A body-force based method to generate supersonic equilibrium turbulent boundary layer profiles

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Abstract

We further develop a simple counterflow body force-based approach to generate an equilibrium spatially developing turbulent boundary layer suitable for Direct Numerical Simulations (*DNS*) or Large Eddy Simulations (*LES*) of viscous-inviscid interactions. The force essentially induces a small separated region in an incoming specified laminar boundary layer. The resulting unstable shear layer then transitions and breaks down to yield the desired unsteady profile. The effects of wall thermal conditions are explored to demonstrate the capability of the method for both fixed wall and adiabatic wall conditions. We then describe an efficient method to select parameters that ensure transition by examining precursor signatures using generalized stability variables. These precursors are shown to be evident in a computational domain spanning only a small region around the trip and can also be detected using 2D simulations. Finally, the method is tested for different Mach numbers ranging from 1.7 to 2.9, with emphasis on flow field surveys, Reynolds stresses, and energy spectra. These results provide guidance on boundary conditions for desired boundary layer thickness at each Mach number. The consequences of using a much lower Reynolds number in computation relative to experiment are evident at the higher Mach number, where a self sustaining turbulent boundary layer is more difficult to obtain.

Keywords: inflow boundary condition; turbulence; transition; implicit large eddy simulation; LES; flat plate

Nomenclature

a	extent of force field in y
b	extent of force field in x
D_c	ratio between electrical and inertial forces
e_c	electronic charge
e_T	total energy
$\mathbf{E}$	electric field vector
f	frequency
$\mathbf{F}_I$	inviscid flux vector
$\mathbf{F}_V$	viscous flux vector
H	shape factor

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$\mathbf{J}$	Jacobian of coordinate transformation
$k_{1,2}$	parameters for electric field drop-off
L	length scale
M	Mach number
Pr	Prandtl number
p	pressure
$\mathbf{Q}_{ht}$	heat conduction vector
Re	Reynolds number
$\mathbf{S}$	source vector
St	Strouhal number
T	temperature
t	time
$\hat{\mathbf{U}}, \hat{\mathbf{F}}_I$	variables in transformed coordinates
$\mathbf{U}$	solution vector
u, v, w	velocity components
$\mathbf{V}$	velocity vector
$_{ref}$	reference values
α	collision efficiency factor; arbitrary flow variable
δ_{99}	boundary layer thickness for $u = 0.99U_\infty$
$\delta_{99,0}$	measured incoming boundary layer thickness
Δt	effective duty cycle of DBD actuator
μ	viscosity
ρ	density
ρ_c	charge density
$\boldsymbol{\tau}$	shear stress tensor
θ	frequency of the applied voltage for DBD; momentum thickness
ξ_d	partial derivative of ξ in direction d
ξ, η, ζ	curvilinear coordinates
$*$	non-dimensional value

1. Introduction

High-fidelity simulations of the interaction of an equilibrium turbulent boundary layer with distortion induced by features such as shock waves or protuberances are greatly facilitated by methods that can effectively and efficiently provide a suitable upstream condition representing the key statistics of the boundary layer. Such methods are necessary components of recent studies of low frequency unsteadiness in shock wave/turbulent boundary layer interactions (STBLIs), which are major performance inhibitors of supersonic and hypersonic systems. In propulsion flowpaths, particularly in inlets and isolators, these interactions cause unsteady boundary layer separation which results in reductions of air intake efficiency and can even induce catastrophic unstart. Their inherently unsteady nature also causes significant pressure fluctuations, potentially compromising the structural integrity of aircraft hardware. STBLIs also cause unsteady vortex shedding and shock/vortex interactions which in turn generate acoustic effects and loads. Reviews of STBLI may be found, for example, by Gaitonde [10], Dolling [5]. The most recent of these [10] summarizes the state of the art in the understanding of STBLI unsteadiness, [6, 14, 13, 21, 32] heat transfer, [12, 3, 15] and control [17, 2, 40].

Since Reynolds Averaged Navier-Stokes (RANS) approaches cannot accurately simulate unsteadiness, the computational methods of choice are Direct Numerical Simulations (DNS) or Large-Eddy Simulations (LES). However, DNS and LES are prohibitively expensive for the real world situation, which may involve multiple successive shock interactions in the presence of three-dimensionality. Many of the key phenomena of interest can however be reproduced in subdomains of the flowfield. For example, SBLI interactions in an isolator shock-train can be fruitfully examined by simply considering one such interaction, which can be examined in a wind tunnel (see *e.g.*, Ref. Webb *et al.* [39]). Even with this reduction to a canonical situation however, the inflow condition remains prohibitively expensive to simulate exactly as it occurs in the wind tunnel *i.e.*, through a turbulence transition process. The difficulty is complicated by the strong sensitivity to a relatively poorly characterized disturbance environment. Furthermore, the simulations are typically carried out at much lower Reynolds numbers than in experiment because grid resolution requirements scale to a high power of the Reynolds number (about 4 for explicit methods) [26] for DNS. Typical computational analyses consider about one tenth the actual Reynolds number (see discussions by Toubert and Sandham [34], Visbal *et al.* [38]). It is thus highly advantageous to specify the incoming unsteady equilibrium turbulent boundary layer profile without actually simulating the entire transition process.

At low Reynolds numbers however, the turbulent profile could potentially contain the signature of the mechanism by which it was generated. Techniques used to specify the boundary layer must therefore ensure that no artificial, energetically-significant time or length scales are introduced into the computation since these may interfere with the subsequent interaction. Several methods have been proposed in the literature to generate desired incoming turbulent profiles. Pronk and Hulshoff [27] document a study of various turbulent inflow generation methods, comparing their effectiveness for computations of turbulent boundary layers.

One of the most popular methods is the recycling-rescaling method of Lund *et al.* [18]. A detailed description and analysis of this approach has been provided by Morgan *et al.* [20], who use it to generate a Mach 2.28 boundary layer over an adiabatic plate. The method is computationally inexpensive but introduces a new length scale comprised of the length of the recycling region, which may manifest itself in the subsequent interaction [27]. Other issues with this approach, discussed for the nominally 2-D case by Morgan *et al.* include the need to address recycling of the inviscid wave structure and unphysical spanwise decorrelation, for which they develop a dynamic reflection technique.

Another common option is the digital filtering method proposed by Klein *et al.* [16] and used extensively for STBLI by Toubert and Sandham [33]. This method requires initiation by knowledge of the mean flow and Reynolds-stress profiles which are seeded with suitable physically consistent perturbations based on filtered random numbers that are then allowed to relax. The computational cost relative to the relaxation process appears to depend on the closeness of the initial profile and the final desired conditions: for example Toubert and Sandham [34] require more than 25δ while Morgan *et al.* [20] tune the upstream condition to obtain equilibrium profiles in about 10δ (where δ is the corresponding incoming boundary layer thickness). Other approaches include blowing/suction [28] in a frequency range, which again are subject to long signatures in the relatively low Reynolds numbers in simulations.

Figure 1: **Schematic of counter-flow body-force for boundary layer tripping**

In our previous efforts [23, 24, 25], a counter-flow force-based method was introduced to generate an equilibrium spatially developing Mach 2.3 turbulent boundary layer. Figure 1 shows the basic scheme. The force field, described in more detail in Section 2, is placed in a specified laminar incoming boundary layer. A small separation region results to yield an unstable shear layer, whose breakdown enhances transition and rapid evolution (relative to the natural process) to turbulence. The method generates very clean boundary layers, *i.e.*, without any signature of the tripping mechanism, which can be difficult to achieve at the low Reynolds numbers that are typically used in this class of simulations. Furthermore, statistical observations, including single and multipoint correlations, match values anticipated from the literature.

Although the advantages of the method, specifically its simplicity and accuracy have been established, no significant effort was made towards efficiency. The results showed particularly that the rapidity with which transition occurred, and thus the streamwise domain extent required to obtain an equilibrium profile depended on mesh resolution. The effects of varying some of the force field parameters in a Mach 2.3 flowfield are extensively described by Mullenix et al. [24]. The most obvious technique, to increase the strength of the force field and thus generate a stronger perturbation, is not effective because of numerical instabilities.

The current work extends this approach in several ways: (i) Transition is accelerated by resolving numerical difficulties associated with higher force fields. This requires an analysis of the work associated with the force, as well as wall boundary conditions. (ii) Issues arising in developing a profile at higher Mach numbers are addressed [19]. This requires consideration of Reynolds numbers effects because of the challenge in sustaining turbulence at the low Reynolds numbers affordable for LES/DNS. (iii) Criteria are developed to quickly determine suitable force field strengths for any given flow condition. (iv) A systematic assessment is made at different Mach and Reynolds numbers to investigate the parametric properties of the method. In particular, we examine the effect of field strength on the statistics of the obtained profiles at different Mach numbers.

The paper is structured as follows. In Section 2, the theoretical and numerical model is presented with the governing equations, force field description, the numerical scheme implemented, and the boundary conditions applied. The grids employed to develop different aspects of the method are specified in Section 3. A summary of previously documented capability of the method is presented in Section 4. The combined effects of the force parameters and wall thermal conditions are summarized in Section 5. Factors related to higher Mach numbers are discussed in Section 6, which includes effects of grid refinement, Reynolds number, force field strength and wall temperature. Precursors based on stability criteria to guide parameter selection are discussed in Section 7. Finally, in Section 8, the performance of the method is considered at three different Mach numbers, considering boundary layer growth and turbulence surveys.

2. Theoretical and Numerical Model

The full compressible Navier-Stokes equations are solved, supplemented by source terms which account for the interaction of the force field with the flow. In non-dimensional form, these may be written as:

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \mathbf{V}^*) = 0 \quad (1)$$

$$\frac{\partial \rho^* \mathbf{V}^*}{\partial t^*} + \nabla^* \cdot (\rho^* \mathbf{V}^* \mathbf{V}^* + p^* \mathbf{I}) - \frac{1}{Re} \nabla^* \cdot \boldsymbol{\tau}^* = D_c q_c^* \mathbf{E}^* \quad (2)$$

$$\frac{\partial \rho^* e_T^*}{\partial t^*} + \nabla^* \cdot \left[(\rho^* e_T^* + p^*) \mathbf{V}^* - \frac{1}{Re} (\mathbf{V}^* \cdot \boldsymbol{\tau}^*) - \frac{1}{(\gamma - 1) Pr M^2 Re} \mathbf{Q}_{ht}^* \right] = D_c \rho_c^* \mathbf{U}^* \cdot \mathbf{E}^* \quad (3)$$

where the superscript $*$ denotes non-dimensional quantities, $\mathbf{V}^* = [u^*, v^*, w^*]^T$ is the velocity vector, ρ^* is the density, p^* is the pressure, t^* is the time, e_T^* is the total energy, $\boldsymbol{\tau}^*$ is the stress tensor and $\mathbf{Q}_{ht}^*$ is the conduction heat transfer vector. The right-hand side terms represent the effects of the body force, details of which are described later in this section.

The non-dimensionalization is accomplished through the following relations:

$$t^* = \frac{t V_{ref}}{L_{ref}}, \quad \rho^* = \frac{\rho}{\rho_{ref}}, \quad \mathbf{V}^* = \frac{\mathbf{V}}{V_{ref}}, \quad p^* = \frac{p}{\rho_{ref} V_{ref}^2}, \quad \mu^* = \frac{\mu}{\mu_{ref}}, \quad T^* = \frac{T}{T_{ref}} \quad (4)$$

where,

$$L_{ref} = \delta_{99,0}, \quad V_{ref} = V_{inf}, \quad \rho_{ref} = \rho_{inf}, \quad \mu_{ref} = \mu_{inf}, \quad T_{ref} = T_{inf}$$

Specific values are provided later. As expected, several non-dimensional parameters appear. They include the Reynolds number $Re = \frac{\rho_{ref} V_{ref} L_{ref}}{\mu_{ref}}$, the Prandtl number $Pr = \frac{\mu_{ref} C_p}{k_{ref}} = 0.72$ (where C_p is the specific heat capacity at constant pressure and k_{ref} is the thermal conductivity) and the Mach number $M = \frac{V_{ref}}{\sqrt{\gamma p_{ref} / \rho_{ref}}}$. The molecular viscosity, μ is obtained from Sutherland's law and the perfect gas assumption is employed. For simplicity, the superscript $*$ will be dropped in the subsequent discussion, and all quantities are non-dimensional unless otherwise noted.

The governing equations in flux vector form are written as:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_I}{\partial x} + \frac{\partial \mathbf{G}_I}{\partial y} + \frac{\partial \mathbf{H}_I}{\partial z} = \frac{\partial \mathbf{F}_V}{\partial x} + \frac{\partial \mathbf{G}_V}{\partial y} + \frac{\partial \mathbf{H}_V}{\partial z} + \mathbf{S} \quad (5)$$

where $\mathbf{U}$ is the solution vector, $\mathbf{U} = [\rho, \rho u, \rho v, \rho w, \rho e_T]^T$, $\mathbf{F}_I$ etc. contain inviscid terms, $\mathbf{F}_V$ etc. contain the viscous effects, and $\mathbf{S}$ contains the body force source terms. For instance:

$$\mathbf{F}_I = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (\rho e_T + p) u \end{bmatrix}, \quad (6)$$

and

$$\mathbf{F}_V = \begin{bmatrix} 0 \\ \frac{1}{Re} \tau_{xx} \\ \frac{1}{Re} \tau_{xy} \\ \frac{1}{Re} \tau_{xz} \\ \frac{1}{Re} (u \tau_{xx} + v \tau_{xy} + w \tau_{xz}) + \frac{1}{(\gamma - 1) Pr M^2 Re} \frac{\partial T}{\partial x} \end{bmatrix}. \quad (7)$$

The transformation $x = x(\xi, \eta, \zeta)$, $y = y(\xi, \eta, \zeta)$, $z = z(\xi, \eta, \zeta)$ can also be used to write these equations in curvilinear coordinates. The strong conservation form is used to obtain:

$$\frac{\partial \hat{\mathbf{U}}}{\partial t} + \frac{\partial \hat{\mathbf{F}}_I}{\partial \xi} + \frac{\partial \hat{\mathbf{G}}_I}{\partial \eta} + \frac{\partial \hat{\mathbf{H}}_I}{\partial \zeta} = \frac{\partial \hat{\mathbf{F}}_V}{\partial \xi} + \frac{\partial \hat{\mathbf{G}}_V}{\partial \eta} + \frac{\partial \hat{\mathbf{H}}_V}{\partial \zeta} + \hat{\mathbf{S}} \quad (8)$$

where $\hat{\mathbf{U}} = \mathbf{U}/\mathbf{J}$, $\hat{\mathbf{S}} = \mathbf{S}/\mathbf{J}$, and $\mathbf{J}$ represents the Jacobian of the transformation, and

$$\begin{aligned} \hat{\mathbf{F}}_I &= \frac{1}{\mathbf{J}} (\xi_x \mathbf{F}_I + \xi_y \mathbf{G}_I + \xi_z \mathbf{H}_I) \\ \hat{\mathbf{F}}_V &= \frac{1}{\mathbf{J}} (\xi_x \mathbf{F}_V + \xi_y \mathbf{G}_V + \xi_z \mathbf{H}_V) \end{aligned} \quad (9)$$

with similar expressions for the remaining flux vectors.

The governing equations are solved with very high-order methods. The inviscid fluxes are obtained with a highly accurate, band-width optimized fifth-order Weighted Essentially Non-Oscillatory (WENO) scheme described by Mullenix and Gaitonde [22]. The viscous flux terms are computed using a sixth-order compact difference, spectral-like scheme [36]. Time integration is performed using the implicit Beam-Warming technique with two sub-iterations and approximate factorization which allows for a relatively large time step of up to $\Delta t^* = 0.002$.

The source terms in Equations 2 and 3 represent the effect of the counterflow body force which acts as a trip. Although a force model could be arbitrarily specified, for convenience we use that developed for Dielectric Barrier Discharge (DBD) actuators [30], and essentially describes a force established by an electrical field.. In Eqn. 10, ρ_c^* is the charge density, e_c is the electronic charge, and $\mathbf{E}^* = [E_x^*, E_y^*, E_z^*]^T$ is the electric field vector. The parameter D_c is the ratio between electrical and inertial forces given by:

$$D_c = \frac{\rho_{c,ref} e_c E_{ref} L_{ref}}{\rho_{ref} U_{ref}^2} \quad (10)$$

The force, $\mathbf{F}_e$, is obtained from [30]

$$\mathbf{F}_e = D_c \theta \Delta t \alpha \rho_c \mathbf{E} \delta_{cr} \quad (11)$$

where θ is the frequency of the applied voltage, Δt is an effective duty cycle to recover an effective mean force, α is a factor for collision efficiency, ρ_c is the charge density, δ_{cr} is a function that has a value of one where the local electric field exceeds the breakdown electric field strength and zero elsewhere, and $\mathbf{E}$ is the electric field vector, whose magnitude is assumed to vary linearly in x and y according to:

$$|\mathbf{E}| = E_0 + k_1 x + k_2 y \quad (12)$$

where $E_0 = V_e/d$, V_e is the applied voltage, and d is the streamwise distance between the two electrodes. This yields a peak value at the wall diminishing to zero beyond a specified distance from the wall. In the current implementation these are not specified directly, but the extent of the force field in y , a , and in x , b , are used to calculate k_1 and k_2

$$k_1 = \frac{1 - E_0}{b}, k_2 = \frac{1 - E_0}{a}. \quad (13)$$

A more significant modification to the Shyy model is that the force is assumed to be parallel to the plate, *i.e.*, only the stream wise component is accounted for. Figure 2 shows the typical location and size of the force relative to the reference computational domain, and also an enlarged region near the trip that shows force vectors and region of application resulting from Eqs. 11 -13.

The initial condition for all interior points is the laminar boundary layer profile obtained from a compressible Blasius solution at the Mach number of interest. The incoming boundary layer is carefully selected, as described in Section 8.1 in order to obtain the desired boundary layer thickness in the interaction region. This profile is also maintained as the inlet boundary condition. A no-slip adiabatic wall boundary condition is applied. However, as noted below, under certain conditions, it is beneficial to use a fixed-wall temperature for small regions around the force field. The dynamic viscosity, μ is defined in the entire flowfield using Sutherland's formula. Periodic boundary conditions are applied in the spanwise direction, and the spanwise spacing of the grid is constant at all locations.

Figure 2: **Schematic showing the relative size of the DBD actuator compared to the domain and the orientation and distribution of the applied force field**

Table 1: **Experimental and Computational Baseline Flow Conditions**

Property	Experimental	Comp. (M=2.3)	Comp.(M=2.9)	Comp.(M=1.7)
M_∞	2.3	2.3	2.9	1.7
U_∞ (m/s)	556	556	692	407
P_∞ (Pa)	23511.12	2351.11	2351.11	2351.11
T_0 (K)	295.6	295.6	295.6	295.6
$\delta_{99,0}$ (m)	5.3×10^{-3}	5.3×10^{-3}	5.3×10^{-3}	5.3×10^{-3}
θ (m)	7.68×10^{-4}	7.68×10^{-4}	7.68×10^{-4}	7.68×10^{-4}
Re_δ	175202.06	17520.21	17520.21	17520.21
Re_θ	25387.77	2538.78	2538.78	2538.78

The nominal conditions, at Mach 2.3, are selected to reproduce those obtained in STBLI experiments at The Ohio State University Gas Dynamics and Turbulence Lab (GDTL) [39]. These are shown in column 2 of Table 1. The corresponding conditions (Mach 2.3) used for the present simulations are shown in column 3 to effectively match the experiment. The computational Reynolds number is a tenth of the experimental value, similar to the practices of other researchers [34, 38]. Because these profiles will be used for STBLIs, the length and velocity scales ($\delta_{99,0}$ and U_∞) of the experimental conditions are also matched in the computation since the frequency spectra of the flow scale with these parameters (see Dussage et al. [8]). The lower Reynolds number is thus achieved by lowering the density, and pressure, by a factor of 10, maintaining Mach number, velocity and boundary layer thickness correspondence with the experiment.

Columns 4 and 5 of Table 1 describe the nominal conditions for other Mach number flows. Note that the Reynolds numbers presented in the table for the Mach 2.9 flow are only used as a starting point (baseline). Because the Mach 2.9 flow is more stable than the Mach 2.3 flow, it needs different tripping parameters which manifest in the Reynolds number, as discussed in Section 6.

3. Computational meshes

Several meshes have been employed to develop the method: these are summarized for ready reference in this section. A schematic of the grid is shown in Fig. 3. The x , y , and z directions refer to the streamwise, wall normal, and spanwise directions, respectively. In the streamwise direction, the grid is divided into several sections: a refined grid near the inflow, constant fine spacing near the force field where the small separation region occurs, evolving to another constant spacing region where turbulence is expected to arise, and finally a stretched mesh section near the outflow. The grid spacing is uniform in the spanwise direction. In the wall-normal direction, the Δy values are governed by a hyperbolic tangent expansion of the spacing, starting with a specified Δy at the wall and ending at the incoming boundary layer height, at which point a power-law distribution is used. The center of the trip is located at $x = 2.6\delta_{99,0}$ downstream of the leading edge, where $\delta_{99,0}$ refers to the measured incoming boundary layer thickness and is used as L_{ref} . It will be dropped from further references to the coordinates. As previously mentioned, the force field is oriented counter to the flow direction and is demarcated by a triangle of height, a , and length, b (where $a = 0.03125$ and $b = 0.125$).

Simulations [23, 24, 25] of the Mach 2.3 flow were carried out on three grids, the finest of which has ~ 65 million points and will be referred to as grid A. Grids B and C have ~ 37 million points and ~ 20 million points respectively. Detailed descriptions of these grids are presented in Table 2. Note that these grids are the same as to be used in the interaction itself *i.e.*, an STBLI simulation does not increase the required number of mesh points.

Grid A is employed essentially as a truth model (see grid study by Mullenix et al. [24]) to compare different results on Grids B and C, which have fewer points and are thus more desirable, if proven to be adequate. The results obtained using each grid for the Mach 2.3 case are summarized in Sections 4 and 5. Several additional grids are employed for Mach 2.9 to explore properties of the method. These are presented here for ready reference. The baseline grid for the Mach 2.9 study is Grid C. Several modified versions of Grid C are also examined: these will be referred to as grids C1, C2, C3, C4, and C5 (see Table 3 for the refined grid parameters). These sub-grids are used in Section 6 to determine the tripping parameters for a Mach 2.9 flow.

Grid C1 is a narrower version of Grid C with only 40% of the original number of points, obtained by reducing the spanwise domain from $5\delta_{99,0}$ to $2\delta_{99,0}$. Grid C2 is a subset of the narrow Grid C1 where only the first half of the streamwise domain is represented: this facilitates examination of the signature of the tripping for different force strengths. Grid C3 is a modification of Grid C2 in which the mesh is refined in the wall normal direction. The wall normal extent is reduced from $25\delta_{99,0}$ to $5\delta_{99,0}$ but the same number of points (167) is used to resolve the domain in this direction, making it relatively finer in y .

Grid C3 is further refined to derive Grids C4 and C5. C4 is obtained by refining Grid C3 in the wall normal direction. The number of grid points is increased by 50% to give 251 points instead of 167 over a $5\delta_{99,0}$ height. To further emphasize the resolution, C4 has 234 points resolving the boundary layer more than on Grid C, which has 101 points. Grid C5 is a modification of Grid C3 in which the x and z directions are refined by increasing the number of grid points by 50% in either direction.

These subsets of Grid C reduce the computational domain by almost an order of magnitude in some cases. Using them for parametric studies nullifies the need to simulate the entire flow field for each case, considerably reducing computational expense. After determining the parameters required for transition on these shorter grids, the entire flow field is simulated for the successful case. As seen later in Section 6, the successful Mach 2.9 case happens to be one where Grid C3 is used. The grid is thus extended to include the full streamwise domain in order to obtain results for the entire flowfield. The full grid used for this is referred to as Grid C3-full. It differs from Grid C in the wall normal direction.

4. Summary of previous results

Results with the counterflow body force method have been extensively documented by Mullenix et al. [23, 24, 25] for a Mach 2.3 flow. This section shows some key results on Grids A, B, and C described earlier in Table 2. In summary, the results indicate that although Grid C is capable of sustaining a turbulent

Figure 3: Schematic of the grid showing the different Δx regions and the clustering of y values near the wall

Table 2: Properties of grids employed for Mach 2.3 upstream boundary layer generation. bf =body force

	A	B	C
$X \times Y \times Z$	$95 \times 25 \times 5$	$95 \times 25 \times 5$	$95 \times 25 \times 5$
$N_x \times N_y \times N_z$	$1301 \times 251 \times 201$	$1101 \times 201 \times 167$	$917 \times 167 \times 133$
N_{total}	6.564×10^7	3.696×10^7	2.025×10^7
$N_{x,bf}$	100	100	100
$X_{bf,center}$	2.6	2.6	2.6
$X_{bf,length}$	1.0	1.0	1.0
$N_{y,bl}$	151	125	101

Table 3: Mesh Properties - Grid C and Modified Versions

Grid	C	C1	C2	C3	C4	C5	C3-full
$X \times Y \times Z$	95x25x5	95x25x2	50x25x2	50x5x2	50x5x2	50x5x2	95x5x5
$N_x \times N_y \times N_z$	917x167x133	917x167x55	642x167x55	642x167x55	642x251x55	961x167x83	917x167x133
N_{total}	2.037×10^7	8.423×10^6	5.897×10^6	5.897×10^6	8.8×10^6	1.332×10^7	2.037×10^7
$N_{x,bf}$	100	100	100	100	100	150	100
$X_{bf,center}$	2.6	2.6	2.6	2.6	2.6	2.6	2.6
$X_{bf,length}$	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$N_{y,bl}$	101	101	101	156	234	156	156

boundary layer, the flow transitions more downstream compared to the other two meshes which in turn affects the properties of the turbulent boundary layer in the region of interest. A qualitative assessment is provided in Fig. 4 which shows Q criterion [11, 9, 7, 4, 1] iso-levels at a time instant. It shows that hairpin vortices which characterize turbulence, are generated further downstream on the nominal grid than on the fine and medium ones.

Figure 5 shows a comparison of the different measures of boundary layer thickness on the fine and nominal grids (Grids A and C respectively). The displacement thickness δ^* and the momentum thickness θ display similar behavior on the two grids. Downstream of $x = 40$, Grid C has consistently lower values for each of the parameters because transition occurs further downstream than for Grid A. This observation is further enforced by Fig. 6 which shows the log law behavior for various x locations on the two grids. The biggest difference between the fine and the coarse grids occurs at $x = 25$ indicating that transition occurs earlier for the fine grid case.

Figure 7 shows the mean flow profiles at various streamwise locations. The u velocity profiles in Fig. 7(a) display the expected laminar behavior at $x = 0.11$, separation and reversed flow at the location of the trip, $x = 2.6$, and transitional flow at $x = 25$ with the fine grid indicating a faster transitioning process. The profiles are almost turbulent at $x = 50$, with the fine grid transitioning faster as before, and fully turbulent at $x = 65$ and $x = 72.5$. Fig. 7(b) shows the v velocity and indicates that at the separated region, $x = 2.6$, Grid C only reaches about 80% of the minimum of the fine grid case and then the two converge to approximately the same constant value. Figures 7(c) and 7(d) show the density and temperature profiles respectively, indicating the evolution from laminar flow to turbulent flow. Except for the separation region where the density and temperature are significantly reduced, the wall values are nearly constant. Fig. 7(e) shows a reduction in the pressure at $x = 2.6$ at the wall and across the boundary layer. The profile recovers quickly and the variations downstream are minimal.

As these results indicate, the nominal grid, Grid C, is sufficient to sustain turbulence but it predicts transition further downstream than Grids A and B. It is preferable to use the nominal grid for the future parametric studies of the STBLI, to keep computational expenses tractable. In fact, as mentioned in Section 3, the work presented in this paper is based on this nominal grid. This motivates the need to move transition upstream so that the ensuing turbulent boundary layer has sufficient streamwise extent to develop before the shock wave impinges without necessitating an increase in the domain. Efforts to move transition upstream for the Mach 2.3 flow on Grid C are the focus of the next section.

5. Force strength and wall thermal conditions

The method was originally developed to generate subsonic turbulent boundary layers [35]. Extending the method to supersonic flows is a more complicated effort, because higher Mach number flows are more stable to disturbances and have more potential to generate a long lasting signature of the shock wave associated with the body force. The challenge is thus to ensure that the disturbance is large enough to trip the more stable boundary layer but small enough to avoid the generation of a persistent shock signature. Since the goal is to encourage transition more rapidly on the smaller meshes, an obvious choice is to increase the strength of the force field by increasing D_c . This however destabilizes the simulation and necessitates extremely

a) $x \leq 75$

b) $50 \leq x \leq 75$

Figure 4: Isosurfaces of $Q=1$ colored by u velocity on Grids A, B and C -Mach 2.3

Figure 5: Boundary layer thickness vs. streamwise location on Grids A and C -Mach 2.3

Figure 6: Velocity profile at various streamwise locations on Grids A (fine) and C (nominal)-Mach 2.3

(a) u

(b) v

(c) ρ

(d) T

(e) p

Figure 7: Mean flow properties at various streamwise locations -Mach 2.3 with different wall thermal conditions

Table 4: Boundary Layer Tripping Test Cases for Mach 2.3 (OSU Conditions)

Name	Grid	D_c	T_w	x_{aw}
A_12.5_aw	A	12.5	T_{aw}	
B_12.5_aw	B	12.5	T_{aw}	
C_12.5_aw	C	12.5	T_{aw}	
C_25_aw	C	25	T_{aw}	
C_25_1.95	C	25	1.95	
C_25_2.5	C	25	2.5	
C_25_1.95x15	C	25	1.95	15

small time steps. In the subsonic flow described by Visbal et al. [37], the source term in the energy equation (Eqn. 3) was ignored and indeed the results of Mullenix et al. [23] do not consider the source term in the energy equation. Since the energy equation is tightly coupled to the momentum and mass conservation equations in supersonic flow, the term was reintroduced into the formulation. Although this recovered the numerical stability for higher force fields, the flow actually relaminarized at the higher D_c .

One possible explanation for this peculiar observation is that although the force is highly localized and has a negligible direct energy contribution to the flow, the localized tripping process which determines the transition process is affected. Energy transfer to the flow yields a relatively colder localized (adiabatic) wall which inhibits transition. Indeed, enforcing the wall thermal condition to be at the expected adiabatic wall temperature or higher recovers the development of turbulent flow as before. Simulations demonstrating this are summarized in Table 4, building on the efforts initiated by Mullenix et al. [24]. The name of each case indicates the grid used, the force strength, and the wall thermal condition. For instance, $C_25_1.95$ represents a solution on Mesh C with $D_c = 25$, and a fixed wall temperature of $T_w = 1.95T_\infty$. Figure 8 shows the growth of the boundary layer for each case and suggests that merely increasing the strength of the force is not sufficient to sustain transition, as a simple increase in D_c leads to a stronger initial separation but later relaminarizes. The restoration of turbulence for hot walls is consistent with their known destabilizing effects. The influence of the thermal condition on the development of turbulent structures is more visible in Fig. 9. Consistent with Fig. 8, using an isothermal wall temperature which is equal to the expected turbulent adiabatic wall temperature ($T_w = 1.95T_\infty$) moves transition furthest upstream suggesting that out of the cases explored, this is the preferred wall thermal condition.

Although it is possible to encourage the tripping process by adjusting the wall thermal condition, many problems are physically consistent with adiabatic wall conditions. In an attempt to recover the adiabatic wall condition in the turbulent region, we investigate the hypothesis that the isothermal condition is only necessary in the neighborhood of the trip. To test this, the wall thermal condition is modified such that only a small portion of the wall ($x < 15$) has a constant wall temperature, after which it is switched to adiabatic, as shown schematically in Fig. 10. It should be noted that the wall temperature condition is changed within one grid point. This abrupt change is not problematic since the change is actually quite small in practice, and the relaxation from one condition to the other is quite smooth.

As before, the isothermal portion is maintained at the expected turbulent adiabatic wall temperature for the specific Mach number, in this case $T_w = 1.95T_\infty$ for the Mach 2.3 flow. The results confirm that this approach facilitates recovery of an adiabatic wall turbulent boundary layer with the larger force field. Figure 11 shows isosurfaces of Q-criterion colored by the streamwise component of velocity. Hairpin vortices similar to those displayed earlier in Fig. 4 are generated. It is also evident that transition occurs at a similar location as on the fine mesh. The similarity is even more obvious in Fig. 12 which presents a more quantitative comparison in the form of streamwise variation of the boundary layer thickness, displacement thickness, and momentum thickness obtained via a spanwise average of the mean flow.

The profiles for the three quantities agree well between the two cases, confirming that this method of recovering the adiabatic wall condition in the turbulent wall region yields the desired result, which is essentially independent of the mesh, thermal condition, and force magnitude (D_c). The boundary layer maintains approximately constant thickness until it interacts with the force field at $x = 2.6$ producing a

Figure 8: Boundary layer thickness along the plate for Mach 2.3 Cases

Figure 9: Isosurfaces of $Q=1$ colored by u velocity for Mach 2.3 Cases

Figure 10: Schematic of wall thermal condition variation

Figure 11: $Q = 0.5$ isosurfaces colored by u -velocity for Mach 2.3 cases

Figure 12: Comparison of boundary layer thickness for Grids A and C for Mach 2.3 cases

sharp localized decrease in δ_{99} , followed by a nearly constant boundary layer thickness until $x \sim 25.0$, after which the boundary layer begins to grow as the flow transitions. The new wall conditions restore the growth rate of the boundary layer thickness, making it similar to that observed on Grid A. Within the fully turbulent region ($x \geq 50$), the shape factor $H = \frac{\delta^*}{\theta}$ lies within the values ($H \sim 1.4$) shown by Smits and Dussage [31] for $3500 \leq Re_\theta \leq 3800$. The increase in δ_{99} subsequent to transition is almost linear — a similar effect is observed at the other Mach numbers simulated. This facilitates a good estimate of the initial laminar boundary layer thickness required as further explained in Section 8.1.

6. Implementation on Mach 2.9 Flowfield

As stated earlier, higher Mach number flows are more stable and as for any other tripping approach are harder to trip effectively. Additionally, the likelihood that the wave generated by the force field is strong enough to exhibit a long lasting signature must also be explored. The difficulty in sustaining turbulence at a given Reynolds number, which is necessarily low to be accessible to high-fidelity simulations, is a physics-based limitation that is a natural part of the current tripping based approach. As such, the method automatically provides an assessment of when the chosen flow parameters are not physical: specifically, a given Reynolds number may be suitable at a lower Mach number, but not at a higher one, if a fully turbulent incoming flow profile is desired. Methods that are initiated with a turbulence profile may hide this effect for the short streamwise distances considered. With this in mind, we employ the above parametric studies to obtain a low Reynolds number Mach 2.9 flow.

Several runs were performed, most focused on the region surrounding the trip for efficiency reasons. Section 7 provides justification for this technique. The conditions applied to each case are documented in Table 5. Because of the large number of cases and the range of parameters involved in this study, a separate numerical naming system is used instead of the one described in Section 5. Each case is discussed systematically below. Collectively, these cases explore the sensitivity of the method to force strength, wall temperature, grid refinement and Reynolds number.

Table 5: Case Parameters for Mach 2.9

Case	Rationale	Grid	T_w	D_c	Re	Transition
1	Baseline	C1	Constant until $x=15$, then adiabatic	25	17520.21	No
2	Downstream influence	C2	Constant until $x=15$, then adiabatic	25	17520.21	No
3	Grid refinement (in y)	C3	Constant until $x=15$, then adiabatic	25	17520.21	No
3A	Compact Differencing in x	C3	Constant until $x=15$, then adiabatic	25	17520.21	No
3B	Compact Differencing in x and z	C3	Constant until $x=15$, then adiabatic	25	17520.21	No
3C	Grid refinement in all 3 directions	C5	Constant until $x=15$, then adiabatic	25	17520.21	No
3D	Even finer grid in y direction	C4	Constant until $x=15$, then adiabatic	25	17520.21	No
4	Lower Re	C2	Constant until $x=15$, then adiabatic	25	10000	No
5	Higher Re	C3	Constant until $x=15$, then adiabatic	25	35000	Yes
6	Moderately higher Re	C3	Constant until $x=15$, then adiabatic	25	22000	No
7	Stronger force	C3	Constant until $x=15$, then adiabatic	40	17520.21	No
8	Hot wall	C2	Constant $T_w=1.5T_{aw}$	25	17520.21	No

Figure 13: **Comparison of Mach 2.9 baseline results on full domain vs short domain**

The results are first analyzed in a high-level visual manner by examining the isolevels of vorticity magnitude colored by the x component of vorticity followed by a more quantitative assessment. Only the flow in the neighborhood of the trip is shown (see for example Fig. 13a discussed below). As will be evident, separation occurs in all cases at the location of the trip but in most cases, the parameters are not sufficient to generate transition, and the flow relaminarizes — hence the absence of coherent structures. This behavior acts as a first filter in determining whether or not the simulation of the full domain will be successful. A more conclusive indicator is described in Section 7 and it is used to make the selection of suitable trip parameters more effective.

The baseline simulation (CASE 1) uses the same parameters for Mach 2.9 as were used earlier for the Mach 2.3 case on the full mesh. As shown in Fig. 13a, the absence of the expected streaks indicates that the flow does not transition *i.e.*, the parameters sufficient to cause transition in a Mach 2.3 flow are inadequate for the higher Mach number flow. The same conclusion can be derived from CASE 2, shown in Fig. 13b which is similar to CASE 1, but only considers the front portion of the domain in the streamwise direction using Grid C2 in Table 5. Section 7 further describes the effect of force parameters on likelihood of transition.

A natural question arises concerning grid resolution with the possibility that the Mach 2.9 case requires a finer mesh. To address this, CASE 3 was set up to be otherwise identical to the baseline case, but on Grid C3, which was more refined in the wall normal direction (see Table 3). The result is shown in Fig. 14a. Although there is increased spanwise variation near the trip, the boundary layer does not break down downstream. The effect of numerical schemes was also explored. For CASE 3A, the sixth-order compact differencing scheme [36], used in all simulations for the viscous terms, is also employed for inviscid fluxes in the x direction (replacing the default more dissipative but robust optimized WENO scheme), while CASE 3B considers the compact scheme in both x and z directions. Figures 14b and 14c indicate that neither of these improves the result. The mesh resolution was further increased for CASE 3C, which employs Grid C5, in which the mesh is isotropically made finer in all three directions. That grid was then further refined in the wall normal direction, (CASE 3D), using 251 points to resolve the wall normal direction on Grid C4 as opposed to Grid C3, which uses 167 points. Judging from Figs. 14d and 14e, neither of these modifications was adequate to yield the desired turbulent boundary layer. Clearly, mesh resolution is not the issue as further demonstrated below.

The effect of Reynolds number variation was investigated next. Higher Reynolds numbers are more likely to transition and sustain turbulence but require much higher mesh resolution. Both higher and lower Reynolds numbers than the nominal 17520.21 were investigated. CASE 4 was set up using a smaller Reynolds number, $Re_\delta = 10000$ on the same grid as CASE 3. Because grid resolution scales to a power of the Reynolds number, a smaller Reynolds number would require a less refined grid. As such, if transition was absent in the baseline because the grid was not sufficiently refined to sustain the turbulence for this Reynolds number, using a lower Reynolds number may be expected to yield better results. Fig. 15a however indicates that this is not the case. CASE 5 also uses Grid C3 but a higher Reynolds number ($Re_\delta = 35000$) than the baseline (17520.21) is employed. As shown in Fig. 15b, in this case the boundary layer transitions to turbulence (detailed statistics discussed below). This suggests not only that the grid resolution is sufficient to

Figure 14: **Effects of mesh refinement and higher order numerical methods-Mach 2.9**

Figure 15: **Effects of Reynolds number-Mach 2.9**

Figure 16: **Effects of DBD strength and wall thermal condition-Mach 2.9**

sustain the turbulence generated, but also that this Reynolds number is high enough to provide a physically consistent boundary layer profile. Since this case yielded a turbulent boundary layer, the domain is extended to include the entire streamwise domain (Grid C3-full). The flow field obtained is examined later in this section and compared to the other Mach number flows in Section 8.

For exploratory purposes, CASE 6 considers a Reynolds number of 22,000 to investigate whether transition and turbulence could be sustained at this level. The value 22,000 is chosen to be directly proportional to the Mach number ratio *i.e.*, $2.9/2.3 \times 17520.21$. As shown in Fig. 15c, although the boundary layer initially appears to transition with the structures being formed, these die out before $x = 7$.

The effect of using a stronger force, represented by a higher D_c value, was investigated by running the baseline case ($Re = 17,500$) with a D_c value of 40 instead of 25 (CASE 6). The hypothesis is that the interaction of a laminar boundary layer with a stronger force may be expected to yield a more unstable shear layer that would be more likely to trip the boundary layer. However, Fig. 16a suggests that merely increasing the strength of the force is not sufficient to sustain transition. This is further confirmation of earlier observations that increasing the strength of the force is not always sufficient to yield a turbulent boundary layer.

Since hotter walls were effective in the Mach 2.3 case, the effect of a hot wall is also investigated. The baseline case (CASE 1) assumes a constant wall temperature equal to the expected adiabatic wall temperature condition up to $x = 15$, after which the wall condition is allowed to develop as adiabatic. The effect of using a constant wall temperature where $T_w = 1.5T_{aw}$ is examined in CASE 7 to examine if a hot wall can provide the required profile. The results obtained (Fig. 16b) indicate that this change in the thermal wall boundary condition is not sufficient to support the development of a turbulent boundary layer at Mach 2.9.

The above results thus suggest that the main issue for higher Mach numbers is the Reynolds number, provided other factors such as mesh resolution, wall thermal conditions and force strength lie within reasonable bounds. Thus, the full Mach 2.9 flow is obtained by fixing the thermal wall condition at the expected adiabatic wall condition, $T_w = 2.51T_{aw}$, in the neighborhood of the trip ($x < 15$) and then switched to adiabatic as described in Section 5. It is important to reiterate that the need for a new set of tripping parameters arises from the fact that the Mach 2.9 flow is inherently more stable and thus harder to trip than Mach 2.3. The specific parameters just described, are thus consistent with the inherent physical nature of the flow. In this light, for even higher Mach number flows ($M_\infty > 4.5$) the trip parameters have to be chosen very carefully, not only because they are more likely to generate shock waves but also because the physics of the flow is inherently different. In this regime, the second mode instability becomes dominant unlike for the Mach number range shown in this work, where the first mode is more crucial. Since the second mode is excited by cooling, a reduction in wall temperature near the trip may be more effective.

A preliminary inspection of the boundary layer indicates that transition occurs for this given set of parameters on the full domain (CASE 5). Figure 17 shows isosurfaces of Q-criterion colored by the wall normal coordinate. Hairpin vortices which are characteristic of such turbulent flows are clearly evident. The heads of the vortices are in the outer part of the boundary layer where the velocity is higher relative to the

Figure 17: Isosurfaces of $Q=1$ colored by wall normal distance for Mach 2.9 flow -CASE 5

Figure 18: **Boundary layer thickness for Mach 2.9 flow -CASE 5**

leg region near the wall. Similar structures have been seen in supersonic flows [11, 9] and in subsonic flows [7, 4, 1].

Figure 18 shows the streamwise variation of span-averaged boundary layer thickness for the Mach 2.9 flow. As with the Mach 2.3 flow field, there is a steep drop in boundary layer thickness at the location of the trip as a result of the separation. The boundary layer quickly recovers and the unstable shear layer later transitions to turbulence at $x = 30$. Consistent with the fact that the Mach 2.9 case is more stable, it tends to transition further downstream than its Mach 2.3 counterpart, which starts to transition at $x = 25$ (Fig. 12).

Figure 19 shows isolevels of $U=0.99$ colored by height above the wall. These surfaces are a three dimensional representation of the boundary layer edge for the mean Mach 2.9 flow. As expected the boundary layer becomes thicker after transition, which occurs at about $x = 30$, as shown in Fig. 18. Succeeding transition, the three dimensionality of the flow is evident, as is expected of a turbulent flow. This is in contrast to the region $x < 30$ where the flow is laminar and does not display as much three dimensionality.

7. Precursor Signature

We now consider the possibility of making the method more efficient by exploring precursors that can provide a quick and straightforward assessment of the effectiveness of the force and thermal parameters. Since the method employs a very small separation region to initiate the transition process, a determining factor is the behavior of the region near the trip, where the separated shear layer is followed by development of the boundary layer succeeding reattachment. It is expected that examining several properties in the vicinity of the actuator trip will enable us to distinguish the tripping parameters that failed (*i.e.* relaminarized) from those that succeeded. We thus examine the Mach 2.3 cases to identify any peculiar features of the relaminarizing Mach 2.3 case, C_{25_aw} which distinguish it from the transitioning cases.

Figure 20 shows a comparison of selected mean properties of the Mach 2.3 cases. The skin friction coefficient is shown in Fig. 20a, the wall temperature is shown in Fig. 20b, and Fig. 20c shows the Reynolds number based on the boundary layer thickness and evaluated with wall properties, $Re_{\delta_{99}}(\rho_w, T_w) = \frac{\rho_w U_\infty \delta_{99}}{\mu(T_w)}$. As expected, all the cases have negative values of C_f at the trip location. This is due to the separation at this location, which is encountered whether or not transition subsequently occurs. While the friction

Figure 19: **Isosurfaces of $U=0.99U_\infty$ colored by wall normal distance**

coefficient does not have any distinguishing characteristics, the wall temperature and Reynolds number show some differences between the re-laminarizing case and the transitioning ones. Relative to the other cases, C_{25_aw} shows higher wall temperatures at the trip location and a slower recovery of the Reynolds number (which depends on wall temperature). While this behavior could be considered a forewarning that transition will not occur, it does not distinguish between cause and effect. Therefore, in order to investigate further, the stability of the region is examined. The generalized inflection variable is inspected as a criterion to characterize the stability of this region. This criterion is defined as [29]:

$$\varphi = \frac{\partial}{\partial y} \left(\bar{\rho} \frac{\partial \bar{u}}{\partial y} \right), \quad (14)$$

Figure 21 shows the generalized stability parameter at several stations shown schematically in Fig. 22, along the plate; near the trip extending from the upstream ($x = 1.0$), within the trip ($x = 2.6$), right after reattachment occurs ($x = 3.1$) and further downstream of the trip ($x = 5.1$). At the inlet, Fig. 21a, the behavior is qualitatively similar for all the cases. It is essential to note that at this location, the inflection parameter is positive at the wall. This is consistent with the fact the boundary layer is still relatively stable and thus lacks an inflection point. At the location of the trip, Fig. 21b, the gradients are significantly higher than at the inlet as a result of the separation. The behavior of the parameter after reattachment is the precursor to transition as it is clearly quantitatively and qualitatively different between the transitioning and the laminarizing cases. Figure 21c shows that there is a change in the sign of the inflection parameter as the boundary layer breaks down. Each of the turbulent cases has a significant negative peak downstream of separation. This is consistent with the introduction of an inflection point in the boundary layer. The unsuccessful simulation (C_{25_aw}) shows very different behavior, with values being positive near the wall (stable boundary layer) until the first zero crossing where it intersects the profiles for the turbulent cases and then becomes negative. This region of intersection occurs at the maximum height of the separation bubble ($y=0.08$) where $\varphi = 0$. The negative value of the peak attenuates with increasing distance from the trip

(a) Skin friction

(b) Wall Temperature

(c) $Re_{\delta_{99}}(\rho_w, \mu_w, U_\infty)$

Figure 20: Average properties in the region of the trip for Mach 2.3 cases

Figure 21: Generalized stability criterion for Mach 2.3 cases at various streamwise stations

Figure 22: **Stations at which generalized stability parameter is examined**

(compare Fig. 21c with Fig. 21d) suggesting that the “sweet spot” is immediately after the reattachment, $x \sim 3.1$.

These above observations show that the characteristics of a successful transition are evident in the near vicinity of the trip, suggesting a rapid method to test tripping properties for new flow parameters. Only a small region downstream of the force need be considered extending a short distance downstream of reattachment, where the Mach number is predominantly supersonic in the boundary layer ($x \sim 10$). Figure 23 shows the location of the $x = 3.1$ station relative to the size of the domain and to the location of the separation. It indicates the station at which the precursor is obtained stipulating the fraction of the domain which needs to be simulated to test trip parameters. Specifically, we consider the possibility of only simulating the region that encompasses the front part of the flow (*i.e* a short distance downstream of the reattachment point) and testing the profile obtained for the above features. Only if the profile shows the desired behavior, is a full simulation necessitated, eliminating the need for expensive trial and error.

First we confirm that transition can be predicted by modeling only the upstream portion of the flow. The Mach $C_{25} 1.95x15$ case described in Section 5, which is known to transition, is modeled on a smaller domain (Grid C2 described in Section 3). Figure 24 shows isosurfaces of vorticity colored by the streamwise component of velocity for this flowfield. The presence of coherent structures, which characterize turbulence, signifies that transition of the Mach 2.3 layer can indeed be detected in the upstream region. This is not entirely unexpected because the spatially developing nature of the boundary layer implies that no significant feed back is encountered from downstream of shear layer attachment, indicating that the events in the upstream region of the boundary layer are not strongly dependent on the streamwise extent of the domain. In like manner, failure to transition on the full domain is replicated on the shorter domain for the $C_{25} aw$ case.

These observations were verified by using the method on a Mach 2.9 boundary layer as shown in Section 6. For all of the parameters tested (CASES 1-8), an initial simulation of a fraction of the domain (Grids C1-C5) was carried out. The behavior of the generalized stability parameter at the reattachment location was investigated as shown in Figure 25. CASE 5 displays the same behavior as the transitioning Mach 2.3 cases, suggesting that the trip parameters used are sufficient to cause transition. The other cases portray the same features as the non-transitioning Mach 2.3 case indicating that the respective boundary layers re-laminarize downstream, precluding the need to simulate the entire flowfield for these cases. This method of selecting the trip parameters significantly increases the efficiency of the method as it minimizes the computational expense that would otherwise accompany the parameter study.

In an effort to further increase efficiency, the ability of 2D simulations to predict transition was investigated. A 2D simulation of each of the Mach 2.9 cases was carried out and the behavior of the generalized stability parameter at $x = 3.1$ was examined (see Fig. 26). These results are clearly different from the 3D

(a) Entire Domain

(b) Zoomed in to show separation region relative to precursor location

Figure 23: Precursor location relative to streamwise extent of domain

Figure 24: Isosurfaces of vorticity colored by u velocity for the C_25_1.95x15 case on a short domain.

Figure 25: **Generalized stability criterion for Mach 2.9 cases**

simulation. However, it is worth noting that the magnitude of the negative peaks in the 3D simulations are recovered in 2D. More importantly, the transitioning CASE 5 still shows markedly different characteristics than the other cases indicating that 2D simulations can indeed be used to predict whether or not a given set of tripping parameters will be sufficient to cause transition at a specified Mach number. The qualitative differences between the 2D and 3D flow fields are due to 3D effects and unsteadiness which are present in the latter. These attributes are evident in Fig. 27 which shows contours of the instantaneous streamwise velocity at $x = 3.1$ in the boundary layer ($y \leq 0.5$). Starting after statistical stationarity has been attained, the snapshots are taken at intervals of 25 characteristic times. It is evident from each of the plots that 3D effects are present in the simulation, unlike the 2D case where $d/dz = 0$. Additionally, each instance in time appears different from the next indicating that the 3D flow is unsteady and as such temporal derivatives are important.

In spite of the inherent physical differences between the 2D and the 3D flows, the authors conclude that the behavior of the generalized stability criterion can be used to distinguish between transitioning and re-laminarizing flows even when using a 2D simulation or very low resolution in the spanwise direction. For this work, the 3D simulations for precursor analysis were carried out only as a proof of concept and could be left out in future practical applications by other researchers.

8. Comparison of Mach 1.7, 2.3 and 2.9 boundary layer profiles

In addition to the Mach 2.3 and 2.9 flows, a Mach 1.7 flow field was also obtained for completeness. The parameters used for the Mach 2.3 flow are sufficient for the Mach 1.7 flow. Namely, Grid C is used for the simulation with $D_c = 25$. The wall temperature was kept constant at the expected turbulent adiabatic wall condition for a Mach 1.7 flow ($T_w = 1.52T_\infty$) in the neighborhood of the trip ($x < 15$), after which the wall condition was switched to adiabatic.

The results obtained for the three Mach numbers are now compared to highlight the performance of the method at different Mach numbers. We examine boundary layer growth for each of the Mach numbers

Figure 26: **Generalized stability criterion for Mach 2.9 two dimensional cases**

Table 6: Boundary Layer Tripping Parameters

M_∞	Grid	D_c	T_w	x_{aw}	Re_δ
1.7	C	25	1.52	15	17520.21
2.3	C	25	1.95	15	17520.21
2.9	C3-FULL	25	2.51	15	35000

and evaluate the velocity profiles and Reynolds stresses in the boundary layer in an effort to characterize turbulence. Finally, the energy spectrum of the flow field is used to confirm that there are no signs of the tripping mechanism -a clean turbulent boundary layer has been obtained. We also examine the mean and statistical behavior of each of the flow fields to ensure that it is in line with theory. Table 6 recapitulates the tripping parameters used for each flowfield, where x_{aw} refers to the location at which the wall thermal condition switches from isothermal to adiabatic.

8.1. Boundary Layer Thickness

The boundary layer thickness is plotted for each case in Fig. 28. As noted earlier, a sharp localized decrease in δ_{99} is evident near the force field, but is then nearly constant boundary layer thickness until the flow transitions after which it grows almost linearly. Transition occurs first for the Mach 1.7 flow at $x = 20$ and then for the Mach 2.3 flow at $x = 25$. Transition occurs further downstream ($x = 30$) for the more stable Mach 2.9 flow, despite the higher Reynolds number used.

Since the inlet boundary condition is a laminar compressible Blasius profile, its $\delta_{99,0}$ governs the thickness of the eventual turbulent profile that will be obtained. Because the growth is approximately linear after transition, Figure 28, this facilitates the choice of incoming thickness in a straightforward fashion for any desired thickness in the interaction region. Additionally, the slope of the boundary layer thickness growth (the rate at which the boundary layer thickness increases with streamwise distance) appears to increase with Mach number. This relationship can be exploited by simple interpolation to determine the slopes for other

(a) $t=0$; First iteration after statistical stationarity is reached

(b) $t=25$

(c) $t=50$

(d) $t=75$

(e) $t=100$

Figure 28: **Comparison of boundary layer thickness at Mach 1.7, 2.3, and 2.9 at Reynolds numbers of 17,500, 17,500 and 35,000 respectively**

Mach numbers of interest. Using the obtained slope and the desired turbulent boundary layer thickness, $\delta_{99,0}$ can be determined in a straightforward fashion.

Figures 29a and 29b show the streamwise velocity profile at various locations for each of the Mach numbers considered. The sharp growth in the boundary layer thickness is reflective of transition. At the inlet, $x = 0.1$, the boundary layer shows distinct laminar behavior for all three cases. The effect of the trip is clearly visible at $x = 2.6$, presenting itself as an inflection point in the velocity profile. As shown in Fig 28, the Mach 1.7 profile is already in the middle of the transition process by $x = 25$ as reflected in the difference between its velocity profile and those for the other two flows which at this location are only slightly departed from a strictly laminar profile. The downstream profiles show that in the region where the shock will impinge, the flow is fully developed and turbulent. The profiles are now independent of the streamwise location.

8.2. Turbulence Surveys

The flow fields obtained are examined closely to confirm that they portray the expected behavior of flat plate turbulent boundary layers. Namely, the velocity profiles are compared against log law predictions in the inner, log, and buffer regions; the Reynolds stresses are compared against values obtained by other researchers, and the spectral content of the boundary layer is investigated.

Figure 30 shows profiles of the law of the wall for all three Mach numbers at various streamwise locations. For each Mach number, the behavior is similar at the inlet ($x = 0.11$) and is consistent with the expected behavior in the inner region. This is expected because the boundary layer is still laminar. At the location of the trip ($x = 2.6$), the values are negative in the inner region due to the reverse flow that occurs as a result of the trip. At $x = 25$, the Mach 1.7 and 2.3 flows are beginning to transition, reflected in the fact that the log law is obeyed by these two flow fields at that location. However, the Mach 2.9 flow still exhibits laminar behavior, consistent with Figure 18 that suggests that transition does not occur until later for the more stable flow. For all three Mach numbers, the profile was observed to be turbulent at $x = 62.25$ and 72.25 which is the region where a shock will eventually impinge to study STBLI.

The density scaled Reynolds stresses, normalized by the shear velocity are given by:

$$R_{ij}^* = \frac{\overline{\rho u'_i u'_j}}{\bar{\rho}_w u_\tau^2}. \quad (15)$$

Figure 29: U velocities before and after transition —Color of curve corresponds to respective axis

Figure 31 shows the normal Reynolds stresses at $x = 30, 40, 62.25$ and 72.25 . For all of the cases, R_{11}^* has the largest values followed by R_{33}^* and then R_{22}^* . At the first two stations transition has begun but the boundary layer is not fully turbulent, as such, there is a streamwise-location dependence. There is not much difference between the values obtained at $x = 62.25$ and $x = 72.25$. This is consistent with the results obtained by Xu and Martin [41], who also show small differences between stations when the Reynolds stresses are scaled by wall parameters, as is the case here.

Figure 32 shows the fast Fourier transform (FFT) of the turbulent kinetic energy (TKE) at three points in the boundary layer of each flowfield. These points are located on the centerline at $x=62.25$ at various heights above the wall in the inner, log, and buffer regions of the boundary layer respectively. In each case, the TKE is obtained from the time history of the velocity components and the Strouhal number is a non-dimensional frequency ($St = f\delta_{99,0}/U_\infty$). As expected for flat plate turbulent boundary layers, the spectra shows roll off which is consistent with theory. It is also evident from these plots that no spurious length scales are introduced by the tripping mechanism. This assures that no artificially imposed frequencies are present which would have otherwise interfered with the eventual study of the STBLI.

9. Conclusions

A simple body-force method to generate a spatially developing equilibrium boundary layer profile is further developed by exploring the effects of wall thermal conditions, grid resolution, numerical scheme, Reynolds and Mach number. Locally constant wall surface thermal conditions can be used to aid the effectiveness of the trip, while relaxing to an adiabatic condition. Precursors based on a generalized stability criterion are shown to contain information on the success or otherwise of chosen trip parameters for any given flow condition. This information can be obtained from 2D simulations increasing the effectiveness of the method. At higher Mach numbers, it is found necessary to increase the Reynolds numbers to sustain turbulence. This physically consistent effect provides insight into lower bounds on the choice of Reynolds numbers typically used for high-fidelity simulations. The flow fields obtained are examined to confirm that they portray the expected behavior of turbulent flows and to ensure that there is no discernible signature

(a) $M = 1.7$

(b) $M = 2.3$

(c) $M = 2.9$

Figure 30: Velocity profiles for Mach 1.7, 2.3, and 2.9

(a) $x = 30$

(b) $x = 40$

(c) $x = 62.25$

(d) $x = 72.25$

Figure 31: Reynolds Stresses for Mach 1.7, 2.3, and 2.9

(a) $M = 1.7$

(b) $M = 2.3$

(c) $M = 2.9$

Figure 32: **FFT of turbulent kinetic energy for Mach 1.7, 2.3, and 2.9. The dashed line represents a $-5/3$ slope**

of the tripping method used. Although transition occurs further downstream for higher Mach numbers, the equilibrium profiles, including velocities, Reynolds stresses and spectra, are consistent with theory. The method is presently being used to specify inflow properties for high-fidelity simulations of STBLIs.

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References

- [1] Adrian, R., 2007. Hairpin vortex organization in wall turbulence. *Physics of Fluids* 19, 041301(2007).
- [2] Ali, M. Y., Alvi, F. S., Venkatakrishnan, L., 2011. Studies on the control of shock wave-boundary layer interaction using steady microactuators. No. AIAA 2011-3425.
- [3] Brown, J., 2013. Hypersonic shock wave impingement on turbulent boundary layers: Computational analysis and uncertainty. *Journal of Spacecraft and Rockets* 50, 96–123.
- [4] Christensen, K., R., A., 2001. Statistical evidence of hairpin vortex packets in wall turbulence. *Journal of Fluid Mechanics* 431, 433–443.
- [5] Dolling, D., 2001. Fifty years of shock-wave/boundary-layer interaction research: What next? *AIAA Journal* 39, 1517–1531.
- [6] Dolling, D. S., Or, C. T., 1985. Unsteadiness of the shock wave structure in attached and separated compression ramp flows. *Experiments in Fluids* 3, 24–32.
- [7] Dussage, J., Smith, R., Smits, A., Fernholz, H., Finley, P., Spina, E., 1996. Turbulent boundary layers in subsonic and supersonic flow. Tr-ag-335, AGARD.
- [8] Dussage, J.-P., P., D., Debieve, J., 2006. Unsteadiness in shock wave boundary layer interactions with separation. *Aerospace Science and Technology* 10 (2), 85–91.
- [9] Elsinga, G. E., Adrian, R. J., Oudheusden, B. W. V., Scarano, F., 2010. Three-dimensional vortex organization in a high-reynolds-number supersonic turbulent boundary layer. *Journal of Fluid Mechanics* 644, 35–60.
- [10] Gaitonde, D., 2013. Progress in shock wave/boundary layer interactions. No. AIAA 2013-2607.
- [11] Ganapathisubramani, B., Clemens, N. T., Dolling, D. S., 2006. Large-scale motions in a supersonic turbulent boundary layer. *Journal of Fluid Mechanics* 556, 271–282.
- [12] Gnoffo, P., Berry, A., Van Norman, J. W., 2013. Uncertainty assessments of hypersonic shock wave-turbulent boundary-layer interactions at compression corners. *Journal of Spacecraft and Rockets* 50, 69–95.
- [13] Gonzalez, J. C., Dolling, D. S., 1993. Correlation of interaction sweepback effects on the dynamics of shock-induced turbulent separation. No. AIAA 93-0776.
- [14] Gramann, R. A., Dolling, D. S., 1990. Detection of turbulent boundary-layer separation using fluctuating wall pressure signals. *AIAA Journal* 28, 1052–1056.
- [15] Holden, M., Moselle, J., Sweet, S., Martin, S., 1996. A database of aerothermal measurement in hypersonic flow for cfd validation. No. AIAA 96-4597.
- [16] Klein, M., Sadiki, A., Janicka, J., 2003. A digital filter based generation of inflow data for spatially developing direct numerical or large eddy simulations. *Journal of Computational Physics* 186, 652–665.
- [17] Lee, S., Loth, E., 2012. Impact of ramped vanes on normal shock boundary layer interaction. *AIAA Journal* 50, 2069–2079.
- [18] Lund, T. S., Wu, X., Squires, K. D., 1998. Generation of turbulent inflow data for spatially-developing boundary layer simulations. *Journal of Computational Physics* 140 (2), 233–258.
- [19] Masad, J. A., Abid, R., 1995. On transition in supersonic and hypersonic boundary layers. *International Journal of Engineering Sciences* 33, 1893–1919.
- [20] Morgan, B., Larsson, J., Kawai, S., Lele, S. K., 2011. Improving low-frequency characteristics of recycling/rescaling inflow turbulence generation. *AIAA Journal* 49 (3), 582–597.
- [21] Muck, K. C., Dussauge, J. P., Bogdonoff, S. M., 1985. Structure of the wall pressure fluctuations in a shock-induced separated turbulent flow. No. AIAA 85-0179.
- [22] Mullenix, N., Gaitonde, D., 2011. A bandwidth and order optimized weno interpolation scheme for compressible turbulent flows. No. AIAA 2011-366.
- [23] Mullenix, N., Gaitonde, D. V., Visbal, M. R., 2011. A plasma -actuator-based method to generate a supersonic turbulent boundary layer inflow condition for numerical simulations. No. AIAA 2011-3556.
- [24] Mullenix, N., Gaitonde, D. V., Visbal, M. R., 2012. Generation of an equilibrium turbulent boundary layer for upstream boundary condition specification. Vol. AIAA-2012-439.
- [25] Mullenix, N., Gaitonde, D. V., Visbal, M. R., 2013. Spatially developing supersonic turbulent boundary layer with a body-force-based method. *AIAA Journal* 51, 1805–1819.
- [26] Pope, S. B., 2011. *Turbulent Flows*.
- [27] Pronk, F. T., Hulshoff, S. J., 2012. A comparison of inflow generation method for large-eddy simulation. *World Academy of Science, Engineering and Technology* 68, 1575–1580.
- [28] Rai, M. M., Gatski, T. B., Erlebacher, G., 1995. Direct simulation of spatially evolving compressible turbulent boundary layers. No. AIAA-95-0583.
- [29] Reed, H., Saric, W., Arnal, D., 1996. Linear stability theory applied to boundary layers. *Annual Review of Fluid Mechanics* 28, 389–428.
- [30] Shyy, W., Jayaraman, B., Andersson, A., 2002. Modeling of glow-discharge induced fluid dynamics. *Journal of Applied Physics* 92 (11), 6434.
- [31] Smits, A. J., Dussage, J.-P., 2006. *Turbulent Shear Layers in Supersonic Flow*, 2nd Edition. Springer Science + Business Media, New York, NY.
- [32] Souverein, L. J., Dupont, P., Debieve, J.-F., Dussage, J.-P., van Oudheusden, J.-P., Scarano, F., 2010. Effect of interaction strength on unsteadiness in turbulent shock-wave-induced separations. *AIAA Journal* 12, 1480–1493.
- [33] Touber, E., Sandham, N. D., 2008. Oblique shock impinging on a turbulent boundary layer: Low-frequency mechanisms. No. AIAA 2008-4170.

- [34] Toubert, E., Sandham, N. D., 2009. Comparisons of three large-eddy simulations of shock-induced turbulent separation bubbles. *Shock Waves* 19, 469–478.
- [35] Visbal, M. R., 2010. Strategies for control of transitional and turbulent flows using plasma-based actuators. *International Journal of CFD* 24 (7), 237–258.
- [36] Visbal, M. R., Gaitonde, D., 2002. On the use of high-order finite-difference schemes on curvilinear and deforming meshes. *Journal of Computational Physics* 181, 155–185.
- [37] Visbal, M. R., Gaitonde, D. V., Roy, S., 2006. Control of transitional and turbulent flows using plasma-body actuators. No. AIAA-2006-3230.
- [38] Visbal, M. R., Rizzetta, D. P., Matthew, J., 2007. Large-eddy simulation of flow past a 3-d bump. No. AIAA 2007-917.
- [39] Webb, N., Clifford, C., Samimy, M., 2011. Preliminary results on shock wave/boundary layer interaction control using localized arc filament plasma actuators. No. AIAA 2011-3426.
- [40] Webb, N., Clifford, C., Samimy, M., 2013. An investigation of the control mechanism of plasma actuators in a shock wave-boundary layer interaction. No. AIAA 2013-402.
- [41] Xu, S., Martin, P., 2004. Assessment of inflow boundary conditions for compressible turbulent boundary layers. *Physics of Fluids* 16 (7), 2623–2639.