Modeling Karst Spring Hydrograph Recession Based on Head Drop at Sinkholes

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Abstract: Spring discharge often responds to rainfall with a rapid increase followed by a slower recession, and the mode of recession is often exponential-like. We propose a new model of the response of spring discharge to rainfall based on the square law for turbulent conduit flow. The new non-exponential model is compared against the exponential model under specific constraints. A hydrograph of St. Marks River in Florida is used to illustrate that when the change in “sinkhole head” (defined as the hydraulic head at the upstream end of the karst conduit connected to the spring) is relatively small, the solution of the new model is close to that of the exponential model, which extends the validity and application of the exponential solution. When the change in sinkhole head is very large, the solutions from the two models clearly differ from each other. Limitations of the non-exponential model are analyzed by simulation of a hydrograph observed downstream of Wakulla Springs. It is concluded that both solutions are applicable when spring response is smaller than or comparable to the base flow, but are nonphysical when the response is much larger than the base discharge.

Keywords: Spring discharge; Sinkhole; Exponential recession; Square law; Turbulent flow

1. Introduction

Karst landscapes and karst aquifers are formed by the dissolution of carbonate rocks by water rich in carbon dioxide. Karst aquifers can be described as a network of solutional conduits and caves embedded in, and interacting with, a rock matrix with intergranular pores and fissures. Therefore, karst aquifers can be described as double-porosity (conduits vs. matrix) or even triple-porosity (conduits, fissures, pores) hydraulic media (Worthington, 2007). Karst landscapes are characterized by solutional landforms, such as karren or dolines, that are genetically related to the aquifer system underneath. Sinkholes and swallow holes are places where surface water sinks underground, into the network of conduits (Ford and Williams 1989).

Karst aquifers are a precious resource because they provide drinking water to about one quarter of the world's population (Ford and Williams 1989), and because they are habitats for many rare and endemic species (e.g. Mahler and Bourgeai 2013). In the United States, over 20% of the land is karst, and the underlying karst aquifers provide about 25% of the country's freshwater supplies (Elliot, 2000). In some European countries, such as Austria or Slovenia, karst water even contributes about 50% to the total freshwater supply (Hartmann et al., 2014). Karst is also extensive in southwest China, where people living in rural areas are typically subjected to untreated water from karst springs for drinking (Guo et al., 2013).

Karst aquifers are typically very productive, because of their ability to convey water through a network of interconnected conduits. These conduits are also preferential pathways for pollutants which can travel rapidly over long distances, making karst aquifers highly vulnerable to contamination, particularly microbial contamination (Field and Nash, 1997; Goldscheider et

al., 2010; Lauber et al., 2014). For example, over the last few decades Wakulla Springs in northwest Florida have been impacted by municipal wastes entering Ames Sink, south of Tallahassee, causing widespread reproduction of algae, which in turn consumes oxygen in water and results in anoxic conditions and the decrease of fishes (Li, 2004). Besides their vulnerability and variable water quality, karst aquifers are also characterized by rapid and marked variations of water table and spring discharge, which further complicates their use as water sources (Bakalowicz, 2005).

In karst aquifers, solution conduits in which rapid and turbulent flow is common are clearly distinctive from the pores and fissures of the matrix in which storage and laminar flow predominates. These two flows can interact with each other, depending on the pressure difference between them (Screaton et al., 2004). Many karst hydrogeology studies focus primarily on qualitative investigations of flow and geochemical analyses (e.g., Martin and Dean, 2001; Moore et al., 2009). Quantitative studies of the physical processes associated with karst aquifers are still challenging because karst aquifers and their conduit networks are complicated systems that are difficult to explore, characterize, and quantify. However, quantitative studies of the physical processes of flow and transport are indispensable for calculating water budgets, predicting climate change impacts and understanding the fate and transport of contaminants. For these reasons, simulations of physical processes are essential for understanding karst-aquifer functioning and for sustainable karst aquifer management.

The linear reservoir model, originally proposed by Maillet (1906), linearly links the discharge at the outlet of a reservoir to its water level. When applied to karst aquifers, the model predicts that a spring's flux is proportional to the aquifer volume. An essential aspect of the

model is the assumption that spring discharge must behave linearly with respect to water-table fluctuation, an essential element of laminar flow (i.e., flux is proportional to pressure difference), because the area of an aquifer is almost fixed and is a quasi-constant. Flow inside pores, relatively tight fractures, and in some instances, very small conduits is typically laminar due to large shear/friction forces caused by the walls such that the linear reservoir model works well for non-karst aquifers. In the case of karst aquifers with large solution conduits that are not well interconnected, flow may be constrained by narrow passages and remain laminar, which would allow application of the linear reservoir model (Rimmer and Hartmann, 2012). However, flow in large solution conduits, such as the coastal karst aquifers in Florida, have been shown to be interconnected (e.g., ramiform), and have been observed to be turbulent, and feed one or more first-magnitude springs. Unfortunately, the difference between laminar- and turbulent-flow dynamics is not always adequately reflected by the exponential model.

An important aspect of linear reservoir model is based on the hydrological mechanism that spring discharge results directly from non-point sources (i.e., areal infiltration), but recharge to karst aquifers often occurs primarily at point locations (e.g., sinkholes). Point recharge in which surface runoff converges very quickly in the horizontal direction results in a response (i.e., change) in spring flux that is caused by transient point recharge. Point-like or localized recharge cannot be replaced by the concept of area-averaged infiltration nor water-table fluctuation because individual recharge locations are singular points from a mathematical perspective.

Hydrologically active sinkholes provide direct recharge to the solution conduits in karst aquifers. Although a water table in these surface features may not be observable in many instances, they actually constitute and represent the direct and quick inlets to the aquifers,

because surface runoff can convey large amounts of precipitation to them and then to the conduits at much higher rates than percolation through the epikarst and vadose zone. In this paper, the term “sinkhole head” (h_s) refers to the hydraulic head at the upstream end of the conduit networks, taking the level of the spring as the reference level ($h = 0$), while the term “sinkhole base head” (h_{SB}) describes the hydraulic head prior to an intense precipitation event.

Spring hydrographs provide a classic approach for analyzing hydrodynamics in karst aquifers that are difficult to access and instrument (e.g., Kovács et al., 2005; Geyer et al., 2008). An intense precipitation event over a short period of time causes a rapid rising limb followed by a slower falling limb in spring hydrographs. The recharge signal can be conceptualized as the Dirac delta function, and thus the resulting spring hydrograph may be treated mathematically as the Green’s function (Strauss, 1992). Green’s function is powerful in that, for linear hydrology, a spring hydrograph resulting from any amount of precipitation with any kind of recharge history can be predicted through the convolution of the input mode with the Green’s function. This has been shown by theory on linear and shift-invariant systems in signal processing (Lathi, 2000), which is the basis of Black-Box models, also referred to as Lumped Parameter Models (Hartmann et al., 2014).

Numerical or distributed models are often capable of producing more detailed results, but they invariably require *a priori* knowledge of the spatial distribution of relevant hydraulic parameters, particularly concerning the spatial configuration and properties of the conduit network. Furthermore, numerical karst aquifer models require substantial efforts to develop the model, to compute the solutions and to conduct uncertainty analyses, which greatly increases the cost of computational efforts (Hartmann et al., 2014).

In this paper, we shall focus on the response of spring discharges to precipitation. The approach adopted is to emphasize the dominant physical processes and simplify the complicated mathematical equations. Li and Field (2014) developed a simple mathematical model to simulate spring discharge and estimate “sinkhole porosity” in a karst aquifer (defined as the ratio of the area of all sinkhole cross-sections to the watershed area in the horizontal plane), based on the assumption of small changes in sinkhole head and the resulting linearization of the flow equation in the conduits. The purpose of this paper is to extend that model and to conduct analysis of a general situation in which the change in sinkhole head is comparable to or much larger than the sinkhole base head. Two spring hydrographs measured in the St. Marks Karst Watershed in northwest Florida are used to test the new model.

2. A Physical Model of Spring Discharge and its Mathematical Solution

Without heavy precipitation, base-level karst springs typically have a base flow originating from the (nearly) steady seepage from the matrix (consisting of fissures and pores) into the conduits. In addition, in many karst systems, the main water storage may occur in the epikarstic zone that allows water to percolate down to the aquifer matrix and to the conduits. The sustainability of base-level springs is due to groundwater in the aquifer matrix draining into the conduits, because rainfall entering a sinkhole and being transmitted down into a conduit bounded by impervious wall is ephemeral. In this ordinary scenario, sinkhole head is almost stable or decreases very slowly. During large precipitation events, however, the sinkhole head can rise very rapidly due to increased inflow from nearby runoff, causing a nearly instantaneous increase

in spring discharge because of the high velocity of the compressional (or acoustic) wave in the water. The compressional wave transmits a change of pressure downstream to rebuild a new pressure gradient between the upstream sinkholes and the downstream spring, resulting in a change of conduit flow and, this, spring discharge. The increased head at sinkholes can also displace water and pollutants from the conduits into the surrounding matrix, which eventually flow back into the conduits when the hydraulic pressures decreases (Goldscheider, 2005; Li et al., 2008). The process and effects of “bank storage” (or “gradient inversion”) in karst aquifers is very complicated but may not be as significant as was once thought (Peterson and Wicks, 2005). For this reason and for the sake of simplicity, we neglect this mechanism and attribute the response of spring discharge to the abrupt flooding of sinkholes and the following natural drop in sinkhole head. Figures 1a and 1b illustrate the basic processes of low flow and high flow, respectively, as described above and Figure 1c schematically illustrates the spring hydrograph for the high-flow case shown in Figure 1b.

Immediately prior to a sudden precipitation event and sinkhole flooding, sinkhole head (water table in one or more sinkholes) is relatively stable or decreases very slowly. As such, we can define a sinkhole base head h_{SB} to represent the water table at this moment, while h_S denotes the sinkhole head at any given time later than this time. The increase in pressure ($h_S - h_{SB}$) causes a change (or response) in spring discharge Q_{sp}^R . The response of spring flux is a function of the change of sinkhole head, i.e.,

$$Q_{sp}^R = f(h_S - h_{SB}), \quad (1)$$

where f represents a generally unknown function. Although the function is unknown, it can still be linearized for relatively small changes in increased pressure because any continuous functions can be well approximated at small ranges by Taylor expansion. However, for large increases in hydraulic pressure, the spring-discharge response is generally a non-linear function of the increased sinkhole head, which is the physical principle addressed in this paper.

For turbulent flow in a phreatic conduit, the square law (Schlichting, 1968) states that increase of discharge from a spring is proportional to the square root of the hydraulic gradient. With the hydraulic gradient being approximated using the sinkhole head, we have

$$\frac{Q_{sp}}{Q_{sp}^B} = \left(\frac{h_s}{h_{SB}} \right)^{1/2}, \quad (2)$$

where Q_{sp}^B denotes the spring base flow and Q_{sp} is the total spring discharge consisting of the base flow and response part of the spring discharge, i.e., $Q_{sp} = Q_{sp}^B + Q_{sp}^R$.

For phreatic (i.e. fully saturated) conduits, the spring response Q_{sp}^R can be related to the rate of the decreased sinkhole head by conservation of water mass

$$Q_{sp}^R = -\phi_s A_s \frac{dh_s}{dt}, \quad (3)$$

where A_s is the recharge area, and ϕ_s is the porosity of sinkholes (i.e., the area of all sinkhole cross-sections within the recharge area divided by the recharge area, a 2D concept). Substituting Equation (3) into Equation (2) yields the governing equation for sinkhole head

$$\frac{dy}{\sqrt{y}-1} = \frac{-Q_{Sp}^B}{\phi_S A_S h_{SB}} dt, \quad (4)$$

where $y = h_s / h_{SB}$ for simplicity and represents dimensionless sinkhole head.

Integrating Equation (4) and using the initial condition, $h_s(t=0) = h_0$, yields

$$\sqrt{y} - \sqrt{h_0/h_{SB}} + \ln \left(\frac{\sqrt{y}-1}{\sqrt{h_0/h_{SB}}-1} \right) = \frac{-Q_{Sp}^B}{2\phi_S A_S h_{SB}} t. \quad (5)$$

With the use of Equation (2), Equation (5) can be transformed to an equivalent equation in the form of spring response Q_{Sp}^R .

$$\frac{Q_{Sp}^R - Q_{Sp}^{Max}}{Q_{Sp}^B} + \ln \left(\frac{Q_{Sp}^R}{Q_{Sp}^{Max}} \right) = \frac{-Q_{Sp}^B}{2\phi_S A_S h_{SB}} t, \quad (6)$$

where Q_{Sp}^{Max} is the maxima of Q_{Sp}^R (or the amplitude above the base flow), which can be obtained from the measured spring hydrograph.

The asymptotic behavior of Equation (5) (when the third term on the left-hand side is dominant over the first and second terms) is

$$\ln \left(\frac{\sqrt{y}-1}{\sqrt{h_0/h_{SB}}-1} \right) \rightarrow \frac{-Q_{Sp}^B}{2\phi_S A_S h_{SB}} t, \quad (7)$$

or equivalently

$$\frac{\sqrt{y}-1}{\sqrt{\frac{h_0}{h_{SB}}}-1} \rightarrow \exp\left[\frac{-Q_{Sp}^B}{2\phi_S A_S h_{SB}} t\right]. \quad (8)$$

Noting that

$$\sqrt{y}-1 = \sqrt{\frac{h_S}{h_{SB}}}-1 = \frac{Q_{Sp}}{Q_{Sp}^B}-1 = \frac{Q_{Sp}^R}{Q_{Sp}^B}, \quad (9a)$$

and that

$$\sqrt{\frac{h_0}{h_{SB}}}-1 = \frac{Q_{Sp}^B + Q_{Sp}^{Max}}{Q_{Sp}^B}-1 = \frac{Q_{Sp}^{Max}}{Q_{Sp}^B}, \quad (9b)$$

Equation (8) can be rewritten as

$$\frac{Q_{Sp}^R}{Q_{Sp}^{Max}} \rightarrow \exp\left[\frac{-Q_{Sp}^B}{2\phi_S A_S h_{SB}} t\right], \quad (10)$$

which is the same as Equation (5) in Li and Field (2014). This is expected, because, as time becomes infinitely large, the increased sinkhole head will correspondingly become very small such that the linearization in Li and Field (2014) is accurate. Therefore, Equations (5) and (6) have significant meaning for the earlier (non-exponential) recession of spring discharge. Equations (8) and (10) state that as time becomes infinitely large, sinkhole head and spring discharge will return to its original steady state and base flow, respectively. Although this model produces an exponential recession at late time, the actual return to base-level discharge can be complicated by another distinctive physical process, the retarded percolation from the ground surface downward to recharge the conduits (Birk and Hergarten, 2010; Li and Field, 2014).

Solving Equations (5) and (6) at a prescribed moment involves implicit functions that can be accomplished by tools such as Mathematica. In this paper, however, we use another quick approach, i.e., first prescribing a value of y in Equation (5) or Q_{sp}^R in Equation (6) and then substituting the value into the left hand side of the equation to get time t on the right hand side.

3. Constrains on Comparison of Two Models

In order to facilitate a comparison between the non-exponential model developed in this paper with the exponential recession model in Li and Field (2014), we must first identify some important parameters or quantities so that the expected comparison will be logical.

The most important constraining parameter is the maximum spring discharge above the base flow Q_{sp}^{Max} . This parameter must be the same for the two models. In order to observe how the non-exponential decrease at the earlier recession differs from the exponential recession, different constraints on different quantities or features will determine how to specify related parameters. In the following, discharge volume of spring response and then its asymptotic behavior is chosen as a second constraint.

3.1. Constraint on the Discharge Volume of Spring Response

According to water mass balance, the precipitation allocated to the sinkholes is

$$\phi_s A_s (h_0 - h_{SB}) = \int Q_{sp}^R dt = V_{sp}^R, \quad (11)$$

where V_{sp}^R is the integral of the response part of spring discharge, or equivalently, the discharge volume of all sinkholes during a flooding period. This quantity may be obtained by integrating the spring response with respect to time.

The exponential recession model (or benchmark model) states (Li and Field, 2014)

$$Q_{sp}^R = Q_{sp}^{Max} \exp \left[\frac{-Q_{sp}^{Max}}{V_{sp}^R} t \right]. \quad (12)$$

The two parameters, Q_{sp}^{Max} and V_{sp}^R completely determine the shape of the spring response in the exponential model.

Equation (6) states that for complete determination of the shape of spring hydrographs, another two quantities Q_{sp}^B and $\phi_S A_S h_{SB}$ (in addition to Q_{sp}^{Max}) are required, which we then correlate with the discharge volume of spring response V_{sp}^R because it is easier to control.

Although obtaining the explicit form of V_{sp}^R from Equation (6) is not straightforward, dimensional analysis of that Equation (6) leads to the relationship

$$\frac{V_{sp}^R}{\phi_S A_S h_{SB}} = F \left(\frac{Q_{sp}^{Max}}{Q_{sp}^B} \right), \quad (13)$$

where F represent the relation (or function) sought for.

Substituting Equation (11) into Equation (13) yields

$$\frac{h_0 - h_{SB}}{h_{SB}} = F \left(\frac{Q_{sp}^{Max}}{Q_{sp}^B} \right). \quad (14)$$

On the other hand, Equation (2) states

$$\frac{Q_{sp}^{Max} + Q_{sp}^B}{Q_{sp}^B} = \left(\frac{h_0}{h_{SB}} \right)^{1/2}. \quad (15)$$

Combining Equations (14) and (15) yields

$$F(x) = x^2 + 2x, \quad (16)$$

which is the function that we are seeking. Thus Equation (13) is

$$\frac{V_{Sp}^R}{\phi_S A_S h_{SB}} = \left(\frac{Q_{Sp}^{Max}}{Q_{Sp}^B} \right)^2 + 2 \left(\frac{Q_{Sp}^{Max}}{Q_{Sp}^B} \right). \quad (17)$$

The above equation is the second constraint of the non-exponential model. Alternatively and equivalently, Equation (15) is the sufficient and necessary condition to assure that the discharge volume of spring response is the same between the two models.

3.2 Constraint on the Asymptotic Behavior

Another comparison is based on the asymptotic behavior of spring response, which is simpler. By comparing Equation (10) with Equation (12) and with use of Equation (11), we obtain

$$\frac{Q_{Sp}^{Max}}{Q_{Sp}^B} = \frac{h_0 - h_{SB}}{2h_{SB}}, \quad (18)$$

which is identical to Equation (16) in Li and Field (2014). This requirement depends only on the extent of the linearization of the square law in Equation (2), i.e.,

$$h_0 - h_{SB} \ll h_{SB} . \quad (19)$$

By setting the maximum spring discharge and asymptotic behavior of the new non-exponential model to be the same as those of the exponential model, the two models are shown to be identical.

In reality, the asymptotic behavior may not be as important as the discharge volume of spring response, because comparison of two negligible values (as time becomes large) is often not a major concern for flow problems. For transport problems, which are not discussed in this paper, the asymptotic behavior will be more meaningful for environmental professionals who have to consider very low concentrations of dangerous pollutants, such as BTEX or MTBE.

4. Illustration of Solutions

While modeling the falling limb of spring hydrographs, the most relevant quantities of the theoretical curve that are often used for model calibration are the maximum discharge above the spring base flow and the discharge volume of the spring response. For the exponential recession model, Equation (12) automatically satisfies these two requirements. For the non-exponential model, Equations (11) and (15) (or equivalently, Equations (11) and (17)) are the sufficient and necessary conditions for these two requirements, which enforces important restraints on the relevant parameters. In our implementation of the non-exponential model, Equations (11) and

(15) are constraints ensured first, and then Equation (6) is used to get the theoretical curve using the quick approach elucidated at the end of section 2.

4.1 Simulation of a Hydrograph with a Response Comparable to Base Discharge.

Li and Field (2014) have shown that if spring response is much smaller than base discharge (a sufficient condition), the non-exponential model will approach the exponential model. Here, we investigate the case when the response is comparable to the base discharge.

An assumption of our model is that conduits must be phreatic (fully flooded). For this reason, St. Marks Karst watershed in northwest Florida is chosen (Figure 2). This watershed is very close to the Gulf of Mexico, so its springs may be severely affected by tidal effects.

The U.S. Geological Survey maintains a gaging station (02326900) on St. Marks River to measure the water flux of this river near Newport (Figure 2). Upstream of this station is the Natural Bridge Sink and a resurging spring. This station is relatively free from tidal influences due to its inland location and higher altitude. The measured discharge data from August 4–6, 2015 (USGS, 2015a) was selected to test the two models to see if spring response is comparable to base discharge.

The base flow at the station during that period was $12.5 \text{ m}^3 \text{ s}^{-1}$, and the maximum response above the base flow was $8.4 \text{ m}^3 \text{ s}^{-1}$. The sinkhole base head was about 1 m above the level of the spring (Li and Field, 2014), and the maximum sinkhole head was 2.8 m. The measured discharge volume of the response part is $158 \times 10^3 \text{ m}^3$, so the discharge volume was set to $100 \times 10^3 \text{ m}^3$ due to the natural drop in sinkhole head (note that a significant part of the measured discharge

volume is from matrix/fissure flow and surface creeks). The sampling location is downstream of the spring so that the actual spring discharge is lower than what is measured at the sampling location. Alternatively, the spring is a large (first magnitude) spring in Florida. For these two reasons, the above discharge volume is regarded as a reasonable approximation. Using the sinkhole porosity (2.9×10^{-4}) estimated by Li and Field (2014) yields a contributing area of 193 km². See Figure 3 for the result of this simulation.

For the non-exponential model the coefficient of determination, $R^2 = 0.99387$ and the Nash-Sutcliffe Efficiency, $NSE = 0.9829$ suggesting a very good model fit. However, although both the exponential and non-exponential model appear to fit the measured data well, theoretically, the governing equation of the non-exponential model does not approach that of the exponential model, because the change in sinkhole head is comparable to the sinkhole base head. The two mathematical solutions (exponential and non-exponential models) are very close because of the constraints on the maximum discharge rates and discharge volumes. This simulation shows that when the change in discharge is comparable to the base flow, the exponential solution is still applicable mathematically (see Li and Field, 2014). However, the physics that the exponential model is based upon is not entirely sound, and the non-exponential solution is much more reasonable from the physical perspective. The non-exponential model is more reasonable because it is based on the square law of turbulent flow in a phreatic conduit (Schlichting, 1968), while the exponential model is based on laminar flow (i.e., Poiseuille flow in a cylindrical pipe) which is often unrealistic for solution conduits.

4.2 Simulation of a Hydrograph with a Response Much Larger than Base Discharge

Wakulla River originates from Wakulla Springs and is an upstream tributary to St. Marks River (Figure 2). The U.S. Geological Survey maintains a gaging station (02327022) on Wakulla River to measure the water flux of this river (Figure 2). Unfortunately, the station is severely impacted by tides due to its location near the coast. We selected measured discharge data from June 24 through July 14, 2012 (USGS, 2015b) to test the exponential and non-exponential models for a case when the response is one order of magnitude larger than the base discharge.

The base flow at the station during that period was $12 \text{ m}^3 \text{ s}^{-1}$, and the maximum response above the base flow was $98 \text{ m}^3 \text{ s}^{-1}$. The sinkhole base head was about 1 m (with respect to the Wakulla Spring), which results in a maximum sinkhole head of 84 m. The measured discharge volume of the response part is $50.4 \times 10^6 \text{ m}^3$, so the discharge volume was set to $45 \times 10^6 \text{ m}^3$ due to the natural drop in sinkhole head. Using the sinkhole porosity (2.9×10^{-4}) estimated by Li and Field (2014) yields a contributing area of 2100 km^2 . This area is comparable to the total area (3030 km^2) of the whole St. Marks watershed (Boning, 2007), which is unreasonable and possibly reflects the effects of a very large precipitation event resulting in overflow of the sinkholes and increased sinkhole porosity (i.e., much larger than 2.9×10^{-4}). The result of the simulation is shown in Figure 4. The $R^2 = 0.8936$ and the NSE = 0.8250 for the non-exponential model to the measured data, which although not as good as for the model fit to the St. Marks River data near Newport, is still quite good.

The solution of the non-exponential model for Wakulla Springs clearly differs from the exponential solution. The difference is apparent in that the non-exponential model has a less steep descending limb at earlier recession and more rapid attenuation at base flow than does the exponential model (Figure 4). This shows that when the change in sinkhole head is much larger than sinkhole base head (the governing equations of the two models differs dramatically), the

two mathematical solutions are quite different despite of the constraints on the maximum discharge rate and discharge volume being set to be the same values for the two models.

Neither of the solutions agrees very well with the measured data, and so neither of the models describes the true physical process in this event. The major problem is that the maximum sinkhole head (84 m) has greatly exceeded the elevation (about 8 m). Another associated problem from the physical perspective is that for such a large sinkhole head, a large amount of sinkhole water would be expected to move into the matrix (Fetter, 2000), violating the assumption that the base/matrix flow is from the matrix into conduits. In practice, neither sinkhole head nor spring discharge has been observed to increase so tremendously in phreatic karst aquifers. The cause is probably a result of the gaging station being located downstream from the spring, with a large portion of the contributed flow originating from surface streams and ambient matrix/fissure flow.

When spring discharge is much larger than the base flow during extraordinary precipitation events, a model reasonable in physics requires consideration of gradient inversion causing bank storage of water between conduits and matrix and/or delayed recharge from the matrix into conduits (or equivalently the increase in groundwater table) due to percolation through the vadose zone.

5. Conclusions

We have proposed and tested a new non-exponential model for the simulation of karst spring recession based on hydraulic head drop at sinkholes. For this model we made use of the (relative)

ratio of spring discharge to spring base flow, so as to avoid the notorious difficulty and uncertainty of the absolute equation for turbulent conduit flow.

The non-exponential recession model based on the square law for turbulent conduit flow is more reasonable than the exponential recession model from the physical perspective. When spring response is smaller than or comparable to base flow, the mathematical solutions of the two models are close to each other and both behave well, favored largely by the constraints that ensure that both the maximum discharge rate and discharge volume remain the same between the two models.

When the spring response is much larger than the base discharge (i.e., during extreme precipitation events), the solutions of the two models clearly differ, but neither is physically reasonable. Therefore, construction of a model sound in physics for extreme high-flow conditions requires detailed quantification of the transient exchange of water between conduits and matrix and/or the change in groundwater table resulting from percolating recharge in the vadose zone.

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Figure Captions

Figure 1. (a) Low flow state of a karst aquifer, without heavy rainfall, groundwater from the matrix feeds the spring via conduit and the head at sinkhole drops slowly. (b) High flow state of the aquifer, with heavy rainfall, runoff recharges the sinkhole much faster than infiltration via the vadose zone to the water table, causing the abrupt rise of the sinkhole head and consequent increase of spring discharge. (c) Schematic spring hydrograph in which the blue bar denotes precipitation within a short period. The rising limb represents the quick transition from the low flow state (a) to the high flow state (b), while the falling limb represents the slow restoration from the high flow state (b) back to the low flow state (a).

Figure 2. The St. Marks Karst watershed in northwest Florida. Wakulla River and St. Marks River are two major rivers within the watershed which immerge at Town of St. Marks and effluent to The Gulf of Mexico. Red Arrows indicate the locations of two USGS stations used in this study.

Figure 3. Simulation of a hydrograph in St. Marks River with the response is comparable to the base flow. Though non-exponential and exponential models are different in physics, their mathematical solutions are close and both well simulate the measured discharge curve. The

reason is that two constraints (the amplitude and total volume of discharge) make the two solutions close.

Figure 4. Simulation of a hydrograph downstream of Wakulla Spring with the response being much larger than the base flow. In this extreme case, the solution of non-exponential model differs significantly from that of exponential model. The former is better in simulating the earlier recession, while the latter is better in simulating the late recession and tailing. Exponential model is based on linearization which is not applicable for the early recession when the response is much larger than the base discharge. Neither of the two models achieves a very good simulation, or describes the true physical process in this event.

(a) Low Flow

(b) High Flow

----- County Boundary	----- Boundary of Water District
----- I10	----- Town of St. Marks
----- Highways	----- Lakes
----- Rivers	----- St. Marks Watershed

(a) Low Flow

(b) High Flow