₁	AS ₂	AS ₃	AS ₄	AS ₅	AS ₆
Serial number	1-20	21-40	41-60	61-80	81-100	100-120	120-140

Fig. 14 Mean value of the subspace distance with μ equaling to different values:
(a) mean value of DI_1 , (b) mean value of DI_2 .

By comparing figure 14 to figure 12, we can see the advantage of the designed ageing index over the existing subspace distances for SHM. The VSS in the case study is with nonlinear behavior due to nonlinearity in viscoelastic material, and this will leads to nonlinear component in structural response signal, such as the nonlinear subspace distribution. To calculate DI_1 and DI_2 , only linear algorithm is used in the singular value decomposition and the definition of distance function, which is not effective to extract

401 nonlinear information for ageing assessment. In the presented method in this paper, subspace sets are treated
402 as manifold and the mapping function φ can transform nonlinear manifold classification to be linear.
403 Quadratic programming in the SVM is performed to find an optimal hyperplane that can maximize the
404 distance between manifolds. Meanwhile, parameter selection strategy in the presented method is utilized to
405 generate an ageing index with obvious increasing trend. With these two merits, the presented Grassmann
406 kernel-based SVM classifier can deal with the nonlinear subspace classification very well, and an ageing
407 index that matches the deepening process of material ageing can be generated.

408 **7. Conclusions**

409 To assess ageing state in VSS, a Grassmann manifold distance is designed by performing SVM on signal
410 manifolds. Traditional SVM is extended to the domain of manifold analysis, and a Grassmann kernel-based
411 SVM classifier is derived by introducing a Grassmann kernel in the SVM algorithm. The result in this study
412 shows the Grassmann kernel-based SVM as an effective method for manifold analysis, and the Grassmann
413 kernel can also be used in other kernel-based manifold analysis methods. An accelerated ageing test on a
414 kind of viscoelastic material is conducted and characteristics of the ageing material are tested. The result
415 shows material characteristics are changed obviously by material ageing. A structural ageing index is
416 designed by utilizing margin in the Grassmann kernel-based SVM classifier, and an optimization method for
417 parameter selection is presented. The obtained ageing index increases with the increase of ageing time. By
418 the assessment result and the comparison, effectiveness of the ageing index for ageing assessment in VSS is
419 shown.

420 **Acknowledgements**

421 Sincerest thanks are expressed to the reviewers and editors for their valuable comments and suggestions to
422 improve our paper. The authors also appreciate support from NSAF of China (Grant No.11176024) and NSF
423 of China (No. 51275382).

424 **References**

- 425 [1] M. Bilasse, L. Azrar, E. M. Daya, Complex modes based numerical analysis of viscoelastic sandwich
426 plates vibrations, *Computers & Structures* 89 (2011) 539-555.
- 427 [2] M. Bilasse, E. M. Daya, L. Azrar, Linear and nonlinear vibrations analysis of viscoelastic sandwich

428 beams, *Journal of Sound and vibration* 329 (2010) 4950-4969.

429 [3] J. Qu, Z. Zhang, T. Guo, X. Luo, C. Sun, and B. Li, State recognition of the viscoelastic sandwich
 430 structure based on the adaptive redundant second generation wavelet packet transform, permutation
 431 entropy and the wavelet support vector machine, *Smart Materials and Structures* 23(2014) 085004.

432 [4] E. Asnaashari, J. K. Sinha, Crack detection in structures using deviation from normal distribution of
 433 measured vibration responses, *Journal of Sound and Vibration* 333 (2014) 4139-4151.

434 [5] W. Fan, P. Z. Qiao, Vibration-based damage identification methods: a review and comparative study,
 435 *Structural Health Monitoring* 10 (2011) 83-111.

436 [6] E. P. Carden, P. Fanning, Vibration based condition monitoring: a review, *Structural Health Monitoring*
 437 3 (2004) 83-111.

438 [7] J. C. Lin, Characterization and numerical evaluation of vibration on elastic–viscoelastic sandwich
 439 structures, *Composite Structures* 92 (2010) 669-675.

440 [8] M. Bilasse, L. Azrar, E. M. Daya, Complex modes based numerical analysis of viscoelastic sandwich
 441 plates vibrations, *Computers & Structures* 89 (2011) 539-555.

442 [9] J. W. Gillespie, L. A. Carlsson, A. A. Gawandi, T. A. Bogetti, Fatigue crack growth at the face sheet-core
 443 interface in a discontinuous ceramic-tile cored sandwich structure, *Composite Structures* 94 (2012)
 444 3186-3193.

445 [10] A. M. G. Lima, S. Lambert, D. A. Rade, E. Pagnacco, L. Khalij, Fatigue reliability analysis of
 446 viscoelastic structures subjected to random loads, *Mechanical Systems and Signal Processing* 43 (2014)
 447 305-318.

448 [11] Y. Liu, A. Chattopadhyay, Low-velocity impact damage monitoring of a sandwich composite
 449 wing, *Journal of Intelligent Material Systems and Structures* 24 (2013) 2074-2083.

450 [12] D. García, I. Trendafilova, A multivariate data analysis approach towards vibration analysis and
 451 vibration-based damage assessment: Application for delamination detection in a composite
 452 beam, *Journal of Sound and Vibration* 333 (2014) 7036-7050.

453 [13] D. A. Tibaduiza, L. E. Mujica, J. Rodellar, A. Güemes, Structural damage detection using principal
 454 component analysis and damage indices, *Journal of Intelligent Material Systems and Structures* in press.

455 [14] A. M. Yan, J. C. Golinval, Null subspace-based damage detection of structures using vibration
 456 measurements, *Mechanical Systems and Signal Processing* 20 (2006) 611-626.

457 [15] M. Döhler, L. Mevel, F. Hille, Subspace-based damage detection under changes in the ambient

458 excitation statistics, *Mechanical Systems and Signal Processing* 45 (2014) 207-224.

459 [16] S. H. Chao, C. H. Loh, M. H. Tseng, Structural damage assessment using output-only measurement:
 460 Localization and quantification, *Journal of Intelligent Material Systems and Structures* 25 (2014)
 461 1097-1106.

462 [17] P. D. Boe, J. C. Golinval, Principal component analysis of a piezosensor array for damage localization,
 463 *Structural health monitoring* 2 (2003) 137-144.

464 [18] G. Dong, J. Chen, N. Zhang, Investigation into on-road vehicle parameter identification based on
 465 subspace methods, *Journal of Sound and Vibration* 333 (2014) 6760-6779.

466 [19] Z. Zhang, J. Fan, H. Hua, Simulation and experiment of a blind subspace identification method, *Journal*
 467 *of Sound and Vibration* 311 (2008) 941-952.

468 [20] P. R. G. Kurka, H. N. Cambraia, Application of a multivariable input–output subspace identification
 469 technique in structural analysis, *Journal of Sound and Vibration* 312 (2008) 461-475.

470 [21] M. Basseville, L. Mevel, M. Goursat, Statistical model-based damage detection and localization:
 471 subspace-based residuals and damage-to-noise sensitivity ratios, *Journal of sound and vibration* 275
 472 (2004) 769-794.

473 [22] C. Sun, Z. Zhang, T. Guo, et al., A novel manifold–manifold distance index applied to looseness state
 474 assessment of viscoelastic sandwich structures, *Smart Materials and Structures* 23 (2014) 065019.

475 [23] E. Gandino, L. Garibaldi, S. Marchesiello, Covariance-driven subspace identification: A complete
 476 input–output approach, *Journal of Sound and Vibration* 332 (2013) 7000-7017.

477 [24] A. R. M. Rao, V. Kasireddy, N. Gopalakrishnan, K. Lakshmi, Sensor fault detection in structural health
 478 monitoring using null subspace–based approach, *Journal of Intelligent Material Systems and Structures*
 479 26 (2015) 172-185.

480 [25] I. Jolliffe, Principal component analysis, John Wiley & Sons, Ltd 2012.

481 [26] Y. Washizawa, S. Hotta, Mahalanobis distance on extended grassmann manifolds for variational
 482 pattern analysis, *IEEE transactions on neural networks and learning systems* 25 (2014) 1980-1990.

483 [27] X. Wang, Z. Li, D. Tao, Subspaces indexing model on Grassmann manifold for image search, *IEEE*
 484 *Transactions on Image Processing* 20 (2011) 2627-2635.

485 [28] C. Cortes, V. Vapnik, Support-vector networks, *Machine learning* 20 (1995) 273-297.

486 [29] A. Widodo, B. S. Yang, Support vector machine in machine condition monitoring and fault
 487 diagnosis, *Mechanical Systems and Signal Processing* 21 (2007) 2560-2574.

488 [30] B. Tang, T. Song, F. Li, L. Deng, Fault diagnosis for a wind turbine transmission system based on
489 manifold learning and shannon wavelet support vector machine, *Renewable Energy* 62 (2014) 1-9.

490 [31] S. Tong, D. Koller, Support vector machine active learning with applications to text classification, *The*
491 *Journal of Machine Learning Research* 2 (2002) 45-66.

492 [32] G. Guo, S. Z. Li, K. L. Chan, Support vector machines for face recognition, *Image and Vision computing*
493 19 (2001) 631-638.

494 [33] J. Hamm, D. D. Lee, Grassmann discriminant analysis: a unifying view on subspace-based learning,
495 *In Proceedings of the 25th international conference on Machine learning*, published by ACM, New
496 York, USA, 2006, pp. 376-383.

497 [34] M. T. Harandi, M. Salzmann, S. Jayasumana, R. Hartley, H. Li, Expanding the family of grassmannian
498 kernels: An embedding perspective, in *European Conference On Computer Vision, Lecture Notes in*
499 *Computer Science*, 8695 (2014) 408-423.

500 [35] P. Van Overschee, B. De Moor, Subspace identification for linear systems: theory, implementation,
501 applications, Kluwer academic publishers, Dordrecht 1996.

502 [36] R. Wang, S. Shan, X. Chen, Q. Dai, W. Gao, Manifold–manifold distance and its application to face
503 recognition with image sets, *IEEE Transactions on Image Processing* 21 (2012) 4466-4479.

504 [37] S. Puntanen, G. Styan, J. Isotalo, et al, Singular Value Decomposition, Matrix Tricks for Linear
505 Statistical Models, Springer, Berlin Heidelberg, 2011.

506 [38] S. Jayasumana, R. Hartley, M. Salzmann, H. Li, M. Harandi, Kernel Methods on Riemannian Manifolds
507 with Gaussian RBF Kernels, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, in press.

508 [39] C. Xu, T. Wang, J. Gao, et al., An ordered-patch-based image classification approach on the image
509 Grassmannian manifold, *IEEE Transactions on Neural Networks and Learning Systems* 25(4) (2014)
510 728-737.

511 [40] Y. Fan, Y. Liu, H. Wu, et al., Discriminant analysis of functional connectivity patterns on Grassmann
512 manifold, *Neuroimage* 56(4) (2011) 2058-2067.

513 [41] M. Maurya, R. Rengaswamy, and V. Venkatasubramanian, Fault diagnosis by qualitative trend analysis
514 of the principal components, *Chemical Engineering Research and Design* 83(9) (2005) 1122-1132.

515 [42] J. Coble, and J. Wesley Hines, Identifying optimal prognostic parameters from data: a genetic
516 algorithms approach, In Annual Conference of the Prognostics and Health Management Society, San
517 Diego, CA, September. 2009.

518 [43] S. H. Chao, C. H. Loh, M. H. Tseng, Structural damage assessment using output-only measurement:
519 Localization and quantification, *Journal of Intelligent Material Systems and Structures*, 25(9) (2014)
520 1097-1106.
521

522

523 [44] Graphical Abstract