Incorporating A Trend Analysis of Large Flow Perturbations into Stochastic

Modeling of Particle Transport in Open Channel Flow

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ABSTRACT: In extreme flow conditions, both the flow carrying capacity and movement of particles may abruptly change from those associated with regular flows. This study investigates movement of sediment particles in response to extreme flow events using a Lagrangian stochastic jump diffusion particle tracking model (SJD-PTM). The study attempts to investigate the frequency change of extreme flow event occurrences and its impact on suspended sediment particle movement. Using the concept of logistic regression, the trend magnitude of extreme flow events can be used as an input of the proposed stochastic jump diffusion particle tracking model with Logistic regression (SJ-PTM_LR) to account for the potential effects of environment change. The predicted frequency change of extreme flows from available data in Chijiawan region in central Taiwan is illustrated in this study. Both ensemble mean and variance of particle trajectory can be quantified under such predicted frequency trend change of extreme flow occurrences via simulations of SJ-PTM_LR. Results show that particle position uncertainty may undergo a significant increase by taking the effect of the predicted flow frequency trend into the consideration. Such probabilistic outcome provides a valuable means for assessing the probability of failure (i.e., risk) resulting from sedimentation processes.

Keywords: flow trend analysis; sediment transport; particle trajectory, stochastic approaches; particle tracking models

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1. INTRODUCTION

Suspended sediment concentrations are typically estimated using either the advection-diffusion equation or sediment rating curves in a deterministic manner. Models based on mass balance have been commonly adopted (e.g., Tsujimoto et al., 1994; Nicholas and Walling, 1997; Middlekoop and Van der Perk, 1998; Fang and Wang, 2000). Use of empirical relationships with other variables (e.g., stream discharge) derived from measurements such as sediment rating curves (e.g., Olive et al., 1980; Walling and Webb, 1981; VanSickle and Beschta, 1983; Lemke, 1990; Kurashige, 1993, 1998) is frequently used. More recently, particle-based approaches for estimating mass concentrations or the number concentrations of soil particles in a tidal estuary or air flow (e.g., Heemink, 1990; Hunter et al., 1993; Harris and Davidson, 2009; Ancey, 2010). Man and Tsai (2007) proposed a stochastic differential equation for the movement of sediment particles with an advection-diffusion equation to quantify the statistical characteristics of sediment concentrations. And more recently, Oh et al. (2015) proposed a stochastic approach for quantifying the probabilistic characteristics of sediment concentrations that can account for the uncertainty associated with flow randomness.

Modeling of suspended sediment particle movement in surface water can be achieved by stochastic particle tracking model (PTM) approaches. In PTM approaches, particle movement is mainly subject to the mean drift flow as well as flow turbulence. In addition to the mean drift flow and randomness found in turbulence, particle movement is augmented by the presence of extreme flow events or large flow perturbations. Consequently, in recent years, increased attention has been paid to the change in occurrence frequencies of these events. In extreme flow conditions, both the flow carrying capacity and movement of the particles change

from those found in normal flow conditions. In this study, we define large flow perturbations as extreme flow events that have a probability of occurrence less than 10%.

Interest in identifying trends in extreme flow events and the associated potential for severe and adverse impacts on human life, civil infrastructure, and ecosystems as well as other far-reaching socioeconomic consequences has recently increased (Frei and Schar, 2001). When observing long term records of extreme events such as temperature, precipitation or flow discharge, one might identify trends in the frequency that are indicative of changes in environment such as land cover change, and climate change etc. Moreover, extreme flow induced sediment transport can accelerate geomorphologic change and severely intensify local erosion and deposition. Therefore, the relationship between the stochastic features of hydrologic events and sediment transport modeling has also gained a higher awareness (Man and Tsai, 2007; Oh and Tsai, 2010).

Many researchers have shown that the increase of extreme events may be attributed to environmental change, such as climate change and land cover change, etc. Zwiers and Kharin (1998) pointed out that climate change may have a significant impact on the frequency and magnitude of extreme events. Birsan et al. (2005) identified significant trends in occurrence in both space and time, and addressed the relationship between the observed change in streamflow, temperature and precipitation. As stated by the Intergovernmental Panel on Climate Change (IPCC), confidence has increased that “some weather events and extremes will become more frequent, widespread or intense during the 21st century” (Solomon, 2007). On the other hand, changes of occurrence frequency of extreme flow events may also be attributed to the changes in land use or land cover. Schilling et al. (2014) proposed

quantitative data to analyze the relationship between downstream flood risks and different land uses for the large, intensely agricultural. Dornblaser and Striegl (2009) stated that the above factors have the potential to alter future suspended sediment loads.

It is found that many environmental factors may lead to changes in the frequency of extreme events, which may also play an important role in sediment transport. Consequently, if the trend in extreme events is not taken into account in future infrastructure, then such changes or variability can lead to underestimation /overestimation of design parameters for water infrastructure, and increase the likelihood of water shortages, water stresses, and agricultural failures (Sen, 2011). Using trend analysis as a means to examine long-term changes in heavy precipitation events, Frei and Schar (2001) concluded that this change of frequency is not reflected in mean values. Based on historical long-term records, trend analyses provided insight into the tendency in magnitude and frequency of hydrologic extreme events during the last several decades. Francipane et al. (2015) stressed that the hydrologic and geomorphic responses of watersheds to changes in climate are increasingly needed. Therefore, more efforts are needed to develop a PTM that accounts for the trends in extreme events that drive sediment transport processes.

As an application of this trend assessment methodology, this study aims to investigate sediment transport and long-term changes in extreme flow event frequency, both in large and small timescales, augmented by environment change. The event frequencies, also called counting events, are determined from historical records, and then linked to a logistic regression model. In the logistic regression model, the count records are used to estimate the trends of the frequency of extreme flow events. The proposed statistical model is based on an estimation of past

occurrence of extreme flow events as opposed to most other methods which tend to focus on the intensity of the extreme flow events or the time series of the extreme quantities such as Hilbert-Huang Transform (HHT) (Huang et al., 1998). This study analyzes frequency trends of large flow perturbations from historical records from the Chijiawan region in central Taiwan. Thus, by incorporating a trend analysis, the jump term in PTM can explicitly account for sediment transport due to extreme flow events and their associated changes in frequency.

In this paper, the introduction is followed by the methodologies which consist of logistic regression and particle tracking models. The stochastic jump diffusion particle tracking model is used to simulate the sediment particle trajectory. Regarded as the input of the SJD-PTM, the logistic regression model is applied to determine the trend magnitude. The third section describes the setting of the simulations of two cases, the large timescale and small timescale flow events. In addition, the selection of the time step and the procedure for the computations are provided. Results and discussions are presented following the simulations. Finally, summaries and conclusions are drawn.

2. METHODOLOGIES

In this paper, we present a conceptual framework for linking the stochastic jump diffusion particle tracking model and the logistic regression model to describe sediment transport affected by changes in the frequency of extreme flow events or large flow perturbations.

2.1 Particle Tracking Model

In recent years, researchers have become increasingly interested in individual particle transport by using the particle tracking models (PTMs). Over last few decades, Lagrangian approaches in simulating the movement of fluid particles have

been used in the field of fluid mechanics. For instance, physics of turbulent flows can be described by using the mathematical properties of the Langevin equation, which is a sample stochastic model and a prototype of the stochastic differential equation (SDE). A few studies such as Sawford and Borgas (1994) and Pope (1994) attempted to model fluid particle properties, including particle position and velocity, in turbulent flows. The Langevin equation has also been adopted in particle tracking models to delineate particle properties, including particle velocity and position (Pope, 1994; Sawford and Borgas, 1994; Spivakovskaya et al., 2005; Man and Tsai, 2007; Oh and Tsai, 2010).

Governing equation of SJD-PTM

The stochastic jump diffusion particle tracking model (SJD-PTM), introduced by Oh and Tsai (2010) and Tsai et al. (2014), addressed the movement of particles in surface flows subject to a sequence of random occurrences of extreme flow events. The stochastic jump diffusion particle tracking model (SJD-PTM) is governed by the stochastic differential equation (SDE), Equation (1). There are three main terms in Equation (1) including a mean drift motion term, a Wiener process representing random turbulent motion, and a Poisson process describing the abrupt movement of particles caused by probabilistic occurrences of the extreme flow perturbations

$$d\mathbf{X}_t = \underbrace{\bar{\mathbf{u}}(t, \mathbf{X}_t)dt}_{\text{drift term}} + \underbrace{\sigma(t, \mathbf{X}_t)d\mathbf{B}_t}_{\text{random term}} + \underbrace{\mathbf{h}(t, \mathbf{X}_t)d\mathbf{P}_t}_{\text{jump term}} \quad (1)$$

where $\mathbf{X}_t = \{x(t), y(t), z(t)\}^T$ represents the trajectory of a particle, and $\mathbf{h}$ is a jump amplification factor. Parameters $\bar{\mathbf{u}}$, σ , which are all continuous functions corresponding to sediment transport, are comparable to the stochastic diffusion process. Based on the relationship between the Fokker-Planck equation and the advection-diffusion equation, Man and Tsai (2007) expressed the drift term $\bar{\mathbf{u}}$ as

follows:

$$\bar{\mathbf{u}}(t, \mathbf{X}_t) = \bar{\mathbf{U}} + \nabla \mathbf{D} = \begin{cases} \bar{U}(t, x, y, z) + \partial D_x / \partial x \\ \bar{V}(t, x, y, z) + \partial D_y / \partial y \\ \bar{W}(t, x, y, z) - w_s + \partial D_z / \partial z \end{cases}$$

where $\bar{U}$, $\bar{V}$ and $\bar{W}$ are the mean drift fluid velocities at $\mathbf{X}_t$ in time t in the streamwise, transverse and vertical directions, respectively. D_x , D_y and D_z are defined as the turbulent diffusivity in the x , y and z directions, respectively. Herein, the diffusion coefficient tensor $\boldsymbol{\sigma}$ is

$$\boldsymbol{\sigma}(t, \mathbf{X}_t) = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

Herein, σ_{ij} , $i = j = x, y, z$ is the diffusion coefficient parallel to the i th axis in the j th-component of a flow. The diffusion coefficients $\boldsymbol{\sigma}$ and the turbulent diffusivities $\mathbf{D}$ are related as:

$$D_i = \sigma_{ii}^2 / 2, i = x, y, z$$

The Poisson process is used to model the frequency of occurrence of extreme flow events.

In this study, the jump of a particle is defined as an abrupt translocation from a state to the other with a discontinuity compared to the state of mean drift motion. Such a jump may be attributed to hydrologic extreme events (e.g. floods or large flow perturbations). Mathematically, the discontinuity point can be divided into two types, namely the removable and jump discontinuity. In stochastic calculus, a branch of mathematics, operating on stochastic processes is primarily concerned with the jump discontinuity. Specifically, if the process $\mathbf{X}_t$ has a jump discontinuity at time t_0 , then both the limits of $\mathbf{X}(t_0^+)$ and $\mathbf{X}(t_0^-)$ exist but are not equal. Thus, the jump at t_0 can be defined as $\Delta \mathbf{X}(t_0) = \mathbf{X}(t_0^+) - \mathbf{X}(t_0^-)$.

The jump term of the proposed model consists of two parts: a jump amplification factor that determines the magnitude of the displacement of the particle and a Poisson process that models the frequency of the occurrence of extreme flow events. The first part of the jump term, the jump amplification factor, can be expressed as follows:

$$\mathbf{h}(t, \mathbf{X}_t) = \int_t^{t+T} \left(\int_0^T \frac{V_{pa} - V_{pb}}{\tau_p} d\tau \right) dt$$

where $\mathbf{h}$ is the jump amplification factor derived by Man (2007), which is associated with properties of the particle and its ambient fluid motion, reflecting the effect of extreme flow events on the particle movement. Specifically, $\mathbf{h}$, can be considered as the change of particle position triggered by the extreme flow event from time t to $t+T$. V_{pb} is the particle velocity immediately before the occurrence of an extreme flow event. In other words, it is the particle velocity in regular flow conditions. V_{pa} is the particle velocity after the occurrence of the extreme flow event; T is the time duration of the extreme flow event. In this study, h is treated as deterministic. While the particle moves from the regular state to the extreme flow state, the relaxation time, τ_p , is a time lag value representing the delay in particle reaction to an extreme flow event as a result of the inertia effects of the particle (Oh and Tsai, 2010). For a particle of larger size or density, it may not follow the fluid exactly due to the delay time during an extreme flow event. By contrast, the relaxation time τ_p approaches zero for a finer particle. Based on the reciprocal of the Dirac-delta function, this study assume that the relaxation time can be expressed as follows:

$$\frac{1}{\tau_p} = \delta\left(\frac{T}{2}\right)$$

By means of the fundamental property of the Dirac-delta function, the change of particle velocity as a result of the extreme flow event within time τ_p obtained as:

$$\int_0^T \frac{V_{pa} - V_{pb}}{\tau_p} d\tau = \int_0^T (V_{pa} - V_{pb}) \delta\left(\frac{T}{2}\right) d\tau = V_{pa} - V_{pb}$$

Therefore, the SJD-PTM for finer particles can be derived as:

$$d\mathbf{X}_t = \bar{\mathbf{u}}(t, \mathbf{X}_t) + \boldsymbol{\sigma}(t, \mathbf{X}_t) d\mathbf{B}_t + \left[\int_t^{t+T} (V_{pa} - V_{pb}) dt \right] d\mathbf{P}_t \quad (2)$$

where $d\mathbf{P}_t$ is defined as the number of extreme flow event occurrences during the simulation time dt , which may come with one or multiple occurrences. In this study, the number of extreme flow event occurrences during simulation time exceeding unity was considered negligible. The definition of $\int_t^{t+T} (V_{pa} - V_{pb}) dt$ is the magnitude of an extreme flow event within the time interval $[t, t + dt]$.

Applications to Stochastic Jump Particle Tracking Model

In modeling sediment transport, the probabilistic fluctuations triggered by turbulence are of smaller consequence relative to the uncertainties due to the random occurrence of large flow perturbations over a long time horizon. In this case, taking the Wiener process into consideration is not significant when modeling the variations of large flow perturbations via the Poisson process. Thus, we aim to employ the stochastic jump process, proposed by Man (2007), Oh and Tsai (2010) and Tsai et al. (2014), to describe movement of particles. Instead of three terms in Equation (1), it is assumed that the main induction of the particle movement would be caused by two terms, namely the mean drift velocity and the occurrence of large flow perturbations, given that they are both sufficiently large as compared to flow turbulence.

The stochastic jump process can be written as

$$d\mathbf{X}_t = \underbrace{\bar{\mathbf{u}}(t, \mathbf{X}_t) dt}_{\text{drift term}} + \underbrace{\mathbf{h}(t, \mathbf{X}_t) d\mathbf{P}_t}_{\text{jump term}} \quad (3)$$

Comparison of PTMs

The stochastic particle tracking model has been modified to more accurately and

realistically describe the movement of sediment particles in open channel flows. Table 1 lists the comparison of various stochastic particle tracking models. The stochastic diffusion particle tracking model (SD-PTM) proposed by Man and Tsai (2007) included drift term and random term to model the movement of particles in a normal two-dimensional flow field. In order to describe the abrupt change of particle position when experiencing the extreme events, the jump term was included in particle tracking model (Tsai et al., 2010). In addition, Tsai et al. (2014) assumed that the movement of particle can be described as stochastic jump particle tracking model (SJ-PTM) when modeling the flood flow events or storm surges of larger time scales. In this study, since the probabilistic fluctuations due to turbulence are of much smaller influence compared to the uncertainties from the random occurrences of extreme flow events over a longer time horizon, the proposed model SJ-PTM_LR incorporates both the SJ-PTM and the logistic regression. The SJ-PTM_LR aims to model the impact of the changed frequencies of extreme events on stochastic movement of suspended sediment particles.

Deposition and Resuspension processes

The study aims to model a sediment particle when it settled down on the bed. The threshold condition of a particle expressed by Einstein (1950) is as follows:

$$F_L > W$$

where F_L is an instantaneous lift force acting on a particle, and W is a submerged weight force of a particle as shown in Figure 1(a). Based on this threshold condition, Cheng and Chew (1998) derived the pickup probability formulation for sediment particle. According to this derivation, Wu and Lin (2002) proposed two approaches employed in the formulation of pickup probability, which assumed that the instantaneous velocity followed the log-normal distribution. More recently, Bose and

Dey (2013) also provided the probability distribution for sediment entrainment based on the exponential or Laplace-type distributions for turbulent velocity fluctuations.

The process of particle deposition and re-suspension can be described using the model proposed in this study, which utilizes the pickup probability proposed by Wu and Lin (2002) for determining whether particle deposition or re-suspension will occur. The instantaneous velocity approaching the particle on the bed, u_b , was treated as a random variable. By comparing the lift force and the submerged weight of the particle, Wu and Lin (2002) obtained the following equation:

$$u_b^2 > \frac{4d}{3C_L} \frac{(\rho_s - \rho)}{\rho} g$$

where d is the particle diameter, C_L is the lift coefficient, ρ and ρ_s is density of fluid and the sediment particle, respectively. Wu and Lin (2002) assumed that the instantaneous velocity u_b follows a log-normal distribution (Figure 1 (b)), and also used the experimental data to verify the pickup probability. As presented in Figure 1 (c) by defining $v_b = \ln(u_b)$, a random variable can be generated by a normal distribution with mean $\bar{v}_b = \ln(5.52u_*)$ and variance $\sigma_v^2 = 0.123$, as approximated by the analytical method presented in Wu and Lin (2002). As a result, deposition and resuspension of sediment particles can be determined by comparing a random variable v_b and the threshold $\ln(\sqrt{4d(\rho_s - \rho)g/(3\rho C_L)})$ where C_L is 0.21. Therefore, the deposition and resuspension pattern of particles becomes realistic by generating v_b according to the flow condition (i.e. shear velocity u_*). If v_b exceeds the value of the threshold, then the particle will start to move based on SJ-PTM.

2.2 Logistic Trend Model

Logistic regression has been widely applied in various fields, including the biological sciences, biomedical research, social sciences and engineering. An

application for trend modeling of rare events was developed by Frei and Schar (2001). Rare events are defined as events with a very low occurrence probability; the number of rare events that occur over a period of time is denoted by n . The Binomial distribution delineates a random process comprising m independence trials with probability p for successful trials n . In this study, it can be said that events occur with probability p , e.g. the threshold exceedence, probability for one year (number of days, $m=365$).

The binomial distribution function is expressed as:

$$B(n; p, m) = \binom{m}{n} \cdot p^n \cdot (1 - p)^{m-n}$$

$$\binom{m}{n} \equiv \frac{m!}{n! \cdot (m - n)!}$$

The expected mean and variance are given as $E[n] = m \cdot p$ and $Var[n] = m \cdot p \cdot (1 - p)$. By means of logistic regression models, the transformed expected value of counts can be expressed as a linear function of time:

$$\eta(p) = \alpha + \beta t$$

Herein, t represents time of year; the regression intercept α and coefficient β can be estimated from historical data; η is called the linear predictor, which can provide a transformation from the value range of $p \in [0,1]$ onto the real axis to ensure its compatibility with the linear model.

The logistic model is defined as:

$$\eta(x) = \text{logit}(x) \equiv \log\left(\frac{x}{1-x}\right)$$

Therefore, the expected value of counts with temporal variations can be obtained:

$$p(t; \alpha, \beta) = \frac{\exp(\alpha + \beta \cdot t)}{1 + \exp(\alpha + \beta \cdot t)}$$

Herein, the odds ratio is the ratio of events against non-events $p/(1 - p)$. The odds

ratio is defined as the relative change in the ratio of odds during the period (t_1, t_2) . Statistically, the odds ratio is designed to determine how the odds increase with certain changes (Walpole, 2007). Thus, by using the regression coefficient β , the magnitude of the trend can be estimated by the odds ratio Θ :

$$\Theta_{(t_2-t_1)} \equiv \frac{p(t_2)}{1-p(t_2)} / \frac{p(t_1)}{1-p(t_1)} = \exp[\beta \cdot (t_2 - t_1)] \quad (4)$$

For instance, $\Theta_{50} = \exp(\beta \cdot 50) = 1.11$, it can be said that odds have been enhanced by a factor of 1.11 when the time spans fifty years.

In this study, the rate of the Poisson process can be quantified as a function of trend magnitude Θ with the given time-average event probability $\bar{p}$:

$$\Theta = \frac{\lambda(\Theta)}{1-\lambda(\Theta)} / \frac{\bar{p}}{1-\bar{p}} \quad (5)$$

where $\bar{p}$ is also defined as an average occurrence of flow events. Hence, the rate of the Poisson process can be calculated by the given trend magnitude (odds ratio Θ) and the average occurrence $\bar{p}$.

Determining the parameter λ is important for simulating the Poisson process. By using the logistic regression model, both the events of the average frequency $\bar{p}$ and also the change frequency $\lambda(\Theta)$ during the observed period, which may be caused by environmental variation, can be calculated. Thus, the purpose of this study is to estimate the frequency change of large flow perturbations and incorporate effects of that change to modeled sediment transport.

Poisson Jump Processes

To incorporate the frequency change of the events into the stochastic jump diffusion particle tracking model, the Poisson process can be modified and derived as follows. The sequence of inter-arrival times between two events $\{T_n, n = 1, 2, \dots\}$ is independent and identically distributed exponential random variables with mean $\mu =$

$1/\lambda(\theta)$, accounts for the effect of changes in trend magnitude. Herein, $T_n, n > 1$ is defined as the elapsed time between the $(n-1)st$ and nth event.

$$T_n = -\mu \ln(U_{RV}) \quad (6)$$

The Poisson process $\{P(t), t \geq 0\}$ with a rate $\lambda(\theta) > 0$ is also called a counting process. According to the condition of Poisson process, the number of events in any interval of length t is following a Poisson distribution with mean $\lambda(\theta)t$. The expected value of Poisson process as a function of trend magnitude θ can be expressed as follows:

$$E[P(t)] = \lambda(\theta)t \quad (7)$$

In this study, $P(t)$ represents the total number of large flow events that occur within the total simulation time t . In this case, the simulation time t is set as a month. Mathematically, $\lambda(\theta)$ is defined as the rate of the Poisson process, which is equal to the reciprocal of an average return period of the events. By setting the trend magnitude θ and the average occurrence of events $\bar{p}$, the frequency change $\lambda(\theta)$ can be calculated using Equation (5). Herein, the trend magnitude, θ , is set as any number between 1 and 6; and $\bar{p}$ is set to be 1/3, 1/6, 1/12 and 1/24.

Using Equation (6), the Poisson process can be simulated via the exponential distribution from a uniformly distributed pseudo-random number generator.

2.3 Illustrative Example - Trend Magnitude Incorporated into PTM

In this section, a two-dimensional flow field case scenario is presented. The purpose of this case scenario is to examine particle movement in open channel flows by PTM (SJD-PTM) and to incorporate trend magnitude of extreme events. Data from Kaftori et al. (1995) can be used to model regular flow conditions and investigate the movement of a particle by the particle tracking model. The experimental system developed in Kaftori et al. (1995) was designed to observe the

motion of particles near the wall in a turbulence boundary layer. The experiments used a polystyrene particle (specific density was 1.05) as a tracer, and were conducted primarily at Reynolds numbers of 5000, 10000, and 14000. In total, there were 33 experiments, 20 of which included particles. For each experiment, details of experimental conditions (e.g. Reynolds number, water depth, and shear velocity etc) for three types of particle sizes (diameter of 100, 275, 900 μm) were provided.

Larger sized particles (900 μm) are particularly prone to settling in a short duration. However, the settling velocity may be overestimated for larger particles if estimated using Stokes' law. The theoretical settling velocity is approximately two times the settling velocity measured by Kaftori et al. (1995). In contrast, for smaller particle size (100 μm), the difference between the estimated settling velocity and the measured settling velocity is insignificant. In addition, Kaftori et al. (1995) observed that it would take smaller particles significantly more time to descend in the water column. Therefore, experimental results for medium sized particles (275 μm) were utilized in this case scenario. Herein, the settling velocity was assumed to be 0.0025 m/s for a particle size of 275 μm .

Extreme flow event conditions are not considered by Kaftori et al. (1995). In this study, it is assumed that the flow velocity during an extreme flow event is 2.5 m/s and 1.5 m/s in the streamwise and vertical direction, respectively. Also, random occurrences of extreme flow events follow the Poisson process with a mean rate of 0.05/s. Additionally, it is assumed that the random occurrences of extreme flow event may increase with the trend magnitudes of 2 and 3 (i.e. $\theta=2, 3$).

Moreover, by using the analytical method proposed by Wu and Lin (2002), we obtain the mean and variance of transformed instantaneous particle velocity (i.e. v_b) for extreme flow events, which was estimated as 0.8675 m/s and 0.0975 m/s,

respectively. This result shows that once the extreme flow events occur, particles on the bed would have a probability of 100% of being re-suspended.

Figure 2 shows both scenarios and the ensemble mean of streamwise and vertical particle position (see Figure 2 (a) and (b)). In this study, it is assumed that particle movement is affected by the extreme flow events that cause an increased frequency of extreme flow occurrences (the trend magnitude of 2 and 3, i.e. $\theta = 2, 3$). Figure 2(b) shows that with the increased frequency of extreme flow events, particles are suspended higher above the bed. Additionally, extreme flow events occurring at a higher frequency cause longer duration particle suspension relative to lower frequency extreme flow events. As a result of the higher suspension above the bed and longer suspension duration resulting from higher frequency of extreme events, particles move further in the streamwise direction. Simulation of particle movement reveals that there is an error of about 11.6% in the stream-wise position between the SD-PTM and SJD-PTM, and there is an error ranging between 7% to 13% relative to the SJD-PTM that does not consider the trend magnitude of 2 and 3.

Figure 2(c) shows the mean cumulative time that particles remained on the bed. As observed, the suspension duration increases when considering the jump term with the increasing frequency of extreme flow events. As a result, the mean time that particles remain on the bed decreases. The SJD-PTM models predicted a mean time that particles remained on the bed which differed by 28%. Furthermore, when the trend magnitude was 2 and 3, values differed by 24% and 40% respectively, without considering changes in frequency.

Comparing a trend magnitude of 3 with magnitudes of 1 and 2, the simulation results indicate that particles move further in the streamwise and higher in the vertical direction. As a result, the trend magnitude might be a dominating factor influencing

the particle movement. Put differently, this study provides a method to examine how an essential input (i.e. trend magnitude θ) affects the particle tracking model and the impact of environmental variation caused changes in extreme flow event frequency on particle movement.

Consequently, the increased frequency of extreme flow events, or trend magnitude θ , can be used to quantify potential impacts on particle movement. Moreover, the model findings imply that increased extreme flow event frequency may cause increased turbidity.

3. SIMULATION

In the simulation, the frequency trend of large flow perturbations is analyzed using historical flow data. This study defines large flow perturbations as extreme flow events having a probability of occurrence of less than 10% (i.e., the events with an average return period of 10 days). By means of this trend analysis, we define the rate of the Poisson process in PTM considering the frequency change of large flow perturbations.

3.1 Case 1: Large Timescale Flow Events

In this case scenario, daily flow discharge collected from 1966 to 2011 at Chijiawan station in central Taiwan is used. The flow events can be classified into four categories according to the different timescales, with an average return period of 10, 30, 90 and 365 days. In this study, the aforementioned flow events can be defined as the large flow perturbations, which have a probability of occurrences of less than ten percent. Herein, the event count is defined as the total number of events exceeding a threshold, tallied over one whole year. The event count is also referred to as annual count records.

Figure 3 shows the event count of daily flow events for different timescales. A

dot is used to represent the number of events, which is the total number of days that daily discharge exceeded the threshold value over the year. Results of the logistic regression model, plotted as a line, show the estimated trend predicting the expected event count relative to the time. Table 2 presents four flow events recorded during the past 45 years. The threshold values are estimated for each event based on the Weibull formula (1951), $T=(n+1)/m$ where T refers to the average return period; n is the number of observation record; and m is the rank of a list ordered by descending magnitude. Also, the odds ratio, which indicates a change of the ratio between the events and non-events in a certain period, is determined by Equation (4). This study is concerned with the probability that the frequency of flow events over 50 years changes. The average frequency and change in frequency during the observed period are also listed in Table 2.

As observed, the evidence suggests that the estimated trend increases for all flow events, and it is obvious that the trend substantially increases with events of higher rarity during the observation period. However, for events having a return period of 10- days, the increase is not as significant.

In this study, the jump term in the PTM is intended to describe how a particle may undergo a significant change of status in the presence of large flow perturbations. One can approximately consider the aforementioned hydrologic flow events with a return period of 10, 30, 90 and 365 days as the weekly, monthly, seasonal and annual flow events, respectively. Through trend estimation, it has been concluded that increasing changes in the frequency of those events may be attributed to environmental factors such as climate change, land use and land cover change. The evidence indicates that events of higher rarity, which have a longer average return period, are likely to have a significant increase in the frequency. This result may have

many important implications to environmental and water resources engineering. For instance, increased occurrence of extreme flow events might cause particles to travel further and faster downstream. Additionally, increased occurrence of extreme flows might induce a higher degree of entrainment and particle resuspension, and consequently more significant bed and bank erosion.

3.2 Case 2: Small Timescale Flow Events

Flow events recorded over a smaller timescale were counted from 20 years of monthly, hourly discharge data from the Chijiawan station in central Taiwan. Flow events of smaller time scales can be classified into four types of events having an average return period of 3, 6, 12 and 24 hours. As shown in Table 3, the details of these events are tabulated from the selected data. The mean event discharge is estimated by averaging the magnitude of the events that exceed the corresponding threshold. By using the logistic regression model, the odds ratio, representing the fractional change of the event over the observed period, can be calculated. In other words, owing to the fact that the frequency of events may increase over the observed period, the odds ratio can be used to quantify the probability of frequency change for a given period of time. The average flow event frequency $\bar{p}$ and the fractional change of event frequency $\lambda(\theta)$ can be computed using the logistic regression over the observed period.

According to the event count, an increase in the frequency for flow events occurring over smaller timescales is also observed. However, the increase in the frequency is greater for events having an average return period of 6 hours relative to 12 hours. This phenomenon can be attributed to the logistic regression model, which was constructed using event count data and does not consider the magnitude of the events. Thus, it is speculated that the increasing trend may be caused by events which

range between an average return period of 6 and 12 hours.

3.3 Selection of time step

Tsai et al. (2014) attempted to simulate the Poisson process using two different time steps for 2,000 runs. The results showed that the difference between the expected value and simulation mean calculated by 2,000 runs is not significant; simulation results are sufficient for 2,000 runs for several different time steps. Thus, it was concluded that the simulation time step did not lead to a significant difference in these two outcomes. However, in this study, it was found that the simulation mean, is underestimated for the same 2,000 runs as compared to the expected value. Little research has been done on selecting the proper time steps in simulating the Poisson process explicitly. Thus, more numerical experiments should be pursued before modeling sediment transport.

The purpose of this numerical experiment was to try three different time steps to simulate the Poisson process and then compare results with the expected value. As shown in Figure 4, four categories of smaller timescale flow events with an average return period of 3, 6, 12 and 24 hours, respectively, were used. By selecting a different time step from $dt= 1, 0.2$ and 0.05 hours, the total simulation steps n increased to $n=720, 3600$ and 14400 , respectively. Herein, the total simulation time is a month, the time step is set as $dt= 1, 0.2$ and 0.05 hours, and 2,000 runs are performed.

In Figure 4 the dotted line represents the expected number of events which can be calculated by Equation (7). The points are the ensemble mean for 2,000 runs simulated by the Poisson process for three different cases, $dt= 1, 0.2$ and 0.05 hours. As observed, by accounting for the effect of environmental variation such as climate change, the expected number of each event increases with the trend magnitude or

odds ratio. In other words, the frequency of flow events increases with the trend magnitude. The evidence suggests that the ensemble means will be underestimated for an increasing frequency. As shown in Figure 4, it is more difficult for the events with a higher frequency of occurrences to be correctly simulated by the Poisson process. In contrast, the number of rare events with higher rarity can be simulated more precisely, especially for the smaller time step with larger total simulation steps.

According to this numerical experiment, it can be concluded that the total simulation steps n should *exceed* 14,400 in order to simulate the Poisson process more precisely within a specific total simulation time. From this numerical experiment, it was found that the total simulation steps play a role in generating the Poisson process. More work is needed to investigate how to effectively determine the appropriate number of simulation steps.

Results reveal long-term trends in historical records for both smaller and larger time scale flow events. Over the observational period, the frequency of these events has increased. Zwiers and Kharin (1998) stated that changes in the frequency or intensity of extreme events will be affected by environmental factors such as climate change, land use and land cover change, which may directly affect temperature and cause indirect effects including floods. According to the trend estimated in this study, it is possible that the increased frequency of events may be due to environmental factors such as climate change, land use and land cover change. As a result, the events may become more uncertain or extreme. Additionally, our findings show that rarer events, which have a longer average return period, are expected to have a more significant increase in occurrence frequency. Hence, it can be concluded that the parameter of the odds ratio in the logistic regression model can provide insights into the frequency change of events. In the following section, the odds ratio, namely the

trend magnitude, is employed to quantify the frequency change of the events. Then, by means of the stochastic jump particle tracking model (SJ-PTM) the effect of changes in event frequency on the movement of particles is described.

3.4 Procedure for the computations

A schematic diagram illustrating the structure of this study is shown in Figure 5.

The procedure for the simulation computations is shown below.

1. (Mean drift term) Compute the fluid velocity in the streamwise direction $\bar{U}$ based on the logarithmic profile

$$\bar{U} = \left(\frac{u_*}{\kappa}\right) \ln\left(\frac{z}{z_0}\right) \quad (8)$$

where the shear velocity can be computed by $u_* = \sqrt{gRS_0}$, the von Karman constant $\kappa = 0.4$ and the zero-velocity level $z_0 = k_s/30$

2. (Mean drift term) Compute the turbulent diffusivity in the vertical direction D_z according to

$$D_z = \kappa u_* z \left(1 - \frac{z}{H}\right)$$

where H is the flow depth

3. (Jump term) Define the extreme event or large flow perturbation, i.e. a probability of occurrence less than 10%. Compute the odd ratio θ by logistic regression based on the historical records.
4. (Jump term) Compute the Poisson process taking the trend of frequency into consideration, which probability can be expressed by $P[\mathbf{P}(dt)] = e^{-\lambda(\theta)dt}$
5. (Turbulence term) Simulate the Brownian motion which is a Gaussian distributed random variable expressed as $\Delta B_t = \sqrt{\Delta t}N(0,1)$
6. Compute the particle position in the streamwise and vertical directions (X_n, Z_n) by substituting Equation (8) into the numerical solutions of Equation (3) solved by

the Euler-Maruyama scheme (Man and Tsai, 2007; Oh, 2011; Tsai et al., 2014)

$$X_n = X_{n+1} + \left(\frac{u_*}{\kappa}\right) \ln\left(\frac{Z}{Z_0}\right) dt + (V_{pa} - V_{pb}) \Delta t dP_t(\Delta t)$$

$$Z_n = Z_{n+1} + \left(\bar{W} - w_s + \kappa u_* \left(1 - \frac{2Z}{H}\right)\right) \Delta t + \sqrt{2 \left(\kappa u_* Z \left(1 - \frac{Z}{H}\right)\right)} \Delta B_t \\ + (V_{pa,z} - V_{pb,z}) \Delta t dP_t(\Delta t)$$

$X_n = X(t_n), Z_n = Z(t_n)$ is the current state of the particle position in the streamwise and vertical direction respectively at $t = t_n$; $X_{n+1} = X(t_{n+1}), Z_{n+1} = Z(t_{n+1})$ is the state of the particle position at time $t = t_{n+1}$; Δt is the simulation time step; V_{pb} and V_{pa} are the particle velocity before and in the presence of the occurrence of extreme event, respectively. In this case, V_{pa} is treated as deterministic.

7. Compute the ensemble means and variances of particle trajectories based on iterative simulation results at least 1,000 times

4. RESULTS AND DISCUSSION

Sediment particle trajectories in flows are simulated in the following two case scenarios by using the stochastic jump particle tracking model (SJ-PTM) and taking into account the effect of changes in flow event frequency. These results can be compared to theoretical means and variances of the stochastic jump processes obtained by $\mathbf{E}[d\mathbf{X}_t] = (\bar{\mathbf{u}} + \lambda \mathbf{h})dt$ and $\mathbf{Var}[d\mathbf{X}_t] = (\sigma^2 + \lambda \mathbf{h}^2)dt$, respectively (Hanson, 2007). The variations of means and variances of particle position caused by the jumps term can also be treated as the difference resulting from the stochastic jump processes as follows:

$$\mathbf{E}[d\mathbf{X}_t]_{\text{jump}} = \lambda \mathbf{h} dt \text{ and } \mathbf{Var}[d\mathbf{X}_t]_{\text{jump}} = \lambda \mathbf{h}^2 dt \quad (9)$$

4.1 Case 1: Large Timescales Flow Events

Herein, the total simulation time is ten years, and the time step is 0.2 days. Table 4 lists the mean drift and large flow conditions. As observed in Figure 6 and Figure 7, the ensemble means and variances of particle position are estimated through 2,000 iterative simulations.

The results in a one-dimensional flow field are presented under the same frequency change: the trend magnitude ranges between 1 to 6 for the four categories of flow events. As expected, with increases in frequency, the ensemble means and variances of particle position also increase for each flow event. A similar trend is observed in theoretical results (see Equation (7)). For all flow events, the ensemble means are in good agreement with the theoretical means. However, the variances of particle position of events with an average return period of 10 days are underestimated for an odds ratio greater than 4.

A limitation of this study is that only one extreme flow event can be simulated in a single time interval. However, the probability of more than one extreme flow event occurring in the Poisson increment is considered negligible. Consequently, it is speculated that the Poisson process may underestimate the actual number of high frequency event occurrences λ . When λ is underestimated, both the ensemble mean and variance of particle trajectory are thus underestimated. Equation (9) demonstrates how such a discrepancy is more significant in the particle trajectory ensemble variance relative to the ensemble mean for large errors in λ .

Trend analysis reveals that the proposed model (SJ-PTM_LR) successfully predicts the ensemble means of particle positions for the next 50 years under the effect of environment change. Events with an average return period of 10 days have lower trend magnitude compared with those with a larger return period, suggesting that the frequency of events having a shorter return period are less effected by

environmental variation. Regarding particle position, Figure 8 shows that the ensemble means of particle positions computed using the proposed model (SJ-PTM_LR) have no evident difference relative to positions computed using the SJ-PTM. By contrast, events with higher rarity (e.g., a return period of 365 days) have a higher trend magnitude, $\theta_{50}=34.99$. As a consequence of the elevated trend magnitude, the number of extreme flow events is increased, which in turn promotes increased particle movement contributed by jump terms. Equation (7) shows how events with a higher trend magnitude have a higher expected value in the Poisson process. As such, a higher number of extreme flow event occurrences are produced by the Poisson process for the given simulation time. Finally, using Equation (2), particle trajectory that considers the abrupt change in the occurrences of flow events can be simulated.

4.2 Case 2: Small Timescales Flow Events

In this case scenario, a time step of 0.05 hours over a total simulation time of one month is used to simulate particle movement. In total, there are 2,000 iterative simulations which apply the mean drift and flow conditions shown in Table 5 to simulate large flow perturbations. As computed using the SJ-PTM, the ensemble means and variances of particle position for the 2,000 iterative simulations, under six different trends, are computed and plotted in Figure 9 and Figure 10.

As suggested by theoretical results (see Equation (9)), trends of the ensemble means and variances of particle position due to the jump term increase as frequency increases. This effect can be estimated using the logistic regression model from historical data.

By using the stochastic jump process, the movement of particles caused by large flow perturbations in large and small timescales flow events has been investigated.

The simulation results indicate that there is a substantial increase in the ensemble means and variances of particle position as a result of an increase in the frequency of extreme flow events. Moreover, consistent with the theoretical results, simulation results demonstrate that the ensemble means of particle position are closer than the variances. Consequently, a change in the frequency of extreme events has a significant impact on the ensemble means and variances of the particle position; particles travel further and faster downstream, though with more uncertainty, under the effect of increased extreme event frequency. For example, Figure 8 shows the final particle position calculated by SJ-PTM (dash line) for a trend magnitude of 0.43%, 2.78%, 8.13% and 18.76% for events with return periods of 10, 30, 90 and 365 days, respectively. The SJ-PTM (dash line) values are higher than those computed by SJ-PTM (dot-dash line).

5. SUMMARY AND CONCLUSIONS

This paper aims to develop a stochastic particle tracking model, namely the SJ-PTM_LR to investigate movement of suspended particles affected by environment induced variations in the frequency of extreme flow events or large flow perturbations. By considering the trend magnitude of large flow perturbations and the resultant effects on flow trends, the proposed model is able to predict the movement of particles more realistically than existing models.

To investigate particle transport in different flow conditions (i.e., extreme flow conditions), this study considered the magnitude, frequency and trend magnitude of extreme flow events to simulate movement of particles using the stochastic jump diffusion particle tracking model (SJD-PTM_LR). The streamwise and vertical positions of the particle relative to time are shown in three-dimensional figures. SJD-PTM_LR, including the effects of trend magnitude, was used to predict the movement

of particles. After considering trend magnitude, particle transport velocity is higher than the velocity modeled without considering the effect of increased trend magnitude. Results also reveal that as the frequency of extreme flow events increases, the duration the particle remains on the bed decreases.

In this study, analysis of historical records offers an insight into the frequency change of large flow perturbations resulting from environmental change. These findings imply that the trend magnitude can be used to quantify the frequency change of events, which can be incorporated into the modeling of sediment transport. In modeling the Poisson jump process with respect to trend magnitude, evidence suggests that the trend magnitude of large flow occurrences might be underestimated. Such discrepancy could be attributed to the selection of numerical time steps. To better understand the effect of the time step on model results, this study applied different time steps to simulate the Poisson process and an appropriate time step was identified.

Case scenarios 1 and 2 modeled movement of particles in a one-dimensional flow field using the stochastic jump particle tracking model (SJ-PTM_LR) for long term predictions. Given that both the mean drift term and a Poisson jump term were sufficiently large compared to a random turbulence term, it was assumed that the turbulence effect resulting from the Wiener process could be considered negligible. Results demonstrated that the ensemble means and variances of the simulated particle position incorporated the effect of increased trend magnitude.

Ensemble means of particle trajectories have significantly increased due to the increased frequency of extreme flow events. Additionally, increased ensemble variances of particle trajectories indicate the particle trajectories will become more uncertain. The variability of particle trajectories can be quantified for an estimated

frequency change of extreme flow events. The ensemble mean and variance of particle positions via simulations are in good agreement with theoretical results.

Additionally, an increased occurrence of extreme flow events induces a higher degree of entrainment and particle resuspension, and consequently more significant bed and bank erosion occurs. These findings are in line with the Molnar (2001), in which they stated that climate change may increase erosion and bed load transport rates.

Compared with deterministic modeling, the proposed SJD-PTM_LR has the capability to describe the probabilistic characteristics of flow and sediment properties attributed to turbulence, random occurrences of extreme flows with probabilistic flow magnitude, and future trends of extreme flow occurrences due to potential climate change, and land use/cover change. One can expect to obtain more comprehensive information on sediment variables from the stochastic modeling of sediment transport such as the statistical moments and the probability distribution function. Such probabilistic outcome provides a valuable means for assessing the probability of failure (i.e., risk) resulting from sedimentation processes. Additionally, this study may offer a potential tool for developing hydraulic structures, facilities, channelization and dredging designs that take flow trends into consideration.

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Table 1. Comparison of SPTMs

Authors	Model	Governing equation
Man and Tsai (2007)	SD-PTM	$d\mathbf{X}_t = \bar{\mathbf{u}}dt + \boldsymbol{\sigma}d\mathbf{B}_t$
Oh and Tsai (2010)	SJD-PTM	$d\mathbf{X}_t = \bar{\mathbf{u}}dt + \boldsymbol{\sigma}d\mathbf{B}_t + \mathbf{h}d\mathbf{P}_t$
Tsai et. al (2014)	SJ-PTM	$d\mathbf{X}_t = \bar{\mathbf{u}}dt + \mathbf{h}d\mathbf{P}_t$
Proposed model	SJ-PTM_LR	$d\mathbf{X}_t = \bar{\mathbf{u}}dt + \mathbf{h}d\mathbf{P}_t$ where $P[P(dt)] = e^{-\lambda(\Theta)dt}$

Table 2. Flow conditions for events with an average return period of 10, 30, 90 and 365 days.

Average return period	10 (days)	30 (days)	90 (days)	365 (days)
Threshold (<i>cms</i>)	11.1100	20.79	37.45	89.67
Mean (<i>cms</i>)	24.9766	45.6338	83.0868	174.2530
Odds ratio θ_{50}	1.11	2.16	7.29	34.99
λ (/days)	0.1000	0.0333	0.0111	0.0027
$\lambda(\theta_{50})$ (/days)	0.1099	0.0693	0.0757	0.0877

Table 3. Flow conditions for events with an average return period of 3, 6, 12 and 24 hours.

Average return period	3 (hours)	6 (hours)	12(hours)	24(hours)
Threshold (<i>cms</i>)	2.4800	3.1200	3.5000	4.2200
Mean (<i>cms</i>)	3.4444	4.0408	4.7387	5.6276
Odds ratio θ_{20}	3.1832	5.948	4.2491	8.2055
λ (/days)	0.3333	0.1667	0.0833	0.0417
$\lambda(\theta_{20})$ (/days)	0.6141	0.5433	0.2786	0.2630

Table 4. Mean drift and flow conditions for flow events having a return period of 10, 30, 90 and 365 days.

Mean drift velocity = 0.2497 m/s			
Events with an average return period of (days)			
10	30	90	365
Mean rate of occurrence of the flow perturbations (/day)			
1/10	1/30	1/90	1/365
Magnitude of the flow perturbations (m/s)			
0.8347	1.2475	1.8601	3.0477

Table 5. Mean drift and flow conditions for flow events having an average return period of 3, 6, 12 and 24 hours, respectively.

Mean drift velocity = 0.1713m/s			
Events with an average return period of (hours)			
3	6	12	24
Mean rate of occurrence of the flow perturbations (/hour)			
1/3	1/6	1/12	1/24
Magnitude of the flow perturbations (m/s)			
0.2228	0.2478	0.2756	0.3091

Figure Caption

Figure 1. (a) Illustration of particle resuspension. (b) The instantaneous velocity ub as log-normal distribution. (c) vb as a normally distributed random variable after logarithm transformation. [*Figure 1(b), (c) are modified figures of Figure 1 in Wu and Lin (2002)*]

Figure 2. Particle trajectory for a particle of diameter of 275 μm simulated by SD-PTM, SJD-PTM and SJD-PTM for $\theta=2, 3$.

Figure 3. Annual event count for flow events having an average return period of 10, 30, 90 and 365 days between 1966 to 2011 at Chijiawan station in central Taiwan.

Figure 4. A comparison of different time steps used to simulate the Poisson process and comparison with the expected value.

Figure 5. Schematic of major components of model development

Figure 6. The ensemble means of particle position subjected to the large flow perturbations for flow events having a return period of 10, 30, 90 and 365 days.

Figure 7. Variances of particle position subjected to the large flow perturbations for events having an average return period of 10, 30, 90 and 365 days.

Figure 8. The ensemble means of particle position relative to relative time, which are subjected to the large flow perturbations for flow events having an average return period of 10, 30, 90 and 365 days.

Figure 9. The ensemble means of particle position subjected to large flow perturbations for events having an average return period of 3, 6, 12 and 24 hours.

Figure 10. Variances of particle position subjected to the large flow perturbations for flow events having an average return period of 3, 6, 12 and 24 hours

Figure 1. (a) Illustration of particle resuspension. (b) The instantaneous velocity u_b as log-normal distribution. (c) v_b as a normally distributed random variable after logarithm transformation. [Figure 1(b), (c) are modified figures of Figure 1 in Wu and Lin (2002)]

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