James Clerk Maxwell: life and science

Philip L. Marston

Physics & Astronomy Department,

Washington State University, Pullman, WA 99164-2814, USA

Email: marston@wsu.edu

Abstract: Maxwell's life and science are presented with an account of the progression of Maxwell's research on electromagnetic theory. This is appropriate for the International Year of Light and Light-based Technologies, 2015. Maxwell's own confidence in his 1865 electromagnetic theory of light is examined, along with some of the difficulties he faced and the difficulties faced by some of his followers. Maxwell's interest in radiation pressure and electromagnetic stress is addressed, as well as subsequent developments. Some of Maxwell's other contributions to physics are discussed with an emphasis on the kinetic and molecular theory of gases. Maxwell's theistic perspective on science is illustrated, accompanied by examples of perspectives on Maxwell and his science provided by his peers and accounts of his interactions with those peers. Appendices examine the peer review of Maxwell's 1865 electromagnetic theory paper and the naming of the Maxwell Garnett effective media approximation and provide various supplemental perspectives. From Maxwell's publications and correspondence there is evidence he had a high regard for Michael Faraday. Examples of Maxwell's contributions to electromagnetic terminology are noted.

Keywords: James Clerk Maxwell, Electromagnetic theory of light, Optical radiation stress, Kinetic theory of gases, Theistic perspectives on science, Maxwell's demon

1. Introduction

James Clerk Maxwell (1831-1879) is frequently remembered in the 21st century for his 1865 publication concluding, “Hence electromagnetic science leads to exactly the same conclusions as optical science with respect to the direction of the disturbances which can be propagated through the field; both affirm the propagation of transverse vibrations, and both give the same velocity of propagation”, [1] and for his extended presentation of electromagnetic theory [2]. These, and his related body of work, served to join the sciences of optics, electricity, and magnetism. This accomplishment should also be remembered in the context of very limited information concerning the subatomic nature of electricity and the absence of a convincing demonstration of electromagnetic wave propagation for most of the decade following Maxwell’s death in 1879 [3, 4]. Maxwell’s prediction in 1873 of the true nature and magnitude of optical radiation pressure [2] was similarly not verified experimentally for several decades. (See Section 4.)

Maxwell’s contributions to other areas of physics were widely appreciated by 1880: his friend Tait [5] asserted that Maxwell “...had no rival...in the whole wide domain of molecular forces...” in connection with his initial and advanced versions of his statistical kinetic theory of gases and his masterful measurements of the viscosity of gases. (See Section 6.) As was the case with Maxwell’s theories of electromagnetic waves and optical radiation pressure, several decades elapsed prior to the experimental verification of velocity distribution for molecules in gases predicted by Maxwell in 1859 [6] and eventually known as the Maxwell-Boltzmann distribution. From the aforementioned contributions to physics and numerous other contributions to science and engineering (of which only selected examples can be mentioned in the present article), Clerk Maxwell’s name is familiar to generations of scientists and engineers. Often lacking, however, is an appreciation for the combined depth and breadth of his thought and personality in the context of 19th century science and culture. The present article introduces and reviews aspects of James Clerk Maxwell for the benefit of those desiring a deeper appreciation. Some previously unaddressed contextual considerations are also examined.

While the *breadth* of Maxwell's interest is implicit in the present Introduction, the *depth* of his thought impressed his peers. In the present discussion of Maxwell and his science, I have endeavored to convey those attributes by including topics I presented at *Electromagnetic and Light Scattering-XV-2015*, in addition to some relevant background. Some of the resources for, and the complexity of, a study of this type are evident from the considerations given below. The task of accessing the majority of Maxwell's publications was simplified by the 1890 compilation (edited by W.D. Niven) of Maxwell's *Scientific Papers* [7] and editions reprinted in the 20th and 21st centuries. Reprint editions of Maxwell's main books [2, 8, 9] have also remained available. From 1990-2002 some additional publications by Maxwell were reprinted together with a considerable amount of his correspondence in volumes edited by the science historian Harman [10, 11, 12]. Even prior to Harman's efforts, Everitt [13] had drawn attention to significant omissions from Niven's compilation. For biographical information, the 1882 *Life* (by Campbell and Garnett)[14] and the shortened 1884 2nd edition [15] remain useful and their acquisition is greatly simplified by the recent availability of reprints. (The 1884 edition corrects some of the minor errors and adds some useful material, but omits some of Maxwell's poetry and Garnett's survey of Maxwell's "Contributions to Science.") The present author remembers the era in which these volumes were "out of print" such that their acquisition was a significant accomplishment and expense. In 1896 a short biography appeared by R.T. Glazebrook [16], one of the students whose research was guided by Maxwell at Cambridge in the late 1870s. (Glazebrook went on to become a major leader in British applied physics for the first third of the 20th century. At the end of his Preface he recalls that Maxwell was "deeply loved by all who knew him.") In the 1960s and early 1970s C.W.F. Everitt, a physicist educated in London, had the foresight to prepare a short modern biography [17]. Harman gave a biographical introduction in each of his volumes [10, 11, 12]. Several monographs pertaining to Maxwell or selected aspects of Maxwell's thought have been published, for example [18, 19], followed by a collection of perspectives concerning Maxwell in 2014 [20].

In addition to the breadth of Maxwell's interest and thought, one of the difficulties in compiling his writings is that he lived in an era in which book reviews and short essays and poems were occasionally published anonymously, and records of authorship are currently unavailable. In an introduction to Niven's compilation [7] Norman Lockyer (an early editor of the journal *Nature*) is acknowledged for assistance in the selection of articles reprinted from *Nature*. Inspection of the original articles show that various book reviews by Maxwell originally appeared anonymously. By implication from the wording in the aforementioned introduction, Maxwell wrote additional items published in *Nature*, not included in Lockyer's selection. It has been found that Maxwell received minor financial compensation for some services rendered to *Nature* [21]. Consequently it is not surprising that there is good evidence that Maxwell wrote a substantial anonymous review published early in 1874 of Balfour Stewart's book on the *Conservation of Energy* [22]. That finding is significant for supplementing other available information concerning Maxwell's synthesis of Christian Biblical awareness of creation with the findings of physical science [21, 23]. See Sections 8 and 9.

2. Chronological summary pertaining to Maxwell's life

To appreciate the context and chronology of Maxwell's research, the following summary of his life will be helpful:

1826: Maxwell's parents John Clerk Maxwell and Frances Cay were married in Edinburgh, Scotland.

1829: Their daughter Elizabeth was born but dies soon in infancy.

1831 (13 June): James is born in Edinburgh.

1830s: John Clerk Maxwell develops the farm of Glenlair acquired through inheritance. (Glenlair is in southwest Scotland, inland from the towns of Dumfries, Dalbeattie, and Castle Douglas.) James is at Glenlair for most of his childhood, and he is often there during summers beyond childhood. The main residence is shown in Fig. 1.

1839 (December): Maxwell's mother Frances dies of painful cancer at the age of 47. Prior to this time James had been educated to some extent by both of his parents. Following his mother's death a tutor is brought in with an unsuccessful outcome.

1841-47: Beginning in autumn of 1841 James is educated at Edinburgh Academy, residing primarily during school terms at the residence of John's widowed sister (Isabella Wedderburn) in Edinburgh. Soon after entering the Academy, James makes the acquaintance of two of his long time friends: his future biographer, Lewis Campbell, and the future physicist, P.G. Tait, who are also enrolled there.

1847 (November): Maxwell becomes a student at the University of Edinburgh, immediately enrolling in a class in Logic from the renowned philosopher and teacher, Sir William Hamilton. While at this university he benefits from the Natural Philosophy instruction of James Forbes and is provided access to instrumentation used in demonstrations. He also benefits from instruction in mathematics from Philip Kelland. Maxwell remains a student at the University of Edinburgh through the spring term of 1850.

1850 (Autumn) – 1854 (Spring): Maxwell enrolls at Cambridge University, initially at Peterhouse College, transferring to the much larger Trinity College after the first term. He completes his Bachelor's degree requirements in spring 1854. In a specialized mathematical examination (the Smith's prize examination of 1854) he is introduced to the integral relation, eventually known as Stoke's theorem. Maxwell ties E. J. Routh for highest honors in that examination.

1854 (Autumn) – 1856 (Spring): Maxwell remains at Trinity College, Cambridge, being elevated to Fellow in 1855 and being awarded an MA degree in 1857.

1856 (3 April): John Clerk Maxwell dies at Glenlair following a progressive illness. James is present and takes on the responsibility associated with Glenlair.

1856 (Autumn) – 1860 (Spring): Maxwell holds the Chair of Natural Philosophy, Marischal College, Aberdeen. His chaired position is designated redundant upon the merging of Marischal College with King's College Aberdeen. (Aberdeen is approximately 170 km north of Edinburgh and 275 km north of Dumfries.)

1858 (2 June): James weds Katherine Mary Dewar (*circa* 1824 - 1886) in Aberdeen [20]. Katherine is the daughter of Principal Daniel Dewar of Marischal College. James and Katherine had no children.

1859: Publication of Maxwell's Adam's Prize award essay *On the Stability of the Motion of Saturn's Rings*.

1860 (Autumn) – 1865 (Spring): Maxwell holds the professorship of Natural Philosophy, King's College London.

1865 (Summer) – 1871 (Summer): Maxwell retires from King's College London, at least partially for reasons of health, though the Maxwells retain their place of residence in London at least through April 1868. Much of the time the Maxwells are at their rural home in Glenlair. In spring 1871 Maxwell accepts the newly established Professorship in Experimental Physics at Cambridge.

1871 (Autumn) – 1879: The Maxwells establish a residence at 11 Scroope Terrace in Cambridge, which is only a short walk (approximately 900 m) from the future location of the Cavendish Laboratory of Physics. The Laboratory, headed by Maxwell, officially opens in June 1874. Maxwell's appearance at about that time is depicted in Figures 2 and 3.

1879 (5 November): Maxwell dies of a painful stomach cancer. Glenlair passes to heir-in-law Andrew Wedderburn-Maxwell (brother of Jemima Wedderburn-Blackburn).

1879 (December) – 1884 (December): Lord Rayleigh replaces Maxwell as the Professor of Experimental Physics, Cambridge.

Maxwell's family background is also discussed elsewhere [14, 15, 17, 24]. Everitt analyzes aspects of Maxwell's personality [24], though information recently released is also helpful. For example, Maxwell's cousin Jemima Blackburn in her *Memoirs* (composed *circa* 1900 but not published until 1988) described Maxwell as follows [25]: "He was truly religious and had the sweetest temper of any man I ever knew. I do not think I ever saw him angry or heard him say a word against anyone." Jemima lived in the same dwelling as Maxwell in the 1840s since her mother was Isabella Wedderburn.

3. Maxwell's research on electromagnetic science and electromagnetic waves

During Maxwell's undergraduate years at Cambridge, mathematical approaches to electricity and magnetism had not been incorporated as a topic suitable for emphasis. Even among British natural philosophers most interested in electricity there was disagreement concerning the applicability of the inverse-square force law in *electrostatics* (commonly known as Coulomb's law of electrical action)[26]. Consequently Maxwell's decision by 1854 to give serious attention to the mathematical aspects of electricity and magnetism involved significant personal initiative. While the former Cambridge student George Green (1793-1841) had made brilliant theoretical contributions, his premature death delayed the impact of his thinking; a reprinting of Green's collected work was not available until 1871. Furthermore, there appears to have been little motivation provided in lectures and a lack of facilities for students to explore experimental aspects of electricity and magnetism.

Everitt has made the case that the underlying British research contributing to Maxwell's development of electromagnetic theory was from Michael Faraday (1791-1867) and William Thomson (Lord Kelvin, 1824-1907) [13]. Everitt aptly describes Faraday as an "accumulative thinker", reporting the results of years of experiments in his multi-volume *Experimental Researches*. Having introduced ideas and diagrams associated with lines of force, Faraday introduced the term "field" in this context, though he lacked the abilities to propose an underlying mathematical theory. (In 1857 Faraday described himself as "only an experimentalist" [27].) By the early 1850s Thomson had related lines of force concepts to available theory of electrostatics and magnetostatics but evidently lacked Maxwell's insight and motivation to develop a suitable theory of coupled electricity and magnetism. Everitt has again aptly described Thomson as an "inspirational thinker", in contrast to Maxwell's significance as an "architectural thinker". When examining Maxwell's importance, notice the evolution of his thought over two decades and the magnitude of the contributions in his successive papers and his 1873 *Treatise*. In the present summary it is only possible to mention some aspects of these papers. Subsequently

in Section 5 some of the difficulties faced by Maxwell and his followers will be mentioned.

1855, 1856: "On Faraday's lines of force" (Transactions of the Cambridge Philosophical Society) [28]: By mid-autumn 1855 Maxwell had made sufficient progress to arrange to read a paper on his results to the Cambridge Philosophical Society on Monday, 10 December 1855. Early in his essay Maxwell remarks on the usefulness of "physical analogies", by which he means "that partial similarity between the laws of one science and those of another which make each of them illustrate each other." In his introduction he goes on to explain his planned analogy between the motion of an incompressible fluid and the "direction" and "intensity" of Faraday's lines of force, and goes on to develop these ideas in Part I for various situations. Part II, subtitled "On Faraday's "Electro-tonic State", was summarized in Maxwell's presentation of Monday 11 February 1856. Here he introduces a mathematical basis for Faraday's electromagnetic induction of currents and associated fields through an extension of Faraday's terminology. Maxwell's "electro-tonic functions" corresponds to the Cartesian components of the modern vector potential $\mathbf{A}$ (terminology not introduced by Maxwell until 1873.) Without the ease of modern vector calculus, Maxwell obtains the modern electrodynamic relation $\mathbf{E} = -\partial \mathbf{A} / \partial t$ for the electric field in situations where there is no scalar potential contribution [29]. Again he goes on to examine applications. He discusses aspects of the research of Wilhelm Weber (1804-1891), whose research was often carried out in Goettingen or Leipzig [30]. Maxwell also mentions Weber's modification of the Coulomb force between moving charges e and e' (measured in electrostatic units) separated by a distance r :

$$\text{Force} = (ee'/r^2)[1 + a (dr/dt)^2 + b r d^2 r/dt^2]. \quad (1)$$

Here Maxwell's notation has been put in modern form, and the coefficients a and b (left unassigned by Maxwell) will be mentioned subsequently. From Maxwell's

correspondence with Faraday late in 1857 [31], Maxwell continued to think about “lines of force” during the following years at a time when his publications concerned other areas of physics. (See Sections 6 and 7.)

1861, 1862: “On Physical Lines of Force” (Philosophical Magazine) [32]: This set of publications introduces concepts associated with a further shift in Maxwell’s thinking towards physically based reasoning. While perhaps the most famous of these is Maxwell’s analogy of electrical vortices and electric particles, the set also introduces the notion of stresses (embodied in the Maxwell stress tensor) and of the displacement current. The vortex analogy results in an elastic medium supporting transverse vibration for which the velocity could be obtained through the comparison of electrostatic and electromagnetic units of charge [10, 17, 32]. This was a more difficult process than conveyed in some 21st century discussions because the best available comparisons were associated with Weber’s research program. The comparison could be expressed in terms of the coefficients a and b in Eq. (1) since those terms were associated with currents and consequently with magnetic interactions. Weber’s formulation of electrodynamics was such that a and b were related to a velocity through the condition - $a = b/2 = 1/c_w^2$, where c_w has the dimensions of a velocity [10]. Expressed here in terms of c_w , the best available measurements cited by Maxwell were from an 1857 publication of Weber and Kohlrausch which gave $c_w = 4.3945 \times 10^8$ m/s. From Maxwell’s model, however, the velocity of the transverse wave was $c_w/\sqrt{2} = 3.1074 \times 10^8$ m/s which was close to the known velocity of light in air [10, 32]. Additional topics addressed by Maxwell’s model include the *Faraday effect* in which the plane of polarization is rotated for optical propagation in certain materials when a static magnetic field is applied along the direction of propagation.

1863: “On the Elementary Relations between Electrical Measurements,” by J. Clerk Maxwell and Fleeming Jenkin (British Association Report) [33]: This significant paper is often overlooked having been omitted both from Niven’s compilation of Maxwell’s papers [7] and Harman’s [10, 11, 12]. This report, co-authored with

Fleeming Jenkin (1833-1885), is an outgrowth of Maxwell's service on the British Association for the Advancement of Science (BAAS) Committee on Electrical Standards (1861-1877). It includes a review of electrical measurements beyond those of Faraday and of Weber (such as those of Ampere) and issues pertaining to electrical measurements raised by Joule, W. Thomson, and Helmholtz. It also introduces the dimensional analysis of electrical quantities in a form similar to the tabulations of Maxwell in subsequent publications.

1865: "A Dynamical Theory of the Electromagnetic Field" (Philosophical Transactions of the Royal Society) [1, 34]: This publication was submitted in October 1864 and read (at least in summary form) to the Royal Society in December 1864. William Thomson's review report is quoted here in Appendix A. The manuscript was sent to the printer in June 1865 [35]. The remembrance and central conclusion of this publication were mentioned in Section 1. In Maxwell's Part VI on the "Electromagnetic Theory of Light" he simplifies his earlier general formulation with the objective: "We now proceed to investigate whether these properties of that which constitutes the electromagnetic field, deduced from electromagnetic phenomena alone, are sufficient to explain the propagation of light through the same substance." His simplification includes the assumption of propagation in a homogeneous isotropic medium not containing sources. In modern notation, if $\mathbf{H}$ denotes the magnetic intensity and $\mathbf{n}$ denotes a unit vector in what is eventually demonstrated to correspond to the direction of wave propagation, Maxwell [in his Eq. (62)] arrives at the transverse field condition $\mathbf{H} \cdot \mathbf{n} = 0$. Making the additional simplification of a "perfect dielectric" having "no true conduction" while retaining what in modern terminology corresponds to displacement currents, Maxwell arrives at a set of wave equations for the components of $\mathbf{H}$ where μ is the relative permeability of the medium. In modern vector notation his Eq. (69) becomes the wave equation:

$$\nabla^2 \mathbf{H} = 4\pi \partial^2(\mathbf{H})/\partial t^2, \quad (2)$$

where the coefficient k involves the ratio of electrostatic and electromagnetic units (previously mentioned and discussed again subsequently) and the “Specific Inductive Capacity” D that corresponds to the modern relative permittivity. Maxwell finds in his Eq. (80) that the refractive index i (in his notation) is such that: $D = i^2/$ in agreement with modern results [29]. Earlier in his Section 97 Maxwell had concluded from the predicted velocity for the case of propagation in air (taking $i = 1$) with the corresponding value of k : “The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.”

In commemorating this paper it is essential to notice that Maxwell does not rely directly on his vortex model of 1861-1862 when formulating his analysis and that for coupled time-dependent electromagnetic processes his formulation is much broader than the specific application to the electromagnetic theory of light in isotropic dielectric media. For example propagation in “Crystallized Medium” (that is, anisotropic crystals) and the relation between “Electric Resistance and Transparency” are examined in his Sections 102-105 and 106-107. Maxwell recognizes that the degree of observed transmission of green light through a thin gold foil greatly exceeds the amount expected according to his formulation. (From a modern perspective his analysis in this case has various deficiencies, though he correctly anticipates that “the rapidity of the vibrations of light” is relevant. He improved his analysis in his *Treatise* [2]. For a modern discussion, see [36].) Maxwell may have considered the case of transmission through a gold foil because in 1857 Faraday had sent Maxwell a copy of his Bakerian lecture concerning the optical properties of gold [27, 31]. In the limit the material resistivity diverges, Maxwell’s Eq. (95) in his Section 106 reduces to the wave equation for the transverse component of the vector potential. Bork [37] gives flow charts pertaining to Maxwell’s derivation of electromagnetic wave equations in this paper and for his subsequent relevant publications. In recent discussions of a historical

nature, Wu and the Nobel laureate C. N. Yang have emphasized Maxwell's insight in associating the vector potential $\mathbf{A}$ with *electromagnetic momentum*, the name assigned to this quantity in the 1865 paper [38].

1868: "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force; With a Note on the Electromagnetic Theory of Light" (Philosophical Transactions of the Royal Society) [39]: The primary objective of this paper was to make a comparison of the aforementioned system of units independent from the measurements of Weber and Kohlrausch by the use of a force balance involving the electromagnetic repulsion between two circular coils and the electrostatic attraction between two conducting disks. This concerns the coefficient k in Eq. (2) and the resulting prediction of the velocity of light in air according to electromagnetic theory was 2.88×10^8 m/s; however, that prediction was directly dependent on the accuracy of the British Association Standard for electrical resistance determined by the committee on which Maxwell was an active member. Rayleigh, during the period in which he succeeded Maxwell's chair at Cambridge, determined that the British resistance standard from the 1860s contained a systematic measurement error [40]. The error affects the predicted value for the velocity, shifting it in closer agreement with the modern value. In a Note near the end of his 1868 paper Maxwell gave an alternative derivation of the wave equation for the transverse component of magnetic intensity in a propagating wave, a result analogous to the one here in Eq. (2). The derivation is important in that it involves reasoning based on Faraday's law and the Maxwell-Ampere law (without conduction currents) in integral form [37]. Though the wave equation for the electric field is not displayed, the results for the components of the propagating wave, Maxwell's Eq. (21) therein, clearly imply that the electric field is orthogonal to the magnetic intensity and to the direction of propagation.

1873: A Treatise on Electricity and Magnetism (Oxford University Press) [2]: This book is partitioned into four parts and bound in two volumes for all of the 19th century editions:

Volume I: Preliminary: On the Measurement of Quantities; Part I: Electrostatics; Part II: Electrokinematics.

Volume 2: Part III: Magnetism; Part IV: Electromagnetism.

Chapter numbers are sequenced separately for each *Part* while *Article* numbers are sequential for the entire publication and uniform for all editions. Modern readers will need to study the notational conventions introduced in Articles 25 and 26 such as the operator ∇^2 being defined as the negative of the modern value (while in [1] ∇^2 takes on the present meaning as here in Eq. (2)). In Chapter III of Part III, Article 528, Maxwell makes his objective in writing the *Treatise* clear. After explaining that it was beneficial for Faraday to have expressed his ideas in “natural, untechnical language” Maxwell explains: “It is mainly with the hope of making these ideas the basis of a mathematical method that I have undertaken this treatise.” In Article 82 when discussing the concept of a “tube of induction” associated with Faraday’s ideas, Maxwell introduced the modern term *solenoid* (from the classical Greek σωλήν for tube or pipe), though the terminology “solenoidal condition” was introduced earlier in Article 21. The term “vector potential” is introduced in Article 405. Material most closely connected with Maxwell’s contributions to dynamical electromagnetics is distributed in Chapters V – XXIII of Part IV. Chapter IX (General Equations of the Electromagnetic Field, Articles 604-619) contains the coupled set of equations for fields and potentials associated with the modern “Maxwell’s equations.” Chapter XX (Electromagnetic Theory of Light) contains the main results pertaining to waves, though Maxwell’s remarks in Chapter XXI (Magnetic Action on Light) reveal his confidence in his formulation by 1873. For example, in Article 830 he downplays his model of the Faraday effect, (based on his molecular vortex model of 1861-1862) in comparison with his electromagnetic wave model (which he takes to be independent of his vortex model). He summarizes: “We must therefore regard any coincidence with observed facts as of much less scientific value in the theory of the magnetic rotation of the plane of polarization than in *the electromagnetic theory of light, which, though it involves hypotheses about the electric properties of media, does not speculate as to the constitution of their molecules.*” (Emphasis added.) By

1873 Maxwell evidently regarded his displacement current to be a “hypotheses about the electric properties of media.”

In Chapter XX Articles 790 and 791 Maxwell illustrates the results of his analysis of “plane-polarized light” propagating in isotropic media, Fig. 4. The diagram shows the spatial relationship of the electromagnetic fields “at a given instant” along a ray. It is perhaps the most important diagram in 19th century theoretical physics. Though the corresponding results were implicit in his aforementioned paper of 1868, he evidently felt that a diagram would be helpful. Wave equations involving the vector potential are derived in Articles 783, 784, and 790. Maxwell’s *Treatise* is unique in its era for the quality of the illustrations he provided. Some of these concern lines of force and the associated curved equipotential surfaces. One of these is reproduced in Fig. 5. Another important innovation in Maxwell’s *Treatise* was his application of Lagrangian techniques to electromagnetic systems, in addition to his prior discussions of his 1865 paper. His formulation is important for incorporating coupling terms in a dynamical theory in a way that minimizes the incorporation of specific physical hypotheses. The *Treatise* contains much more than the formulation associated with the electromagnetic theory of light as reflected in numerous calculations of inductance and associated instrumental methods pertinent to absolute electrical standards.

4. Maxwell stresses, radiation pressure, and radiation torque

In Articles 105-111 and 641-646 of his *Treatise* [2] Maxwell expands upon the topic of electromagnetic stresses beyond the level of his 1861-1862 papers. In Articles 792 and 793 Maxwell applies the results of Articles 107 and 643 to electromagnetic waves “falling on a thin metallic disk, delicately suspended in a vacuum” and concludes that the “stress of radiation” “might perhaps produce an observable mechanical effect”. He correctly anticipates the difficulty of such experiments; unambiguous observations demonstrating optical radiation forces finally became available in about 1900 [41, 42], and interest in using forces of that type to manipulate small objects has grown by using beams from continuous-wave lasers

[43]. Relatively recently it has been realized that for some objects in beams intersecting with a sufficiently large intersection angle, the momentum transfer is such that the object is attracted along the axis of the specially designed beam (as opposed to the usual case of repulsion) [44, 45]. This is fully consistent with Maxwell's formulation. An analogous situation has been predicted and observed for acoustic beams [46, 47, 48].

During Maxwell's final years, John Henry Poynting (1852-1914) carried out research in Cambridge under Maxwell's direction [48, 49]. In 1884 Poynting published his now famous general analysis of energy flow in electromagnetic fields [50], though he didn't make the effort to document all of the examples he had considered [40]. By 1909 Poynting suggested that a beam of circularly polarized light carries angular momentum along the beam's axis [51, 52], and Beth experimentally demonstrated that in 1936 [53]. The radiation torque on a sphere illuminated by circularly polarized light has been evaluated using Maxwell's stress tensor [54], and the torque has been interpreted in terms of projections of electromagnetic spin and orbital angular momentum [48, 55, 56]. These results are consistent with Maxwell's formulation and make use of a related approach introduced by Humblet in 1943 [57]. In the case of acoustical wavefields in fluids, the transport of angular momentum along the axis of a beam is associated with "acoustical vortices" and partially analogous effects have been predicted and observed [58, 59]. Maxwell stresses have also been considered for complicated "optical vortices" [60]. Reasoning based on Maxwell stresses also remains important in the *teaching* of electrodynamics [61].

5. Early electromagnetics after Maxwell, the Maxwellians, and "Maxwell's equations"

When considering the difficulties British researchers had in the 1880s in understanding Maxwell's formulation of electromagnetics, it is helpful to remember the issues and ideas that were unresolved in 1879. The physical processes associated with the conduction of electricity in metals were not understood even

classically until after the discovery of the small mass of the mobile charge carrier (the electron) and formulation in about 1900 of the Drude-Lorentz model of the conductivity of metals [36, 62, 63]. While Maxwell was clearly curious about the underlying physics behind *Ohm's law* (*Treatise* [2], Article 241), no record has survived suggesting he attempted to quantitatively model methods for *generating* electromagnetic waves. When formulating propagation problems in terms of potentials, Maxwell had neglected to utilize the simplification associated with (in the terminology of the late 20th century) the Lorenz gauge condition on the vector potential $\mathbf{A}$ [29, 64, 65, 66]. (By 1873 Maxwell had noticed Lorenz's 1867 publication [64] that, while not mentioning Maxwell, arrives at the gauge condition by a line of reasoning differing greatly from modern presentations [29, 65]. For Maxwell's awareness, see Article 805 of [2].) Though Maxwell had anticipated and contributed to some of the developments related to vector calculus, probably because of the influence of his friend P. G. Tait he had expressed aspects of his formulation in his *Treatise* [2] in terms of quaternions. The American Willard Gibbs (1839-1903) realized that the vector portions of the quaternions were especially relevant to Maxwell's formulation and by the early 1880s Gibbs had started to develop vector analysis [67]. Concerning the physics of dielectric media, Maxwell probably never saw the 1879 discussion by Clausius of the Clausius-Mossotti relation [68]; in addition, the modern understanding of dispersion relations for dielectric media, relevant to testing electromagnetic wave theory, was arrived at many decades later [29, 68]. From the comparison of "optical" with quasistatic "electrical" properties of dielectrics in Articles 788 and 789 of the *Treatise* [2] Maxwell expressed concerns associated with the frequency dependence of those properties. He correctly asserted, "... our theories of the structure of bodies must be much improved before we can deduce their optical properties from their electrical properties."

In the 1880s much of the innovative electromagnetic research in Britain was led by G. F. FitzGerald (1851-1901), Oliver Lodge (1851-1940), and Oliver Heaviside (1850-1925) [4, 30, 66, 69, 70]. Heaviside introduced the term "Maxwellians" to describe those involved in developing and extending Maxwell's ideas concerning

electromagnetics [66, 69]. They began work subject to the difficulties noted in the previous paragraph and others not considered here, and Heaviside became influential for having recognized the value of incorporating vector notation [67]. Inspection of Lodge's popular overview [71] suggests that Lodge failed to appreciate Maxwell's transition *away from* the molecular vortex models of 1861-1862. Lodge would have been unaware of Maxwell's 1867 letter to Tait indicating that [72] "the nature of this [molecular vortex] mechanism is to the true mechanism what an orrery is to the solar system" and that his 1865 paper "is built on Lagranges Dynamical Equation and is not wise about vortices." The identification and development of the modern "Maxwell's Equations" can to some extent be credited to Heaviside and Heinrich Hertz (1857-1894) [3, 30, 66]. Additionally, Maxwell had difficulties with signs that have been discussed both by early and recent investigators [18, 30, 66, 73].

6. Maxwell's research on kinetic theory of gases, the dynamics of molecules, and "Maxwell's demon"

By spring of 1859 Maxwell noticed a recent paper by Clausius, "On the mean length of the paths of separate molecules of gaseous bodies...", and undertook a probabilistic approach to the kinetic theory of gases. (Perhaps coincidentally, in 1857 Maxwell had been studying the 18th century Christian classic *Butler's Analogy* [21, 74] having a strong emphasis on qualitative probabilistic reasoning.) Maxwell described his approach in a letter to Stokes (at Cambridge) as [75] "an exercise in mechanics." It is important to remember that during much of the 19th century the physical reality of atoms and molecules was in doubt, especially on the European continent. Thus, for example, we find in the translation of 1900 of Pfeffer's *Physiology of Plants* (1899) the assertion [76]: "All ideas of molecular structure rest on a hypothetical basis, and indeed atoms and molecules are simply convenient mental abstractions and may have no actual existence." (Pfeffer of Leipzig, having been trained in physics, revolutionized botany through the development of physical measurements.) It happened that the BAAS meeting of 1859 was held in Aberdeen

in September, the city of Maxwell's faculty appointment. Maxwell presented the initial results of his analyses in which according to an abstract he emphasized [77]: (a) the non-uniformity of the velocity of the gas particles (which eventually became known as the Maxwell-Boltzmann velocity distribution); (b) for two different sets of particles in equilibrium the distribution is such that the average kinetic energies are equal; (c) the mean distance travelled between consecutive collisions; and (d) applications to the diffusion of gases. Entry in this area of research started for Maxwell a series of publications during the 1860s having a lasting influence.

1860 (January and July): "Illustrations of the dynamical theory of gases" (Philosophical Magazine) [78]: Containing the published version of his 1859 presentation, this goes beyond the work in the aforementioned abstract so as to include the prediction that the shear viscosity (described as the "coefficient of internal friction") is predicted to be "independent of the density" of the gas. The analysis is based on the collision of perfectly elastic spheres, and the associated analysis resembles in some way modern discussions of the differential cross section. Maxwell went on to examine what in modern terminology corresponds to the transport coefficients associated with the diffusion of gases and the conduction of heat. (By 1862 Clausius published corrections to Maxwell's analysis of heat conduction in gases which Maxwell later admitted were "oversights" [79].) Concerning the limitations of Maxwell's approach, his final page clearly indicates a major deficiency: the predicted ratio of specific heat at constant pressure to that at constant volume is inconsistent with overall values for gases, a result not resolved until the 20th century advent of quantum theory. The Nobel laureate Pauli gave a concise introduction to Maxwell's results useful for teaching [80].

1866: "On the viscosity or internal friction of air and other gases, The Bakerian lecture" (Philosophical Transactions of the Royal Society) [81]: Read February 1866, this paper contains Maxwell's measurements confirming the lack of dependence of the shear viscosity on the density of the gas. He also examined the temperature

dependence of that coefficient. The experiments were carried out in London in his dwelling with the assistance of his wife.

1866: "On the dynamical theory of gases" (Philosophical Transactions of the Royal Society) [82]: Read May 1866, this paper contains a complete reformulation of his prior kinetic theory in which the force between molecules is extended beyond the case of rigid spheres previously considered. The formulation allows for the evaluation of the temporal rate of change of molecular quantities through a conservation relation commonly referred to as the Maxwell-Boltzmann transport equation for much of the 20th century [83]. Maxwell, however, was primarily interested in steady-state situations and to facilitate a solution, he postulated a particular power law, radical dependence for the repulsion between molecules, that was eventually realized to be unrealistic. Being based on assumptions quite different from those of his 1860 paper, his analysis provided an early alternative derivation of the velocity distribution function and alternative expressions for the transport coefficients. The analytic generalization to the case of more realistic molecular force laws was a significant and difficult accomplishment of kinetic theorists during the first half of the 20th century [84]. This paper is also important for Maxwell's introduction of one of the primary models of viscoelastic media.

The kinetic theory of gases and the associated molecular physics continued to occupy appreciable aspects of Maxwell's research as illustrated by the following partial list of publications (all reprinted by Niven [7]) which includes the relatively elementary issue of the estimation of molecular sizes from macroscopic observables:

1873: "On Loschmidt's experiments on diffusion in relation to the kinetic theory of gases" (Nature).

1876: "Diffusion of gases through absorbing substances" (Nature).

1878: "On Boltzmann's theorem on the average distribution of energy in a system of material points" (Transactions of the Cambridge Philosophical Society).

1879: "On stresses in rarified gases arising from inequalities of temperature" (Philosophical Transactions of the Royal Society).

During the 1860s Maxwell became interested in the second law of thermodynamics and by 1867 he had conceived of the consequence of a super human “agent” having the ability to sense the speed of a molecule and admit or block admission into a given volume. While this agent in Maxwell’s thought experiment (eventually known as Maxwell’s demon) was not the central issue in any of his research publications, the idea profoundly impacted the development of 20th century information theory [85, 86] and remains of considerable interest [87, 88]. It is plausible that when conceiving the consequences of such an agent, the breadth of his reading had influenced Maxwell. For example, in the 18th century classic previously noted, *Butler’s Analogy* [74] (four copies of which were in Maxwell’s personal library [21]), in the third paragraph of *Butler’s Preface* we find the suggestive statement: “Probable evidence, in its very nature, affords but an imperfect kind of information, and is to be considered as relative only to beings of limited capacities; for nothing which is the possible object of knowledge, whether past, present, or future, can be probable to an infinite intelligence, since it cannot but be discerned absolutely as it is in itself, certainly true, or certainly false ; but to us probability is the very guide of life.”

7. Examples of Maxwell’s contributions to other areas of research

To gain a proper appreciation for the breadth of the impact of Maxwell’s research it is important to remember that he made highly original lasting contributions to numerous other areas [20]. Some of these areas are listed here, together with the year of publication. Interested readers can consult Niven’s edition of Maxwell’s *Scientific Papers* [7] for the full publication title for situations where quotation marks are not used in this list:

1850: “On the Equilibrium of Elastic Solids”

1854: Gradient index fish-eye lens

1855, 1857, 1860, 1861: Perception of color

1859: “On the stability of the motion of Saturn’s rings”

1864, 1870, 1872: Equilibrium of frames, reciprocal figures, force diagrams and stress functions

1868: "On governors"

1873: "On double refraction in a viscous fluid in motion"

1876: "Capillary action"

Maxwell also made contributions to thermodynamics embedded in other papers.

Another research area important in the discussions that follow concerns Maxwell's interest in the optical spectrum of atoms. This is not evident from Niven's compilation because Maxwell's analysis of the effect of stellar motion on spectral lines (now commonly known as the Doppler effect) was not included in [7]. That analysis appears in an addendum to a discussion of observations by the astronomer William Huggins [89].

8. Maxwell's broader British Association addresses of 1870 and 1873 having philosophic and religious overtones and related issues

At the 1870 BAAS meeting (held that year in Liverpool) Maxwell served as president of the Mathematical and Physical Section and consequently had the responsibility of presenting an assessment of developments pertinent to that Section. Maxwell's 1870 Address [90, 91] is often remembered for his innovative suggestions pertaining to the physical basis for standards of measurement. Maxwell also examined some culturally relevant aspects of science, indicating that he continued to view science in a religious context. For example, after discussing the similarity of the optical spectra of "terrestrial hydrogen" in comparison to "light of the sun and stars," Maxwell remarks concerning atoms: "when we reflect that no power in nature can now alter in the least either the mass or the period of any one of them, we seem to have advanced along the path of natural knowledge to one of those points at which we must accept the guidance of that faith by which we understand that 'that which is seen was not made of things which do appear.'" The phrase at the end that Maxwell placed in quotation marks is from the Hebrews 11:3 of the *King James Version* (the translation of the New Testament commonly used in that era) which reads: "Through faith we understand that the worlds were framed by the word of God, so that things which are seen were not made of things which do appear." (The modern *New International Version* reads, "what is seen was not made

out of what was visible” at the end.) Newspaper coverage was favorable, having quoted this section of the address [92]. George Gabriel Stokes was evidently in the audience and a newspaper [93] reported his reaction that “the paper contained matters of thought which ordinary minds could not conceive, and which demand deep consideration even on the part of members of the association.” In a book for general audiences written in 1872 (and published in 1873), J. J. Murphy conveyed the importance of Maxwell’s biblical quotation [94]. By the early 1950s, however, some scientists favorable to Christian concepts categorized similar approaches to Christian apologetics as being limited to the “God of the Gaps” [95]. While it is difficult to fully evaluate how Maxwell’s remark was perceived in 1870, it may be relevant to examine how James Moffatt (1870-1944), a scholar trained in 19th century Presbyterian Scottish theology, viewed the corresponding chapter of the New Testament book of Hebrews in a commentary drafted by 1914 but delayed in publication. (Maxwell himself had an appreciation of Scottish theology and the present author has determined that Maxwell had a large library of related volumes.) Moffatt indicates concerning Hebrews chapter 11 [96]: “It is faith as the reflex of eternal realities or rewards promised by God which is fundamental in this chapter, the faith by which a good man lives.” If that was Maxwell’s intent, it illustrates beliefs beyond the “God of the Gaps”. Maxwell continued his discussion by examining the dissipation of energy and irreversible processes, concluding: “This idea of a beginning is one which the physical researches of recent times have brought home to us, more than any observer of the course of scientific thought in former times would have had reason to expect.”

It was not until after Maxwell’s BAAS address of 1873 that his positions became an object of appreciable public interest and occasional public criticism. Maxwell had agreed to give a prestigious invited lecture at the British Association September 1873 meeting in Bradford but looked into being relieved of the obligation. As scheduled, however, he delivered his lecture “On Molecules” on Monday evening, 22 September [97, 98]. Again, his lecture was extremely broad, containing a wide range of historical information and recent developments, the publication of which was appended by tables of molecular data and diffusion

coefficients. His controversial positions, however, pertain to Maxwell's perspectives associating physics and the creation of matter, some examples of which are as follows: "On the other hand, the exact equality of each molecule to all others of the same kind gives it, as Sir John Herschel has well said, the essential character of a manufactured article, and precludes the idea of its being eternal and self-existent. ... Science is incompetent to reason upon the creation of matter itself out of nothing. We have reached the utmost limit of our thinking faculties when we have admitted that because matter cannot be eternal and self-existent it must have been created ... But that there should be exactly so much matter and no more in every molecule of hydrogen is a fact of a very different order. We have here a particular distribution of matter—a collocation—to use the expression of Dr. Chalmers, of things which we have no difficulty in imagining to have been arranged otherwise." [97, 98]

Newspaper coverage was favorable and the implications easily evident: "Hence, he argued the necessity for a Creator was absolutely proved by these infinitely less than microscopic points of matter." Significant public criticism of Maxwell's 1873 Address began in spring of 1874 when a mathematician and friend of Maxwell, William Kingdon Clifford (1845-1879) addressed the London Sunday Lecture Society in April [99]. Clifford objected to Maxwell's position against applying evolutionary thought to molecules and objected to thermodynamic inferences of a beginning. (Clifford's lecture was presented at the kind of meeting that had been characterized during that era as "recreative religion" [100].) The attention given to Clifford's criticism was minor in comparison to the alternative perspective conveyed when John Tyndall addressed the British Association as its President in Belfast in August 1874. The controversy associated with Tyndall's "Belfast Address" continued for over a decade and greatly increased the amount of attention given to Maxwell's 1873 Address (which is occasionally confused in the early literature with his 1870 Address). One consequence is that one can easily find dozens of references to Maxwell's religious position prior to about 1900 [21, 23]. In 1878 there appeared a book using the pseudonym *Physicus* characterizing Maxwell's position as [101] "an atrocious piece of arrogance", though nothing has survived indicating Maxwell became aware of that criticism. Here *Physicus* is Latin for

natural philosopher or scientist. Much later the author was identified as the biologist George John Romanes (1848-1894), having returned to Christian faith only late in life [21, 23].

Maxwell's Christian perspective was evident in the verse placed in Latin over the Cavendish Laboratory entrance prior to the opening in June 1874 [21]. In translation, the verse Psalm 111:2 reads "Great are the works of the LORD, studied by all who have pleasure in them" and that verse happened to be an epigraph of a book in Maxwell's library [102]. Late in 1874 Maxwell expanded on some of the themes of his 1873 Address in an article on "Atoms" for the *Encyclopedia Britannica* [103]. It is also noteworthy that in one of Maxwell's last essays, on Faraday (also for the *Encyclopedia Britannica*), Maxwell elected to quote a section of Faraday's 1854 lecture on mental education that including the proclamation: "Yet even in earthly matters I believe that 'the invisible things of Him from the creation of the world are clearly seen, being understood by the things that are made, even His eternal power and Godhead'." Here Faraday quoted Romans 1:20 from the New Testament [104].

9. Eranus Club friends and various accounts of Maxwell's death

When considering Maxwell's thoughts in the 1870s, it is helpful to consult perspectives by members of the informal Eranus Club of Cambridge which was organized by the Regius Professor of Divinity Brooke Foss Westcott (1825-1901) who happened to reside in Scroope Terrace, the same building where the Maxwells moved to in Cambridge in 1871 [21, 23, 88]. Two other of the members, like Westcott, were renowned scholars of historical Greek documents of the New Testament: Joseph Barber Lightfoot (1828-1889) and Fenton J. A. Hort (1828-1892). Residing in St. Peter's Terrace, Hort was also Maxwell's neighbor. The Club facilitated the presentation of essays by members concerning serious philosophic issues. Perspectives concerning Maxwell are available through other Eranus Club members, most notably the Lucasian Professor of Mathematics George Gabriel Stokes (1819-1903) [21].

Lightfoot: Having departed from Cambridge following nearly three decades of service early in 1879 to become the Bishop of Durham (in northeast England), many of Lightfoot's personal papers are archived in the Durham Cathedral library. Lightfoot appears to have retained significant amounts of correspondence, the archive containing at least 7 letters from Maxwell. These reveal a closer relationship than evident from Campbell and Garnett's *Life*. For example, in June 1857 Maxwell addressed Lightfoot as "my tutor", suggesting Maxwell had been tutored in classics by Lightfoot at Trinity College, much as J.W. Strutt (Lord Rayleigh) had been a decade later [40]. More important, however, is Maxwell's letter to Lightfoot of May 1875. Lightfoot, having recently published his commentary *St. Paul's Epistles to the Colossians and to Philemon*, evidently arranged for Maxwell to receive a copy. Maxwell's reply is thoughtful [105]:

"Dear Prof. Lightfoot

Many thanks for your new volume on St Pauls Epistle. May you ever partake of the grace spoken of in the companion Epistle, Ephes iii.8-12.

Yours very truly

J. Clerk Maxwell

11 Scroope Terrace

11 May 1875"

To appreciate Maxwell's referenced blessing, notice Paul's epistle reads in part: Ephesians 3:8-9, "Unto me, who am less than the least of all saints, is this grace given, that I should preach among the Gentiles the unsearchable riches of Christ; And to make all men see what is the fellowship of the mystery, which from the beginning of the world hath been hid in God, who created all things by Jesus Christ." It was previously known that Maxwell referred favorably to the discussion in Lightfoot's commentary concerning Christ as "the beginning, middle and end of creation" conveyed in the first chapter of Colossians [21, 24] though prior to the discovery of the aforementioned letter, Lightfoot's gift and Maxwell's reaction were not known. In another letter of that period Maxwell replies to Lightfoot concerning ancient pumping technology, an issue of interest to Lightfoot while preparing a commentary on the writing of the *Apostolic Fathers* (which remains a highly

regarded work of scholarship). We can be certain that to some degree Lightfoot was aware of public criticisms of Maxwell's positions. For example the Durham Cathedral Archives contain the only surviving essay by Lightfoot for the Eranus Club [106]. That essay, while mainly concerned with the late John Stuart Mill's *Three Essays on Religion*, also discusses Tyndall's *Belfast Address* as determined by the present author through visual inspection. In addition, in Lightfoot's *personal copy* (containing Lightfoot's book-plate) of the *Proceedings of the 1874 Church Congress* in Brighton, a favorable discussion of Maxwell's positions was observed by the present author to be underlined.

Westcott: Since much of what may be learned concerning Maxwell by studying Westcott has been discussed by the present author elsewhere [21, 24], the summary here concerns the most surprising aspect. Having succeeded Lightfoot as Bishop of Durham in 1890, Westcott eventually published some of his theological Cambridge lectures as *The Gospel of Life*. Therein Westcott quotes short sections of Maxwell's BAAS addresses of 1870 pertaining to the principle of the dissipation of energy and the beginning of the universe and of 1873 which (in Maxwell's words) "precludes the idea [of each molecule] being eternal and self-existent." More important is Westcott's attribution to Maxwell of an essay in *Nature* [22] that upon inspection is found to be anonymously published. Importantly, that essay concludes from thermodynamics concerning what became known as the heat death of the universe: "... the available energy has been diminishing, and the unavailable increasing by a process as irresistible and as irreversible as Time itself. The duration of the universe according to the present order of things is therefore essentially finite, both *à parte ante* and *à parte post*." (In simpler language: Maxwell had argued that the existing type of universe had a beginning and will have an end.) Other evidence of Maxwell's authorship is the duplication of phrases from one of Maxwell's Eranus essays not yet published in 1874 [21, 24, 107].

Hort: Pages 417-421 of the *Life* [14] contain some of Hort's impressions and recollections concerning Maxwell, sometimes recalling (evidently from memory) his

words during his final month, attributing to Maxwell the statement: “The only desire which I can have is like David to serve my own generation by the will of God, and then fall asleep.” This takes on a deeper significance (certainly evident to the Biblical scholar Hort) in the context of the New Testament account of Saint Paul’s first missionary journey, Acts 13:35-36: “For David, after he had served the purpose of God in his own generation, fell asleep, and was laid among his fathers and underwent decay; but He [Christ] whom God raised did not undergo decay.” (For ease of reading the *New American Standard Bible* translation is given.) Hort recalled Maxwell’s “freedom from the mental dualism often found in distinguished men who are absorbed chiefly in physical inquiries.” He indicated that Maxwell “carried into every thought a perfect fidelity to the divine proverb [...] ‘The lip of truth shall be established forever,’” indicating that Proverbs 12:19 was one example of “sacred verses” important to Maxwell. It is also helpful to observe Hort’s letter to his eldest daughter (9 November 1879, four days after Maxwell’s death): “He was one of the greatest men living, and, I believe it is not too much to say, one of the best; his rare powers being united to a high type of pure and simple Christian character. There is no one who can take his place here” [23].

Stokes: The revelations pertaining to Maxwell are provided both by his observations and those of Stokes’ wife Mary who was a dear friend of Maxwell’s wife Katherine. Both Mary and Stokes’ daughter Isabella helped to care for Maxwell and Katherine as Maxwell approached death late in October and early in November 1879 [21]. Stokes himself in correspondence of November 1899 described Maxwell as a “deeply religious Christian” [108].

Other accounts of Maxwell’s personality and religious preferences by eyewitnesses of his later years are consistent with those reported here. Some of these can be found in [21], [23], and in the *Life* [14]. Maxwell’s own words to his wife Katherine in December 1873 (*Life*, p. 387) reads: “I am always with you in spirit, but there is One who is nearer to you and to me than we ever can be to each other, and it is only through Him and in Him that we can ever really get to know

each other. Let us try to realise the great mystery in Ephesians v., and then we shall be in our right position with respect to the world outside, the men and women whom Christ came to save from their sins.” Those words are similar to those also addressed to her when he remained in London to work in June 1864 while she had gone to the healthier climate of Glenlair, letters that were written in a period when Maxwell attended Baptist chapels in London [23]. Also significant is Maxwell’s attention to religious verse, the poetry of his own composition, and the known content of his personal library [14, 21, 88].

Among the Maxwell archives in Cambridge is an undated note in which Maxwell had written by hand [21], from the commonly used translation of the New Testament during Maxwell’s era, the verse Philippians 4:8 –
 “Finally, brethren, whatsoever things are true, whatsoever things are honest, whatsoever things are just, whatsoever things are pure, whatsoever things are lovely, whatsoever things are of good report; if there be any virtue, and if there be any praise, think on these things.”

Acknowledgement

The transcription of Maxwell’s May 1875 letter to J.B. Lightfoot given here in Section 9 is reproduced by kind permission of the Chapter Library of Durham Cathedral. I am grateful to the Lightfoot scholar Geoffray R. Treloar (presently of Deakin University, Australia) for bringing the existence of uncatalogued letters from Maxwell in the Durham Cathedral to my attention and to the assistance from my wife Trude Marston in finding the letter of 1875. Thomson’s report in Appendix A is reproduced by kind permission of the Royal Society of London. Concerning the content of Maxwell’s personal library I am grateful for assistance from Chris Jakes (Cambridgeshire Libraries) in 2010. Archivists at several libraries have been helpful, especially at the Department of Manuscripts and University Archives, Cambridge University. Several scholars have been helpful including C.W.F. Everitt (Stanford University), Brian Bowers (retired, Victoria and Albert Science Museum, London), and the late Cyril Domb. The present proprietor of Glenlair, Duncan Ferguson provided the photograph in Fig. 1 and permission to reproduce it here.

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Figure Captions

Fig. 1. Glenlair house, James Clerk Maxwell's rural family dwelling as it appeared in summer 2015. There was additional dwelling space nearby for farm workers during Maxwell's era. The present proprietor Duncan Ferguson provided this photograph.

Fig. 2. Portrait of Maxwell from the front piece of the 1882 printing of the *Life* [14] scanned from a copy in the possession of the present author. This copy had evidently once been owned by a physics Nobel laureate, the following "Luis W. Alvarez, Univ. Calif. 1938" appearing adjacent to the opening page.

Fig. 3. Statue on Edinburgh's George Street of James Clerk Maxwell. Maxwell is shown holding a disc-shaped "color top." He used the disc early in his investigation of color perception by spinning it rapidly so as to obtain adjustable mixtures of colors. (Photographed in July 2012.)

Fig. 4. Maxwell's diagram showing the spatial relationship of the electromagnetic fields "at a given instant" along a ray from his 1873 *Treatise* [2]. This diagram was scanned from an 1892 printing of the *Third Edition* available to the present author.

Fig. 5. Maxwell's diagram (as in his 1873 *Treatise* [2]) showing the electric field lines and orthogonal equipotential curves for a point charge placed at A in a previously uniform vertical electric field and plotted in the plane containing the charge and the curved field lines. This diagram was scanned from an 1881 printing of Maxwell's *An Elementary Treatise on Electricity* (University Press, Oxford, 1881) in the possession of the present author.

Lines of Force and Equipotential Surfaces.

electric displacement $\times$

magnetic force $\times$