A galloping quadruped model using left–right asymmetry in touchdown angles

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Abstract

Among quadrupedal gaits, the galloping gait has specific characteristics in terms of locomotor behavior. In particular, it shows a left–right asymmetry in gait parameters such as touchdown angle and the relative phase of limb movements. In addition, asymmetric gait parameters show a characteristic dependence on locomotion speed. There are two types of galloping gaits in quadruped animals: the transverse gallop, often observed in horses; and the rotary gallop, often observed in dogs and cheetahs. These two gaits have different footfall sequences. Although these specific characteristics in quadrupedal galloping gaits have been observed and described in detail, the underlying mechanisms remain unclear. In this paper, we use a simple physical model with a rigid body and four massless springs and incorporate the left–right asymmetry of touchdown angles. Our simulation results show that our model produces stable galloping gaits for certain combinations of model parameters and explains these specific characteristics observed in the quadrupedal galloping gait. The results are then evaluated in comparison with the measured data of quadruped animals and the gait mechanisms are clarified from the viewpoint of dynamics, such as the roles of the left–right touchdown angle difference in the generation of galloping gaits and energy transfer during one gait cycle to produce two different galloping

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gaits.

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1. Introduction

Quadruped animals use various gaits depending on the locomotion speed. They use a walking gait in the lowest range of locomotion speed, and this changes to a trotting gait as the locomotion speed increases. In the highest range of locomotion speed, they use a galloping gait. These gaits are characterized by footfall sequence [32]. During gaits used at slow speeds, such as a walking gait, at least one limb is in contact with the ground, that is, in the stance phase. In contrast, gaits used at higher speeds, such as a galloping gait, have a flight phase during which all four limbs are in the air, that is, in the swing phase. These gaits have been investigated from mechanical, energetic, kinematic, and kinetic viewpoints to clarify the underlying mechanism for the use of such different gaits depending on the locomotion speed [1, 13, 22, 23, 24, 31].

Among these quadrupedal gaits, the galloping gait has characteristic properties. Differing from the walking and trotting gaits, the galloping gait is asymmetric [1, 22, 23]. More specifically, the relative phase of the movements between the left and right limbs is away from the antiphase, unlike the walking and trotting gaits, as shown in Fig. 1A. In addition, as the locomotion speed increases, the relative phase decreases and approaches the in-phase as in a bounding gait, which is not generally used by large, cursorial quadrupeds [26]. There are two types of galloping gaits in quadruped animals, the transverse gallop and the rotary gallop, and the two gaits have different footfall sequences (Fig. 1B) [22]. The transverse gallop is the preferred gait of horses, and the foot contacts take place in the order of a hindlimb, the contralateral hindlimb, the ipsilateral forelimb, and the contralateral forelimb. The rotary gallop is the preferred gait of dogs and cheetahs, and the foot contacts occur in the sequence of a hindlimb, the contralateral hindlimb, the contralateral forelimb, and the ipsilateral forelimb.

Both gallops have a flight phase after the liftoff of the forelimbs. In contrast, the fast rotary gallop of dogs and cheetahs has another flight phase after the liftoff of the hindlimbs, unlike the transverse gallop of horses [5, 23] (some species show a rotary gallop with just one flight phase at low speeds [25]). Although these specific characteristics in quadrupedal galloping gaits and the dependence on the locomotion speed and species have been observed and described in detail [1, 5, 22, 23], the underlying dynamic mechanisms remain unclear.

Locomotion in humans and animals involves moving the center of mass (COM) of the whole body using the limbs. The essential contribution of a limb in locomotion dynamics can be represented by a spring. To explain the locomotion mechanisms from a dynamic viewpoint, spring-loaded inverted pendulum models have been used [6, 7, 8, 9, 11, 16, 29, 30, 36]. In particular, for human running, the dependence of stability on touchdown angles has been clarified [17, 19, 38]. A simple model having mass and two springs has been used to explain the characteristic difference between human walking and running, which appears in the vertical ground reaction forces: a double-peaked shape in human walking, and a single-peaked shape in human running [18]. For quadrupedal locomotion, a rigid body with two springy legs has shown the stability characteristics of a bounding gait [10, 35]. The difference in the energy levels between the trotting, bounding, and galloping gaits also has been examined [33]. Although simple physical models with leg springs have been used for the quadrupedal galloping gait [21, 26, 28, 33, 40], they do not explain the above-mentioned specific characteristics. In this paper, we use a simple physical model with a rigid body and four massless springs. The simulation results show that our model produces stable galloping gaits for certain combinations of model parameters, and explains these specific characteristics in the quadrupedal galloping gait. The results are then evaluated in comparison with quadrupedal animals and the gait mechanisms are discussed from the viewpoint of dynamics.

55 2. Materials and Methods

56 2.1. Physical model

57 In this paper we use a physical model, which consists of a rigid body and
 58 four massless springs in two dimensions (Fig. 2), as used in [33]. x and y are,
 59 respectively, the horizontal and vertical positions of the COM of the body, and
 60 θ is the pitch angle. m and I are, respectively, the mass and moment of inertia
 61 around the COM. l is the distance between the COM and the root of the spring.
 62 g is the gravitational acceleration. $+x$ is the locomotion direction. The front
 63 two springs and the rear two springs represent the forelimbs and hindlimbs,
 64 respectively. The spring constant is k . In the forelimbs, the anterior limb during
 65 the swing phase is the leading forelimb (LF) and the posterior limb is the trailing
 66 forelimb (TF). Similarly, the anterior and posterior hindlimbs during the swing
 67 phase are the leading hindlimb (LH) and trailing hindlimb (TH), respectively.

68 During the swing phase, the spring length remains the neutral length l_0 and
 69 the angle relative to the vertical line keeps the specific value γ_i^{TD} ($i = \text{LF, TF,}$
 70 LH, TH), which corresponds to the touchdown angle ($\gamma_{\text{LF}}^{\text{TD}} \geq \gamma_{\text{TF}}^{\text{TD}}$, $\gamma_{\text{LH}}^{\text{TD}} \geq \gamma_{\text{TH}}^{\text{TD}}$).
 71 We assumed $\gamma_{\text{LF}}^{\text{TD}} + \gamma_{\text{TF}}^{\text{TD}} \geq 0$ and $\gamma_{\text{LH}}^{\text{TD}} + \gamma_{\text{TH}}^{\text{TD}} \geq 0$ so that the trailing limbs
 72 contact the ground earlier than the leading limbs, as observed in quadruped
 73 animals. When a spring tip reaches the ground, it is constrained on the ground
 74 and behaves as a frictionless pin joint. When the spring length returns to the
 75 neutral length after the compression, the tip leaves the ground. Because the
 76 touchdown and liftoff occur at the neutral length, this physical system is energy
 77 conservative.

78 2.2. Governing equations

79 In our model, the four limbs have no influence on body dynamics during the
 80 swing phase. In contrast, during the stance phase, they work as springs and
 81 influence body dynamics through their compression. The motion of our model
 82 is governed by the equations of motion of x , y , and θ , which are given by

$$m\ddot{x} = \sum_{i=\text{LF,TF,LH,TH}} -f_i \sin \gamma_i$$

$$\begin{aligned}
m\ddot{y} &= \sum_{i=\text{LF,TF,LH,TH}} f_i \cos \gamma_i - mg \\
I\ddot{\theta} &= \sum_{i=\text{LF,TF}} f_i l \cos(\gamma_i - \theta) - \sum_{i=\text{LH,TH}} f_i l \cos(\gamma_i - \theta)
\end{aligned} \tag{1}$$

where

$$f_i = \begin{cases} -k(l_i - l_0) & \text{stance phase} \\ 0 & \text{swing phase} \end{cases}$$

l_i and γ_i ($i = \text{LF, TF, LH, TH}$) are the spring length and the angle relative to the vertical line, respectively, which are determined by x , y , θ , l , and the touchdown position ($l_i = l_0$ and $\gamma_i = \gamma_i^{\text{TD}}$ during the swing phase).

By using m , l_0 , and $\sqrt{l_0/g}$ as the characteristic mass, length, and time scale of our model, the state variables and parameters become dimensionless as $x^* = x/l_0$, $y^* = y/l_0$, $(\dot{})^* = (\dot{})\sqrt{l_0/g}$, $I^* = I/(ml_0^2)$, $k^* = kl_0/(mg)$, $l^* = l/l_0$, and $l_i^* = l_i/l_0$, which yields the dimensionless equations of (1) by

$$\begin{aligned}
\ddot{x}^* &= \sum_{i=\text{LF,TF,LH,TH}} -f_i^* \sin \gamma_i \\
\ddot{y}^* &= \sum_{i=\text{LF,TF,LH,TH}} f_i^* \cos \gamma_i - 1 \\
I^* \ddot{\theta}^* &= \sum_{i=\text{LF,TF}} f_i^* l^* \cos(\gamma_i - \theta) - \sum_{i=\text{LH,TH}} f_i^* l^* \cos(\gamma_i - \theta)
\end{aligned} \tag{2}$$

where

$$f_i^* = \begin{cases} -k^*(l_i^* - 1) & \text{stance phase} \\ 0 & \text{swing phase} \end{cases}$$

2.3. Generation of galloping gait using the left-right asymmetry of touchdown angles

An important function of limbs in locomotion dynamics is to produce translational and rotational forces for the whole body through interaction with the ground. Such forces are generated through the movements of the limb tip relative to the limb root (length and orientation). During a steady gait, the limb tip produces a periodic trajectory relative to the body. When the relative phase

99 between the left and right limb movements is antiphase, as in walking and trot-
 100 ting gaits, the left and right limbs move alternately relative to the body and
 101 the periodic trajectories are almost identical between the left and right limbs.
 102 When the relative phase is in-phase, as in a bounding gait, the left and right
 103 limbs move simultaneously and the periodic trajectories are also almost identi-
 104 cal between the left and right limbs. In contrast, during a galloping gait, the
 105 relative phase is not in-phase or antiphase as shown in Fig. 1A and the periodic
 106 trajectories differ between the left and right limbs, which leads to the difference
 107 in touchdown positions relative to the body as shown in Fig. 3 [21]. That is,
 108 the touchdown angles are different between the left and right limbs.

109 In our model, when we use identical touchdown angles between the left
 110 and right limbs ($\gamma_{LF}^{TD} = \gamma_{TF}^{TD}$, $\gamma_{LH}^{TD} = \gamma_{TH}^{TD}$), foot contact occurs simultaneously
 111 between the left and right limbs, which produces a bounding gait and a zero
 112 relative phase between the left and right limbs. In contrast, different touchdown
 113 angles ($\gamma_{LF}^{TD} \neq \gamma_{TF}^{TD}$, $\gamma_{LH}^{TD} \neq \gamma_{TH}^{TD}$) induce different foot contact timings and
 114 positions between the left and right limbs and yield a galloping gait. In addition,
 115 an increase in the touchdown angle difference indicates an increase in the relative
 116 phase between the left and right limbs.

117 2.4. Search of periodic solutions and stability analysis

118 We define the Poincaré section when $\dot{y}^* = 0$ and when all four limbs are in the
 119 swing phase (the model is in the flight phase). We find a periodic solution, which
 120 corresponds to a gait, by searching for a fixed point in the Poincaré section,
 121 where we neglect x^* because the horizontal position monotonically increases
 122 during locomotion. In addition, we assume that all four limbs have to experience
 123 the stance phase at least once before the intersection with the Poincaré section so
 124 that the solution explains a gait. When we set $z^* = [y^* \theta \dot{x}^* \dot{\theta}^*]^T$, the Poincaré
 125 map P is written as

$$z_{n+1}^* = P(z_n^*, u^*) \quad (3)$$

where z_n^* is the value of z^* at the n th intersection with the Poincaré section and u^* is the parameter set. When we denote $\hat{z}^*$ for the fixed point on the Poincaré section, we obtain $\hat{z}^* = P(\hat{z}^*, u^*)$.

In this paper, because the left-right touchdown angle differences are not so different between the forelimbs and hindlimbs during galloping gaits [21], we find gaits in which the differences in the left and right touchdown angles are identical between the forelimbs and hindlimbs ($\gamma_{\text{LF}}^{\text{TD}} - \gamma_{\text{TF}}^{\text{TD}} = \gamma_{\text{LH}}^{\text{TD}} - \gamma_{\text{TH}}^{\text{TD}}$). In this condition, we can write the touchdown angles using the difference δ (≥ 0) by $\gamma_{\text{LF}}^{\text{TD}} = \bar{\gamma}_{\text{F}}^{\text{TD}} + \delta/2$, $\gamma_{\text{TF}}^{\text{TD}} = \bar{\gamma}_{\text{F}}^{\text{TD}} - \delta/2$, $\gamma_{\text{LH}}^{\text{TD}} = \bar{\gamma}_{\text{H}}^{\text{TD}} + \delta/2$, and $\gamma_{\text{TH}}^{\text{TD}} = \bar{\gamma}_{\text{H}}^{\text{TD}} - \delta/2$ ($\bar{\gamma}_{\text{F}}^{\text{TD}}, \bar{\gamma}_{\text{H}}^{\text{TD}} \geq 0$), and the parameter set of this model is given by $u^* = [I^* k^* l^* \bar{\gamma}_{\text{F}}^{\text{TD}} \bar{\gamma}_{\text{H}}^{\text{TD}} \delta]^T$. We used the following four constraints: $\hat{y}^* = y_0^*$, $\hat{\theta} = \theta_0$, $\hat{\theta}^* = \dot{\theta}_0^*$, and $Fr = Fr_0$, where Fr is the Froude number defined by

$$Fr = \left\{ \frac{1}{\tau^*} \int_0^{\tau^*} \dot{x}^* dt^* \right\}^2 \quad (4)$$

and τ^* is a dimensionless one gait cycle [1]. We determined $I^* = 0.1$ and $l^* = 0.6$ based on the physical parameters of such quadruped animals as horses, dogs, cheetahs, and goats [15, 20, 25, 39], and used $y_0^* = 0.94$ and $\theta_0 = 0.018$. To clarify the dynamic characteristics of galloping gait of our model, we used various values for k^* , $\dot{\theta}_0^*$, and Fr_0 and searched $\hat{x}^*$, $\bar{\gamma}_{\text{F}}^{\text{TD}}$, $\bar{\gamma}_{\text{H}}^{\text{TD}}$, and δ that satisfy $\hat{z}^* = P(\hat{z}^*, u^*)$, where we used the `fsolve` function of MATLAB.

When we found periodic gaits, we investigated the local stability from the eigenvalues of the linearized Poincaré map around the fixed points. Because our model is energy conservative, the gait is asymptotically stable when all the eigenvalues except for one eigenvalue of 1 are inside the unit circle (these magnitudes are less than 1).

3. Results

We obtained various periodic gaits depending on Fr_0 , k^* , and $\dot{\theta}_0^*$. Figures 4A and B show the angle difference δ for Fr_0 and k^* for the obtained gaits with $\dot{\theta}_0^* = 0.92$ and 1.65, respectively. The gray regions show unstable

153 gaits and the white regions show stable gaits. The obtained gaits have different
 154 sequences of the stance status (Sequences A–E), depending on the parameters,
 155 as shown in Fig. 5. The obtained gaits in Fig. 4**A** have Sequences A–E, whereas
 156 those in Fig. 4**B** have only Sequence E. Sequences B and E correspond to the
 157 transverse and rotary gallops, respectively, in Fig. 1**B**. In both Figs. 4**A** and
 158 **B**, as the locomotion speed increases, δ decreases and approaches 0 (but, did
 159 not reach 0). At slow speeds (small Froude number), the obtained gaits are
 160 unstable, and when the locomotion speed increases, the gaits become stable.
 161 The dimensionless spring stiffness was estimated as 7 [14] or 12 [27] for horses,
 162 11 [14] for dogs, and 16 [14] for goats. The left–right touchdown angle difference
 163 during galloping gaits was observed around 5 to 15° [21]. The Froude number
 164 of various quadruped animals is shown in Fig. 1**A** and about 3 during the trans-
 165 verse gallop of goats [14], and is seen to be greater than 50 during the rotary
 166 galloping of cheetahs [5]. Our simulation results are comparable with biological
 167 data.

168 To evaluate the biological relevance of the obtained gaits, we compared the
 169 vertical COM movement of our simulation results with data measured during
 170 quadrupedal galloping gaits. In Fig. 6**A**, we used $\dot{\theta}_0^* = 0.92$, $Fr_0 = 14.4$,
 171 and $k^* = 5.1$ (open dot in Fig. 4**A**) and compared the simulation result with
 172 data measured during a transverse gallop by a horse (the Froude number is
 173 around 18) [34]. The simulation result shows a single sinusoidal curve, and the
 174 magnitude is similar to that of the measured data. The dimensionless spring
 175 stiffness of a horse transverse gallop was estimated by 12 [27], and the simulation
 176 result is fairly consistent with the measured data. In Fig. 6**B**, we used $\dot{\theta}_0^* = 0.92$,
 177 $Fr_0 = 42.1$, and $k^* = 5.5$ (black dot in Fig. 4**A**) and $\dot{\theta}_0^* = 1.65$, $Fr_0 = 25.7$, and
 178 $k^* = 7.0$ (black dot in Fig. 4**B**) and compared these simulation results with data
 179 measured during a rotary gallop by a dog (the Froude number is about 20) [4].
 180 The dimensionless spring stiffness of dogs was estimated by 11 [14]. When
 181 $\dot{\theta}_0^* = 0.92$, the simulation result is very different in shape and magnitude from
 182 the measured data. In contrast, for $\dot{\theta}_0^* = 1.65$, although the magnitude is slightly
 183 lower than the measured data, the simulation result shows a double sinusoidal

184 curve and has a similar shape to the measured data. This simulation result is
 185 consistent with the measured data. See supplementary movies in Appendix A
 186 for simulated locomotor behaviors.

187 To clarify the dynamical difference between the obtained transverse and
 188 rotary gallops, we investigated the energy transfer during one gait cycle. Fig-
 189 ures 7A and B show the ratio of four components of the conservative mechanical
 190 energy (the gravitational energy (zero at the bottom of the vertical COM move-
 191 ment), forelimb and hindlimb spring energies, and kinetic energy) for $\dot{\theta}_0^* = 0.92$,
 192 $Fr_0 = 14.4$, and $k^* = 5.1$ (transverse gallop), and $\dot{\theta}_0^* = 1.65$, $Fr_0 = 25.7$, and
 193 $k^* = 7.0$ (rotary gallop), respectively. In the transverse gallop, the gravitational
 194 and kinetic energies move to the hindlimb spring energy during the first half
 195 of the hindlimb stance phase. The energy transitions into the kinetic energy
 196 during the last half of the stance phase, and then changes to the forelimb spring
 197 energy due to the touchdown of the forelimbs. Finally, it moves back to the
 198 gravitational and kinetic energies. In the rotary gallop, the gravitational and
 199 kinetic energies transition into the hindlimb spring energy during the first half
 200 of the hindlimb stance phase, similarly to the transverse gallop. However, the
 201 energy returns to the gravitational and kinetic energies during the last half of
 202 the stance phase unlike the transverse gallop. The similar energy transfer occurs
 203 in the forelimb stance phase. Although the rotary gallop has the energy transfer
 204 to the gravitational energy between the hindlimb and forelimb stance phases,
 205 the transverse gallop does not have such an energy transfer. This difference
 206 produces two types of galloping gaits; the transverse gallop with a single flight
 207 phase and the rotary gallop with two flight phases.

208 4. Discussion

209 In this paper, we produced galloping gaits using a simple model with a rigid
 210 body and four massless springs, and showed that the model can explain specific
 211 characteristics in a quadrupedal galloping gait, such as dependence of left-right
 212 asymmetry in gait parameters on the locomotion speed, from the viewpoint of

213 dynamics.

214 *4.1. Asymmetric gait parameters*

215 In quadrupedal galloping gaits, the relative phase between the left and right
216 limbs is away from the antiphase, irrespective of species, as shown in Fig. 1A [1].
217 In addition, the relative phase decreases and approaches in-phase as the loco-
218 motion speed increases; however, it does not reach complete in-phase. These
219 characteristics reflect the left–right asymmetry in touchdown angles, as shown
220 in Fig. 3 [21]. We focused on this asymmetry to produce the galloping gait of
221 our model. As the locomotion speed increased, the left–right touchdown an-
222 gle difference decreased and approached 0 (but did not reach 0), as shown in
223 Fig. 4. This means that the relative phase difference decreased and approached
224 in-phase. This trend is consistent with the observations in quadrupedal gallop-
225 ing gaits. Furthermore, the fact that there is no solution for the zero left–right
226 touchdown angle difference means that this asymmetry in touchdown angles
227 allows the model to produce periodic solutions for gaits, which may suggest an
228 important role of left–right asymmetry in touchdown angles in the locomotion
229 dynamics of galloping gaits.

230 *4.2. Transverse and rotary gallops*

231 There are basically two types of galloping gaits (transverse and rotary gal-
232 lops) in quadruped animals, and they have different footfall sequences as shown
233 in Fig. 1B. The transverse gallop observed often in horses has a single flight
234 phase after the liftoff of the forelimbs. In contrast, the fast rotary gallop ob-
235 served often in dogs and cheetahs has another flight phase after the liftoff of
236 the hindlimbs in addition to the flight phase after the liftoff of the forelimbs.
237 Our simulation results showed that our model produced various galloping gaits
238 depending on the parameters, which had different sequences of the stance condi-
239 tion (Sequences A–E), as shown in Fig. 5. As locomotion speed and spring stiff-
240 ness increased, the sequence of the stance condition changed from Sequence A
241 to E. Sequences B and E corresponded to the transverse and rotary gallops,

242 respectively (because we cannot determine left and right limbs due to the two-
 243 dimensional nature of our model, we decided these gaits from the number of
 244 flight phases). However, even when the sequences of the stance condition were
 245 identical, the locomotor behavior of the obtained gaits, such as the vertical COM
 246 movement, differed depending on the parameters, as shown in Fig. 6. Depending
 247 on the model parameters, both the sequences of stance condition and locomotor
 248 behavior of the obtained gaits were comparable to those in quadruped animals,
 249 which was evaluated by comparing our simulation results with measured data of
 250 quadruped animals. Although quadruped animals use these two different gaits
 251 depending on the locomotion speed and species, our simple model can explain
 252 these different gaits using only a few parameters.

253 The transverse gallop has a small relative phase between the forelimbs and
 254 hindlimbs, while the rotary gallop has a large relative phase, as shown in Fig. 1B.
 255 In our simulation results, different parameters produced different relative phase
 256 between the forelimbs and hindlimbs, as shown in Fig. 5, which changed the
 257 number of flight phases and induced different galloping gaits. While the relative
 258 phase between the left and right limbs depends on the left–right touchdown angle
 259 difference δ , our modeling has no constraint on the relative phase between the
 260 forelimbs and hindlimbs, which were only determined through gait dynamics
 261 with parameters. The dynamic mechanism that creates two different gaits was
 262 ascertained from the energy transfer during one gait cycle in Fig. 7. It has
 263 been suggested that horse transverse galloping and human skipping gaits have a
 264 similarity in the energy transfer between the leading and trailing limbs [30]. We
 265 also intend to investigate the mechanism by improving our model in the future.

266 4.3. *Gait stability*

267 Gait stability is an important factor in dynamic locomotion, as investigated
 268 in humans [17, 19, 38] and quadrupeds [2, 3, 10, 12, 35, 37]. Our simulation
 269 results showed that the galloping gait of our model was stable depending on the
 270 Froude number (Fig. 4). That is, our model had self-stability for a particular
 271 locomotion speed. More specifically, our model generated stable galloping gaits

only at fast locomotion speeds ($Fr_0 > 5$), which is consistent with the observation in quadrupedal galloping gaits as shown in Fig. 1A [1]. At slow speeds, the obtained galloping gaits became unstable. This limitation of gait stability due to the decrease in locomotion speed suggests a change of the gait to an alternative gait, such as a trotting gait, to improve gait stability.

4.4. Limitation of our model and future work

In this study, we used a very simplified physical model for quadrupedal galloping gaits. For example, the galloping quadruped animal was modeled by a single rigid body and four massless springs, and symmetric assumptions between the forelimbs and hindlimbs were used in the model parameters, such as the left-right touchdown angle difference δ and dimensionless spring stiffness k^* . Such simplifications resulted in quantitative differences in locomotion parameters from actual animals. However, it is clear that our model showed similar trends in the asymmetric gait parameters for the locomotion speed and in the generation of two different galloping gaits, which are characteristic for quadrupedal galloping gaits, as was confirmed by the comparisons with quadruped animals. This suggests that our simple model is capable of capturing the essential aspects needed to generate the galloping gait in quadruped animals from the viewpoint of dynamics. To further clarify the underlying mechanisms in quadrupedal galloping gaits, we intend to develop a more sophisticated and plausible model by incorporating important dynamical factors, such as muscle actuators, frictional dissipation, and neuromechanical interactions, in future studies.

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304 References

- 305 [1] Alexander, R.McN., and Jayes, A.S., 1983. *A dynamic similarity hypothesis*
306 *for the gaits of quadrupedal mammals*, J. Zool., Lond., 201:135–152.
- 307 [2] Aoi, S., Yamashita, T., and Tsuchiya, K., 2011. *Hysteresis in the gait transi-*
308 *tion of a quadruped investigated using simple body mechanical and oscillator*
309 *network models*, Phys. Rev. E, 83(6):061909.
- 310 [3] Aoi, S., Katayama, D., Fujiki, S., Tomita, N., Funato, T., Yamashita, T.,
311 Senda, K., and Tsuchiya, K., 2013. *A stability-based mechanism for hystere-*
312 *sis in the walk–trot transition in quadruped locomotion*, J. R. Soc. Interface,
313 10(81):20120908.
- 314 [4] Bertram, J.E.A. and Gutmann, A., 2009. *Motions of the running horse*
315 *and cheetah revisited: fundamental mechanics of the transverse and rotary*
316 *gallop*, J. R. Soc. Interface, 6:549–559.
- 317 [5] Biancardi, C.M. and Minetti, A.E., 2012. *Biomechanical determinants of*
318 *transverse and rotary gallop in cursorial mammals*, J. Exp. Biol., 215:4144–
319 4156.
- 320 [6] Blickhan, R., 1989. *The spring–mass model for running and hopping*, J.
321 Biomech., 22:1217–1227.
- 322 [7] Bullimore, S.R. and Burn, J.F., 2006. *Dynamically similar locomotion in*
323 *horses*, J. Exp. Biol., 209:455–465.

- [8] Bullimore, S.R. and Burn, J.F., 2006. *Consequences of forward translation of the point of force application for the mechanics of running*, J. Theor. Biol., 238:211-219.
- [9] Bullimore, S.R. and Burn, J.F., 2007. *Ability of the planar spring-mass model to predict mechanical parameters in running humans*, J. Theor. Biol., 248:686-695.
- [10] Cao, Q. and Poulakakis, I., 2013. *Quadrupedal bounding with a segmented flexible torso: passive stability and feedback control*, Bioinspir. Biomim., 8:046007.
- [11] Cavagna, G.A., Heglund, N.C., and Taylor, C.R., 1977. *Mechanical work in terrestrial locomotion: two basic mechanisms for minimizing energy expenditure*, Am. J. Physiol., 233(5):R243-R261.
- [12] Diedrich, F.J. and Warren Jr., W.H., 1998. *Why change gaits? Dynamics of the walk-run transition*, J. Exp. Psychol. Hum. Percept. Perform., 21:183-202.
- [13] Farley, C.T. and Taylor, C.R., 1991. *A mechanical trigger for the trot-gallop transition in horses*, Science, 253:306-308.
- [14] Farley, C.T., Glasheen, J., and McMahon, T.A., 1993. *Running springs: Speed and animal size*, J. Exp. Biol., 185:71-86.
- [15] Fischer, M.S., Lilje, K.E., Lauströer, J., and Andikfar, A., 2011. Dogs in motion, *Dortmund: VDH-Service GmbH*.
- [16] Full, R.J. and Koditschek, D.E., 1999. *Templates and anchors: neuromechanical hypotheses of legged locomotion on land*, J. Exp. Biol., 202:3325-3332.
- [17] Geyer, H., Seyfarth, A., and Blickhan, R., 2005. *Spring-mass running: simple approximate solution and application to gait stability*, J. Theor. Biol., 232:315-328.

- [18] Geyer H., Seyfarth, A., and Blickhan, R., 2006. *Compliant leg behaviour explains basic dynamics of walking and running*, Proc. R. Soc. B, 273:2861–2867.
- [19] Ghigliazza, R.M., Altendorfer, R., Holmes, P., and Koditschek, D., 2003. *A simply stabilized running model*, SIAM J. Appl. Dyn. Syst., 2(2):187–218.
- [20] Grossi, B. and Canals, M., 2010. *Comparison of the morphology of the limbs of juvenile and adult horses (*Equus caballus*) and their implications on the locomotor biomechanics*, J. Exp. Zool., 313A:292–300.
- [21] Herr, H.M. and McMahon, T.A., 2001. *A galloping horse model*, Int. J. Robot. Res., 20(1):26–37.
- [22] Hildebrand, M., 1977. *Analysis of asymmetrical gaits*, J. Mammal., 58(2):131–156.
- [23] Hildebrand, M., 1989. *The quadrupedal gaits of vertebrates*, BioScience, 39(11):766–775.
- [24] Hoyt, D.F. and Taylor, C.R., 1981. *Gait and the energetics of locomotion in horses*, Nature, 292:239–240.
- [25] Hudson, P.E., Corr, S.A., and Wilson, A.M., 2012. *High speed galloping in the cheetah (*Acinonyx jubatus*) and the racing greyhound (*Canis familiaris*): spatio-temporal and kinetic characteristics*, J. Exp. Biol., 215:2425–2434.
- [26] Marhefka, D.W., Orin, D.E., Schmiedeler, J.P., and Waldron, K.J., 2003. *Intelligent control of quadruped gallops*, IEEE/ASME Trans. Mechatron., 8(4):446–456.
- [27] McGuigan, M.P. and Wilson, A.M., 2003. *The effect of gait and digital flexor muscle activation on limb compliance in the forelimb of the horse *Equus caballus**, J. Exp. Biol., 206:1325–1336.

- 377 [28] McMahon, T.A., 1985. *The role of compliance in mammalian running gaits*,
378 J. Exp. Biol., 115:263–282.
- 379 [29] McMahon, T.A. and Cheng, G.C., 1990. *The mechanics of running: how*
380 *does stiffness couple with speed?*, J. Biomech., 23(Suppl. 1):65–78.
- 381 [30] Minetti, A.E., 1998. *The biomechanics of skipping gaits: a third locomotion*
382 *paradigm?*, Proc. R. Soc. Lond. B, 265:1227–1235.
- 383 [31] Minetti, A.E., Ardigo, L.P., Reinach, E., and Saibene, F., 1999. *The rela-*
384 *tionship between mechanical work and energy expenditure of locomotion in*
385 *horses*, J. Exp. Biol., 202:2329–2338.
- 386 [32] Muybridge, E., 1957. *Animals in motion*, New York: Dover Publications.
- 387 [33] Nana, P. and Waldron, K., 1995. *Energy comparison between trot, bound,*
388 *and gallop using a simple model*, ASME J. Biomech. Eng., 117:466–473.
- 389 [34] Pfau, T., Witte, T.H., and Wilson, A.M., 2006. *Centre of mass movement*
390 *and mechanical energy fluctuation during gallop locomotion in the Thor-*
391 *oughbred racehorse*, J. Exp. Biol., 209:3742–3757.
- 392 [35] Poulakakis, I., Papadopoulos, E., and Buehler, M., 2006. *On the stability*
393 *of the passive dynamics of quadrupedal running with a bounding gait*, Int.
394 J. Robot. Res., 25(7):669–687.
- 395 [36] Raibert, M.H., 1986. *Legged robots that balance*, Cambridge: MIT Press.
- 396 [37] Schöner, G., Jiang, W.Y., and Kelso, J.A.S., 1990. *A synergetic theory of*
397 *quadrupedal gaits and gait transitions*, J. Theor. Biol., 142:359–391.
- 398 [38] Seyfarth, A., Geyer, H., Günther, M., and Blickhan, R., 2002. *A movement*
399 *criterion for running*, J. Biomech., 35:649–655.
- 400 [39] Taylor, C.R., Shkolnik, A., Dmi’el, R., Baharav, D., and Borut, A., 1974.
401 *Running in cheetahs, gazelles, and goats: energy cost and limb con-*
402 *figuration*, Am. J. Physiol., 227(4):848–850.

403 [40] Waldron, K., Estremera, J., Csonka, P., and Singh, S., 2009. *Analyz-*
404 *ing bounding and galloping using simple models*, ASME J. Mech. Rob.,
405 1(1):011002.

406 Figure legends

407 Figure 1. Characteristics of the quadrupedal galloping gait. **A**: relative
 408 phase between left and right forelimbs of quadrupeds depending on locomotion
 409 speed (Froude number), modified from [1]. Although it remains almost
 410 antiphase in the walking and trotting gaits, it is away from the antiphase and
 411 approaches the in-phase in the galloping gait as the locomotion speed increases.
 412 **B**: footfall diagrams of the transverse gallop of horses and the rotary gallop of
 413 dogs and cheetahs, modified from [22]. The transverse gallop has a single flight
 414 phase after the liftoff of the forelimbs, while the rotary gallop has two flight
 415 phases after the liftoff of the hindlimbs and forelimbs. TH: trailing hindlimb,
 416 TF: trailing forelimb, LH: leading hindlimb, and LF: leading forelimb.

417
 418 Figure 2. Physical model of galloping consisting of a rigid body and four
 419 massless springs in two dimensions.

420
 421 Figure 3. Different touchdown angles of (**A**) trailing and (**B**) leading hindlimbs
 422 in the horse galloping gait.

423
 424 Figure 4. Touchdown angle difference δ of obtained gaits for Froude num-
 425 ber Fr_0 and spring stiffness k^* with (**A**) $\dot{\theta}_0^* = 0.92$ and (**B**) $\dot{\theta}_0^* = 1.65$. Gray
 426 regions show unstable gaits, and white regions show stable gaits. **A** has five
 427 gaits (Sequences (Seqs.) A–E) depending on Fr_0 and k^* , while **B** has one gait
 428 (Sequence E). Sequences A–E have different sequences of the stance condition,
 429 as shown in Fig. 5. The open dot (in **A**) and the black dots (in **A** and **B**) are
 430 the parameter sets used in Fig. 6 to compare vertical COM movement with the
 431 measured data for transverse and rotary gallops, respectively.

432
 433 Figure 5. Schematic sequences of stance condition of obtained gaits: Se-
 434 quences (Seqs.) A–E. TH: trailing hindlimb, TF: trailing forelimb, LH: leading
 435 hindlimb, and LF: leading forelimb.

436

437 Figure 6. Comparison of vertical COM movement between the simulation
438 results and measured data for one gait cycle between touchdowns of trail-
439 ing hindlimbs during quadrupedal galloping gaits. **A**: transverse gallop using
440 $\dot{\theta}_0^* = 0.92$, $Fr_0 = 14.4$, and $k^* = 5.1$. Measured data are based on [34] from a
441 horse. **B**: rotary gallop using $\dot{\theta}_0^* = 0.92$, $Fr_0 = 42.1$, and $k^* = 5.5$ and $\dot{\theta}_0^* = 1.65$,
442 $Fr_0 = 25.7$, and $k^* = 7.0$. Measured data are based on [4] from a dog.

443

444 Figure 7. Energy transfer during one gait cycle for (**A**) transverse gallop
445 using $\dot{\theta}_0^* = 0.92$, $Fr_0 = 14.4$, and $k^* = 5.1$ and (**B**) rotary gallop using $\dot{\theta}_0^* = 1.65$,
446 $Fr_0 = 25.7$, and $k^* = 7.0$.

figure 1b

figure 2

figure 3

figure 4

figure 5

figure 6

figure 7