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Gaussian-Windowed Frame Based Method of Moments Formulation of Surface-Integral-Equation for Extended Apertures[☆]

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Abstract

Scattering or coupling of electromagnetic beam-field at a surface discontinuity separating two homogeneous or inhomogeneous media with different propagation characteristics is formulated using surface integral equation, which are solved by the Method of Moments with the aid of the Gabor-based Gaussian window frame set of basis and testing functions. The application of the Gaussian window frame provides (i) a mathematically exact and robust tool for spatial-spectral phase-space formulation and analysis of the problem; (ii) a system of linear equations in a transmission-line like form relating mode-like wave objects of one medium with mode-like wave objects of the second medium; (iii) furthermore, an appropriate setting of the frame parameters yields mode-like wave objects that blend plane wave properties (as if solving in the spectral domain) with Green's function properties (as if solving in the spatial domain); and (iv) a representation of the scattered field with Gaussian-beam propagators that may be used in many large (in terms of wavelengths) systems.

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1. Introduction

The problem of the electromagnetic beam-wave scattering from, or coupling into, a complex medium is important for a number of applications. One example is to provide accurate analysis of light coupled through an "imaging fiber bundle," an array of parallel fibers, where each fiber is a high-index multimode dielectric waveguide surrounded by a lower index cladding. Such fiber bundles provide a non-imaging transfer of the spatial distribution of light between two surfaces, with applications including endoscopic relay imaging [1, 2] and image sensing of a curved input surface [3]. Accurate modeling of the spatial resolution of such **fiber bundles** is complicated by energy transfer between modes [4–6]. Such modeling has become increasingly important, especially as optical imaging applications emerge where the spatial period of the fiber structure and the image sensor are both on the order of a few microns [7]. Other type of applications involves the coupling of fields into an open ended aperture or waveguide **or the scattering from**

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cavities [8–20]. Both of these applications involves a rich variety of physical phenomena, which should be properly described in computational modeling.

An infinite **fiber bundle** is a periodic structure that can be analyzed within the Floquet-Bloch framework by, for example, an efficient rigorous coupled wave analysis (RCWA), see in [21–24]. The **fiber bundle** can be viewed as a volume grating with a periodically discontinuous dielectric constant (see, e.g., [25–29]). However, the perfectly periodic case does not account for the finite structure size and does not naturally lead to extensions involving non-periodic **fiber bundles**.

This paper concerns with the problem of an electromagnetic beam coupling into a finite **fiber bundle**, which is a finite array of adjacent parallel dielectric fibers (that can be termed “unit-cells”) that are fused together and can be periodic or non-periodic (see in Fig. 1). Due to the finite size and possibly non-periodicity the Floquet-Bloch framework does not directly apply to this case. Instead, we treat the whole structure as an electromagnetic scattering problem with PMCHWT-type (Poggio, Miller, Chew, Harrington, Wu and Tsai) surface integral equation without direct recourse to the periodicity [30–33]. We employ a spectral-spatial (phase-space) analysis and synthesis within a method of moment (MoM) Gabor-based Gaussian window frame formulation. In the *MoM Gabor-based Gaussian window frame formulation* the rigorous frame theory [34, 35] was used with Gaussian window functions to establish an *overcomplete* Gabor frame set to be applied in a computational electromagnetics wave field framework. The Gabor frames for electromagnetic wave field expansions was used in, e.g., [36–43] to provide more numerical robustness and stability as compared to the *complete* Gabor expansion with bi-orthogonal functions (see, e.g., [44–46]). The Gabor expansion with Gaussian windows (either complete or overcomplete) provides a direct access to wave field propagation in terms of the Gaussian beam summation method (GBSM) [36–49] where a Gaussian beam is the propagator function corresponding to a Gaussian window source. Moreover, if the propagation away from the interface is in a smooth non-uniform domain, Gaussian beams are natural and easy to traced propagators [50, 51]. Another beam summation formulation with exact propagators is given by using complex point sources [52, 53]. The Gabor formulation in computational electromagnetic MoM problems of coupling through slots connecting two half spaces was addressed in [37, 44]. In [44] the complete bi-orthogonal Gabor *basis* formulation was used, which render the problem to be somewhat numerically unstable. A stable Gabor type MoM formulation was demonstrated, for the first time, in [37] for the slot antenna problem with the use of the Gabor *frame set* formulation. In the following contribution, the problem of scattering/coupling due to an “extended” (very wide in terms of wavelength, possibly an infinite in size without any predetermined restriction) “aperture-like” surface separating two propagation media with *different* characteristics is addressed (i.e., free space vs. guiding structure). This formulation uses coupled surface integral equations that arise from satisfying *both* of the electric and magnetic boundary conditions on the surface with two auxiliary currents (electric and magnetic) unlike the single source formulations (e.g., in the slot problem).

In the following formulation, the generality of the fiber bundles is accommodated by the use of their corresponding modal function sets to represent the guided propagation [10, 54]. Applying the Gabor-based Gaussian window frame expansion to the PMCHWT surface integral equation, as discussed above, yields an overdetermined MoM linear system of equations [37]. This system can be solved as a least square problem. It should be noted that: (i) the use of the Gabor frames provides an inherent spatial-spectral phase-space localization [38, 45] of the scattering operator; and (ii) an appropriate setting of the frame parameters (redundancy, phase-space lattice axial ratio, etc.) gives rise to mode-like wave-objects that blends plane wave properties (as if solving in the spectral domain) with Green’s function properties (as if solving in the spatial domain). Furthermore, it casts the PMCHWT surface integral equation and the associate MoM to be identified within a transmission line framework as relating mode-like wave-objects of one medium with mode-like wave-objects of the other medium (e.g., plane waves or asymptotic Gaussian beams of one media with guided modes of the other media).

The paper is organized as follows: Sec. 2 gives the problem layout and the PMCHWT formulation. In Sec. 3 the set of integral equations are discretized with the Gabor-type frame set with Gaussian windows and the identification within a **transmission line** mode matching is discussed, followed by the introduction of the MoM system and its solution. Section 4 discusses the scattered fields as a summation of Gaussian beam contributions within the GBSM. Numerical demonstrations for validation and application are given in Sec. 5. Summary and concluding remarks are provided in Sec. 6. Two appendices supplement the discussion: Appendix A gives a detailed derivation of the PMCHWT that are stated on Sec. 2 for the case of two media with different propagation characteristics, while Appendix B provides additional details regarding Gabor’s frame (Sec. 3.1).

Figure 1. Physical layout of the structure. Medium “2” display truncated fiber-bundle structure

2. Problem layout and formulation

The physical layout of the considered problem is a 2-dimensional domain (2D) with an $x - z$ coordinate system such that the point $\mathbf{r} = (x, z)$, see Fig. 1. The half space $z > 0$, designated as “1”, is a homogeneous space with the permittivity of ϵ_1 and permeability of μ_1 , whereas the half space $z < 0$, designated as “2”, is a linear propagation domain comprised of parallel dielectric waveguides with an inhomogeneous permittivity $\epsilon_2 = \epsilon_2(x)$ and constant permeability μ_2 . An exciting **wave field** $\mathbf{E}_{1,2}^i$ can impinge on the interface at $z = 0$ from $z \leq 0$. The electromagnetic fields in the two half spaces are given by: $\mathbf{E}_{1,2}(\mathbf{r}) = \mathbf{E}_{1,2}^i(\mathbf{r}) + \mathbf{E}_{1,2}^s(\mathbf{r})$ and $\mathbf{H}_{1,2}(\mathbf{r}) = \mathbf{H}_{1,2}^i(\mathbf{r}) + \mathbf{H}_{1,2}^s(\mathbf{r})$, where $\mathbf{E}_{1,2}^s(\mathbf{r})$ and $\mathbf{H}_{1,2}^s(\mathbf{r})$ are the scattered electric and magnetic fields propagating away from the interface at $z = 0$ into the two separate half spaces.

The formulation of the scattering problem follows that of the PMCHWT coupled **surface integral equation** (see, e.g., [30, 31, 33]). This formulation uses one set of auxiliary equivalent surface currents, $\{\mathbf{J}_1(\mathbf{r}), \mathbf{M}_1(\mathbf{r})\}$, to represent the scattered fields in both $z > 0$ and $z < 0$. The derivation of the PMCHWT that is used here is given in Appendix A for both TE and TM polarizations. In the following discussion we use the TE polarization $\mathbf{E}_{1,2} = \hat{\mathbf{y}}E_{1,2}$, $\mathbf{H}_{1,2} = \hat{\mathbf{x}}H_{1,2,x} + \hat{\mathbf{z}}H_{1,2,z}$, with the auxiliary surface electric currents $\mathbf{J}_1(\mathbf{r}) = \hat{\mathbf{y}}J_{1,y}$ and magnetic current $\mathbf{M}_1(\mathbf{r}) = \hat{\mathbf{x}}M_{1,x}$. The associate **surface integral equations** are:

$$\begin{aligned} \mathbf{E}_{1,y}^i(\mathbf{r}) + \int_{S_\infty} d\mathbf{r}' \left\{ \frac{1}{j\omega\epsilon_1} G_{yy}(\mathbf{R}; \mathbf{r}') J_{1,y}(\mathbf{r}') - \frac{\partial}{\partial z} G(\mathbf{R}; \mathbf{r}') M_{1,x}(\mathbf{r}') \right\} \\ = \mathbf{E}_{2,y}^i(\mathbf{r}) + \sum_{\ell} \left(\frac{1}{2} \right) \left[\langle M_{1,x}, e_{y_\ell} \rangle + Z_\ell^{TE} \langle J_{1,y}, e_{y_\ell} \rangle \right] e_{y_\ell}(x), \quad z = 0, \end{aligned} \quad (1a)$$

$$\begin{aligned} \mathbf{H}_{1,x}^i(\mathbf{r}) + \int_{S_\infty} d\mathbf{r}' \left\{ \frac{1}{j\omega\mu_1} G_{xx}(\mathbf{R}; \mathbf{r}') M_{1,x}(\mathbf{r}') - \frac{\partial}{\partial z} G(\mathbf{R}; \mathbf{r}') J_{1,y}(\mathbf{r}') \right\} \\ = \mathbf{H}_{2,x}^i(\mathbf{r}) + \sum_{\ell} \left(\frac{1}{2} \right) \left[Y_\ell^{TE} \langle M_{1,x}, e_{y_\ell} \rangle + \langle J_{1,y}, e_{y_\ell} \rangle \right] e_{y_\ell}(x), \quad z = 0. \end{aligned} \quad (1b)$$

Here, $G_{pp} = \hat{\mathbf{p}} \cdot (\nabla \nabla + k_1^2 \mathcal{I}) \cdot G \hat{\mathbf{p}}$, $\hat{\mathbf{p}} = \hat{\mathbf{x}}, \hat{\mathbf{y}}$, $G(\mathbf{R}, \mathbf{r}')$ with $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ is the scalar Green function of an unbounded infinite medium with the properties of medium 1 and $\mathcal{I} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}}$ is the identity operator. In the present discussion only guided modes are considered for $z < 0$, assuming that the plane wave (radiation) spectrum has a weak contribution

for large propagating distances along the **fiber bundle** (extensions for including the radiation modal wave propagation in dielectric fibers can also be obtained [55]). To that end, the electric transverse orthonormal modal function set in $z < 0$ is $\{\hat{\mathbf{y}}e_{y_\ell}(x)\}$, with L propagating modes $\ell = 1, \dots, L$, modal propagation coefficient $\{\beta_\ell\}$, and TE impedance $Z_\ell^{TE} = [Y_\ell^{TE}]^{-1} = \omega\mu_2/\beta_\ell$. Additionally, $\langle a(x), b(x) \rangle = \iint_{S_\infty} ds' a(x) b^*(x)$ is the scalar product, the asterisk indicates complex conjugation and S_∞ is the $z = 0$ plane.

A solution of the PMCHWT equations for the two auxiliary current sources is carried out by formulating an MoM matrix equation as discussed in the following section.

3. Matrix formulation of the PMCHWT equations

The solution of the coupled **surface integral equations** (1) for the unknown auxiliary currents $J_{1y}(\mathbf{r})$ and $M_{1x}(\mathbf{r})$ are formulated within the framework of the MoM. For the considered 2D space $\mathbf{r} = \hat{\mathbf{x}}x$ for $\mathbf{r} \in S_\infty$. As the first step in the MoM, the unknown currents are expanded using an appropriate set of basis functions and the system of equations is discretized. In the following, the current expansions and discretization are performed on a discrete spatial-spectral **phase-space** lattice within the Gabor's-type frame-based (Windowed Fourier Transform frame) formulation with Gaussian windows [37–40, 56, 57]. Section Sec. 3.1 provides a brief review of the use of the Gabor frame formulation and further details are given in Appendix B while the discretization follows in Sec. 3.2.

3.1. Gabor's-type frame-based formulation

The Gabor frame (windowed Fourier) formulation is structured upon a discrete lattice in the spatial-spectral **phase-space** $\mathbf{X} = (x, \xi)$, where ξ is a dual normalized spectral component with respect to the spatial-spectral Fourier Transform (for a general function $f(x)$)

$$\tilde{f}(\xi) = \int dx f(x) e^{jk\xi x}, \quad f(x) = \frac{k}{2\pi} \int d\xi \tilde{f}(\xi) e^{-jk\xi x}. \quad (2)$$

The spectral quantities are designated with an over-tilde and recall that $k_x = k\xi$ is the spectral coordinate associated with x . The discrete **phase-space** lattice is defined as $\mathbf{X}_p = (x_m, \xi_n)$, $\mathbf{p} = (m, n)$ with $x_m = m\bar{x}$, $\xi_n = n\bar{\xi}$, $\bar{x}$ and $\bar{\xi}$ are spatial and spectral **phase-space** unit cell dimensions, and $m, n \in \mathbb{Z}^2$. In the Gabor frame formulation $\mathbf{X}_p$ is an overcomplete (redundant) by a factor of ν^{-1} , ($\nu < 1$) lattice with respect to the standard critically complete Gabor lattice ($\nu = 1$), i.e., $k\bar{x}\bar{\xi} = 2\pi\nu$.

Assigned to $\mathbf{X}_p$, a proper window function $\psi(x)$ is chosen with its dual $\varphi(x)$ (see the in Eq. (B.1)). Proper spatial shifts and spectral modulations of the dual windows give rise to Gabor's-frame set [34, 35]

$$\psi_p(x) = \psi(x - x_m) e^{-jk\xi_n(x - x_m)}, \quad \varphi_p(x) = \varphi(x - x_m) e^{-jk\xi_n(x - x_m)}. \quad (3)$$

Note that the spectral counterpart, via Eq. (2), is $\tilde{\psi}_p(\xi) = \tilde{\psi}(\xi - \xi_n) e^{jk\xi_1 \xi x_m}$ with a similar expression for φ_p .

Following the frame set construction, the frame representation of the auxiliary currents is given by

$$J_{1y}(x) = \sum_p J_p \psi_p(x), \quad M_{1x}(x) = \sum_p M_p \psi_p(x), \quad (4)$$

where $J_p = \langle J_{1y}, \varphi_p \rangle$ and $M_p = \langle M_{1x}, \varphi_p \rangle$ are discrete coefficients. Note that (i) by exchanging between $\{\psi_p\}$ and $\{\varphi_p\}$ in Eq. (4) and Eq. (B.1), a valid alternative representation is obtained; and (ii) a representation of the type in Eq. (4) has many analytical and numerical desired properties [35].

Using, Eq. (1) with Eq. (4) formulate the integral equations in terms of the discrete sets of unknowns $\{J_p\}$, and $\{M_p\}$ that will be solved for in the following sections.

3.2. Discretization of the surface integral equations

Following the discussion in the previous section, the **surface integral equations** are discretized using the Gabor-frame formulation on the **phase-space** lattice $\mathbf{X}_p$.

3.2.1. Frame function formulation

The formulation starts with expressing J_{1_y} and M_{1_x} in terms of the frame expansion of Eq. (4) with $\{J_p\}$ and $\{M_p\}$, $m, n = \mathbb{Z}^2$ as sets of unknown coefficients. Insertion of the currents expansion Eq. (4) into the integral equations Eq. (1) yields: (i) on the right hand side (RHS): algebraic expressions similar to those on Eq. (1), with terms involving $\langle \psi_p, e_{y_\ell} \rangle e_{y_\ell}(x)$; (ii) on the left hand side (LHS): integral terms involving the Green function, of the form $\int dx' G_{qq}(R; x') \psi_p(x')$ with $q = x, y$ and $\int dx' \left[\frac{\partial}{\partial z} G(\mathbf{R}; \hat{\mathbf{x}}x') \right]_{z=0} \psi_p(x')$ with $R = |x - x'|$ and $\mathbf{R} = \mathbf{r} - \hat{\mathbf{x}}x'$. These integral on the left hand side, can be evaluated on either of the spatial or spectral domains. Since $\psi(x)$ is a Gaussian windows (see the discussion in Sec. 3.1 with Sec. Appendix B and Sec. Appendix A) it is convenient to evaluate the integral in the spectral (ξ) domain. Using Eq. (2), it can be shown that $\tilde{G}(\xi; x, z) = \frac{1}{2jk_1\zeta} e^{-jk_1\zeta|z| - jk_1\xi x}$, where $\zeta = \sqrt{1 - \xi^2}$ and $\text{Im}\zeta < 0$ [58]. Inserting into the LHS of Eq. (1) and performing some algebraic manipulations we are left with:

$$LHS(\text{Eq. (1a)}) = \mathbf{E}_{1_y}^i(x) - \frac{1}{2} \sum_p \left\{ J_p \frac{k}{2\pi} \int d\xi \frac{\eta_1}{\zeta} \tilde{\psi}_p(\xi) e^{-jk_1\xi x} - M_p \right\}, \quad (5a)$$

$$LHS(\text{Eq. (1b)}) = \mathbf{H}_{1_x}^i(x) - \frac{1}{2} \sum_p \left\{ M_p \frac{k}{2\pi} \int d\xi \frac{\zeta}{\eta_1} \tilde{\psi}_p(\xi) e^{-jk_1\xi x} - J_p \right\}, \quad (5b)$$

where $\eta_1 = \sqrt{\mu_1/\epsilon_1}$.

The two Fourier transform integrations in Eq. (5) have an integrand with a similar form $(\eta_1/\zeta)^{\pm 1} \tilde{\psi}_p(\xi)$ with $\zeta = \zeta(\xi)$. Inserting the spectral frame function $\tilde{\psi}_p(\xi) = \tilde{\psi}(\xi - \xi_n) e^{jk_1\xi x_m}$ with $\tilde{\psi}(\xi) = (2\pi b/k_1)^{1/2} e^{-k_1 b \xi^2/2}$ (see Appendix B) and performing some algebraic manipulations yields:

$$\frac{k}{2\pi} \int d\xi \left(\frac{\eta_1}{\zeta} \right)^{\pm 1} \tilde{\psi}_p(\xi) e^{-jk_1\xi x} = \psi_p(x) \cdot \begin{cases} \mathcal{Z}_p^{TE}(x; b) & \text{upper sign;} \\ \mathcal{Y}_p^{TE}(x; b) & \text{lower sign,} \end{cases} \quad (6a)$$

with

$$\mathcal{Z}_p^{TE}(x; b) = \sqrt{\frac{kb}{2\pi}} \int d\xi \frac{\eta_1}{\zeta} e^{-kb(\xi - \xi_p^s)^2}, \quad \mathcal{Y}_p^{TE}(x; b) = \sqrt{\frac{kb}{2\pi}} \int d\xi \frac{\zeta}{\eta_1} e^{-kb(\xi - \xi_p^s)^2}, \quad (6b)$$

and $\xi_p^s = \xi_n - j(x - x_m)/b$. Note that Eq. (6a) implies that given x and b , the frame function $\psi_p(x)$ acts as an eigenfunction of an integral operator with an eigenvalue $\mathcal{Z}_p^{TE}(x; b)$ or $\mathcal{Y}_p^{TE}(x; b)$.

As a final step insertion of Eq. (6) with Eq. (5) into Eq. (1) yields the “frame”-like version of the surface integral equations:

$$\mathbf{E}_{1_y}^i(x) - \frac{1}{2} \sum_p \left[J_p \mathcal{Z}_p^{TE}(x; b) \psi_p(x) - M_p \psi_p(x) \right] = \mathbf{E}_{2_y}^i(x) + \sum_p \sum_\ell \left(\frac{1}{2} \right) \left[M_p + \mathcal{Z}_\ell^{TE} J_p \right] \langle \psi_p, e_{y_\ell} \rangle e_{y_\ell}(x), \quad (7a)$$

$$\mathbf{H}_{1_x}^i(x) - \frac{1}{2} \sum_p \left[M_p \mathcal{Y}_p^{TE}(x; b) \psi_p(x) - J_p \psi_p(x) \right] = \mathbf{H}_{2_x}^i(x) + \sum_p \sum_\ell \left(\frac{1}{2} \right) \left[\mathcal{Y}_\ell^{TE} M_p + J_p \right] \langle \psi_p, e_{y_\ell} \rangle e_{y_\ell}(x). \quad (7b)$$

3.2.2. Discussion

This section provides some highlights regarding the derivation of Eq. (7):

- The integrations of $\mathcal{Z}_p^{TE}$ and $\mathcal{Y}_p^{TE}$ on Eq. (6b) may be calculated numerically in the complex ξ plane along the steepest descent path (SDP) or for collimated type of windows $kb \gg 1$, asymptotically, with the stationary phase evaluation. In both cases, ξ_p^s is the integrand's stationary point [58]. For $kb \gg 1$, the asymptotic stationary phase integration yields $\mathcal{Z}_p^{TE} = [\mathcal{Y}_p^{TE}]^{-1} = \frac{\eta_1}{\zeta_p^s}$ with $\zeta_p^s = \sqrt{1 - (\xi_p^s)^2}$. Two x -regimes can be considered:

1. For $(x - x_m)/b \ll \xi_n$, $\zeta_p^s = \sqrt{1 - \xi_n^2} = \zeta_n$: identify $\xi_n = \sin \theta_n$ and $\zeta_n = \cos \theta_n$ (θ_n is the propagation angle with respect to the z axis of a Gaussian-beam associated with the source assigned to the **phase-space** nodal point $\mathbf{X}_p$, see further discussion on Sec. 4 and also in [38]). It follows that $\mathcal{Z}_p^{TE} = [\mathcal{Y}_p^{TE}]^{-1} = \eta_1 / \cos \theta_n$, i.e., the impedance of a TE polarized plane-wave propagating at angle θ_n within a transmission-line model

of propagation [10, 54]. Note that this type of contributions are dominated in the vicinity of the X_p **phase-space** nodal point.

In an analogy to the **transmission-line model**, and in view of Eq. (5) with Eq. (6), a “beam-like” transmission-line model for beam propagation in an angle θ_n to the z -axis can be established. This model is only valid and applicable for boundary surfaces where “voltage/current”-type boundary or continuity conditions are imposed ($z = 0$ plane, here) with impedance $Z_p^{TE}(x; b)$ at $x \in S_\infty$.

2. For $(x - x_m)/b \gg \xi_n$, i.e., away from the nodal point X_p , the integral contributions on Eq. (5) are dominated by a “point source”-like contributions $\sim (x - x_m)^{-1}$.
- Following the discussion on Sec. 3.1 and Sec. Appendix B, changing b changes the Gaussian window width and the **phase-space** lattice and eventually the frame expansion (see, also, on Fig. 2). Furthermore, following items 1-2 the balance between “plane-wave” and “point-source” contributions can be controlled by varying the b to give alternative mathematical representations for the same fields and currents.
- The coupled equations in Eq. (7b) for the coefficients of the unknown auxiliary currents $\{J_p, M_p\}$ are formulated with characteristic functions of two “propagation”-type operators: (i) For $z < 0$ the propagation medium is a guiding structure (waveguide) equipped with the characteristic – modal – functions $\{e_\ell(x)\}$; (ii) For $z > 0$ the propagation environment is a free space with Gaussian-beams as propagating objects that correspond to the Gaussian-window sources $\{J_p\psi_p(x)\}$ and $\{M_p\psi_p(x)\}$. Though the Gaussian-beams are not strictly eigenfunctions of the field propagation operator (Eq. (A.1)–Eq. (A.2)), for $z \rightarrow 0$ the Gaussian-beams render the Gaussian windows (ψ_p) . Therefore, following the discussion on Eq. (6), they can be identified as characteristic functions or “mode”-like propagators. Consequently, the coupled equations can be recognized within a general *mode-matching* formulation, and Eq. (7) can be understood in view of a generalized **transmission line** theory of combining two sets of “waveguides-like” structures at an interface.

3.2.3. Discretization into a MoM system

As a final step towards achieving a discrete set of equations for the current’s coefficients, $\{J_p, M_p\}$ the set of equation Eq. (7) are projected on the dual set of functions $\{\varphi_q(x)\}$ that corresponds to $\{\psi_q(x)\}$ with $\mathbf{q} = (m', n')$, $m', n' = \mathbb{Z}^2$, see in Eq. (3), to yield

$$\langle \mathbf{E}_{1_y}^i, \varphi_q \rangle - \frac{1}{2} \sum_p \left[J_p \langle Z_p^{TE} \psi_p, \varphi_q \rangle - M_p \langle \psi_p, \varphi_q \rangle \right] = \langle \mathbf{E}_{2_y}^i(x), \varphi_q \rangle + \frac{1}{2} \sum_p \sum_\ell \left[M_p + Z_\ell^{TE} J_p \right] \langle \psi_p, e_{y_\ell} \rangle \langle e_{y_\ell}, \varphi_q \rangle, \quad (8a)$$

$$\langle \mathbf{H}_{1_x}^i, \varphi_q \rangle - \frac{1}{2} \sum_p \left[M_p \langle Y_p^{TE} \psi_p, \varphi_q \rangle - J_p \langle \psi_p, \varphi_q \rangle \right] = \langle \mathbf{H}_{2_x}^i, \varphi_q \rangle + \frac{1}{2} \sum_p \sum_\ell \left[Y_\ell^{TE} M_p + J_p \right] \langle \psi_p, e_{y_\ell} \rangle \langle e_{y_\ell}, \varphi_q \rangle. \quad (8b)$$

It should be noted that:

1. Generally, since the function set that is used for the expansion $\{\psi_p, \varphi_p\}$ is overdetermined by v^{-1} , the resulting set of equations Eq. (8), is also overdetermined by v^{-1} (see in Sec. 3.1).
2. The Gabor-frame set with Gaussian windows is of infinite dimension, which is further inherits the set of equations in Eq. (8) rendering it to be of an infinite dimension. However, from physics point of view, the scattering scenarios that we consider are of relatively finite spatial extent (which can be large). Therefore, the $\mathbf{p} = (m, n)$ summations should be truncated for $|m| \geq M$ and $|n| \geq N$, where M and N are scalars. **The truncation of the spatial and spectral indexes m and n , respectively, should provide a coverage of the dominant phase space contributions (see the discussion in [38]). Therefore, m should be set to allow a coverage of the entire coupling aperture, with some additional margin of several points beyond the physical aperture. Similarly, the spectral index n should allow the coverage of the dominant part of the spectral domain. In periodic and semi-periodic physical structures, where the spectral characteristics introduce some dominant periodic behavior even beyond the visible spectrum, n should be set to provide a corresponding spectral coverage ($\xi n > 1$) for the inclusion of the dominant spectral features (see Fig. 8). Thus, one can truncate the domain of interest to a finite subset of X_p of dimension $(2M - 1) \times (2N - 1)$ with a finite range of the index $\mathbf{p}$. Consequently, Eq. (8) can be cast as a matrix equation:**

$$\begin{bmatrix} \mathbf{Q}_{\varphi e} \mathbf{Q}_{e\psi} - \mathbf{Q}_{\varphi\psi} & \mathbf{Q}_{\varphi Z} + \mathbf{Q}_{\varphi e} Z^{wg} \mathbf{Q}_{e\psi} \\ \mathbf{Q}_{\varphi Y} + \mathbf{Q}_{\varphi e} Y^{wg} \mathbf{Q}_{e\psi} & \mathbf{Q}_{\varphi e} \mathbf{Q}_{e\psi} - \mathbf{Q}_{\varphi\psi} \end{bmatrix} \begin{bmatrix} \mathbf{M}_\varphi \\ \mathbf{J}_\varphi \end{bmatrix} = 2 \begin{bmatrix} \mathbf{E}_\varphi \\ \mathbf{H}_\varphi \end{bmatrix}, \quad (9a)$$

where, the block matrices are

$$\mathbf{Q}_{\varphi\psi} = [\langle \psi_p, \varphi_q \rangle], \quad \mathbf{Q}_{e\psi} = [\langle \psi_p, e_{y_\ell} \rangle], \quad \mathbf{Q}_{\varphi e} = [\langle e_{y_\ell}, \varphi_q \rangle] \quad (9b)$$

$$\mathbf{Z}^{wg} = [\mathbf{Z}^{wg}]^{-1} = [\text{diag} \{Z_\ell^{TE}\}], \quad \mathbf{Q}_{\varphi Z} = [\langle Z_p^{TE} \psi_p, \varphi_q \rangle], \quad \mathbf{Q}_{\varphi Y} = [\langle \mathcal{Y}_p^{TE} \psi_p, \varphi_q \rangle]$$

and the column vectors $\mathbf{M}_\varphi = [M_p]$, $\mathbf{J}_\varphi = [J_p]$, $\mathbf{E}_\varphi = [\langle \mathbf{E}_{1_y}^i - \mathbf{E}_{2_y}^i, \varphi_q \rangle]$, and $\mathbf{H}_\varphi = [\langle \mathbf{H}_{1_x}^i - \mathbf{H}_{2_x}^i, \varphi_q \rangle]$. Equation (9a) is the final matrix equation that will be used for the solution of the auxiliary currents.

3. If the sets of functions $\{e_\ell\}$ expanding the waveguide's modal field and $\{\psi_p, \varphi_p\}$ expanding the surface currents have an overlapping range (expanding the same subspace of $\mathbf{X}$), then the diagonal block matrices have the property $\mathbf{Q}_{\varphi e} \mathbf{Q}_{e\psi} - \mathbf{Q}_{\varphi\psi} = \mathbf{0}$, i.e., a null matrix, and Eqs. (8a)– (8b) are uncoupled. This is the case if, for example, medium “2” is a uniform infinite free space domain. Otherwise, the diagonal block matrices are non-zero.
4. The norm of the different matrix blocks in Eq. (9a) differ drastically [while the norm of the diagonal blocks is ~ 1 , that of the upper-right block is $\sim O(\eta_0) \gg 1$, and that of the lower-left block is $\sim O(\eta_0^{-1}) \ll 1$]. These properties follow since these are equations of different type: one is electric (“voltage”) while the other is magnetic (“current”). This difference may cause some numerical instability. To overcome this difficulty, the second equation may be multiplied by some reference impedance $\bar{\eta}$ (such as $\eta_{0,1} = \sqrt{\mu_{1,2}/\epsilon_{1,2}}$) and Eq. (9a) can be re-formulated for the unknown vector elements $\mathbf{M}$ and $\bar{\eta}\mathbf{J}$ (which renders the two equations to have the same dimensions of “voltage”).
5. As was already noted in item 1 the matrix equation is overdetermined and is non-invertible. The solution for the current coefficients can be thought as a solution of a corresponding least square problem (or regularized or constrained least square). [The convergence of the solution to the “true” \(physical\) solution can be verified by the approval the conditions: \(i\) the boundary conditions \(e.g., \$E_{1_y}^i + E_{1_y}^s = E_{2_y}^s\$ \); and \(ii\) the conservation of energy balance. The approval of these conditions and thus of the convergence is deeply related to the phase space truncation \(see the discussion in item 2\), i.e., the truncated phase space lattice should retain the dominant features of the spatial-spectral representation of the currents to guaranty the convergence and satisfy conditions \(i\) and \(ii\).](#)

4. The scattered field

Once the system (Eq. (9)) is solved and the expansion coefficients $\{J_p\}$ and $\{M_p\}$ are at hand, the auxiliary sources can be reconstructed by Eq. (4) to give $\mathbf{M}(x) = \hat{\mathbf{x}}M_x(x)$ and $\mathbf{J}(x) = \hat{\mathbf{y}}J_y(x)$ and the scattered field follows by Eq. (A.1) for the free space ($z > 0$) or with Eq. (A.3) within the waveguiding system ($z < 0$), see Appendix A. The rest of this section concerns with the representation of the scattered wave-field ($z > 0$) by summation of Gaussian beam contributions.

The Gabor frame formulation belongs to a class of discrete **phase-space** representations, which uses both the spatial and spectral domains to yield a field decomposition. The particular choice of Gaussian windows adds additional degree of freedom to the formulation of both the sources (see the discussion on the previous sections) and the radiation mechanism. In that sense two complementary perspectives can be gained, the first is the a **Source point of view** where the currents (J_{1_y}, M_{1_x}) are reconstructed by **phase-space** summation of Gaussian contributions that are weighted by **phase-space** coefficients $\{J_p\}$ and $\{M_p\}$ of the discrete **phase-space** lattice $\mathbf{X}_p = (x_m, \xi_n)$. This, in turn, implies that the p 'th **phase-space** node corresponds to a current element source with Gaussian distribution that is centered at x_m on the physical $z = 0$ plane and radiates (Gaussian beam) *dominantly* in the $\theta_n = \arcsin \xi_n$ direction along “radiation lines” (see in [38, 45]). This further recalls for the **Radiation point of view** for which the scattered (radiated) fields are formulated by **phase-space** summation of Gaussian beam propagators, also termed *Gaussian beam summation* method (GBSM), see [38–43, 45, 46, 49]. In the GBSM, fields are reconstructed by summing up Gaussian beam propagators that correspond to sources at the nodes of the discrete **phase-space** lattice $\mathbf{X}_p$, i.e., the beams are initiated from the x_m 'th point on the $z = 0$ plane in direction $\theta_n = \arcsin \xi_n$ and in magnitude that is weighted by the p 'th current. These beams are the “radiation pattern” of the Gaussian source distribution. It should be pointed out that the Gaussian windowed Gabor frame formulation naturally lends itself to the GBSM upon propagation.

For the GBSM field formulation, Eq. (4) is inserted into Eq. (A.1), to yield the scattered field beam summation representation at $z > 0$:

$$\mathbf{E}_1^s(\mathbf{r}) = \sum_p \left[J_p \mathbf{B}_p^{e,j}(\mathbf{r}) + M_p \mathbf{B}_p^{e,m}(\mathbf{r}) \right], \quad (10a)$$

$$\mathbf{H}_1^s(\mathbf{r}) = \sum_p \left[J_p \mathbf{B}_p^{h,j}(\mathbf{r}) + M_p \mathbf{B}_p^{h,m}(\mathbf{r}) \right], \quad (10b)$$

where $\mathbf{B}_p^{e,j}(\mathbf{r})$, $\mathbf{B}_p^{e,m}(\mathbf{r})$, $\mathbf{B}_p^{h,j}(\mathbf{r})$, $\mathbf{B}_p^{h,m}(\mathbf{r})$ are sets of electromagnetic vector Gaussian beams assigned to the **phase-space** node X_p (e, h correspond to contributions to the electric or magnetic fields, while, j, m correspond to the source current type magnetic or electric). These beams follow from Eq. (A.1)-Eq. (A.2) with electric and magnetic Gaussian window surface source distributions ψ_p :

$$\begin{aligned} \mathbf{B}_p^{e,j}(\mathbf{r}) &= \hat{\mathbf{y}} \frac{k_1^2}{j\omega\epsilon_1} B_p(\mathbf{r}), & \mathbf{B}_p^{e,m}(\mathbf{r}) &= -\nabla B_p(\mathbf{r}) \times \hat{\mathbf{x}}, \\ \mathbf{B}_p^{h,j}(\mathbf{r}) &= \nabla B_p(\mathbf{r}) \times \hat{\mathbf{y}}, & \mathbf{B}_p^{h,m}(\mathbf{r}) &= \frac{1}{j\omega\mu_1} (\nabla \nabla + k_1^2 \mathcal{I}) \cdot \hat{\mathbf{x}} B_p(\mathbf{r}), \end{aligned} \quad (10c)$$

where

$$B_p(\mathbf{r}) = \int_{S_\infty} d\mathbf{r}' G(\mathbf{R}, \mathbf{r}') \psi_p(\mathbf{r}') \quad (10d)$$

is a scalar generic Gaussian beam wave-object that initiates from the p^{th} **phase-space** node with a collimation distance b . A similar formulation for currents on a curved surfaces is given in, e.g., [59]

It should be noted that in the presented GBSM formulation the beam sources ($\{J_p \psi_p\}$, $\{M_p \psi_p\}$) are distributed on the $z = 0$ plane (at x_m), which forms an angle θ_n with the direction of propagation $\hat{\mathbf{x}}_{\xi_n} + \hat{\mathbf{z}}_{\xi_n}$, i.e., the source distribution (beam's source disk) is not, generally, perpendicular to the direction of beam propagation. Upon propagation the beam propagators (scalar (10d) or vectorial (10c)), maintain their *asymptotic* Gaussian-like structure *only* for collimated conditions ($kb \gg 1$). Otherwise, the beam propagators can be somewhat deformed for higher propagation angles (see the discussion in [38] and also in [60], where a remedy to that deformation was proposed).

5. Demonstrations

In this section we provide examples to demonstrate the use of the frame formulation in solving **surface integral equations** and the calculation of the surface currents. Two examples are considered, the first example in Sec. 5.1 is of the scattering of a collimated incident Gaussian beam field from a wide (in terms of wavelength) open ended multi-modal dielectric homogeneous waveguide (Fig. 3). The second example, in Sec. 5.2, simulates scattering of a Gaussian beam from an open ended dielectric **fiber bundle** (with a finite number of fibers), where each of the fibers supports multi-modal propagation (Fig. 6). Note that the example in Sec. 5.2 can be viewed as scattering/coupling into a volume grating (see, e.g., [25–29, 61] for a sinusoidal modulated grating).

In both examples, a pre-calculation of the modal functions in the waveguiding section, $\{e_{\nu\ell}(x)e^{\pm j\beta_\ell z}\}$, and the dual frame sets functions, $\{\psi_p, \varphi_p\}$, are performed. The waveguide's modal functions can be obtained by the *ABCD* matrix formulation [10, 54] or the transverse resonance technique [58]. The dual frame set $\{\psi_p, \varphi_p\}$ is obtained from the dual window set $\{\psi, \varphi\}$. Thus, given the Gaussian window $\psi(x) = e^{-kx^2/2b}$, its dual φ is obtained via the solution of the duality relation Eq. (B.1). One solution approach to Eq. (B.1) is by expanding φ by the frame itself, $\varphi(x) = \sum_p \alpha_p \psi_p x$ with expansion coefficients α_p , and equating terms on both sides of the equation [38, 62]. This procedure yields the canonical dual frame. Another approach follows from the Wexler-Raz identity [63]. Note that (i) while ψ depends only on b , the dual window function φ depends also on the **phase-space** unit cell size $2\pi\nu$, i.e., on $(\bar{x}, \bar{\xi})$ as outlined in Sec. 3.1 and hence for the same b but different values of ν there are different dual windows (see in Fig. 2 below); and (ii) from the Wexler-Raz identity [63] it follows that the windows are scaled such that $\langle \psi, \varphi \rangle = \nu$ in addition to other duality relations on a reciprocal **phase-space** lattice. For high redundancy of the frame ($\nu < 0.4$) it is convenient to use $\varphi(x) \approx \nu \psi(x)/\|\psi\|^2$ (see in [38]). But for large problems this convenience in obtaining φ comes at the expense of an increase size of the matrix equation Eq. (9).

In the following examples we use the same window $\psi(x)$ with $b = 0.1$, $k = 666.667\pi[1/m]$ but with two values of oversampling $\nu = 0.4, 0.75$ and $\rho = 1$ that gives different dual functions $\varphi(x)$. Figure 2 depicts $\psi(x)$ (full line) and its dual $\varphi(x)$ for $\nu = 0.4$ (dashed line) and $\nu = 0.75$ (dotted line). Note that for $\nu = 0.4$, $(\bar{x}, \bar{\xi}) = (0.011, 0.1095)$ and for $\nu = 0.75$, $(\bar{x}, \bar{\xi}) = (0.015, 0.15)$, i.e., different **phase-space** lattices. Furthermore, while $\psi(x)$ remain the same, the dual functions differ by their general characteristics (shape, magnitude and localization) such to maintain, for example, $\langle \psi, \phi \rangle = \nu$. These changes in lattices and function characteristics further results in different **phase-space** layout of the surface currents as will be shown below.

Figure 2. The frame set functions: $\psi(x)$ (full line), $\varphi(x)$ for $\nu = 0.4$ (dashed line) and $\varphi(x)$ for $\nu = 0.75$ (dotted line)

5.1. Open ended dielectric waveguide scattering

In the first example the scattering of a $-z$ propagated Gaussian beam perpendicular incidence on an open ended dielectric “slab-like” waveguide is considered, see in Fig. 3. This case is addressed for validation of the Gabor frame use for solving surface integral equations and to demonstrate the effect of different **phase-space** parameter on the solution. The beam field is a TE polarized perpendicularly incident Gaussian beam with beam-width of $7.333\lambda_1$ at the incidence ($z = 0$) plane. The region $z > 0$ is characterized as free space with $\epsilon_1 = \epsilon_0$ and $\mu_1 = \mu_0$. The region $z < 0$ is an open ended “slab-like” waveguide of width $L = 28\lambda_1$ where $\lambda_1 = 3mm$ is the free space wavelength at $z > 0$ (at the operating frequency $100GHz$) with $\mu_2 = \mu_0$ and dielectric profile: $\epsilon_2(x) = 2\epsilon_0$ for $|x| \leq L/2$ and $\epsilon_2(x) = 1.25\epsilon_0$ for $|x| > L/2$, see in Fig. 3.

The **phase-space** analysis was performed for two lattices with $\nu = 0.4$ and $\nu = 0.75$ both with $\rho = 1$ and $b = 0.1$ (see Fig. 2 for the corresponding φ). Figure 4 depicts two sets of normalized **phase-space** layouts of the surface currents on a logarithmic scale, Fig. 4(a) for $\{J_p\}$ and Fig. 4(b) for $\{M_p\}$ both for $\nu = 0.4$ and similarly Fig. 4(c) and Fig. 4(d) for the $\nu = 0.75$ case. As can be observed the two sets of **phase-space** layout in Fig. 7 differ by: (i) the spatial and spectral resolutions for $\nu = 0.4$ is higher than that of $\nu = 0.75$, this comes due to the different **phase-space** lattice dimensions $(\bar{x}, \bar{\xi})$ (see in the preceding section); (ii) while for the same ν , $\{J_p\}$ and $\{M_p\}$ have similar features like localization and spreading (around a central point), the particular details are different for the different ν . Specifically, for $\nu = 0.4$ the distribution is more concentrated at the centre around the **phase-space** point $\mathbf{p} = (0, 0)$ than for $\nu = 0.75$. This comes due to the locality and span of the dual functions, as can be seen on Fig. 2 by comparing $\varphi(x)$ for $\nu = 0.75$ (dotted line) which is less localized than for $\nu = 0.4$ (dashed line). The non-locality and wider spreading of $\varphi(x)$ for $\nu = 0.75$ (as compared to $\nu = 0.4$) suggest that the **phase-space** coefficients $J_p = \langle J_1, \varphi_p \rangle$ and $M_p = \langle M_1, \varphi_p \rangle$ (see in Eq. (4)) encapsulate contributions from far away x point along the aperture. In both cases the localization and concentrated spreading around $\mathbf{p} = (0, 0)$ imply that the dominant surface sources contribute to radiation in the back direction ($\hat{z}$, backscattering) due to specular reflection. Nevertheless, as is next depicted on Fig. 5, the scattered field is the same.

Figure 3. Physical layout of the open ended dielectric waveguide structure

Figure 4. Normalized phase-space layout distribution of the surface currents J_p in 4(a) and M_p in 4(b) with $\nu = 0.75$, and J_p in 4(c) and M_p in 4(d) with $\nu = 0.4$

Figure 5 depicts the real (Fig. 5(a)) and imaginary parts (Fig. 5(b)) of the scattered field E_{1y}^s at the surface $z = 0$ for $\nu = 0.75$ (dashed line), $\nu = 0.4$ (full line). Since the exciting Gaussian beamwidth is much narrower than the width of the slab an approximation for the actual scattered field can be calculated by assuming that the whole half space $z < 0$ is a homogeneous medium with $\epsilon_2 = 2\epsilon_0$ and using an SDP integration of a scattered field spectral integral [58] (this field is also depicted in dotted line). For both real and imaginary parts of the fields, a visually undistinguished matching is obtained (the absolute error is less than 0.5 along the span of excitation). Similar matching is also obtain in E_{2y}^s . This matching demonstrates that though the frame sets are different due to different redundancy ratio (giving rise to different number of frame elements by nearly 2 times more functions for $\nu = 0.4$ than for $\nu = 0.75$), the calculated fields are the same, which further validates that the formulation is insensitive to the frame parameters.

Figure 5. Real (Fig. 5(a)) and imaginary (Fig. 5(b)) parts of the scattered field E_{1y}^s at the surface $z = 0$ for $\nu = 0.75$ (dashed line), $\nu = 0.4$ (full line) and the an SDP calculated scattered filed. The fields are undistinguishable.

5.2. Fiber-bundle scattering

In the second example the scattering of a perpendicularly incident Gaussian beam on a finite **fiber bundle** with 63 fibers (unit-cells) adjacent to each other is demonstrated, see in Fig. 6. This scattering structure can also be identified as a layered aggregation of slab waveguides. The beam-width at incidence is $2.6\lambda_1$, with the same homogeneous medium for $z > 0$ as in the first example and $\lambda_1 = 3\text{mm}$. For the **fiber bundle** side ($z < 0$), each fiber width is $L_{uc} = 2\lambda_1$, the magnetic permeability is $\mu_2 = \mu_0$ while the dielectric permittivity profile of each **unit-cell** along the transversal coordinate is given by

$$\epsilon_{uc}(x') = \begin{cases} 3.2761\epsilon_0 & |x'|/L_{uc} \leq 0.42 & \text{“core”,} \\ 2.1904\epsilon_0 & 0.42 < |x'|/L_{uc} \leq 0.48 & \text{“cladding”,} \\ 2.3716\epsilon_0 & 0.48 < |x'|/L_{uc} \leq 0.5 & \text{“separation”,} \end{cases} \quad (11)$$

assuming that the **unit-cells** is centered at $x' = 0$ with a symmetric profile. There are 63 such **unit-cells** fused together symmetrically around $x = 0$ such that the **fiber bundle** occupies the lateral span of $|x| \leq 31.5L_{uc}$, for $|x| > 31.5L_{uc}$ $\epsilon_2(x) = 1.5\epsilon_0$, see the **fiber bundle** layout on Fig. 6 and the permittivity profile of a single **unit-cell**.

The **phase-space** construction used a dual frame set with $\nu = 0.75$, $\rho = 1$ and $b = 0.1$ (Fig. 2). The modal wave function set in the **fiber bundle** was calculated, numerically, using the *ABCD* matrix formulation [10, 54] to give 315 propagating modes. Note that (i) the modal field beyond the bundle ($|x| > 31.5L_{uc}$) decays exponentially. As a result, the x – calculation domain was truncated further away than the **fiber bundle** boundaries, where the modal fields decay sufficiently. (ii) Since the **fiber bundle** is a periodic structure it dominantly scatters fields in an infinite spectrum of discrete angular directions. Truncation dictates that the scattered contributions away from the specular direction

Figure 6. Physical layout of the fiber-bundle structure (Fig. 6(a)) and permittivity profile of a single fiber – unit-cells – (Fig. 6(b))

($\xi = 0$ direction) decay. Therefore, the ξ – calculation domain was also truncated to $|\xi| \gg 1$ (evanescent contributions) such as to satisfy the boundary conditions and consequently provide numerical stability.

Figure 7 depicts the normalized **phase-space** layout distribution of the auxiliary surface currents $\{J_p\}$ and $\{M_p\}$ in Fig. 7(a) and Fig. 7(b), respectively. As expected, the two figures are generally similar in their features with slight differences. The dominant dark spot at the centre corresponds to the specular back reflection contribution due to the exciting Gaussian beam, i.e., back scattered contribution, while the dominant contributions further away correspond to additional non-perpendicular beam contributions due to the periodicity. The weak horizontal “line”-like contributions at $|x| \sim 31.5L_{uc}$ correspond to a weak diffraction contribution from the edges of the **fiber bundle**.

Figure 7. Normalized **phase-space** layout distribution of the surface currents J_p in 7(a) and M_p in 7(b) on a logarithmic scale

Figure 8 depicts the real (Fig. 8(a)) and imaginary (Fig. 8(b)) parts of the fields at the surface $z = 0$ (E_{1y}^i , E_{1y}^s and E_{2y}^s in dotted, dashed, and full line, respectively). It can be observed that the boundary condition at $z = 0$ is indeed satisfied ($E_{1y}^i + E_{1y}^s = E_{2y}^s$). Additionally, following the discussion in item 5 at the end of Sec. 3.2, the MoM system was solved such that it satisfies the conservation of energy to less than 0.25%

The final part of the **fiber bundle** example, in Fig. 9, concerns with the propagation of the coupled wave-field from the $z = 0$ plane, given in Fig. 8, into and along the **fiber bundle** waveguiding structure. Figure 9 depicts the magnitude of the electric field as it propagates $50\lambda_1$, $100\lambda_1$, $200\lambda_1$ and $500\lambda_1$ into the **fiber bundle** in the $-z$ direction, in dashed, “x-x”, o-o and $\diamond-\diamond$ lines, and the aperture field (zero propagation) in a solid line for two cases with the same number of **unit-cells** and L_{uc} . The first case, in Fig. 9(a), is the propagation along a **fiber bundle** with the **unit-cell** profile as

Figure 8. Real (Fig. 8(a)) and imaginary (Fig. 8(b)) parts of the fields at the surface $z = 0$: E_{1y}^i , E_{1y}^s and E_{2y}^s (dotted line, dashed line and full line, respectively).

in Eq. (11) and Fig. 6(b) with the aperture field of Fig. 8. The second, on Fig. 9(b), is the propagation along a **fiber bundle** with a narrower core of width $0.7L_{uc}$ and a wider cladding of width $0.26L_{uc}$ (in comparison with $0.84L_{uc}$ and $0.12L_{uc}$, respectively, of Fig. 9(a)) with the corresponding aperture field that was obtained by re-solving for this case. In both cases, it can be observed that as the field propagates along the **fiber bundle**, it first exhibits a localized in the centre core, while further away it loses its dominance and localization in the centre core and “leaks” to the adjacent cores. Since the cladding section for Fig. 9(b) is wider than that of Fig. 9(a) it provides more isolation between the cores. As a result the field spreading into the adjacent cores occurs at larger propagation distances, as can be compared in Fig. 9. This localization/delocalization phenomenon agrees with the Bessel type behavior of the field amplitude in an infinitely periodic **fiber bundle** (see, e.g.[64, 65]).

Figure 9. The magnitude of the electric field propagated a distance of $50\lambda_1$, $100\lambda_1$, $200\lambda_1$ and $500\lambda_1$ into the **fiber bundle** in the $-z$ direction, in dashed, “x-x”, “o-o” and “diamond-diamond” lines, and the aperture field (zero propagation) in a solid line for two cases with the same number of unit cells and L_{uc} : (a) is for the **fiber bundle** with the **unit cell** profile as in Eq. (11) and Fig. 6(b) with the aperture field of Fig. 8; (b) is for the **fiber bundle** with a narrower core of width $0.7L_{uc}$ and a wider cladding of width $0.26L_{uc}$ (in comparison with $0.84L_{uc}$ and $0.12L_{uc}$, respectively, of Fig. 9(a)) with a corresponding aperture field for this case.

6. Summary and conclusions

This paper presented a MoM solution for the problem of scattering and coupling of wave fields at an interface between two propagation media: a homogeneous half-space on one side and an arbitrary guiding wave structure (e.g., wave guiding slab or a fiber bundle) on the other side. The problem was solved by projecting the surface integral equation, within a PMCHWT framework, on an overcomplete Gabor-type frame set with Gaussian windows (see in Sec. 3). On the homogeneous medium side, this expansion set renders the scattered field as a phase-space summation of Gaussian beams (within the GBSM, see in Sec. 4). On the waveguide side, the expansion set describes the local coupling to the waveguide modes. The use of a frame set renders an overdetermined MoM matrix equations on a discrete phase-space lattice. Validation and demonstration of the solution for wave field coupling into different guiding structure was given in Sec. 5 to highlight also the basic features of phase-space based solution.

It was shown that the phase-space Gaussian frame based formulation provides that (i) the surface integral equation renders a simple mode matching structure; (ii) the scattered fields are easily and directly obtained as a summation of canonical prescribed Gaussian beam contributions within the GBSM formulation; and (iii) The spatial spectral phase-space formulation provides a “built in” physically-based transparent view of “radiation centers”, i.e., spatial localized regions on the aperture, where radiation in dominant spectral directions takes place, and energy concentrates (see the dark spots on the phase-space layouts on Fig. 4 and Fig. 7). Moreover, though the frame based phase-space distribution is sensitive to variations in the parameters (Gaussian window width and redundancy), the surface currents and scattered fields render the same solution. Finally, though the approach of using Gabor-type basis has been used in the past it has been shown to suffer from numerical instability and non-locality problems (see also the discussion in the Introduction, Sec. 1). The use of Gabor frames with Gaussian windows (an overcomplete/overdetermined set of function) provides a remedy to these problems. Furthermore, since the overcompleteness (ν) is also a design parameter of the frame-set, the phase-space locality can also be controlled by varying ν . In that respect Figs. 4 (Sec. 5.1) demonstrates the phase space locality of the coefficients of the currents for $\nu = 0.4$, which is somewhat more localized than for $\nu = 0.75$. Recall also the corresponding “projecting” (φ) windows in Fig. 2 where φ for $\nu = 0.4$ is more localized than for $\nu = 0.75$ for the same essentially localized, ψ . In the extreme case of $\nu \rightarrow 1$, the “critically complete” bi-orthogonal Gabor case is recovered (as used in [44, 45]), which, following the Balian-Low theorem [35, Theorem 4.1.1], cannot be localized in both the spatial and spectral domains, rendering spreading of the phase-space coefficients.

The formulation presented here calculates the scattering and coupling of an incident wave-field into a dielectric fiber bundle-type of structure that is modeled by its guided modal propagator set. As such, it may be used for any structure: regular (fiber bundle) or irregular (or random, see in [66, 67]) provided that its transversal modal function set is known (by for example the ABCD method [11, 54] or coupled wave-modes [55] or numerically). Moreover, the presented formulation can also be used in other coupling and scattering problems, such as for cavities, open ended waveguides, horns, etc. (see, e.g., [13, 15–18]), where it provides an integral equation based solution paradigm (note that in the case of scattering from a cavity structure in waveguide, e.g., the loading of the cavity on the aperture can be modeled by the input impedance of the cavity). An extension to three-dimensional (3D) layouts may follow along the same lines of derivation to formulate the solution within the MoM Gabor-based Gaussian window frame formulation. In a Cartesian 3D case, a valid Gabor frame is obtained by a multiplication of two one-dimensional Gabor frames, each corresponding to a different Cartesian coordinate transversal to the z direction ([38], note that ν need not be equal for the two frame sets). Correspondingly, the phase space lattice multiplies its size, which results in an increased MoM matrix equation. The GBSM can also be used with 3D beam propagators [38, 50].

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Appendix A. Appendix A: Derivation of the PMCHWT equations

The presentation in this section is aimed towards the derivation of the PMCHWT surface integral equations, Eq. (1), for the general layout depicted in Fig. 1. Derivations of these equations can readily be found in, e.g., [33].

However, a detailed derivation is provided here for the case, where the 2D space domain is divided into two sub-domains for $z \leq 0$ with different propagation characteristics: $z > 0$ (medium “1”) is a free space with ϵ_1 and μ_1 ; and $z < 0$ (medium “2”) can be any linear propagation domain, e.g., plane-waves or, modes in waveguiding system. To that end, μ_2 is assumed constant while possibly $\epsilon_2 = \epsilon_2(x)$ (truncated periodic for the **fiber bundle**). The excitations $\mathbf{E}_{1,2}^i$ can either impinge on the interface at $z = 0$ from $z \leq 0$, respectively.

The electromagnetic fields in the two half spaces are given by: $\mathbf{E}_{1,2}(\mathbf{r}) = \mathbf{E}_{1,2}^i(\mathbf{r}) + \mathbf{E}_{1,2}^s(\mathbf{r})$ and $\mathbf{H}_{1,2}(\mathbf{r}) = \mathbf{H}_{1,2}^i(\mathbf{r}) + \mathbf{H}_{1,2}^s(\mathbf{r})$, where $\mathbf{E}_{1,2}^s(\mathbf{r})$ and $\mathbf{H}_{1,2}^s(\mathbf{r})$ are the scattered electric and magnetic fields propagating away from the interface at $z = 0$ into the two separate half-spaces. In the PMCHWT, the scattered fields results from two sets of equivalent (auxiliary) electric and magnetic currents $\{\mathbf{J}_1(\mathbf{r}), \mathbf{M}_1(\mathbf{r})\}$ and $\{\mathbf{J}_2(\mathbf{r}), \mathbf{M}_2(\mathbf{r})\}$ for $z > 0$ and $z < 0$, respectively. Each of the current sets satisfy a different equivalent problem **as depicted in Fig. A.10**: (i) $\{\mathbf{J}_1(\mathbf{r}), \mathbf{M}_1(\mathbf{r})\}$ assume that the entire space is uniform and occupied with the properties of the free space at $z > 0$ and that for $z < 0$, $\mathbf{E}_2(\mathbf{r}) = \mathbf{H}_2(\mathbf{r}) = 0$ while for $z > 0$ the fields $\mathbf{E}_1(\mathbf{r})$ and $\mathbf{H}_1(\mathbf{r})$ remain the same (see in Fig. 10(a), the light grey lines on the background for $z > 0$ serve to remind the original structure of medium 2); (ii) $\{\mathbf{J}_2(\mathbf{r}), \mathbf{M}_2(\mathbf{r})\}$ assume that the entire space is characterized as in $z < 0$, i.e., if the medium is uniform, then it is similar to item (i) while in the case of guiding structure, it is extended also to $z > 0$ with, $\mathbf{E}_1(\mathbf{r}) = \mathbf{H}_1(\mathbf{r}) = 0$ while for $z < 0$ the fields $\mathbf{E}_2(\mathbf{r})$ and $\mathbf{H}_2(\mathbf{r})$ remain the same (see in Fig. 10(b)). Within this setup the equivalent currents are given by $\mathbf{J}_{1,2}(\mathbf{r}) = \hat{\mathbf{n}}_{1,2} \times \mathbf{H}_{1,2}(\mathbf{r})$, $\mathbf{M}_{1,2}(\mathbf{r}) = \mathbf{E}_{1,2}(\mathbf{r}) \times \hat{\mathbf{n}}_{1,2}$, for $\mathbf{r} = (x, z = 0)$ with $|x| < \infty$ and $\hat{\mathbf{n}}_2 = -\hat{\mathbf{n}}_1 = -\hat{\mathbf{z}}$ are the normals to the interface. The actual tangential boundary conditions at the surface $z = 0$ imply that $\mathbf{J}_1 = -\mathbf{J}_2$ and $\mathbf{M}_1 = -\mathbf{M}_2$. The PMCHWT set of equation is followed from the boundary conditions of Maxwell’s equations at $z = 0$: $[\mathbf{E}_1^i(\mathbf{r}) + \mathbf{E}_1^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)] \times \hat{\mathbf{n}} = [\mathbf{E}_2^i(\mathbf{r}) + \mathbf{E}_2^s(\mathbf{r}; \mathbf{J}_2, \mathbf{M}_2)] \times \hat{\mathbf{n}}$ and $\hat{\mathbf{n}} \times [\mathbf{H}_1^i(\mathbf{r}) + \mathbf{H}_1^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)] = \hat{\mathbf{n}} \times [\mathbf{H}_2^i(\mathbf{r}) + \mathbf{H}_2^s(\mathbf{r}; \mathbf{J}_2, \mathbf{M}_2)]$ with $\hat{\mathbf{n}} = \hat{\mathbf{z}}$. Note that the explicit dependence of the scattered fields on the equivalent currents was inserted. The final PMCHWT equations with a single set of unknown currents either $\{\mathbf{J}_1, \mathbf{M}_1\}$ or $\{\mathbf{J}_2, \mathbf{M}_2\}$ is obtained by using the field – auxiliary equivalent currents relations that are given next.

Figure A.10. Decomposition of the physical problem, depicted in Fig. 1, into two “equivalent problems” for $z \geq 0$ for the formulation of the set of integral equations. The light grey lines on the background in Fig. 10(b) serve to remind the original structure of medium 2.

Following item (1) of the previous section, the scattered fields at $z > 0$, $\{\mathbf{E}_1^s, \mathbf{H}_1^s\}$, due to the current sources $\{\mathbf{J}_1, \mathbf{M}_1\}$ are given by:

$$\mathbf{E}_1^s(\mathbf{r}) = \frac{1}{j\omega\epsilon_1} \left(\nabla \nabla \cdot \mathbf{A}_1(\mathbf{r}) + k_1^2 \mathbf{A}_1(\mathbf{r}) \right) - \nabla \times \mathbf{F}_1(\mathbf{r}) \quad (\text{A.1a})$$

$$\mathbf{H}_1^s(\mathbf{r}) = \nabla \times \mathbf{A}_1(\mathbf{r}) + \frac{1}{j\omega\mu_1} \left(\nabla \nabla \cdot \mathbf{F}_1(\mathbf{r}) + k_1^2 \mathbf{F}_1(\mathbf{r}) \right) \quad (\text{A.1b})$$

with the electric and magnetic potentials given by:

$$\mathbf{C}(\mathbf{r}) = \int_{S_\infty} d\mathbf{r}' G(\mathbf{R}; \mathbf{r}') \mathbf{c}(\mathbf{r}'), \quad (\text{A.2})$$

with $\mathbf{C} = \mathbf{A}_1$ and $\mathbf{c} = \mathbf{J}_1$ for the magnetic or $\mathbf{C} = \mathbf{F}_1$ and $\mathbf{c} = \mathbf{M}_1$ for the electric, S_∞ is the $z = 0$ plane, and $G(\mathbf{R}, \mathbf{r}')$ with $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ is the scalar Green function of an unbounded infinite medium with the properties of medium 1 and $k_1 = \omega \sqrt{\mu_1 \epsilon_1}$. Following item (ii), the scattered fields at $z < 0$, $\{\mathbf{E}_2^s, \mathbf{H}_2^s\}$, are the fields in waveguiding structure due to current sources $\mathbf{J}_2$ and $\mathbf{M}_2$ at $z = 0$ (interface plane). Two polarization cases should be consider, TE and TM with respect to propagation direction z . In TE: $\mathbf{E}_2 = \hat{y}E_{2y}$, $\mathbf{H}_2 = \hat{x}H_{2x} + \hat{z}H_{2z}$, and respectively $\mathbf{J}_2 = \hat{y}J_{2y}$, $\mathbf{M}_2 = \hat{x}M_{2x}$ while in TM: $\mathbf{E}_2 = \hat{x}E_{2x} + \hat{z}E_{2z}$, $\mathbf{H}_2 = \hat{y}H_{2y}$, and respectively $\mathbf{J}_2 = \hat{x}J_{2x}$, $\mathbf{M}_2 = \hat{y}M_{2y}$. For the TE case, assume electric transverse orthonormal modal function set $\{\hat{y}e_{y_\ell}(x, y)\}$ and modal propagation coefficient $\{\beta_\ell\}$, with L propagating modes $\ell = 1, \dots, L$. Expressing the scattered fields by the surface equivalent currents, and using the modes orthogonality and symmetry yield,

$$\mathbf{E}_2^s(\mathbf{r}) = \sum_{\ell} \left(-\frac{1}{2}\right) \left[\langle M_{2x}, e_{y_\ell} \rangle + Z_\ell^{TE} \langle J_{2y}, e_{y_\ell} \rangle \right] e_{y_\ell}(x, y) e^{j\beta_\ell z} \hat{y}, \quad (\text{A.3a})$$

$$\mathbf{H}_2^s(\mathbf{r}) = \sum_{\ell} \left(-\frac{1}{2}\right) \left[\langle M_{2x}, e_{y_\ell} \rangle + Z_\ell^{TE} \langle J_{2y}, e_{y_\ell} \rangle \right] \left[Y_\ell^{TE} e_{y_\ell} \hat{x} - \hat{z} \frac{1}{j\omega\mu_2} \frac{\partial}{\partial x} e_{y_\ell} \right] e^{j\beta_\ell z}, \quad (\text{A.3b})$$

where $\langle a(x, y), b(x, y) \rangle = \iint_{S_\infty} ds' a(x, y) b^*(x, y)$ is the scalar product, the asterisk indicates complex conjugation and $Z_\ell^{TE} = [Y_\ell^{TE}]^{-1} = \omega\mu_2/\beta_\ell$ is the TE impedance of the ℓ 'th mode. For the TM case, assume orthonormal magnetic transverse modal function set $\{\hat{y}h_{y_\ell}(x, y)\}$ with $\{\beta_\ell\}$ to similarly obtain:

$$\mathbf{E}_2^s(\mathbf{r}) = \sum_{\ell} \frac{1}{2} \left[\frac{\langle M_{2y}, h_{y_\ell} \rangle}{\langle Z_\ell^{TM} h_{y_\ell}, h_{y_\ell} \rangle} + \langle J_{2x}, h_{y_\ell} \rangle \right] \left[Z_\ell^{TM} h_{y_\ell} \hat{x} + \frac{1}{j\omega\epsilon_2} \frac{\partial}{\partial x} h_{y_\ell} \hat{z} \right] e^{j\beta_\ell z}, \quad (\text{A.4a})$$

$$\mathbf{H}_2^s(\mathbf{r}) = \sum_{\ell} \frac{1}{2} \left[\frac{\langle M_{2y}, h_{y_\ell} \rangle}{\langle Z_\ell^{TM} h_{y_\ell}, h_{y_\ell} \rangle} + \langle J_{2x}, h_{y_\ell} \rangle \right] e^{j\beta_\ell z} \hat{y}, \quad (\text{A.4b})$$

where $Z_\ell^{TM} = \beta_\ell/\omega\epsilon_2$. Note that Z_ℓ^{TM} is a function of x in the case of the dielectric **fiber bundle**. For homogeneous and uniform medium $z < 0$ where $\epsilon_2(x) = \epsilon_2$, Z_ℓ^{TM} is constant and, therefore, $Z_\ell^{TM} = [Y_\ell^{TM}]^{-1}$.

Next, applying the symmetry relations of the equivalent currents into Eq. (A.3)-Eq. (A.4) it is noted that due to linearity of the scalar product, $\mathbf{E}_2^s(\mathbf{r}; \mathbf{J}_2, \mathbf{M}_2) = -\mathbf{E}_2^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)$ and $\mathbf{H}_2^s(\mathbf{r}; \mathbf{J}_2, \mathbf{M}_2) = -\mathbf{H}_2^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)$. Now that the two sets of scattered fields are given in terms of a single set of two unknowns, either $\{\mathbf{J}_2, \mathbf{M}_2\}$ or $\{\mathbf{J}_1, \mathbf{M}_1\}$, the PMCHWT equations are given by: $[\mathbf{E}_1^i(\mathbf{r}) + \mathbf{E}_1^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)] \times \hat{n} = [\mathbf{E}_2^i(\mathbf{r}) - \mathbf{E}_2^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)] \times \hat{n}$ and $\hat{n} \times [\mathbf{H}_1^i(\mathbf{r}) + \mathbf{H}_1^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)] = \hat{n} \times [\mathbf{H}_2^i(\mathbf{r}) - \mathbf{H}_2^s(\mathbf{r}; \mathbf{J}_1, \mathbf{M}_1)]$. Insertion of the expressions for the scattered fields in Eq. (A.1) and Eq. (A.3)-Eq. (A.4) yields the two PMCHWT integral equations for the auxiliary currents. The corresponding equations for the TE case, that is the main concern here, are given in Eq. (1) to form the starting point for the discussion on Sec. 3.

Appendix B. Appendix B: Gabor's-type frame-based formulation

This section provides some additional details regarding the construction of Gabor's-frame set given on Sec. 3.1.

Following the setting of the **phase-space** lattice $\mathbf{X}_p$, the window $\psi(x)$ and the frame set $\{\psi_p(x)\}$, the frame operator $\mathcal{S}$ is defined (see in Eq. (B.1)). The corresponding dual window, $\varphi(x)$, is defined by a duality relation $\mathcal{S}\varphi(x) = \psi(x)$:

$$\mathcal{S}\varphi(x) = \sum_{\mathbf{p}=(m,n) \in \mathbb{Z}^2} \langle \varphi, \psi_p \rangle \psi_p(x) = \psi(x), \quad (\text{B.1})$$

where the summation should be understood as double summation over m and n . The duality expression Eq. (B.1), provides a relation for the calculation of the canonical dual window φ associated with the window ψ (see in [38, 62]). This dual window (and frame) is termed the “canonical dual”. An alternative approach follows from the Wexler-Raz identity [63].

Generally, $\psi_p(x)$ should be localized in X around the p 'th lattice points $\mathbf{X}_p$ with $v < 1$. This locality in X ($\mathbf{X}_p$) also depends on a “matching” parameter $\rho = (\Delta_x/\bar{x})/(\Delta_\xi)/\xi$ that describes the window fitting to the **phase-space** unit cell with $\Delta_{x,\xi}$ being the spatial and spectral effective width of the window $\psi(x)$ [38]. For $\rho = 1$ the **phase-space**

unit-cell coverage is balanced (referred to as “snug” frame). In the following discussion we use a Gaussian window: $\psi(x) = \psi_{00}(x) = e^{-kx^2/2b}$, and $\tilde{\psi}(\xi) = \sqrt{\frac{2\pi b}{k_1}} e^{-k_1 b \xi^2/2}$ with $b > 0$ (i.e. $\mathbf{p} = (0, 0)$ in Eq. (3)). For this Gaussian window, $\Delta_x = (b/k)^{1/2}$ and $\Delta_\xi = (kb)^{-1/2}$ it follows that with $\rho = 1$ and $k\bar{x}\bar{\xi} = 2\pi\nu$: $b = \bar{x}/\bar{\xi}$, $\bar{x} = \sqrt{2\pi\nu b/k}$ and $\bar{\xi} = \sqrt{2\pi\nu/bk}$ and the frame elements and lattice are set (for a more extended discussion see [38]).

A more extensive discussion about the Gabor-type frame procedure can be found in [34, 35, 37–39, 56].

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