**Assessment of Input Uncertainty by Seasonally Categorized Latent Variables Using
SWAT**

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Abstract

Watershed processes have been explored with sophisticated simulation models for the past few decades. It has been stated that uncertainty attributed to alternative sources such as model parameters, forcing inputs, and measured data should be incorporated during the simulation process. Among varying uncertainty sources, input uncertainty attributed to precipitation data exhibits a dominant role, as it is the source driving most hydrologically-related processes. In previous studies, latent variables (normally distributed random noise) have been implemented to explicitly incorporate input uncertainty from precipitation data. However, it may not be appropriate to apply the same set of latent variables throughout temporal series without considering seasonal effects. In this study, seasonally categorized latent variables were defined to investigate potential effects on model predictions and associated predictive uncertainty. Results show that the incorporation of seasonal latent variables resulted in better statistical solutions (NSE, Nash-Sutcliffe Efficiency coefficient) for both calibration ($0.58_{\text{[streamflow]}}$ / $0.73_{\text{[sediment]}}$ / $0.59_{\text{[ammonia]}}$) and validation ($0.57_{\text{[streamflow]}}$ / $0.45_{\text{[sediment]}}$ / $0.53_{\text{[ammonia]}}$) periods. Alternative definitions of Dry/Wet seasonality (two definitions are defined in this study) also affected model predictions. In addition, it was determined that predictive uncertainty can be enhanced by incorporating more latent variables during model calibration. The implementations of proposed seasonal latent variables have further substantiated the importance of incorporating seasonal effects when conducting comparable approaches. Applications of latent variables on future work should evaluate potential effects on model predictions before performing associated scientific studies or relevant decision making processes.

Keywords: Input uncertainty; Latent variables; Model calibration; IPEAT; SWAT

1. Introduction

In the past two decades, sophisticated watershed simulation models have been developed to investigate relevant issues of hydrologic and nutrient processes (e.g., Soil and Water Assessment Tool, SWAT (Arnold et al., 2012); Agricultural Policy/Environmental Extender Tool, APEX (Williams et al., 2012); and Hydrological Simulation Program-Fortran, HSPF (Bicknell et al., 1997)). The associated high-dimensional calibration work required is challenging. The uncertainty attributed to different sources such as model parameters, forcing inputs, and measured data should be incorporated in hydrologic and nutrient modeling research (Balin et al., 2010; Yen et al., 2014b). In addition, the issue of equifinality could also be a concern since similar model predictions may be generated through alternative parameter sets (Beven, 2006; Ficklin and Barnhart, 2014). While several studies have addressed the issue broadly (Wang et al., 2014; White et al., 2015), additional detailed investigations have been conducted on specific uncertainty sources including: parameters, forcing inputs, model structure, and measured calibration/validation data (Ajami et al. 2007; Salamon and Feyen, 2009; Balin et al., 2010; Yen et al., 2014b). Biased results, in terms of model predictions, are often generated when one or more uncertainty sources are omitted during calibration. Consequently, prejudiced information may lead to poor judgment during decision making activities (McMillan et al., 2011).

Among varying uncertainty sources, input uncertainty associated with precipitation data exerts a significant role (Balin et al., 2010, McMillan et al., 2011). Precipitation data is typically measured by tipping bucket rain gauges whereas mean areal precipitation records may be comprised of both spatial and temporal data (Balin et al., 2010). Input data uncertainty associated with precipitation may be caused by mechanical issues, evaporation losses, wind disturbance and other random errors (McMillan et al., 2011). Most hydrological and nutrient processes are driven by surface and subsurface flow resulting from precipitation. It is therefore difficult to provide

reliable model predictions when input uncertainty associated with precipitation data is not identified using scientifically credible approaches (McMillan et al., 2011).

Studies exploring uncertainty associated with precipitation data model input are still uncommon. The mainstream approach implements latent variables, where input uncertainty attributed to precipitation is incorporated explicitly, during the calibration process (Kavetski et al. 2002; Ajami et al. 2007, Yen et al., 2015a). Precipitation data is altered by multiplying random numbers (i.e., white noise) from a normally distributed pool with a pre-defined mean (μ) and variance (σ^2). The mean and variance are calibrated, along with other model parameters, in the following generation. The first use of latent variables assigned random numbers to each and every time step of precipitation data (Kavetski et al. 2002; Kavetski et al. 2006). However, the calibration requirements become extremely difficult with increasing data record length because the number of parameters requiring calibration increases correspondingly. A more feasible approach, proposed by Ajami et al. (2007), requires only a small set of latent variables (i.e., single values for both mean and variance) applied during the calibration process. The justification for their default mean (0.9 to 1.1) and variance (10^{-5} to 10^{-3}) ranges is however unexplained. Additionally, there is no guarantee that model predictions will be improved by using these default settings. Yen et al. (2015a) investigated several alternative latent variable ranges and found they altered the associated calibration and/or the uncertainty analysis results.

Latent variable use can provide more comprehensive knowledge on model predictions and ultimately better guidance for decision making processes, however, more studies are required to develop the method's robustness. For example, the seasonal effect of latent variables has not been considered in other studies. The goal of this study was to explore the effect and magnitude of latent variable applications upon model predictions by categorizing latent variables (i.e.,

precipitation data) on a seasonal basis. Two general questions were proposed: 1) does a single set of latent variables, representing the whole simulation period provide statistically sufficient results? and, 2) how much do model predictions results change when different sets of latent variables are assigned? Specifically, the following objectives were examined: (i) quantify and compare model predictions (streamflow rate, sediment, and ammonia loads) by implementing different categories of latent variables; and (ii) quantify corresponding predictive uncertainty while the latent variable approach is conducted to incorporate input uncertainty on precipitation data. In this study, SWAT was employed, with precipitation latent variable scenarios, to conduct watershed simulations for the Arroyo Colorado Watershed (ACW) in Texas, USA. Five scenarios were examined and compared.

2. Materials and Methods

2.1. The SWAT Model

SWAT was developed in early 1990s by the United States Department of Agriculture – Agricultural Research Service (USDA-ARS), Temple, Texas, to evaluate the impact of land management on water, sediment and pollutant yields (Gassman et al., 2007; Arnold et al., 2012). SWAT is a continuous hydrologic model that has the capability to run on a daily time-step at a watershed scale. Major components of the SWAT model include surface runoff, return flow, reach routing, meteorology, erosion, soil temperature, crop growth, nutrients, pesticides and agricultural management. In addition, SWAT is a semi-physically based, spatially distributed model that disaggregates the watershed into several hydrologic response units (HRUs) with homogeneous land use, soil and topographic characteristics. Over the years, the SWAT model has been successfully applied to model water quality issues including sediments, nutrients and

pesticides in watersheds (Lin et al., 2013; Zhenyao et al., 2013; Yen et al., 2015b). More than 2,000 peer-reviewed journal articles are available for users as references in implementing relevant topics.

2.2. Study Area & Model Inputs

The study area was the Arroyo Colorado watershed (ACW) located in southern Texas along the border of USA and Mexico (see Figure 1). The Arroyo Colorado River flows eastwards to the Gulf of Mexico and drains the watershed with an area of 1,692 km². The predominant land use type in ACW is agriculture (54%) and range land (18.5%), followed by urban areas (12.5%) along the mainstream of the river. The soil types in the watershed are mostly clays, fine loams, and clay loams (clay: 20.4%, fine loam: 13.2%, clay loam: 7.4%, fine silt: 6.3%, silty clay: 4.0%, fine clay: 4.0%) with soil depths ranging between 1.6 and 2.0 m (USDA, 1972). The climate in the watershed is semi-arid with annual precipitation ranging from about 530 to 680 mm, and mean monthly temperatures ranging from 14.5 °C in January to 28.9 °C in July.

Figure 1 shows spatial positions for three weather stations and a stream gauge. Daily meteorological data including precipitation, minimum and maximum air temperature were compiled from these three stations covering 2000 to 2005. This meteorological data was obtained from Texas State Climatologist Office located at Texas A&M University at College Station. International Boundary and Water Commission provided daily streamflow data for two stations, one near the estuary of the river and the other near Llano Grande at middle of the reach. Streamflow data near the estuary is influenced by the diurnal fluctuations of tidal waves. Therefore, 2002 to 2005 streamflow data from the station near Llano Grande (marked in Fig. 1) was used to calibrate the SWAT model.

A digital elevation model, with a resolution of 30 meters, was downloaded from the National Elevation Dataset of the U.S. Geological Survey (USGS) (Gesch et al., 2009) (accessible online at <http://ned.usgs.gov/>; last accessed on September 18, 2013). Land use data was created from remote sensing data and field surveys for 2004 to 2007 were used to represent land cover conditions. Soil properties were acquired from the SSURGO soil database of USDA-NRCS (Natural Resources Conservation Service). Based on the topography and river network, the ACW was divided into 17 subbasins. The subbasins were further partitioned into 475 HRUs according to the land use and soil properties.

The ACW has 21 permitted point source discharges, 16 of which are municipal, three are industrial, and two are shrimp farms. The discharge permit limits the municipal plants range from 0.4 to 10 million gallons per day. The shrimp farms discharge infrequently (Rains and Miranda, 2002). Water quality data from limited grab samples were obtained for suspended sediment (SS) and ammonia (NH_4^{+-}N). The grab-samples were converted, using the Maximum Likelihood Estimation (MLE) method, to time series load by the LOAD ESTimator (LOADEST) developed by USGS (Runkel et al. 2004). The time series water quality data, with monthly average within 95% confidence interval, was used as observation data for calibration of the SWAT model. Additional ACW details can be found in Seo et al. (2014).

2.3. Examining Input Uncertainty using Seasonally Defined Latent Variables

In general, “forcing inputs” consist of data that exert critical control on model simulations. Classic examples include precipitation and temperature. More and more studies have taken the approach of using latent variables to incorporate forcing input uncertainty explicitly into watershed modeling (Kavetski et al., 2002; Ajami et al., 2007; Yen et al., 2015a). The original scheme of implementing input uncertainty, proposed by Kavetski et al. (2002) (Bayesian total

error analysis, BATEA), assigns latent variables to each time step. It starts from pre-defined, estimated, values which are calibrated, along with other model parameters. In this case, the number of latent variables, and associated calculations, increases rapidly, thereby complicating the modeling process. An alternative approach, developed by Ajami et al. (2007) (Bayesian uncertainty estimator, IBUNE), applies one set of latent variables throughout the entire simulation period. In this study, precipitation was chosen as the source of input uncertainty and the associated equation is shown as follows:

$$R_i^{adjusted} = k \times R_i^{observed} \quad (1)$$

Where, $R_i^{adjusted}$ is the adjusted precipitation (mm); $R_i^{observed}$ is the observed precipitation (mm); and k is a normally distributed random number or noise with latent variables mean (θ) and variance (σ^2). In this study, input uncertainty settings use the approach implemented in IBUNE, latent variables are predefined parameters which require further calibration within the specific ranges; $\theta \in [0.9 \sim 1.1]$; $\sigma^2 \in [10^{-5} \sim 10^{-3}]$.

2.4. Case Scenario Descriptions

The input uncertainty scheme used in IBUNE assumes that latent variables are applied uniformly throughout a simulation period. However, it may not be reasonable to apply the same set of latent variables without considering seasonal effects upon model predictions. In this study, two seasonal definitions were considered. Definition I averages monthly precipitation data combined from three gauge stations (Figure 2A). Definition II averages monthly precipitation from individual stations (Figures 2B, 2C, and 2D). The definition of wet and dry seasons can be subjective. In this study, the three rain gauges in the watershed each show distinct patterns of monthly average precipitation. Accordingly, dry and wet seasons are defined at each individual

gauge station according to monthly precipitation (Figure 2). The six months with the highest precipitation comprise the wet season, while dry season consists of the remaining months. Latent variables were implemented based on whether dry or wet seasons are defined by individual gauge stations or as an average of all three gauge stations.

A total of five different model scenarios were evaluated (Table 1). Scenario 01 did not utilize latent variables. Scenario 02 followed standard latent variable usage defined by Ajami et al. (2007). Scenarios 03 and 04 implemented two sets of latent variables and two Dry/Wet season definitions. In Scenario 04, Dry/Wet seasons are defined using Definition II. Monthly precipitation from individual gauges provides more rigorously defined Dry/Wet seasons. However, there are still only two sets of latent variables assigned for Dry/Wet seasons, but different stations have different months categorized into Dry or Wet seasons. In Scenario 05, Definition II is used and precipitation data from each gauge station is assigned independent sets of latent variables. In this case, a total of six latent variable sets are implemented for varying gauge stations, and Dry/Wet seasons. By comparing different definitions of Dry/Wet seasons and precipitation averages (combined or individual stations), the potential impact on model predictions, caused by seasonal effects, can be further explored.

2.5. Model Calibration/Validation

The SWAT model calibration was conducted using Integrated Parameter Estimation and Uncertainty Analysis Tool (IPEAT), developed by Yen et al. (2014b). IPEAT uses a Dynamically Dimensioned Search (DDS) (Tolson and Shoemaker, 2007) to perform an auto-calibration process. DDS is an implementation of the Bayes theorem that has been shown to deliver superior solutions and convergence speed performance (Yen et al., 2014a) over other widely-applied algorithms such as Differential Evolution Adaptive Metropolis (DREAM)

(Vrugt et al., 2009), Shuffle Complex Evolution (SCE-UA) (Duan et al., 1992), and Metropolis-Hastings algorithm (MHA) (Metropolis et al., 1953). IPEAT has been implemented on large-scale, high-resolution watershed modeling efforts in prediction of streamflow, sediment loads, and nutrient losses such as the Conservation Effects Assessment Project - Wildlife (The Nature Conservancy, 2014). The framework of IPEAT and the incorporation of input uncertainty, on precipitation data, are shown in Figure 3. The calibration and validation periods of ACW are from 2002 to 2003 and from 2004 to 2005 respectively. Data from years 1999 to 2000 were utilized for model “warm up”. Daily streamflow, monthly sediment load, and monthly ammonia load output responses were utilized for model calibration/validation. As the ACW is close to the Gulf of Mexico, ocean tides (e.g., periodical waves or diurnal fluctuations) exert a direct effect upon measured observation data. In this case, observation ammonia data measured at the southernmost station near Weslaco (near Llano Grande at FM 1015) was used to minimize tidal effect.

The Nash-Sutcliffe Efficiency coefficient (NSE, Nash and Sutcliffe, 1970) was used to formulate the objective function during model calibration. NSE is a broadly implemented statistical measurement (ASCE 1993; Servat and Dezetter 1991) with potential ranges from $-\infty$ to one (note: perfect match is indicated between observed and simulated responses when NSE equals one). As suggested in previous work (Seo et al., 2014), the objective function is revised for the model calibration and must be minimized in order to perform the automated calibration process. In Equation (2), y_i^{Obs} is the observation data at time step i ; y_i^{Sim} is the simulated model responses at time step i ; y_i^{Mean} is the averaged observation data at time step i ; and N is the total time steps.

$$NSE = 1 - \frac{\sum_{i=1}^N (y_i^{Obs} - y_i^{Sim})^2}{\sum_{i=1}^N (y_i^{Obs} - y_i^{Mean})^2} \quad (2)$$

$$OF = \sum_{v=1}^V (1 - NSE_v) \quad (3)$$

The revised objective function is shown in Equation (3). Where: OF is the aggregated objective function; NSE_v is the NSE value of output variable v (e.g., streamflow rate, sediment load, and ammonia losses in this study); and V is the total number of output variables (e.g., $V = 3$ in this study). The best possible value of the objective function is zero when all NSE_v are equal to one. To calibrate streamflow, sediment, and ammonia processes, 23 parameters were included in auto-calibration of all case scenarios. Model parameters and the recommended ranges are presented in Appendix A.

In this study, the Relative Percentage Difference (RPD) is defined to provide supplementary indices in categorizing relative differences among various statistical results. By the use of RPDs, users may have better understanding in making decisions upon available scenarios. RPDs among individual variables (streamflow, sediment, NH4) and scenarios (1-5) were calculated as:

$$RPD(\%) = \frac{|V_1 - V_2|}{\frac{V_2 + V_1}{2}} * 100 \quad (4)$$

Where, V_1 is the NSE result of an individual scenario; V_2 is the best NSE result among all scenarios. The value of RPD is zero when the current state represents the best scenario. In addition, uncertainty analysis was conducted after the statistical evaluations. In this study, inclusion rate and spread are used to identify the relative level of uncertainty in each scenario. Inclusion rate is the percentage of observation data falling within a 95% confidence interval of simulated outputs. Spread is the average width of the 95% confidence interval along simulation time series. Relevant details can also be found in Yen et al. (2014b).

2.6. Behavior Definition

To explore the quality of model predictions in different applications of latent variables, the General Performance Ratings (GPR) was implemented to assess the success rate of calibration results (details of GPR can be found in Moriasi et al., 2007). GPR is currently considered to be the most widely accepted statistical guideline (or behavior definition) in the field of water resources and agricultural engineering (more than 1,900 citations since 2007). In this study, results of model predictions can be characterized into conceptual categories such as Satisfactory ($NSE > 0.5$) and Unsatisfactory ($NSE \leq 0.5$).

3. Results and Discussion

3.1. General Comparisons of Model Performance

The general performance (convergence of objective function values versus model iterations), of all scenarios, is shown in Figure 4. The auto-calibration convergence patterns were similar for each scenario after ~3,000 runs and final objective function values were limited to a range of 1.19 to 1.40 (Scenario 01: 1.19; Scenario 02: 1.29; Scenario 03: 1.33; Scenario 04: 1.29; Scenario 05: 1.40;). This result was not unexpected as Yen et al. (2015a) found latent variable variation does not considerably alter the final converged values of the objective function.

Model statistics were evaluated by considering NSE values and RPD, among calibration/validation periods and precipitation scenarios. Results are summarized in Table 2. NSE values > 0.5 were considered acceptable (Moriasi et al. 2007) with the scenario closest to 1 considered best. In general, NSE values were > 0.5 for all variables and scenarios in the calibration period. In the validation period, NSE values were mostly lower than 0.6 especially in sediment and NH_4 predictions. In comparing each case scenario for the calibration period,

Scenario 02 performed the best in two of three variables (NSE = 0.61 and 0.64 for streamflow and NH_4) where Scenario 01 generated the better performance for sediment (NSE=0.78). However, statistical differences in the calibration period are not far apart among all scenarios and output variables. RPD for streamflow, sediment, and NH_4 is only 12%, 7%, and 13%, which indicating compatible performance in terms of model predictions. In the validation period, Scenario 05 outperformed other scenarios in all output variables (NSE=0.57, 0.45, and 0.53 for streamflow, sediment, and NH_4). RPD for streamflow (15%) is still not showing substantial difference, however, much higher RPD was derived for sediment (50%) and NH_4 (64%).

By using the default settings of latent variables defined by Ajami et al. (2007), the best results were delivered for two of the three variables. However, Scenario 02 also generated the worst statistical results for the validation period. On the other hand, Scenario 05 provided compatible results in both calibration and validation periods in all three output variables evaluated. Overall, Scenario 05 that incorporated seasonal effect from multiple stations provided the best solution. As mentioned previously, variation did not considerably alter the final converged values of the objective function because the objective function was insensitive to the use of latent variables (Yen et al., 2015a). In addition to NSE for evaluation, percent bias (PBIAS) was also calculated for each scenario (as supplementary information) and showed fairly consistent results. Scenario 02 performed well in the calibration period but was the worst in validation; however, Scenario 05 performed well in both calibration and validation periods compared to other scenarios. In the calibration period, PBIAS (%) for Scenario 01~05 are 10.31/13.15/19.44/19.47/13.30 (streamflow), 15.52/11.06/12.93/16.93/11.43 (sediment), and 2.60/1.15/4.39/3.14/3.81 (ammonia). In the validation period, PBIAS (%) for Scenario 01~05 are 26.37/28.70/28.39/30.79/22.28 (streamflow), 22.42/28.45/27.46/26.63/21.13 (sediment), and

3.78/7.68/2.17/2.70/1.12 (ammonia). The marginal differences in the PBIAS values are not as significant as those in NSE, but they do demonstrate the advantage of Scenario 05.

As mentioned previously that model performance can also be grouped into different categories by pre-defined statistical threshold such as GPR by Moriasi et al. (2007). Calibration results were grouped into two major clusters in Figure 5. It is clear to see that the use of default latent variables (Scenario 02 by Ajami et al. (2007)) has provided considerably more results within the Unsatisfactory category. Percentage of behavior solutions between Scenario 03 and 04 was not far apart from Scenario 01 without any latent variables applied. In addition, Scenario 05 again deployed better performance in terms of more behavioral solutions. As mentioned previously, statistical results in the calibration period are relatively comparable in all scenarios but not the same case for validation (Scenario 05 performed the best in the validation period). By the application of GPR, the use of seasonally categorized latent variables (with detailed definition of Dry/Wet seasons) can not only provide a better single solution, but it can also generate more quality results in the targeted watershed.

The results show that Scenario 02, using default latent variable settings, was less stable than other scenarios considering seasonal effect. Therefore, the application of Ajami et al. (2007) defined latent variable default settings, without considering seasonal effect, may be questionable. Also, combining latent variables with seasonal effects can produce better model responses in impulsive hydrological cycles such as peak flow in spring or summer. It may also imply that other sources of uncertainty, such as precipitation, temperature, etc., should be considered simultaneously for future investigation.

3.2. Evaluation of Latent Variables and the Adjusted Precipitation Data

Latent variable convergences (θ) are shown in Figure 6(A), 6(B), 6(C), 6(D), and 6(E). Variances (σ^2) are not presented, due to their minimal effect on model predictions (Yen et al., 2015a)). Scenario 01 did not apply latent variables to seasonality (Figure 6(A)). The converged value of θ is 1.01 throughout the simulation time series indicating precipitation data, for all time periods, increased slightly. Figure 6(B) and 6(C), utilized the same number of latent variables (four each) with differing definitions of Dry/Wet seasonality (I & II). In these two scenarios, latent variable θ converged to similar values in the dry season ($\theta_{Dry}^{Scenario\ 03} = 1.075$; $\theta_{Dry}^{Scenario\ 04} = 1.088$) but not in the wet season ($\theta_{Wet}^{Scenario\ 03} = 0.902$; $\theta_{Wet}^{Scenario\ 04} = 1.077$). The sum of adjusted precipitation data in Scenario 03 should therefore be higher than Scenario 02 (also see Figure 7(A)). In Figure 6(D), 6(E), and 6(F), three sets of latent variables were assigned to each precipitation gauge station (G01, G02, and G03). G01 and G03 have similar latent variables convergence (Dry season: $\theta_{Dry,G01}^{Scenario\ 05} = 1.020$, $\theta_{Dry,G03}^{Scenario\ 05} = 1.039$; Wet season: $\theta_{Wet,G01}^{Scenario\ 05} = 0.982$, $\theta_{Wet,G03}^{Scenario\ 05} = 1.001$) where precipitation data was adjusted, within limited order. Dry/Wet season latent variables exhibit substantial differences at G02 ($\theta_{Dry,G02}^{Scenario\ 05} = 1.093$; $\theta_{Wet,G02}^{Scenario\ 05} = 0.909$) where precipitation values increased after convergence (Figure 7(B)).

Results showed the dry season latent variable θ has consistently higher values than the wet season, regardless of seasonal definition (i.e., I or II). Different definitions in categorizing Dry/Wet seasons while implementing latent variables will cause impact on precipitation data and the associated model predictions. It also indicates that precipitation data in the dry period tends to be adjusted more than the wet period. Definition I that categorizes Dry/Wet seasons by averaged precipitation data in all stations cannot reveal the regional characters among varying

stations. Latent variables for G02 are noticeably different than G01 or G03 (See Figure 6(D), 6(E), and 6(F)). The drawback of combining latent variables (non-seasonal effect considered, single set of latent variables for all stations) may become a major concern in watersheds with gauge stations located far apart.

3.3. Applications of Latent Variables and Uncertainty Analysis

Latent variable use not only affects overall model predictions but the predictive uncertainty as well. Quantifying predictive uncertainty magnitude is not straightforward and requires further evaluation. In this study, inclusion rate and spread were used to perform uncertainty analysis in each scenario. The unit of inclusion rate is percentage whereas the spread units should reflect output response units (i.e., m^3/s for streamflow, etc.). High inclusion rates, or low spreads, represent a lower level of predictive uncertainty. However, it is not definitive that one watershed modeling result uncertainty is more or less superior to another. Quantified uncertainty analysis simply provides supplemental information to assist scientists or decision makers in the decision making process.

As shown in Table 3, substantial influences of uncertainty in terms of inclusion rate and spread, attributed to seasonal latent variable use, was noted in streamflow predictions and reflected in ammonia and sediment predictions. This may be because streamflow responses are directly associated with precipitation data. Also, uncertainty magnitude is correlated with the number of latent variables used. In general, higher uncertainty is expected with the incorporation of seasonal latent variables and scenarios with more latent variables exhibit higher levels of predictive uncertainty. Therefore, Scenario 03 and 04 have more uncertainty than Scenario 02 and Scenario 05 uncertainty is higher than all other scenarios. One of the major reasons to incorporate input uncertainty during calibration process is because it has been shown to be

inappropriate by not considering other uncertainty sources while conducting watershed modeling (Ajami et al., 2007). On the other hand, it is still not clear how to alleviate the impact on model predictions by including other sources either. Intuitively, the incorporation of input uncertainty using seasonally categorized latent variables should increase predictive uncertainty since more variables are involved (which is true in this study). However, more impact can be found to be placed on streamflow predictions instead of sediment and ammonia. This important finding has pointed out a good direction for future work whereas more studies are required to further verify the statement.

4. Conclusion

In this study, seasonal latent variables were applied to input uncertainty, in the form of precipitation data, to determine the effect on model predictions and the associated predictive uncertainty. Latent variable applications, without considering seasonal effects, produced reasonable statistical solutions for the calibration period but not for the validation period. The use of seasonal latent variables resulted in better statistical solutions for both calibration and validation periods. Also, varying Dry/Wet seasonality definitions affected model predictions as well. It is, however, difficult to judge whether a higher level of uncertainty is better than a lower level. Scientists and engineers generally prefer to evaluate model simulations for a long term period to ensure the quality of model predictions. In the case study at the Arroyo Colorado Watershed, four years of data are available for calibration and validation periods on streamflow, sediment, and ammonia outputs. However, it is not clear whether the results will be affected because the approach proposed in this study is not based on how well the model represents the hydrologic system, but how different schemes of latent variables affect model optimization. More studies should be conducted to address detailed evaluations on predictive uncertainty.

In previous studies, the use of latent variables was shown to have improved model predictions compared to those not considering input uncertainty on precipitation data (Ajami et al., 2007). The use of latent variables does not guarantee better statistical solutions if not applied in the default settings (Yen et al., 2015a). In this study, seasonal latent variables demonstrated the importance of seasonal effects by comparing differing input data combinations. Meanwhile, the approach performed in this study represented as a semi-arid watershed. Future work can be further conducted on less or more humid regions to compare differences and the associated impacts toward model predictions. Implementations of latent variables on future work should pre-verify the potential impact towards model predictions before performing associated scientific studies or relevant decision making processes.

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Appendix A

Calibration parameters for all case scenarios

Parameters	Input file	Units	Range	Description
ADJ_PKR	.bsn	-	0.5 – 2	Peak rate adjustment factor for sediment routing in the subbasin (tributary channels)
CMN	.bsn	-	0.001 – 0.003	Rate factor for humus mineralization of active organic nitrogen
EPCO	.bsn	-	0 – 1	Plant uptake compensation factor
NPERCO	.bsn	-	0 – 1	Nitrogen percolation coefficient
PRF	.bsn	-	0 – 2	Peak rate adjustment factor for sediment routing in the main channel
SPCON	.bsn	-	0.0001 – 0.01	Linear parameter for calculating the maximum amount of sediment that can be re-entrained during channel sediment routing
SPEXP	.bsn	-	1 – 1.5	Exponent parameter for calculating sediment re-entrained in channel sediment routing
SURLAG	.bsn	Day	1 – 24	Surface runoff lag time
ALPHA_BF	.gw	1/Day	0 – 1	Baseflow alpha factor
GW_DELAY	.gw	Day	0 – 500	Groundwater delay
GW_REVAP	.gw	-	0.02 – 0.2	Groundwater "revap" coefficient
GWQMN	.gw	mm H ₂ O	0 – 5000	Threshold depth of water in the shallow aquifer required for return flow to occur
SLSUBBSN	.hru	M	10 – 150	Average slope length
CN_F	.mgt	%	±10	Initial SCS CN II value
CH_COV2	.rte	-	-0.001 – 1	Channel cover factor
CH_K2	.rte	mm/hr	-0.01 – 500	Effective hydraulic conductivity in main channel alluvium
CH_N2	.rte	-	-0.01 – 0.3	Manning's "n" value for the main channel
SOL_AWC	.sol	%	±10	Available water capacity of the soil layer
SOL_K	.sol	%	±10	Saturated hydraulic conductivity
CH_K1	.sub	mm/hr	0 – 300	Effective hydraulic conductivity in tributary channel alluvium
CH_N1	.sub	-	0.01 – 30	Manning's "n" value for the tributary channels
RS3	.swq	mg/m ² -day	0 – 1	Benthic source rate for NH ₄ -N in the reach at 20°C
RS4	.swq	1/day	0.001 – 0.1	Rate coefficient for organic N settling in the reach at 20°C

Parameter values for CN_F, SOL_AWC, SOL_K, USLE_K, and USLE_C are the changes of fraction from default values.

Table 1: Case scenarios descriptions including: identification, number of latent variables, and specific definitions

Case Scenarios	Number of Latent Variables	Scenario Descriptions
Scenario 01	-	Case scenario with no latent variables applied
Scenario 02	x 2	Same set of latent variables on both Dry/Wet seasons and all stations (default settings of latent variables as the in Ajami et al. (2007))
Scenario 03	x 4	Latent variables are applied in Dry/Wet seasons (based on seasonal Definition I) respectively, all three stations are sharing the same set of latent variables
Scenario 04	x 4	Latent variables are applied in Dry/Wet seasons (based on seasonal Definition II) respectively, all three stations are sharing the same set of latent variables
Scenario 05	x 12	Latent variables are applied in Dry/Wet seasons (based on seasonal Definition II) respectively, all three stations are using different sets of latent variables

Seasonal Definition I: Dry/Wet seasons are average monthly precipitation from all gauging stations combined (see Figure 2)

Seasonal Definition II: Dry/Wet seasons are average monthly precipitation from individual gauge stations (see Figure 2)

Table 2: Nash Sutcliffe Efficiency (NSE) statistics and Relative Percentage Difference (RPD) for stream flow, sediment, and ammonia Scenarios 1 to 5 (results for calibration and validation are shown on the left and right side of the “/” sign).

Scenarios	Streamflow		Sediment		Ammonia	
	NSE	RPD (%)	NSE	RPD (%)	NSE	RPD (%)
Scenario 01	0.54*/0.55**	12/4	0.78/0.45	0/0	0.61/0.26	5/51
Scenario 02	0.61/0.56	0/2	0.75/0.27	4/50	0.64/0.19	0/64
Scenario 03	0.57/0.52	7/9	0.76/0.44	3/2	0.56/0.39	13/26
Scenario 04	0.57/0.49	7/15	0.76/0.39	3/14	0.59/0.47	8/11
Scenario 05	0.58/0.57	5/0	0.73/0.45	7/0	0.59/0.53	8/0

*: Error statistics for the calibration period (from 2002 to 2003)

***: Error statistics for the validation period (from 2004 to 2005)

- RPD is defined to provide supplementary indices in categorizing relative differences among various statistical results.

- The value of RPD is zero when the current state represents the best scenario.

Table 3: Inclusion rate of observed streamflow, sediment, and ammonia, within the 95% confidence interval and corresponding spread for the simulation period 2002 to 2003.

Scenario	Inclusion Rate (%)			Spread		
	Streamflow	Sediment	Ammonia	Streamflow	Sediment	Ammonia
Scenario 01	49.59	62.50	91.67	1.815	0.044	0.147
Scenario 02	46.99	70.83	79.17	2.664	0.054	0.182
Scenario 03	31.92	62.50	75.00	1.436	0.044	0.153
Scenario 04	23.29	50.00	75.00	1.178	0.044	0.146
Scenario 05	22.19	45.83	70.83	1.135	0.043	0.131

Inclusion rate (%): Percentage of observed data points located within the 95% confidence interval.

Spread: Average width of the corresponding uncertainty band along the predicted time series. The units for streamflow, sediment, and ammonia are cms (cubic meter per second), ton/ha (tons per hectare), and kg/ha (kilogram per hectare) respectively.

Figure 1. Arroyo Colorado Watershed stream gauge and weather station locations.

Elevation, streams, sub-basins, and counties are shown. (Seo et al., 2014)

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Figure 2. Average monthly precipitation used to categorize Dry/Wet seasons in case study:
(A) all gauge stations; (B) gauge station #1; (C) gauge station #2; and (D) gauge station #3

Figure 3. Organogram of IPEAT and the incorporation of input uncertainty on precipitation data

Figure 4. General model performance, as convergence of objective function values versus model iterations, in five scenarios

Figure 5. Rate of behavior solutions by implementing the General Performance Ratings (Moriassi et al., 2007) in categories of (i) satisfactory; and (ii) unsatisfactory, of five case scenarios

Definition I: Definition of Dry/Wet seasons is using average monthly precipitation for all stations (also see Figure 2)
 Definition II: Definition of Dry/Wet seasons is using average monthly precipitation for each gauge station (also see Figure 2)
 G: Gauge station

Figure 6. Convergence patterns of latent variables. Scenario 02, 03, and 04 are cases with inclusion of latent variables during calibration): (A) same set of latent variables in both Dry/Wet seasons; (B) latent variables applied to Dry/Wet seasons (based on Definition I) respectively on all three gauge stations, using same set of latent variables; (C) latent variables applied in Dry/Wet seasons (based on Definition II) respectively on all three gauge stations (using same set of latent variables); and, (D)~(F) latent variables applied in Dry/Wet seasons (based on Definition II) respectively on all three gauge stations (using three sets of latent variables).

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*: To better compare adjusted precipitation (precipitation is usually compared in average annual fashion), the adjusted seasonal precipitation was projected to be in a full year length. For example, the adjusted average daily precipitation of Scenario 03 on wet season is 2.99 (mm/day). Therefore, the projected average annual precipitation is $2.99 \times 365 = 1093$ (mm/day).

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Figure 7. Original (Scenario 01) and adjusted average annual precipitation depth for Scenarios 1-5: (A) average annual precipitation; and (B) projected average annual precipitation in dry and wet seasons.