1 Power law breakthrough curve tailing in a fracture: The 2 role of advection.

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9 Abstract

10 We offer an explanation of the strongly tailed solute breakthrough curve
11 typically observed when a tracer test is conducted in fractured bedrock. In
12 this example, we limit the model to a single planar fracture of varying aper-
13 ture. Flow heterogeneity derives from variable fracture aperture, which im-
14 plies variable transmissivity (T). The analysis employs a physically based
15 model well-suited to strong heterogeneity and relies only upon advective
16 transport. The purely advective model is able to explain a power-law trend
17 of magnitude -2 to -3 in the breakthrough curve tail; a range that has been
18 found in field tracer experiments. The principle cause of this trend is the com-
19 paratively slow transport in zones of small transmissivity (tight aperture).
20 Slow advection occurs when either heterogeneity (variance of $\ln T$) is strong
21 or when the assumed heterogeneity distribution is non-Gaussian. Thus, we
22 link breakthrough tailing to the statistical parameters for the transmissivity
23 field.

24 1. Introduction

25 When a non-reactive tracer is injected into a naturally or artificially flow-
26 ing groundwater system, the late-time arrival of tracer at a detection point
27 is extended with respect to the early arrival. This observation is referred to

28 as a tracer breakthrough curve (BTC) tailing and often exhibits a power law
 29 decline over time. A -1.5 exponent of the tail is often observed in fractured
 30 media, which is generally attributed to matrix diffusion and/or sorption ki-
 31 netics (e.g. Reimus et al., 2003; Cvetkovic, 2007). Exponents smaller than
 32 -1.5 , in the range -3 to -2 , have been also often found in the past (see,
 33 e.g., Becker and Shapiro, 2000; Becker and Shapiro, 2003, Kosalowski, 2004;
 34 more reference can be found in the literature review of Willman et al., 2008),
 35 leading to the so-called "anomalous transport". As noted in the review by
 36 Zhou et al (2007), considering only pore-scale matrix diffusion will lead to an
 37 underestimation of breakthrough tailing at the field scale, and explanations
 38 other than matrix diffusion have been offered in the past. The power-law
 39 BTC has been explained with many theoretical models including continu-
 40 ous time random walk (Berkowitz et al., 2006), mobile-immobile exchange
 41 (Haggerty and Gorelick, 1995; Haggerty et al., 2000), fractional-order ad-
 42 vection/dispersion (Benson et al, 2000), among other approaches (Zhang et
 43 al., 2007, 2009). Most of those explanations invoke some local or non-local
 44 hydrodynamic dispersion and/or diffusion to help explain the behavior. By
 45 contrast, field examples have shown that power-law tailing can be observed
 46 in the absence of diffusion effects in highly heterogeneous velocity systems
 47 such as fractured bedrock (Becker and Shapiro, 2000; Becker and Shapiro,
 48 2003). Recent direct numerical simulations (Wang and Cardenas, 2014) have
 49 shown a similar behavior at the core scale, observing an increasing deviation
 50 from the Gaussian behavior of the BTC for increasing fracture heterogeneity.

51 In this article, we examine the possibility that local dispersion, diffusion
 52 and the presence of eddies can be removed from the transport model entirely,
 53 and still account for strong tailing of the BTC similar to power-law. That
 54 is, power-law tailing with exponents in the range $-3 \div -2$ in tracer BTC's
 55 can be explained by advection only. Similar problems were addressed by
 56 Wang and Cardenas (2014) through detailed numerical simulations, while
 57 our aim here is investigate them through a theoretical, physically based, an-
 58 alytical approach, which is more amenable to generalizations. The key to
 59 our examination is the high degree of heterogeneity often encountered in the
 60 subsurface, i.e. we consider a wide range in advective velocities that result
 61 from a highly heterogeneous conductivity field. We consider here fractured
 62 bedrock, in which advective velocities are expected to vary widely. The the-
 63 oretical and mathematical construct developed here, however, is applicable
 64 to any highly heterogeneous transport system.

65 The reason most often cited for the large variation in advective velocities
 66 in bedrock fractures is that flow is constrained to the two-dimensional domain
 67 of the fracture aperture. This invokes the “local cubic-law” (LCL) of fluid
 68 flow which dictates average flow rate at any location within a fracture is
 69 proportional to the cube of the aperture (Oron and Berkowitz, 1998). In
 70 this case, a local effective transmissivity (T) that relates local flow rate to
 71 local hydraulic gradient, is also expected to vary as the cube of the aperture.
 72 The LCL is not strictly valid under all circumstances. At large Reynolds
 73 or Peclet numbers (Detwiler et al., 2000; Konzuk and Kueper, 2004), in the

74 presence of asperities or inclusions in the aperture that may create eddies
75 (Konzuk and Kueper, 2004; Liu and Fan, 2012; Oron and Berkowitz, 1998;
76 Qian et al., 2012) or under conditions of hydromechanical dialation and/or
77 normal stress (Cornet et al., 2003; Gentier et al., 2013; Liu et al., 2013;
78 Witherspoon et al., 1980), the rate of flow may not scale with the cube of
79 the local aperture. However, for the development herein we do not need to
80 invoke the LCL directly; we assume only small variations in aperture will
81 cause large variations local advective velocity.

82 Fracture apertures are expected to vary from zero (wall contact) to about
83 a mm in most competent bedrock systems. This variation can occur over the
84 scale of cm, however, leading to highly variable velocity fields even within a
85 single fracture. Reimus, for example found estimated 8 orders of magnitude
86 in local permeability variation in tuff samples on the order of 10x10 cm in size
87 (Reimus, 1995). The greater the normal stress on the fracture, the greater
88 the contact area between the fracture walls (Gentier et al., 2013; Liu et al.,
89 2013; Watanabe et al., 2008). Increased contact area (Watanabe et al., 2008;
90 Witherspoon et al., 1980) and/or an increase in the coefficient of variation in
91 aperture (Lee et al., 2003) will likely lead to an increase in hydromechanical
92 dispersion and/or channeling.

93 There is unfortunately little field evidence of velocity distribution in
94 bedrock fractures (Cvetkovic and Gotovac, 2013). Shapiro and Hsieh (1998)
95 performed injection tests over short intervals in a crystalline granite/gneiss
96 in central New Hampshire, USA, and found up to 5 orders of magnitude vari-

97 ation in transmissivity. Rutqvist et al. (1998) measured hydraulic apertures
 98 in granite between 8 and 164 microns, implying 4 orders of magnitude of T
 99 variability. Zhou et al. (2007) reviewed field tracer tests in fractured bedrock
 100 and found ranges in equivalent aperture ranging between 0.06 mm and 2.9
 101 mm among different tests (implying 5 orders-of-magnitude variation in trans-
 102 missivity). There is direct evidence that flow channeling occurs in bedrock
 103 fractures (Bourke, 1987; Moreno et al., 1985; Neretnieks, 1983; Neretnieks,
 104 1987; Neretnieks et al., 1982). In some cases, flow channeling has been im-
 105 aged using ground penetrating radar (Becker and Tsoflias, 2010; Day-Lewis et
 106 al., 2006; Day-Lewis et al., 2003; Talley et al., 2005). The similarity between
 107 flow channeling observed in cores (e.g. Reimus, 1995; Watanabe et al., 2008)
 108 and in the field (e.g. Day-Lewis et al., 2002; Talley et al., 2005) suggests that
 109 velocity gradients can be large and occur at all scales in bedrock fractures.

110 Fracture apertures are often assumed to be lognormally distributed (Tsang
 111 and Neretnieks, 1998) but near gaussian distributions have also been mea-
 112 sured (Bauget and Fourar, 2008; Lee et al., 2003). Konzuk and Kueper
 113 (2004) measured apertures in a fractured dolomite sample and initially found
 114 a lognormal distribution. After the distribution was de-trended, however,
 115 the apertures appeared to have a gaussian distribution. This suggests that
 116 other reported distributions may be affected by non-stationarity as well. For
 117 theoretical studies normal, log-normal, and tailed distributions (e.g. Painter,
 118 2001; Painter et al., 2002) have been employed. Some authors have been able
 119 to reproduce laboratory breakthrough experiments by measure apertures and

120 using small-perturbation theory to predict transport (Keller et al., 1999; Lee
 121 et al., 2003). However, these samples had relatively small coefficients of
 122 variation in aperture (between 0.4 and 0.82) and were conducted over short
 123 distances (16- 30 cm) which may have contributed to the approximate agree-
 124 ment with small perturbation theory. Other researchers have suggested that
 125 Fickian processes are not sufficient to explain transport in fractures (Bauget
 126 and Fourar, 2008; Wang and Cardenas, 2014).

127 In the following we present a model of pure advection in a single, hetero-
 128 geneous planar fracture. Flow heterogeneity derives from variable fracture
 129 aperture, which implies variable transmissivity (T). We employ a simpli-
 130 fied but physically based analytical approach suited for highly heterogeneous
 131 fields. The model is therefore aimed at analyzing the BTC tailing and its
 132 relation to relevant measurable features of the transmissivity field.

133 **2. Modelling approach**

134 Similar to what done for aquifers (see, e.g., Dagan, 1989; Rubin, 2003),
 135 we model the medium as a stationary, two-dimensional (2D) random log-
 136 transmissivity ($Y = \ln T$) field, of distribution $f(Y)$, with mean $\langle Y \rangle = \ln T_G$
 137 (the geometric mean), variance σ_Y^2 and isotropic two-point covariance; the
 138 transmissivity integral scale I is finite and is assumed to be much smaller
 139 than the characteristic length scale of the flow domain and of the solute
 140 plume. As stated in the Introduction, the variance σ_Y^2 can be very large in
 141 fractured systems, and thus a model suited to high heterogeneity is needed.

142 The analytical model employed here is a 2D variant of the Self Consistent
 143 model (SCA) model which in the recent years was extensively analyzed and
 144 tested against accurate, large-scale numerical simulations (Jankovic et al.,
 145 2006; Fiori et al., 2006; Fiori et al., 2007) as well as experimental data (Fiori
 146 et al., 2012; Fiori et al., 2013). In this method, the first-passage travel time
 147 (t) distribution $f(t, x)$ is determined at a given control plane at a distance, x ,
 148 from the source. For a non-reactive tracer $f(t, x)$ corresponds to the BTC
 149 at the same control plane. The calculation of the travel time distribution
 150 through the SCA method was originally proposed for 3D transport, and it
 151 is extended here for the first time to 2D flows. The main developments are
 152 briefly summarized in the following, and for further details of the general
 153 procedure developed for 3D flows the reader may refer to Fiori et al (2006)
 154 or Cvetkovic et al (2014).

155 The approach is to describe the transport medium as dense set of circular
 156 inclusions of radius R and random $Y = \ln T$ submerged in a homogeneous
 157 matrix of effective transmissivity T_{eff} . The velocity field in the heterogeneous
 158 medium is represented as the sum of the perturbation velocities associated
 159 with each isolated inclusion. The neglected nonlinear interactions among
 160 the inclusions are mimicked through the background effective transmissivity
 161 $T_{eff} = T_G$, the geometric mean on transmissivity (Dagan, 1989). Along these
 162 lines, the travel time t of a particle moving from from $x = 0$ to x is the sum of
 163 travel time disturbances τ_{R_j} associated with the generic inclusion j , which

164 can be written as

$$t = \langle t \rangle + \sum_{j=1}^{j=N} \tau_{Rj} \quad (1)$$

165 where $\langle t \rangle = x/U$ is the mean and the residuals τ_{Rj} are independent random
 166 variables pertaining to the $N > 1$ blocks encountered by the particle. The
 167 random residual τ_R is derived by integration of the velocity field along the
 168 trajectory, as follows.

169 We consider uniform flow of velocity U at infinity, aligned with the lon-
 170 gitudinal coordinate x_1 , past a circular, isolated inclusion of radius R and
 171 transmissivity T , submerged in a medium of conductivity T_{eff} . The velocity
 172 field V is given exactly by Eq. (1) of Fiori et al (2003), while the travel time
 173 residual τ_R is calculated by integration of dx/V_x , as described by Eq. (2) of
 174 Fiori et al (2006).

175 Examination of the behavior of τ_R for different streamlines shows that
 176 the contribution of streamlines that do not cross the sphere is negligible, and
 177 following the approach of Fiori et al (2006) τ_R can be obtained by a simple
 178 interpolation of the residual travel time τ_M pertaining to the centerline; τ_M
 179 can be determined analytically as follows

$$\frac{\tau_M R}{U} = \frac{1}{\kappa} - 1 + 2\sqrt{\frac{1-\kappa}{1+\kappa}} \tanh^{-1} \sqrt{\frac{1-\kappa}{1+\kappa}} \quad (2)$$

180 Following the same steps as Fiori et al (2006) we arrive at the probability
 181 density function for τ_R for circular inclusions

$$f(\tau_R) = \tau_R \int_0^{\eta(\tau_R)} \frac{2\kappa}{1+\kappa} \frac{(1 - \tau_R^2/\tau_M^2)^{-1/2}}{\tau_M^2(\kappa)} f(\kappa) d\kappa \quad (\tau_R)0) \quad (3)$$

$$f(\tau_R) = -\tau_R \int_{\eta(\tau_R)}^{\infty} \frac{2\kappa}{1+\kappa} \frac{(1 - \tau_R^2/\tau_M^2)^{-1/2}}{\tau_M^2(\kappa)} f(\kappa) d\kappa \quad (\tau_R \langle 0)$$

182 where $\kappa = \eta(\tau_M)$ is obtained by inverting the relationship $\tau_M(\kappa)$ (2) with
 183 $0 \langle \kappa \langle \infty$.

184 The total travel time of a particle results from summation of τ_R pertinent
 185 to the $N > 1$ inclusions crossed by the trajectory, along (1). In general, $N \simeq$
 186 $nx/\langle \ell \rangle$ where n is the volume fraction and $\langle \ell \rangle = \pi R/2$ is the average length
 187 of a segment of the streamline intercepting a block. Since the τ_R involved in
 188 the sum are independent, $f(t, x)$ is given by a multiple convolution, which
 189 can be written in the Fourier space, as follows

$$f(\tau, x) = \text{FT}^{-1}[(\tilde{f}_{\tau_R})^N] \quad ; \quad \tilde{f}_{\tau_R} = \text{FT} [f_{\tau_R}(\tau)] \quad (4)$$

190 where FT stands for Fourier transform and FT^{-1} for its inverse. Since $f(\tau_R)$
 191 is given in an analytical form by (3), in terms of the pdf of K , $f(\tau, x)$ can be
 192 determined by using fast Fourier Transform (FFT) for any given conductivity
 193 distribution.

194 The kernel (3) requires the definition of the marginal distribution $f(\kappa)$,
 195 or equivalently the distribution $f(Y)$ for the log-transmissivity Y . In view
 196 of the relatively large influence of the tail of $f(Y)$, we consider examining
 197 the impact of non-Gaussian distributions of Y . To this aim, because of its
 198 flexibility, we have adopted the subordination model for the transmissivity
 199 distribution, which is a generalization of the log-normal one. The model
 200 suggested by Painter (2001) for the pdf of Y is a mixture of Gaussian dis-

tributions with different variances. The mathematical expression for the subordination model is the following

$$f(Y) = \int_0^\infty f_s(Y; \langle Y \rangle, s) g_s(s; \nu, I_s) ds \quad (5)$$

where $f_s(Y; \langle Y \rangle, s)$ is the Gaussian distribution of Y , with mean $\langle Y \rangle$ and standard deviation s . In turn, g_s is a lognormal distribution of s , with mean ν and standard deviation I_s . The resulting distribution, that depends on the three parameters $\langle Y \rangle$, ν and I_s , is flexible and converges to the Gaussian for $I_s = 0$, and displays significant and increasing tailing as I_s grows, similar to the Levy distribution. For given total variance σ_Y^2 , we have $\nu = \ln \sigma_Y^2 / 2 - I_s^2$ and the model (5) depends only on the parameters $\langle Y \rangle$ and I_s .

3. Analysis of the BTC tail

We show here a few results based on the analytical approach previously outlined. We focus on the BTC and its tail, as function of a few relevant parameters, like e.g. the shape of the distribution (5) and the log-transmissivity variance σ_Y^2 . The main scope is to show the emergence of a power-law like tail of the BTC, especially for high heterogeneity and broad Y distributions.

Figure 1 displays a few examples of travel time distributions $f(t, x)$ calculated by the analytical, SCA method; $f(t, x)$ is evaluated at the distance $x = 50I$, for two rather different levels of heterogeneity ($\sigma_Y^2 = 2, 32$, i.e. moderate and very high), for normal ($I_s = 0$) and nongaussian logtransmissivity ($I_s = 0.6$). The Figure shows that the BTCs are generally very skewed and non-Gaussian, characterized by a strong preferential flow, which increases

222 with the degree of heterogeneity (σ_Y^2). Our interest here is in the power-law
 223 like tail, which emerges in all cases except perhaps $\sigma_Y^2 = 2; I_s = 0$, for which
 224 the tail - although persistent - exhibits deviations from power-law behavior
 225 which increase with time (the behaviour is hardly visible in the figure because
 226 of the t axis limitations). When the log-transmissivity has a broad distri-
 227 bution ($I_s = 0.6$) the slope of the tail in the log-log plot does not seem to
 228 depend much on the degree of heterogeneity σ_Y^2 . In such cases, the behavior
 229 of the tail is dictated by the particular shape of the log-transmissivity pdf,
 230 and in particular its lower tail, and a small perturbation approach may give
 231 reasonable results. These results show that a fully advective model is able
 232 to predict a heavy and persistent tailing of the BTC, similar to power law.
 233 The strong tailing is mainly determined by slow advection in low transmis-
 234 sivity areas, the distribution of which is crucial for the tail determination.
 235 In particular, it is easy to see that the residual travel time τ_R (2) diverges
 236 for $\kappa \rightarrow 0$, meaning that the solute particles which enter such low- T areas
 237 experience a slow velocity field and remain trapped there for a very long
 238 time. We emphasize that, for a lognormal T ($I_s = 0$), the BTC will always
 239 tend to gaussian at large distances, although the rate of convergence can be
 240 very gradual for large σ_Y^2 . We also note that it is often quite difficult to infer
 241 a genuine power-law behavior from field or numerical data (see, e.g., Clauset
 242 et al., 2007). With those reservations in mind we discuss in the following the
 243 exponents of the power-law BTC tail.

244 We examine here the absolute value ϵ of the exponent of the power law

245 tail $f(t, x) \sim t^{-\epsilon}$ as function of the control plane distance x . This is shown in
 246 Figure 2, for $I_s = 0$ (normal Y), $I_s = 0.6$ (non-gaussian Y with fat tails) and
 247 increasing values of σ_Y^2 (2, 4, 8, 16, 32, 64). All exponents, ϵ , display a slow
 248 increase with normalized distance, x/I , which may indicate a gradual ten-
 249 dency to gaussianity due to the repeated convolutions of the strongly tailed
 250 kernel, $f(\tau_R)$. When $I_s = 0$, the slope is generally large when heterogene-
 251 ity is low/moderate, but it rapidly tend to smaller values for increasing σ_Y^2 .
 252 There seems to be a limit value $\epsilon = 2$ to which the exponent tends when
 253 heterogeneity is strong ($\sigma_Y^2 \gg 1$). In turn, the exponent ϵ pertaining to the
 254 nongaussian transmissivity distribution ($I_s = 0.6$) always stays in the range
 255 $2 < \epsilon < 3$, even for mildly heterogeneous media. This behavior is determined
 256 by the relatively thick tail of $f(Y)$ for $I_s > 0$, in particular at the $T \rightarrow 0$
 257 limit.

258 Because of the mild increase of ϵ with distance, the exponent can be
 259 directly derived from the kernel $f(\tau_R)$ (eq. 3), and the behaviour of ϵ as
 260 function of σ_Y^2 is represented in Figure 3, for $I_s = 0, 0.6$. For strong hetero-
 261 geneity ϵ converges to values between 2 and 3, regardless of the deviation of
 262 $f(Y)$ from gaussianity. Conversely, for smaller values of σ_Y^2 the difference
 263 between the gaussian and non gaussian $f(Y)$ becomes larger.

264 An asymptotic analysis of the BTC tail, focusing on possible anomalous
 265 behavior of transport, can be easily developed through the SCA model devel-
 266 oped in Section 2. Transport is considered anomalous when, after the multi-
 267 ple convolutions of the kernel, the BTC does not become gaussian. Anoma-

lous transport may occur, consequently, when the kernel has a power law tail (i.e., is a "broad" distribution in the physics nomenclature) and the central limit theorem may not hold (Bouchaud and Georges, 1990). The kernel may exhibit a power law tail when $f(T) \sim T^\delta$ for $T \rightarrow 0$, with $-1 \leq \delta \leq 0$. Classic statistical physics (see, e.g., Bouchaud and Georges, 1990) dictates that the power law distribution $f(T) \sim T^\delta$ leads to anomalous transport when $\delta < 0$. The "non-defective random walk" requires that $\delta \geq -1$, i.e., $\int_0^\infty f(T) dT = 1$. In the limit $T \rightarrow 0$, it is $\tau_R U/I \sim 1/\kappa$, $\eta(\tau_R) = \tau_R^{-1}$ and the first expression of (3) becomes

$$\begin{aligned} f(t) &\approx 2\tau_R \int_0^{1/\tau_R} T^{3+\delta} (1 - \tau_R^2 T^2)^{-1/2} dT \\ &= \frac{\sqrt{\pi}\Gamma(2 + \delta/2)}{\Gamma((5 + \delta)/2)} t^{-(3+\delta)} \approx t^{-\epsilon} \quad \left(\frac{\tau_R U}{I} \gg 1 \right) \end{aligned} \quad (6)$$

The bounding of $-1 \leq \delta \leq 0$, implies $2 \leq \epsilon \leq 3$. This results confirms the range of the exponent of the power law BTC tail previously discussed. Hence, anomalous transport (if present) is characterized by exponents which are similar to those pertaining to strongly heterogeneous systems. We note that the above result is similar with what found by Fiori et al (2007) for 3D transport, hence the range of exponents do not seem to depend on space dimensionality.

4. Conclusions

We developed a purely advective model for analyzing BTC tailing in a single fracture of highly variable aperture (and therefore transmissivity).

287 The model relates BTC tailing to the transmissivity structure in the absence
 288 of dispersion or diffusion or other local mechanisms. Advection alone is
 289 sufficient to explain the power law behavior of the BTC tail, which has been
 290 often observed in field experiments. According to our analysis, the principal
 291 cause of BTC tailing is slow advection in zones of small transmissivity (tight
 292 aperture). As such, the slope of the power-law tail is related to the magnitude
 293 of heterogeneity (σ_Y^2) of log-transmissivity $Y = \ln T$. Our analysis shows that
 294 the power law exponent should be in the range of -3 to -2, which is consistent
 295 with what has been found in field experiments and numerical simulations.
 296 This range is found with either (1) heterogeneity is strong ($\sigma_Y^2 \gg 1$) and the
 297 log-transmissivity distribution $f(Y)$ is gaussian, or when (2) heterogeneity
 298 is moderate and $f(Y)$ is non-gaussian and characterized by a strong tail. It
 299 also implies that the BTC will not become Gaussian even over large distances
 300 when transmissivity distribution exhibits a power-law distribution at $T \rightarrow 0$.

301 Our results indicate that, when heterogeneity is strong ($\sigma_Y^2 \gg 1$), the
 302 power law slope approaches -2 . Becker and Shapiro (2003) found -2 power
 303 law slopes in tracer BTC's from tests conducted between multiple well pairs
 304 and different flow configurations (dipole, convergent) in crystalline bedrock.
 305 Those experiments suggest heterogeneity in log-transmissivity may be quite
 306 large in bedrock systems. They also imply that measurement of strong het-
 307 erogeneity using tracers may be insensitive to the imposed flow field.

308 The relationship between heterogeneity in transmissivity and BTC tailing
 309 has probably not been captured in the past because theoretical linear models

310 of transport are unable to cope with large variances in transmissivity and/or
311 tailing behavior of the log-transmissivity distribution. Previous efforts to
312 understand BTC tailing, particularly power-law behavior, have focused on
313 mass transfer processes. These mass transfer processes are typically diffusion
314 based, but mimic strong contrasts between relatively slow and fast advection.
315 The association between mass transfer processes and highly heterogeneous
316 transmissivity structure is still not clear. Here, however, we give an example
317 of how slow advection may lead to a power law BTC tail, and how the
318 power-law exponents of the BTC tail can be linked directly to the statistical
319 parameters of the transmissivity field.

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495 Figure captions

496 Figure 1. The BTC at a control plane distance of 50 logtransmissivity
497 integral scales, for medium ($\sigma_Y^2 = 2$) and large ($\sigma_Y^2 = 32$) heterogeneity, for
498 gaussian ($I_s = 0$) and non-gaussian ($I_s = 0.6$) logtransmissivity distribution.

499 Figure 2. The exponent ϵ of the BTC tail ($f(t, x) \sim t^{-\epsilon}$) as function
500 of the control plane distance x , for increasing values of heterogeneity ($\sigma_Y^2 =$
501 $2, 4, 8, 16, 32, 64$), for $I_s = 0$ and $I_s = 0.6$

502 Figure 3. The exponent ϵ of the power-law tail of the BTC kernel $f(\tau_R)$
503 (eq. 3), as function of σ_Y^2 , for $I_s = 0$ and $I_s = 0.6$

Figure 1

Figure 2

Figure 3