Peregrinations through topics in light scattering and radiative transfer

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Abstract

In this van de Hulst essay, I have taken the liberty to present a journey through some topics in light scattering and radiative transfer which I feel were major contributions to the field but the number of topics I would like to cover is far more numerous than I have the time or the space to present. I also wanted to share with the reader some heartwarming memories I have of my wonderful friend and truly distinguished colleague Hendrik Christoffel van de Hulst (affectionately known to his colleagues as “Henk”) whom I consider to be one of the preeminent scientists of his era.

1. Introduction

It is indeed an honor to have been selected as the co-recipient of the 2015 Hendrik C. van de Hulst Award along with my distinguished colleague Dan Mackowski. This is the third time this prestigious award has been given, the first one going to Joop Hovenier and the second one going to Michael Mishchenko. Both of these distinguished colleagues have worked in the areas of light scattering and radiative transfer and have presented essays for their awards. I would also like to extend my sincerest gratitude to the van de Hulst Awards Committee, for all the hard work they did in bringing this years award to fruition. I should also like to thank José Stoop, who represented Elsevier, for her contributions to the ELS XV Conference.

Since this article is more of an essay, I would like to touch on some topics which I have been involved in over the past forty years and present one which directly involved correspondence and collaboration with my dear friend and colleague H. C. van de Hulst whom I also regard as one of the truly great humanists of his time. I would first like to begin this essay with a couple of wonderful stories about Henk when he visited me at Texas A&M University in the fall of 1995 under a Distinguished Foreign Scientist Program sponsored by the Office of Naval Research (see Fig. 1). We spent two wonderful weeks together discussing a plethora of problems and new approaches to solving them. I wanted to share with the readers of this essay a couple of stories about Henk that transpired on his visit.

One morning as we were riding down the elevator and a few students who had apparently just left their physics class and were intently discussing the fact that if the elevator were to suddenly free fall , then they could jump up just before the elevator hit the ground and escape death! I recall Henk started to smile and there was a little twinkle in his eye and he said to one of them “ young man, even if you could jump up that fast the damage to your head would be enormous because you will surely hit it against the ceiling and you might expire even faster”. When the elevator reached the ground level, the student looked at Henk, scratched his head in puzzlement and walked away.

On another occasion my wife and I decided to have a party in honor of Henk’s visit, so we invited several faculty members and graduate students along with their wives to my home for an evening of good fellowship and conversation. My wife was making hors d’oeuvres and out the corner of my eye I saw Henk reaching into a bowl holding fragrant potpourri and was fixing to put a large handful into his mouth and I had to loudly shout “No Henk! That’s potpourri don’t eat it! Not only was Henk embarrassed, but so was every one in the room who kind of chuckled and basically went back to their chatter and ignored the "old" guy but not in a disrespectful way. Henk with a very red face sheepishly put down the potpourri and sat very quietly. Everyone was looking as if to say, how could such a very brilliant man mistake potpourri for hor d’oeuvres? One of my colleagues tried to break the hushed silence by posing a puzzle about the shape of a light beam after having passed through a dome roof and while people were starting to heatedly debate the question, out of the corner of the room Henk loudly shouted, it’s a circle! It’s obvious! This quick response showed a skeptical group that the “old man” was as sharp as ever.

2. Matrix Operator theory and Invariant imbedding

The historical evolution of invariant imbedding and matrix operator theory is quite fascinating since it appears in quite diverse areas of science and due to this diversity it cannot be attributed to a single individual. This is quite fortunate in a way since it avoids being subjected to Stigler’s Law of eponymy[1] which states that “no scientific discovery is named after its original discoverer”.

One of the earliest references to scattering and transmission in multilayered media where the principle of imbedding was used dates back to Stokes[2]in which he considered transmission and reflection through a stack of identical glass plates. He noted the fact that the resultant reflection and transmission for the stack were invariant with respect to the particular way the plates were combined. He then obtained a set of commutativity relations. The continuous problem was first considered by McClelland[3]and later by Schmidt[4] who continuously varied the thickness of his transmission medium to obtain a set of differential

equations. Schmidt's method was actively pursued by Bellman and colleagues (extensive references to their work are given by Bellman *et al.* [5] and later in his book[6]). The first attempt to apply these new ideas to radiative transfer theory was made by Ambarzumian[7] where he obtained an equation for reflection from a semi-infinite atmosphere by noting that the reflection operator is left *invariant* upon addition of a new layer. Using elegant mathematical techniques Chandrasekhar [8] generalized Ambarzumian's analysis and was able to obtain reflection and transmission operators for a finite layer. During the period 1944-1949, R. Redheffer [9], began a study of microwave transmission problems, and was able to construct an algebra for the reflection and transmission operators that contained the Stokes commutativity relations, the Ambarzumian invariance principle, and the general functional equations covering a variety of transfer procedures. Preisendorfer[10] used the concepts of reflection and transmission operators to make significant contributions to the area of radiative transfer. Van de Hulst [11] formulated a set of doubling equations for the reflection and transmission operators which have proven to be very useful. Twomey *et al.* [12] formulated a matrix formalism which was essentially the same as that of Van de Hulst. Later Grant and Hunt[13] utilizing the algebra developed by Redheffer and the radiative transfer methods developed by Preisendorfer, rederived the matrix theory previously published by others but added a term which represented the contribution from internal sources. In particular, the fundamental equation, which relates the transmission and reflection operators for a combined layer to those of its parts was originally derived by Redheffer [9] and he also included the concept of the star semigroup. Plass, et. al [14] showed how the theory could be obtained in such a way to give a complete physical understanding of each of the equations. We will present a brief derivation of the main equations used in matrix operator theory, neglecting internal sources, along with a complete physical description of the equations. A more complete derivation, including internal sources, can be found in Plass. et. al.[14] and Kattawar [15]. Referring to Fig. 2, let the top boundary of layer 1 be denoted by x and the bottom denoted by y . Let the contiguous layer 2 have upper boundary y and lower boundary z . These regions can be either homogeneous or inhomogeneous. We will now show how to “add” these two layers together to obtain the reflection and transmission operators for the combined layer knowing these operators for the individual layers. Let $I^+(x)$ denote the radiance at the boundary x in the upward direction and $I^-(x)$ denote the radiance at the boundary x in the downward direction. Let $r(x,y)$ denote the reflection operator for radiation incident on the x boundary and reflected by the region volume whose other boundary is y . Similarly, $r(y,x)$ is the reflection operator for radiation incident on boundary y and reflected by the region volume whose other boundary is x . Similar definitions are used for the transmission operators denoted by $t(x,y)$ and $t(y,x)$. It should be noted that for inhomogeneous regions in

general $r(x,y) \neq r(y,x)$ and the same is true for the transmission operators. Now referring to Fig. 2, we can write the following equations for region 1:

$$I^+(y) = t(x,y)I^+(x) + r(y,x)I^-(y) \quad (1a)$$

$$I^-(x) = r(x,y)I^+(x) + t(y,x)I^-(y) \quad (1b)$$

Now considering region 2, we can write similar equations

$$I^+(z) = t(y,z)I^+(y) + r(z,y)I^-(z) \quad (2a)$$

$$I^-(y) = r(y,z)I^+(y) + t(z,y)I^-(z) \quad (2b)$$

Now multiply Eq (1a) by $r(y,z)$ from the left, add this result to Eq (2b) and multiply the resulting equation by $[E - r(y,z)r(y,x)]^{-1}$ where E is the identity operator or identity matrix, we then obtain:

$$I^-(y) = [E - r(y,z)r(y,x)]^{-1} [t(z,y)I^-(z) + r(y,z)t(x,y)I^+(x)] \quad (3a)$$

A similar manipulation yields:

$$I^+(y) = [E - r(y,x)r(y,z)]^{-1} [t(x,y)I^+(x) + r(y,x)t(z,y)I^-(z)] \quad (3b)$$

Substituting Eq (3b) into Eq (2a) for $I^+(y)$ and comparing this equation with the equation for the combined layer:

$$I^+(z) = t(x,z)I^+(x) + r(z,x)I^-(z) \quad (4a)$$

and comparing coefficients, we obtain:

$$t(x,z) = t(y,z)[E - r(y,x)r(y,z)]^{-1}t(x,y) \quad (5a)$$

$$r(z,x) = r(z,y) + t(y,z)[E - r(y,x)r(y,z)]^{-1}r(y,x)t(z,y) \quad (5b)$$

Doing similar manipulations we can obtain the remaining two operators:

$$r(x,z) = r(x,y) + t(y,x)[E - r(y,z)r(y,x)]^{-1}r(y,z)t(x,y) \quad (6a)$$

$$t(z,x) = t(y,x)[E - r(y,z)r(y,x)]^{-1}t(z,y) \quad (6b)$$

The physical meaning of each of these terms can easily be explained. We will illustrate by using Eq (5a) and expanding the operator $[E - r(y,x)r(y,z)]^{-1}$ in a power series

$$[E - r(y,x)r(y,z)]^{-1} = \sum_{n=0}^{\infty} [r(y,x)r(y,z)]^n \quad (7)$$

This equation can now be interpreted in the following way. For $n=0$, $t(x,z) = t(y,z) t(x,y)$ which says that the radiation is first transmitted through the region from x to y and then directly

transmitted from the region from y to z with no reflections between the layers. For $n=1$, the second term in the expansion is $t(x,z)=t(y,z)r(y,x)r(y,z)t(x,y)$ which shows that now the radiation is transmitted through the region from x to y and reflected from the layer from z to y and then reflected from the layer from x back to y and then transmitted from y to z. It is important to note at this point that the so-called “doubling” equations are contained within this formalism. From the above equations, we will show how to get sets of ordinary differential equations that will give us a very powerful way to build a small layer of high order accuracy to use in the “doubling” equations. It should be noted that the added layer can be a reflecting layer such as the ground or an ocean surface.

Let the layer from y to z be of infinitesimal thickness Δ ($z=y+\Delta$) and let the variable η be such that $y \leq \eta \leq y+\Delta$. Substituting these values into Eq (5b) we get:

$$r(y+\Delta, x) = r(y+\Delta, y) + t(y, y+\Delta)[E - r(y, x)r(y, y+\Delta)]^{-1}r(y, x)t(y+\Delta, y) \quad (8)$$

where

$$r(y+\Delta, y) = \Gamma^{+-}(\eta)\Delta + O(\Delta) \quad (9a)$$

$$r(y, y+\Delta) = \Gamma^{-+}(\eta)\Delta + O(\Delta) \quad (9b)$$

$$t(y+\Delta, y) = E - \Gamma^{--}(\eta)\Delta + O(\Delta) \quad (9c)$$

$$t(y, y+\Delta) = E - \Gamma^{++}(\eta)\Delta + O(\Delta) \quad (9d)$$

The term $O(\Delta)$ is defined such that $\lim_{\Delta \rightarrow 0}[O(\Delta)/\Delta] \rightarrow 0$ and the Γ operators or matrices are defined in terms of the single scattering phase function (see ref [15] for complete details). Now expanding both sides of Eq (8) and using Eqs (9a)-(9d) and taking the limit as $\Delta \rightarrow 0$ we obtain the following partial differential equation (PDE) for $r(y, x)$:

$$\frac{\partial r(y, x)}{\partial y} = \Gamma^{+-}(y) - \Gamma^{++}(y)r(y, x) - r(y, x)\Gamma^{--}(y) + r(y, x)\Gamma^{-+}(y)r(y, x) \quad (10a)$$

This type of PDE is known as a Ricatti differential equation which is nonlinear but has a very simple initial condition $r(x, x) = 0$ and is the same one derived by Bellman, *et al.*[16] using invariant imbedding methods. A rigorous analysis of the correct way to get good starting values for use in the doubling equations using this formulation can be found in Kattawar and Plass [17]. The remaining differential equations for the reflection and transmission operators are obtained in a similar manner and are:

$$\frac{\partial r(x, y)}{\partial y} = t(y, x)\Gamma^{-+}(y)t(x, y) \quad (10b)$$

$$\frac{\partial t(y, x)}{\partial y} = t(y, x)[\Gamma^{-+}(y)r(y, x) - \Gamma^{--}(y)] \quad (10c)$$

$$\frac{\partial t(x,y)}{\partial y} = [-\Gamma^{--}(y) + r(y,x)\Gamma^{++}(y)]t(x,y) \quad (10d)$$

To obtain the PDEs for the other independent variable, one simply interchanges x and y which is equivalent to inverting the layer.

3. Exact solution for small angle scattering

At a remote sensing meeting ca 1992, Henk and I were having coffee together and he was quite excited over the problem of obtaining the spread function for a pulsed collimated beam in a scattering medium with small angle scattering. I told him I would be delighted to collaborate with him on this problem since I had been following some of the work in the Russian literature on this very subject for quite some time. When he returned to Leiden, we started a very lengthy correspondence on this problem that culminated into a beautiful solution and publication [18]. I will present a brief review of the genesis and evolution of the solution. Prior to 1963 there were many papers published on this subject and they were listed in an excellent review paper by Scott [19]. The approach we took was to include the temporal dependence of a pulsed beam as well as the spatial dependence. The problem is formulated for the case of small random deviations from rectilinear propagation in a totally scattering medium; however, the inclusion of absorption is quite straightforward.

Referring to Fig. 3, we consider a pulsed collimated beam in an (x_1, x_2, z) orthogonal coordinate system where $\hat{x}_1 \times \hat{x}_2 = \hat{z}$ where the caret denotes unit vector. The angles θ_1 and θ_2 denote projected angular deviations from the positive z direction in the x_1 -z, x_2 -z planes respectively (see Fig. 3). The symbol t will be used to denote the path *detour* from the straight line path OP shown in Fig. 3. To get the actual time detour, one simply divides this t by the phase velocity of light in the medium. As the beam propagates, a spreading will occur in all five variables and our task was to compute the joint distribution function $f(x_1, x_2, \theta_1, \theta_2, t; z)$ for this stochastic situation. A parameter we denoted by s which has units of reciprocal length is introduced in the following way. Radiation leaving the origin at $z=0$ and arriving at distance z with an angular distribution in θ with respect to the z axis such that

$$\langle \theta^2 \rangle = 4sz \quad (11)$$

where θ is in radians and the parameter s must be related to the inherent optical properties of the medium. The $\langle \dots \rangle$ symbol denotes a statistical or ensemble average. Consider a medium with scattering coefficient β and a phase function $P(\theta)$ subject to the normalization condition

$$\frac{1}{2} \int_0^\pi P(\theta) d(\cos \theta) = 1 \quad (12)$$

and asymmetry factor g defined by

$$\frac{1}{2} \int_0^\pi P(\theta) \cos(\theta) d(\cos \theta) = g \quad (13)$$

Next let us assume that $P(\theta)$ is virtually zero beyond small angle scattering, then a single scattering event will produce on average

$$\langle \theta^2 \rangle = 2 \langle 1 - \cos(\theta) \rangle = 2(1 - g) \quad (14)$$

Multiplying Eq(14) by the average number of scattering events βz over the path length z and equating this result to Eq (11) we get

$$4s = 2(1 - g)\beta \quad (15)$$

It is important to note that the quantities β and g are not what are important but it is the product $(1 - g)\beta$ that is the key player in the formulation. Another caution is due here, and that is the condition that the second moment $\langle \theta^2 \rangle$ exists which is violated by two commonly used phase functions; namely, the Henyey-Greenstein phase function and the Fraunhofer diffraction pattern which occurs in the Lorenz-Mie scattering solutions for large spheres. Since we are considering optically thick media, the beam must still be concentrated near the forward direction after undergoing a large number of scattering events and is guaranteed provided that $sz \ll 1$.

If we can obtain the projected distribution $f_1(x_1, \theta_1, t_1; z)$ then the distribution function in the other variable must be identical by symmetry. Now it can be shown that within the approximations used, $\theta^2 = \theta_1^2 + \theta_2^2$ and $t = t_1 + t_2$. The complete solution can then be expressed as a convolution in the t variable as follows:

$$f(x_1, x_2, \theta_1, \theta_2, t; z) = \int_0^t f_1(x_1, \theta_1, t - t_2; z) f_1(x_2, \theta_2, t_2; z) dt_2 \quad (16)$$

Using invariance arguments, the partial differential equation for the projected distribution was obtained:

$$\frac{\partial f_1}{\partial z} + \theta_1 \frac{\partial f_1}{\partial x_1} - s \frac{\partial^2 f_1}{\partial^2 \theta_1^2} + \frac{1}{2} \left(\theta_1 - \frac{x_1}{z} \right)^2 \frac{\partial f_1}{\partial t_1} = 0 \quad (17)$$

subject to the initial condition:

$$f(x_1, \theta_1, t_1; 0) = \delta(x_1) \delta(\theta_1) \delta(t_1) \quad (18)$$

Yang[20] who considered the path length of electrons in foils and Weinman and Shipley [21] using the time dependent equation of transfer derived a similar equation but used a different reference point for the path detour. To solve this formidable PDE we Fourier transformed the variables x_1 and θ_1 and then Laplace transformed the variable t_1 as follows where the subscript 1 will be suppressed:

$$F(u, v, w; z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} d\theta \int_0^{\infty} dt \exp(iux + iv\theta - wt) f(x, \theta, t; z) \quad (19)$$

and the inverse transformation is given by

$$f(x, \theta, t; z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_0^{\infty} dt \exp[-(iux + iv\theta)] L^{-1}[F(u, v, w; z)] \quad (20)$$

where the operator $L^{-1}[F]$ denotes the inverse Laplace transform of F . We were able to obtain the solution to the transformed PDE; however, we were not able to analytically invert the Laplace transform. However, in many situations where we have a continuous source or need only the signal integrated over the pulse length we were able to obtain the contracted distribution function:

$$f(x, \theta; z) = \frac{\sqrt{3}}{2\pi s z^2} \exp\left(-3 \frac{x^2}{s z^3} + 3 \frac{x\theta}{s z^2} - \frac{\theta^2}{s z}\right) \quad (21)$$

It is interesting to note that this distribution function is Gaussian-like in the variables x and θ ; however, there is a cross term which implies a correlation between the variables x and θ . Using Eq. (21) it is easy to show that $\langle \theta \rangle = 3x / 2z$ and $\langle x \rangle = z\theta / 2$ and from this it follows that the correlation between these variables is $3/4$ which is quite significant. We were also able to give explicit expressions for many of the moments of the distribution function which is often times all that one needs. The solutions obtained in this study have many important applications in lidar, atmospheric and oceanic optics, and biomedical optics just to mention a few.

4. Expanding the DDA to handle problems in metamaterials and cloaking

After reading the wonderful papers on cloaking by John Pendry and colleagues [22-24], it occurred to me and my postdoc (Yu You) at the time that perhaps the DDA formulation could be extended to include the magnetic susceptibility as well as the polarizability. The reason being that the coordinate transformations used to exclude the electric and magnetic fields in a region of free space induce transformations in both the permittivity and susceptibility tensors. The basic idea of the method is that if one thinks of electric field lines attached to a rectangular coordinate grid, then if a coordinate transformation excludes a certain volume of a region the field lines effectively “stick” to the coordinates and are excluded as well. However, to do this requires an inhomogeneous and anisotropic permittivity and susceptibility tensor. We will illustrate the concept later in this section; however, now we must show how to implement the magnetic susceptibility into the DDA formalism. To do this we incorporated the ideas from Lakhtakia [25] who showed it was necessary to account for the magnetic response to the incident electromagnetic field. Following the notation we used in earlier papers on this subject [26,27] we proceed as follows. For non-zero magnetic susceptibility $\chi_m = \mu - 1$ (where μ is the

permeability) the magnetization $\vec{M} = \chi_m \vec{H}$ is also non vanishing and therefore magnetic dipoles must also be introduced. This was reported The equation to connect the magnetic susceptibility and the permeability in the zero frequency limit $kd \rightarrow 0$, where k is the wavenumber of the incident radiation and d is the interdipole separation is given by [28] in the coordinate system where the permeability and susceptibility are diagonalized

$$\chi_{m,j}^{ll} = \frac{3}{4\pi n} \frac{\mu_j^{ll} - 1}{\mu_j^{ll} + 2} \quad (22)$$

for the diagonal components ll , where $n=N/V$ is the number density of dipoles and N is the number of dipoles and V is the particle volume. Once we know the polarizability and susceptibility tensors denoted by $\tilde{\alpha}_j$ and $\tilde{\chi}_{m,j}$ respectively at each of the lattice sites j , then a set of self-consistent equations can be written for the electric and magnetic dipole moments P_j, M_j respectively ($j=1, \dots, N$) as

$$\begin{bmatrix} \tilde{\alpha}_j^{-1} P_j \\ \tilde{\chi}_{m,j}^{-1} M_j \end{bmatrix} = \begin{bmatrix} E_{inc,j} \\ H_{inc,j} \end{bmatrix} - \sum_{k \neq j} \begin{bmatrix} A_{jk}^{(ee)} & A_{jk}^{(eh)} \\ A_{jk}^{(he)} & A_{jk}^{(hh)} \end{bmatrix} \begin{bmatrix} P_k \\ M_k \end{bmatrix} \quad (23)$$

where

$$\begin{bmatrix} E_{inc,j} \\ H_{inc,j} \end{bmatrix} = \begin{bmatrix} E_0 \exp(i\vec{k} \cdot \vec{r}_j - i\omega t) \\ H_0 \exp(i\vec{k} \cdot \vec{r}_j - i\omega t) \end{bmatrix} \quad (24)$$

are the electric and magnetic fields of the incident radiation at lattice site j with the convention that $\vec{E}_0 \times \vec{H}_0 = \vec{k}$ where $\vec{k}$ is the wave vector of the incident radiation. It should be noted that the terms $A_{jk}^{(ee)} P_k$ and $A_{jk}^{(eh)} M_k$ are the contributions to the electric field at position j due to the electric and magnetic dipoles at position k and these expressions can be found in [28]. Similarly the expressions $A_{jk}^{(he)} P_k$ and $A_{jk}^{(hh)} M_k$ are the contributions to the magnetic field at position j due to the electric and magnetic dipoles at position k . The essence to cloaking is that the electric and magnetic dipoles are now coupled to each other. We showed in [25] that Eq. [23] can be written in the form;

$$\sum_{k=1}^N \begin{bmatrix} A_{jk}^{(ee)} & A_{jk}^{(eh)} \\ A_{jk}^{(he)} & A_{jk}^{(hh)} \end{bmatrix} \begin{bmatrix} P_k \\ M_k \end{bmatrix} = \begin{bmatrix} E_{inc,j} \\ H_{inc,j} \end{bmatrix} \quad (25)$$

which is now a set of $2N$ inhomogeneous linear complex vector equations. Due to certain symmetries in the $\mathbf{A}$ matrices both the FFT (Fast Fourier Transform) method and the CCG

(complex-conjugate gradient) method can be used to solve for the P_j 's and M_j 's just as in conventional DDA methodology.

Now let us consider how the coordinate transformations are done in ordinary Cartesian space where a point in this space is described by its Cartesian components x^i ($i=1,2,3$). Let us now consider a coordinate transformation that maps a point x^i to a point $x^{i'}$ (x^i) in which case a rank 2 contravariant tensor T^{ij} is transformed as:

$$T^{i'j'} = \left| \det(\Lambda_i^{i'}) \right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} T^{ij} \quad (26)$$

where

$$\Lambda_i^{i'} = \frac{\partial x^{i'}}{\partial x^i} \quad (27)$$

is the Jacobian transformation matrix. Since the permittivity and permeability are 2nd rank tensors, they will transform as:

$$\epsilon^{i'j'} = \left| \det(\Lambda_i^{i'}) \right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \epsilon^{ij} \quad (28a)$$

$$\mu^{i'j'} = \left| \det(\Lambda_i^{i'}) \right|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij} \quad (28b)$$

Pendry *et al.*[22] posited that there were two ways to interpret the above two equations. One was the topological interpretation where $\epsilon^{i'j'}$ and $\mu^{i'j'}$ are the properties of the same material in the transformed coordinate system, or the material interpretation where they can be considered as the permittivity and permeability of a “metamaterial” in a flat Cartesian coordinate system.

In [26] we considered a spheroidal region shown in green in the x-z plane with the rays in red and the orthogonal wave fronts in blue which has both vacuum inside and outside the region. We then applied a coordinate transformation similar to the one used by Schurig, et al.[23] and transformed the region shown in Fig. 4a into the region shown in Fig. 4b. Using the transformed permittivity and permeability tensors in the modified DDA code we obtained the following solution shown in Fig. 5. As can be seen from the figure, the cloaked region is completely hidden from the external field and the wave fronts are completely reconstructed which will yield total invisibility of the object! Some nice videos of this process in slow motion can be seen at <https://drive.google.com/folderview?id=0B9tMvcOZYzL7LVgteFRQN3ktTVU&usp=sharing>. It should be noted that the permittivity can have values much less than unity in the cloaked region which implies that the phase velocity of light can far exceed the speed of light in vacuum! This will be seen in the videos if you concentrate on the inner boundary of the cloak.

5. Radiative transfer in a coupled atmosphere-ocean systems

To conclude this peregrination, I would like to discuss several aspects of ocean optics that are highly relevant to the preservation and health of our oceans. We have done a great deal of calculations using Monte Carlo techniques to couple the atmosphere-ocean system using a realistic stochastic interface with a complete Stokes vector-Mueller matrix treatment. One of the first of these was done by Kattawar and Adams [29]. Some researchers ask why do we want to complicate the situation by using the full vector treatment of radiative transfer (RT). The simple answer is that it is the only *correct* way to do the RT calculations. If one simply does scalar RT then the results are virtually assured to be incorrect and the degree of error really depends on how strong the polarization properties are of the single scattering phase or Mueller matrix. Discrepancies in the two types of calculations were first done for Rayleigh scattering by Adams and Kattawar [30] using invariant imbedding techniques to show that substantial errors could be incurred if one simply did scalar RT. In fact errors in excess of 11% could be incurred in the reflected radiance and in excess of 17% in the transmitted radiance for certain situations.

I had the privilege and pleasure to be selected by the Office of Naval Research to participate in a very large program called RaDyO (Radiance in a Dynamic Ocean which officially began Oct. 1, 2005). The program manager was Steve Ackleson who was at ONR at the time and the lead-scientist was Tommy Dickey both extremely competent ocean optics researchers. The objectives of the program were to a) investigate the time-dependent radiance distribution induced by the stochastic boundary layer, b) to explore the effects of foam, bubbles, etc. on the surface on the upper layer radiance distributions, and c) to see how one could invert the data to get boundary layer properties. My group's charge was to be able to model the dynamic ocean surface and then be able to calculate the time dependent equation of transfer for this complicated system. This necessitated a good 3D vector RT model capable of doing calculations on a millisecond time scale. We first developed an impulse response method that was capable of calculating the impulse response for both the atmosphere and the ocean models. The *raison d'être* was the fact that on the timescale of milliseconds, only the wave surface layer changed while the atmosphere and ocean layers remained constant. The impulse response for the surface waves was calculated using the Fresnel equations and then the impulse response for the three coupled layers was computed using a 3D extension of the 1D matrix operator method which was discussed in section 2 of this essay. We termed this method the Hybrid Matrix Operator-Monte Carlo method (HMOMC) [31,32]. To insure that the finite 3D volume accurately simulated the real system, we imposed periodic boundary conditions for the volume element so that radiation leaving the side of the region was allowed to reenter the opposite side in the same direction. A code of this complexity needs rigorous validation so we set up the following test case. We took an atmosphere of optical depth $\tau=0.25$ and single scattering albedo

$\omega=0.50$ overlying an ocean with optical depth $\tau=1.0$ and single scattering albedo $\omega=0.50$ with both layers exhibiting scattering according to a Rayleigh phase matrix. The ocean bottom albedo was taken as $A=1.0$ to maximize bottom reflections. The interface was a smooth ocean surface with zero wind velocity and refractive index $n=1.338$. The model is shown in Fig 6 where a solar incident angle of 60° was used. We chose the multicomponent approach, which is a 1D model, to check the validity of the results [33]. The comparisons are shown in Fig 7 for both the I and Q components of the Stokes vector. As can be seen the agreement is very good between the two very disparate calculations. We were now able to use the matrix operator theory alluded to earlier in this essay to add the dynamic ocean surface layer in a very fast and accurate way.

The quantity that was measured in the experiment was the down-welling plane irradiance which we denoted by $E_d(t)$ and complete details of the calculations and model can be found in [34]. In the ocean model, a three-component system was assumed for the ocean layer with pure seawater suspended particles and colored dissolved organic matter (CDOM) (also known as gelbstoff). The inherent optical properties (IOPs) of the ocean used as input to all simulations were derived from measurements in the Santa Barbara Channel on 11 September 2008 and included the beam attenuation coefficient of pure seawater, the single scattering albedo of pure seawater, the particulate and CDOM beam attenuation coefficient, the single scattering albedo of particles and CDOM, and the particulate backscattering ratio, b_{bp}/b_p . The Fournier–Forand scattering phase function with the corresponding value of b_{bp}/b_p was used for the particulate scattering in the ocean [35]. In modeling the wave surface we did not include swell nor capillary waves with wavelength less than 1 cm. When wavelengths less than 1 cm were included, no significant change in the irradiance statistics were detected. In Fig 8 we show a comparison of the modeled and measured probability density functions for the normalized down-welling irradiance $E_d(t)/\langle E_d \rangle$ for a wavelength of 532 nm and at surface depths of 0.86 m, 1.70 m, and 2.84 m. As can be seen the agreement is quite good considering the fact that a lot of the data used in the model came from wave slope simulations derived from wind speeds. I consider this a marvelous achievement of a symbiosis between the Monte Carlo method and matrix operator theory.

6. Conclusion

This essay was meant to highlight some topics I thought were notable and to show the respect and admiration I had for Henk who inspired my way of thinking about many physical problems. I would also like to make a point of note for all young researchers and that would be to take the time to read some of the papers and books that were written by the “giants” in our field and to not simply copy references without reading them. If one does this then one will soon discover

that there are very few “new” ideas in any field but more often than not just variations on a theme. This point was driven home to me by the famous Peter Debye when he was in his early eighties and I had the privilege of having a one-on-one discussion with him about some of my research. As I was excitedly explaining my “new” ideas to him he looked like he had fallen to sleep so I just stopped talking and he opened his eyes and said: “Young man what you consider as new ideas were ideas that can be found in articles by”... and he started to rattle off in almost encyclopedic order the researchers and the old German literature where those articles could be found with the same “new” ideas I was explaining to him. This was a shock to my ego and I will always remember those prophetic remarks!

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Figures

Fig. 1 A picture of Henk and me at my home when he visited TAMU ca 1994

Fig. 2 A schematic representation of the reflection and transmission operators for a two layer system as well as the downward Γ^+ and upward Γ^- radiances at the boundaries of the two layers

Fig. 3 Schematic showing the projected coordinates used in the formulation of the distribution function

Fig. 4. Application of the coordinate transformation to a spheroidal region subject to a plane-wave radiation propagating in the z -direction. Shown in the figures is the situation in the x - z plane, with the x - and z -axes as depicted in Fig. (a). The scales are in unit of λ . Figure (a) shows the closed region (green region) in Cartesian coordinates, with vacuum both inside and outside. The light rays (dark red lines) and wave fronts (dark blue lines) of the incident radiation are also shown. Figure (b) shows the transformed region with aspect ratio of 2, with the material properties in the cloaking region determined by Eq.(28a) and Eq.(28b). The light rays and wave fronts transform accordingly.

Fig. 5 Results of the DDA calculation showing how effectively the wave front is reconstructed beyond the cloaked region rendering the region enclosed by the cloaking “invisible”

Fig. 6 A schematic representation of the three layer model (atmosphere, ocean interface, and ocean) used to test the validity of the HMOMC model we developed

(a)

(b)

Fig. 7. (a) Diffuse radiance distribution for the three layer system compared to the MCA method. The detector is placed below the dielectric interface to measure the downwelling radiation. The optical thickness between the dielectric interface and the detector is 0.1. (b) The Stokes parameter Q for the same system.

Fig. 8. Modeled and measured probability density functions of the normalized downwelling irradiance $E_d(t)/\langle E_d \rangle$ at $\lambda = 532$ nm for the three near-surface depths as indicated.