Dynamic instability and bifurcation of electrically actuated circular nanoplate considering surface behavior and small scale effect

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Abstract In this paper, the dynamic pull-in instability and bifurcation characteristics of circular nanoplate subjected to electrostatic and Casimir forces are studied. Surface effect originates from high surface/volume ratio of nanostructures, where atoms at a free surface experience distinct local environments with respect to those in the bulk material. Thus, the surface free energy being negligible in classical elastic theory, becomes significant in dynamic behaviors of nanostructures. Based on Eringen's nonlocal elasticity and Gurtin-Murdoch surface model, the nonlinear governing equation of electrically actuated circular nanoplate is derived in polar coordinate. The closed-form solution of dynamic frequency and electrostatic voltage is obtained by utilizing the homotopy perturbation method (HPM). Furthermore, the coupling effects of nonlocal parameter and surface characteristics on the dynamic pull-in instability of circular nanoplate are investigated, and the nonlinear dynamics behaviors, time histories and phase diagrams of electrically actuated circular nanoplate are discussed. Some new results obtained in this work could be helpful in design of 2-D circular nanoplate-type actuator considering size-dependency and quantum vacuum fluctuation effects.

Keywords: Dynamic pull-in, Circular nanoplate, Bifurcation, Small scale effect, Surface behaviors, Homotopy perturbation method

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1. Introduction

The continuing miniaturization of electromechanical devices promotes the wide application of micro-/nano-electromechanical systems (MEMS/NEMS) in sensors [1, 2], actuators [3-7], resonators [8-10], switches [11-13], tweezers [14], oscillators [15, 16] and gyroscopes [17]. However, dynamic pull-in phenomenon as one critical electromechanical instability might lead actuated devices to easily lose stability and reliability, even result in adhesion, stiction, wear and failure [18]. Therefore, how to clearly understand the dynamic pull-in characteristics of actuated devices and effectively control the pull-in instability has become urgent in the design of MEMS/NEMS-based devices.

In general, typical electrically actuated devices are composed of a parallel-plate capacitor, where the movable plate deforms towards to the substrate as electrostatic voltage increases. As the size of devices reduces to nanometer scale, dispersion forces such as van der Waals force and Casimir force become significantly important in electromechanical coupling behaviors of devices [19-25]. Both of van der Waals force and Casimir force originate from fluctuation of electromagnetic field, but based on different separation regimes. Furthermore, van der Waals force generally describes that a fluctuating dipole p_1 induces a fluctuating electromagnetic dipole field, which in turn induces a fluctuating dipole p_2 on a neighbor particle. The van der Waals force has two crucial approximations: (i) quasi-static approximation to ignore wave effect, and (ii) ignores multiple scattering if there are more than two particles [21, 26]. However, when the separation is much larger than the typical wavelength, the finite wave propagation speed of light must be taken into account. Also, the multiple scattering can be negligible for a sufficiently dilute gas or for weak polarizability, but it becomes very significant for interactions between two solid bodies. Therefore, the multiple scattering effects combined with wave retardation yield only the Casimir force [26], where the effect of van der Waals force may be ignored in studying the dynamic instability of electrostatically actuated nanoplate system with a pair of conducting parallel plates.

Lin and Zhao [27] utilized a one degree of freedom mass-spring model to investigate the effect of Casimir force on static and dynamic pull-in instability of nano-actuators, and indicated that the movable beam would collapsed onto the substrate without any voltage at the two electrodes due to Casimir effect. Batra et al. [28] developed reduce-order models for electrically actuated circular and rectangular micro-plate with geometrical nonlinearity and Casimir force, where the vibrational behaviors of micro-plate subjected to Coulomb and the Casimir forces were analyzed and results showed that the first natural frequency equals zero at the pull-in voltage.

At the nanoscale, the dependence of material behaviors on size becomes a crucial factor in designing such nano-devices. The size-dependent properties of nanostructures have been confirmed in experimental observation and theoretical analysis[29, 30]. Classical elastic theory cannot capture the size-dependent characteristics of nanostructures; thus, some modified models with size-dependency are proposed. Among these, Eringen's nonlocal elastic theory[31, 32] has been widely employed in nanotechnology, in which the stress at a referenced point depends on the strains at all points in elastic body, [33-35]. Based on Eringen's nonlocal elastic theory, the size-dependent constitutive model depends on an internal characteristic length of materials (e.g., lattice parameter, granular distance), an external characteristic length (e.g., crack length, wave length), and their comparative ratios [36]. For macro structures, the internal characteristic length of materials and the ratio between internal and external characteristic lengths are too small, so that the effect of size dependency is negligible. Nevertheless, when the size of structure enters to nanometer scale and the ratio is comparative, the small scale parameters become significant. One key factor for utilizing the scale parameters in Eringen's nonlocal elastic theory to describe the scale effect of nanoscale materials is how to determine the correct scale parameter e_0a . Eringen [36] presented a method to determine the small scale parameter by comparing the results of plane waves from Born-Karman model of lattice dynamics and nonlocal elasticity theory. Zhang et al. [37] gave different scale parameters to solve the frequency of SWCNTs with different length-to-diameter ratios

and chiral angels by comparing the flexural vibrations of single walled carbon nanotubes (SWCNTs) from Timoshenko beam model and MD simulation. By comparing the dynamic behaviors of bilayer graphene sheets (BLGSs) from MD simulations and nonlocal elastic models, Shen et al. [38] determined a scale parameter to describe the scale effect in solving the natural frequency of BLGSs.

In the MEMS/NEMS-based devices regime, Mousavi et al. [12] employed nonlocal Euler-Bernoulli beam theory to study the static pull-in instability of nano-switches under electrostatic and intermolecular forces, and showed that the nonlocal cantilever nano-beam displays stiffening effects whilst doubly-clamped nanobeam shows softening effect owing to the small scale effect. Arani et al. [39] considered the nonlocal beam model and geometric nonlinearity to investigate the static pull-in instability of boron nitride nano-switches (BNNSs) subjected to electrostatic and van der Waals forces. Wang et al. [40] used nonlocal plate model to study the small scale effect on the dynamic pull-in instability and frequency of rectangular graphene sheets under electrostatic and intermolecular forces.

Furthermore, the surface effect is one of the significant molecular effects at the atomic scale, showing size-dependency of nanostructures [41-44]. Because of distinct environment conditions, atoms locating at or near the free surface have different equilibrium requirements than those in the bulk of material. This difference gives rise to additional surface energy in the surface layer of material and surface residual stress. Gurtin and Murdoch[45, 46] developed an analytical model to incorporate the surface effects to characterize the mechanical response of nanostructures. Ansari et al. [47] used Gurtin-Murdoch model to analyze the static pull-in instability behaviors of hydrostatically and electrostatically actuated circular nanoplates considering surface behaviors. Yang et al. [48] incorporated nonlocal beam theory and surface effect to model carbon nanotubes-reinforced cantilever nano-actuator and analyzed the effect of Casimir force on static pull-in instability and free standing behaviors. In another paper, Yang et al. [49] studied the coupling impacts of surface stress, nonlocal parameter and thermal effect on pull-in of nano-switches. Recently, Sahmani and Bahrami[50] utilized a high-order shear deformation plate model to study the dynamic

pull-in instability and free vibration properties of circular nanoplates including surface effect. They also tried to implement the nonlocal elasticity theory into the governing equation, but it is unfortunate that they cannot derive an explicit governing equation including surface stress and small scale effect, even the impact of Casimir force cannot be considered.

This work aims to employed nonlocal elasticity theory and surface effects to present a size-dependent analytical model of electrically actuated circular nanoplate and investigate the coupling effects of nonlocal parameter, surface elasticity modulus and surface residual stress on dynamic pull-in instability. The dynamic frequency and bifurcation characteristics of circular nanoplate subjected to electrostatic and Casimir forces are also studied. The nonlinear nonlocal governing equation is analytically solve by the homotopy perturbation method (HPM) [51] widely used to solve nonlinear physical problems [52-54]. Finally, to verify the accuracy of HPM, the comparison of results obtained by using the HPM, experiment and other numerical approaches is performed, and the results from different methods show a good agreement.

2. Theory and model

To capture the size-dependent characteristics of nanostructures, Eringen's nonlocal elasticity containing the long range force among atoms in body is utilized, in which the stress-strain relationship can be expressed as [55]

$$\sigma_{ij}^{nl}(x) = \int H(|x-x'|, e_0 a/l) C_{ijkl} \varepsilon_{kl}(x') dV(x'), \quad \forall x \in V \quad (1)$$

where σ_{ij}^{nl} , ε_{kl} and C_{ijkl} denote the nonlocal stress, strain and fourth-order elasticity tensors, respectively. The term $H(|x-x'|, e_0 a/l)$ is the principal attenuation kernel function related to the nonlocal effects in terms of the Euclidean distance between the point x and neighboring points x' within a domain V . $|x-x'|$ is the distance between the two reference points x and x' . $e_0 a/l$ represents nonlocal effects, e_0 denotes a material constant, a is the internal characteristic length of material and l

is the external characteristic length of the material. The internal characteristic length represents the scale parameter such as length of C-C bond, lattice spacing or granular distance.

For the homogenous isotropic body without body forces, the nonlocal elasticity is reduced to following form as [55]

$$\left[1 - (e_0 a)^2 \nabla^2\right] \sigma^{nl} = C : \varepsilon \quad (2)$$

where $(e_0 a)$ represents the nonlocal parameter, ∇^2 represents the Laplace differential operator. For axisymmetric deformation of circular nanoplate, the size-dependent constitutive equation based on nonlocal elasticity is expressed in polar coordinates, as follows:

$$\sigma_r^{nl} - (e_0 a)^2 \nabla^2 \sigma_r^{nl} = \frac{E}{1 - \nu^2} (\varepsilon_r + \nu \varepsilon_\theta) \quad (3a)$$

$$\sigma_\theta^{nl} - (e_0 a)^2 \nabla^2 \sigma_\theta^{nl} = \frac{E}{1 - \nu^2} (\nu \varepsilon_r + \varepsilon_\theta) \quad (3b)$$

where r is the radial variable, σ_r^{nl} and σ_θ^{nl} denote the nonlocal radial and circumferential counterparts of stress tensor, ε_r and ε_θ are radial and circumferential strains, and ν represents the Poisson's ratio, respectively. The corresponding displacements of circular nanoplate are described as

$$u_z(r, t) = w(r, t), \quad u_r(r, t) = -z \frac{\partial u_z}{\partial r}, \quad u_\theta(r, t) = 0 \quad (4)$$

where u_z , u_r and u_θ are the displacements in the transverse z-direction, radial r direction and circumferential direction, respectively. Based on the Kirchhoff plate theory, the radial and circumferential strains of axisymmetric circular nanoplate are expressed as

$$\varepsilon_r = -z \frac{\partial^2 w(r, t)}{\partial r^2}, \quad \varepsilon_\theta = -\frac{z}{r} \frac{\partial w(r, t)}{\partial r}, \quad \varepsilon_{r\theta} = 0 \quad (5)$$

Moreover, the circular nanoplate is considered to have an elastic surface layer with zero thickness, and the surface layer has specific material characteristics incorporating surface energy effects and is assumed to be perfectly bonded to the bulk material, as shown in **Fig. 1**. According to Gurtin-Murdoch model [47], the constitutive relations

of surface layer are expressed as

$$\sigma_{\alpha\beta}^{s\pm} = \tau^{s\pm} \delta_{\alpha\beta} + \lambda^{s\pm} \varepsilon_{\alpha\beta}^{s\pm} + 2\mu^{s\pm} \varepsilon_{\alpha\beta}^{s\pm}, \quad \sigma_{\gamma z}^{s\pm} = \tau^{s\pm} \frac{\partial w}{\partial \gamma} \quad (6)$$

where $\sigma_{\alpha\beta}^{s\pm}$, $\varepsilon_{\alpha\beta}^{s\pm}$, $\tau^{s\pm}$ denote the stress, strain, residual tension of surface layers; $\lambda^{s\pm}$, $\mu^{s\pm}$ are the surface Lamé constants; $\delta_{\alpha\beta}$ is the Kronecker delta. In the above equations, the Greek subscripts represent the values of r and θ . From Eqs. (5) and (6), the surface stresses can be expressed as

$$\sigma_r^{s\pm} = \tau^s \mp \frac{E^s h}{2(1-\nu^2)} \left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r} \frac{\partial w}{\partial r} \right) \quad (7a)$$

$$\sigma_\theta^{s\pm} = \tau^s \mp \frac{E^s h}{2(1-\nu^2)} \left(\nu \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (7b)$$

$$\sigma_{rz}^{s\pm} = \tau^s \frac{\partial w}{\partial r} \quad (7c)$$

where E^s represents the surface modulus and h is the thickness of circular nanoplate. Using the following stress resultants for the derivation of nonlocal circular nanoplate considering surface effects, leads to

$$M_r = \int_{-h/2}^{h/2} \sigma_r^{nl} z dz + \frac{h}{2} (\sigma_r^{s+} - \sigma_r^{s-}) \quad (8a)$$

$$M_\theta = \int_{-h/2}^{h/2} \sigma_\theta^{nl} z dz + \frac{h}{2} (\sigma_\theta^{s+} - \sigma_\theta^{s-}) \quad (8b)$$

where M_r and M_θ denote radial and circumferential bending moments, respectively.

Substituting Eqs.(7a~c) into Eqs.(8a,b) in conjunction with Eqs.(3a,b), the nonlocal stress resultants is given by

$$M_r - (e_0 a)^2 \nabla^2 M_r = -D^* \left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r} \frac{\partial w}{\partial r} \right) \quad (9a)$$

$$M_\theta - (e_0 a)^2 \nabla^2 M_\theta = -D^* \left(\nu \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (9b)$$

where

$$D^* = k^s D, \quad k^s = 1 + \frac{6E^s}{Eh}, \quad \text{and } D = \frac{Eh^3}{12(1-\nu^2)} \quad (10)$$

In the above formula, k^s represents the ratio of surface rigidity versus bulk and D is the flexural rigidity of circular nanoplate. Note that the stress resultants in Eqs. (9a, b) are reduced to the classical circular plate model when the surface elastic modulus E^s and nonlocal parameters are set as zero. It is seen that when bulk thickness decreases into the nanoscale, the ratio k^s becomes significant, so that the surface elasticity has a hardening effect on flexural rigidity of nanoplate. The equilibrium equations of axisymmetric circular nanoplate is expressed as [56]

$$\frac{\partial M_r}{\partial r} + \frac{M_r - M_\theta}{r} + \sigma_{rz}^{s+} - \sigma_{rz}^{s-} = Q_r \quad (11a)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r Q_r) + \frac{N_r}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + F_{dis} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (11b)$$

where F_{dis} is the sum of transverse distributed loads; ρ is the mass density of circular nanoplate. For circular nanoplate under uniform radial mechanical loading coupling with environment temperature change, the radial in-plane force N_r is expressed as

$$N_r = N_{me} + N_T \quad (12)$$

where N_{me} and N_T are the radial forces induced by the mechanical loading and the thermal effect from the environment temperature change, respectively. According to the thermal elasticity theory[57], the thermal radial force is given by

$$N_T = - \left(\frac{E \alpha h}{1 - \nu} \right) \Delta T \quad (13)$$

where α and ΔT are the coefficient of thermal expansion (CTE) and temperature change. Decoupling Eqs.(9a,b) and Eqs.(11a,b) leads to the nonlocal dynamic governing equation of circular nanoplate considering surface elasticity and surface residual stress[58], as follows:

$$\begin{aligned}
& D^* \left(\frac{\partial^4 w}{\partial r^4} + \frac{2}{r} \frac{\partial^3 w}{\partial r^3} - \frac{1}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^3} \frac{\partial w}{\partial r} \right) - (e_0 a)^2 \left[D^* + (e_0 a)^2 (N_r + 2\tau^s) \right] \frac{\partial^6 w}{\partial r^6} \\
& - (N_r + 2\tau^s) \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \frac{7}{r} (e_0 a)^2 \left[D^* + (e_0 a)^2 (N_r + 2\tau^s) \right] \frac{\partial^5 w}{\partial r^5} \\
& + (e_0 a)^2 \left\{ 2(N_r + 2\tau^s) - \frac{1}{r^2} \left[D^* (3 + 2\nu) + 5(e_0 a)^2 (N_r + 2\tau^s) \right] \right\} \frac{\partial^4 w}{\partial r^4} \\
& + \frac{2}{r} (e_0 a)^2 \left\{ 4(N_r + 2\tau^s) + \frac{1}{r^2} \left[D^* (1 + 2\nu) + 3(e_0 a)^2 (N_r + 2\tau^s) \right] \right\} \frac{\partial^3 w}{\partial r^3} \\
& + \frac{1}{r^2} (e_0 a)^2 \left\{ 2(N_r + 2\tau^s) + \frac{3}{r^2} \left[D^* (1 - 2\nu) - (e_0 a)^2 (N_r + 2\tau^s) \right] \right\} \frac{\partial^2 w}{\partial r^2} \\
& - \frac{1}{r^3} (e_0 a)^2 \left\{ 2(N_r + 2\tau^s) + \frac{3}{r^2} \left[D^* (1 - 2\nu) - (e_0 a)^2 (N_r + 2\tau^s) \right] \right\} \frac{\partial w}{\partial r} \\
& + mh \left[(e_0 a)^4 \frac{\partial^6 w}{\partial r^4 \partial t^2} + (e_0 a)^4 \frac{6}{r} \frac{\partial^5 w}{\partial r^3 \partial t^2} - (e_0 a)^2 \frac{1}{r^2} (2r^2 - 3\mu) \frac{\partial^4 w}{\partial r^2 \partial t^2} \right. \\
& \left. - (e_0 a)^2 \frac{3}{r^3} (2r^2 + (e_0 a)^2) \frac{\partial^3 w}{\partial r \partial t^2} + \frac{\partial^2 w}{\partial t^2} \right] - (e_0 a)^4 \frac{\partial^4 F_{dis}}{\partial r^4} - (e_0 a)^4 \frac{6}{r} \frac{\partial^3 F_{dis}}{\partial r^3} \\
& - (e_0 a)^2 \frac{1}{r^2} \left[2r^2 - 3(e_0 a)^2 \right] \frac{\partial^2 F_{dis}}{\partial r^2} + (e_0 a)^2 \frac{3}{r^3} \left[2r^2 + (e_0 a)^2 \right] \frac{\partial F_{dis}}{\partial r} - F_{dis} = 0
\end{aligned} \tag{14}$$

For the electrically actuated circular nanoplate, the gap between the electrodes is small compared to nanoplate radius, thus the fringing field effect can be ignored and the distributed electrostatic force can be written as [27]

$$F_e = \frac{\varepsilon_0 \bar{V}_e^2}{2[g_0 - w(r)]^2} \tag{15}$$

where $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$ is the permittivity of vacuum, $\bar{V}_e$ is the applied electrostatic voltage, and g_0 is the initial gap between circular nanoplate and substrate.

Moreover, Casimir force as the macro effect of quantum field fluctuation is defined as [48]

$$F_c = \frac{\pi^2 \hbar c_0}{240[g_0 - w(r)]^4} \tag{16}$$

where $\hbar = 1.055 \times 10^{-34} Js$ is the Plank's constant divided by 2π and

$c = 3 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ is the speed of light. The transverse distributed load F_{dis} in Eq. (14) is the sum of electrical actuation force and Casimir force exerted on the circular nanoplate, which is written as $F_{dis} = F_e + F_C$.

For further parametrical investigation, non-dimensional variables are given by

$$W = \frac{w}{g_0}, \quad \rho = \frac{r}{R}, \quad \mu = \frac{e_0 a}{R}, \quad N_0 = \frac{N_r R^2}{D}, \quad \tau_0^s = \frac{\tau^s R^2}{D},$$

$$\chi = \frac{t}{R^2} \sqrt{\frac{D}{\rho h}}, \quad K_2 = \frac{\varepsilon_0 \bar{V}_e^2 R^4}{2 g_0^3 D}, \quad K_4 = \frac{\pi^2 \hbar c_0 R^4}{240 g_0^5 D} \quad (17)$$

Substituting Eq. (17) into Eq. (14) leads to the non-dimensional nonlocal dynamic governing equation of circular nanoplate, as follows

$$\begin{aligned} & k^s \left(\frac{\partial^4 W}{\partial \rho^4} + \frac{2}{\rho} \frac{\partial^3 W}{\partial \rho^3} - \frac{1}{\rho^2} \frac{\partial^2 W}{\partial \rho^2} + \frac{1}{\rho^3} \frac{\partial W}{\partial \rho} \right) - (N_0 + 2\tau_0^s) \left(\frac{\partial^2 W}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial W}{\partial \rho} \right) \\ & - \mu^2 \left[k^s + \mu^2 (N_0 + 2\tau_0^s) \right] \frac{\partial^6 W}{\partial \rho^6} - \frac{7\mu^2}{\rho} \left[k^s + \mu^2 (N_0 + 2\tau_0^s) \right] \frac{\partial^5 W}{\partial \rho^5} \\ & + \mu^2 \left\{ 2(N_0 + 2\tau_0^s) - \frac{1}{\rho^2} \left[k^s (3 + 2\nu) + 5\mu^2 (N_0 + 2\tau_0^s) \right] \right\} \frac{\partial^4 W}{\partial \rho^4} \\ & + \frac{2\mu^2}{\rho} \left\{ 4(N_0 + 2\tau_0^s) + \frac{1}{\rho^2} \left[k^s (1 + 2\nu) + 3\mu^2 (N_0 + 2\tau_0^s) \right] \right\} \frac{\partial^3 W}{\partial \rho^3} \\ & + \frac{\mu^2}{\rho^2} \left\{ 2(N_0 + 2\tau_0^s) + \frac{3}{\rho^2} \left[k^s (1 - 2\nu) - \mu^2 (N_0 + 2\tau_0^s) \right] \right\} \frac{\partial^2 W}{\partial \rho^2} \\ & - \frac{\mu^2}{\rho^3} \left\{ 2(N_0 + 2\tau_0^s) + \frac{3}{\rho^2} \left[k^s (1 - 2\nu) - \mu^2 (N_0 + 2\tau_0^s) \right] \right\} \frac{\partial W}{\partial \rho} \\ & + \mu^4 \frac{\partial^6 W}{\partial \rho^4 \partial \chi^2} + \frac{6\mu^4}{\rho} \frac{\partial^5 W}{\partial \rho^3 \partial \chi^2} - \frac{\mu^2}{\rho^2} (2\rho^2 - 3\mu^2) \frac{\partial^4 W}{\partial \rho^2 \partial \chi^2} \\ & - \frac{3\mu^2}{\rho^3} (2\rho^2 + \mu^2) \frac{\partial^3 W}{\partial \rho \partial \chi^2} + \frac{\partial^2 W}{\partial \chi^2} - \mu^4 \frac{\partial^4 \hat{F}_{dis}}{\partial \rho^4} - \frac{6\mu^4}{\rho} \frac{\partial^3 \hat{F}_{dis}}{\partial \rho^3} \\ & - \frac{\mu^2}{\rho^2} (2\rho^2 - 3\mu^2) \frac{\partial^2 \hat{F}_{dis}}{\partial \rho^2} + \frac{3\mu^2}{\rho^3} (2\rho^2 + \mu^2) \frac{\partial \hat{F}_{dis}}{\partial \rho} - \hat{F}_{dis} = 0 \end{aligned} \quad (18)$$

where $\hat{F}_{dis}$ denotes the sum of non-dimensional transverse distributed forces.

3. Solution procedure

3.1 Nonlinear dynamic equations

Utilizing Eigen function expansion, the solution of Eq. (18) is considered as

$$W(\rho, T) = \sum_{m=1}^N \eta_m(\chi) \phi_m(\rho) \quad (19)$$

where $\phi_m(\rho)$ is the m -th mode shape function, $\eta_m(\chi)$ is the m -th coefficient function for the m -th shape function, and N is the number of chosen shape functions. As N approaches a convergence value, the approximation in Eq.(19) becomes exact. For the axisymmetric circular plate satisfying the clamped boundary conditions, the linear undamped modes of shape functions $\phi_m(\rho)$ is expressed as[59]

$$\phi_m(\rho) = \left(\frac{J_0(\rho\sqrt{\omega_m})}{J_0(\sqrt{\omega_m})} - \frac{I_0(\rho\sqrt{\omega_m})}{I_0(\sqrt{\omega_m})} \right) \quad (20)$$

where J_0 is the Bessel function of the first kind, I_0 is the modified Bessel function of the first kind, and ω_m is the known m -th frequency. The mode shapes are orthonormal, satisfying

$$\int_0^1 \rho \phi_m(\rho) \phi_n(\rho) d\rho = \delta_{mn} \quad (21)$$

where δ_{mn} represents the Kronecker delta. The fourth-order Taylor's series expansion of distributed forces is proved to have excellent accuracy to analytically study dynamic pull-in instability of nano-devices [60]. Substituting Eq. (19) into Eq. (18), multiplying updated Eq. (18) with ρ^4 to avoid the singularity of integration, then multiplying every term by $\rho \phi_n(\rho)$ over $\rho \in [0,1]$, and using the orthogonality condition of Eq. (21), lead to the governing equation of electrically actuated circular nanoplate as

$$\begin{aligned} & \sum_{k=1}^N \Psi_0 \ddot{\eta}_k(\chi) + \Psi_1 + \sum_{k=1}^N \Psi_2 \eta_k(\chi) + \sum_{k=1}^N \sum_{m=1}^N \Psi_3 \eta_k(\chi) \eta_m(\chi) + \sum_{k=1}^N \sum_{m=1}^N \sum_{l=1}^N \Psi_4 \eta_k(\chi) \eta_m(\chi) \eta_l(\chi) \\ & + \sum_{k=1}^N \sum_{m=1}^N \sum_{l=1}^N \sum_{n=1}^N \Psi_5 \eta_k(\chi) \eta_m(\chi) \eta_l(\chi) \eta_n(\chi) = 0 \end{aligned} \quad (22)$$

where $\Psi_k, k \in [0, 5]$ are given in **Appendix A**.

3.2 Application of homotopy perturbation method

It is well known that well-established numerical techniques in literatures can be easily applied to solve the problems about dynamic instability of nanostructures. However, it is difficult to get generalizations of the problem from numerical results due to the results obtained from the numerical techniques are limited to fixed calculation parameters. In order to clearly describe the roles of some parameters of interest in a physical process, the homotopy perturbation method can be utilized to serves this purpose well in this paper, which is an effectively approximate method. The roles of some important controlled parameters for the dynamic instability and bifurcation of electrically actuated circular nanoplate, such as small scale parameter, surface effect, applied voltage, thermal environment, etc., can clearly be revealed from the present solution.

For Eq. (22), considering the dynamic behaviors of circular nanoplate mainly depend on the first term of eigen function expansion gives

$$\ddot{\eta}_1(\chi) + \psi_1 + \psi_2 \eta_1(\chi) + \psi_3 [\eta_1(\chi)]^2 + \psi_4 [\eta_1(\chi)]^3 + \psi_5 [\eta_1(\chi)]^4 = 0 \quad (23)$$

where $\psi_k = \Psi_k / \Psi_0$, $k \in [1, 5]$. Based on the basic idea of HPM (homotopy method) [51], the homotopy equation can be constructed to solve Eq. (23), as follows:

$$\mathcal{H}(\eta_1, p) = (1 - p)[\ddot{\eta}_1 + \psi_1 \eta_1] + p[\ddot{\eta}_1 + \psi_0 + \psi_1 \eta_1 + \psi_2 \eta_1^2 + \psi_3 \eta_1^3 + \psi_4 \eta_1^4] = 0 \quad (24)$$

Assume the solution of Eq.(23) has the following form as

$$\eta_1(\chi) = \eta_{1,0}(\chi) + p \eta_{1,1}(\chi) + p^2 \eta_{1,2}(\chi) + \dots \quad (25)$$

where the coefficient of η_1 is expanded into a series of p , as follows

$$\psi_1 = \omega^2 - p \omega_1 - p^2 \omega_2 - \dots \quad (26)$$

Substituting Eq. (25) and Eq. (26) into Eq. (24), and equating the terms with the identical powers of p ,

$$p^0: \eta_{1,0}(\chi) + \omega^2 \eta_{1,0}(\chi) = 0, \quad \eta_{1,0}(0) = A, \quad \dot{\eta}_{1,0}(0) = 0, \quad (27a)$$

$$p^1: \eta_{1,1}(\chi) + \omega^2 \eta_{1,1}(\chi) = \omega_1 \eta_{1,0}(\chi) - [\psi_0 + \psi_2(\eta_{1,0}(\chi))^2 + \psi_3(\eta_{1,0}(\chi))^3 + \psi_4(\eta_{1,0}(\chi))^4],$$

$$\eta_{1,1}(0) = 0, \quad \dot{\eta}_{1,1}(0) = 0, \quad (27b)$$

$$p^2: \eta_{1,2}(\chi) + \omega^2 \eta_{1,2}(\chi) = \omega_1 \eta_{1,1}(\chi) + \omega_2 \eta_{1,0}(\chi) - [2\psi_2 \eta_{1,0}(\chi) \eta_{1,1}(\chi) + 3\psi_3(\eta_{1,0}(\chi))^2 \eta_{1,1}(\chi) + 4\psi_4(\eta_{1,0}(\chi))^3 \eta_{1,1}(\chi)], \quad \eta_{1,2}(0) = 0, \quad \dot{\eta}_{1,2}(0) = 0$$

$$, \quad (27c)$$

Firstly, the solution of Eq. (27a) is written as

$$\eta_{1,0}(\chi) = A \cos(\omega \chi) \quad (28)$$

Because the solution of Eq. (27b) should not incorporate the secular term $\cos(\omega \chi)$,

substituting Eq.(28) into Eq. (27b) yields

$$\begin{aligned} \eta_{1,1}(\chi) + \omega^2 \eta_{1,1}(\chi) = & \left(\omega_1 A - \frac{3}{4} \psi_3 A^3 \right) \cos(\omega \chi) - \left(\frac{1}{2} \psi_2 A^2 + \frac{1}{2} \psi_4 A^4 \right) \cos(2\omega \chi) \\ & - \frac{1}{2} \psi_2 A^2 - \frac{3}{8} \psi_4 A^4 - \psi_0 - \frac{1}{4} \psi_3 A^3 \cos(3\omega \chi) - \frac{1}{8} \psi_4 A^4 \cos(4\omega \chi) \end{aligned} \quad (29)$$

Letting the secular terms of Eq. (29) be zero, leading to

$$\omega_1 = \frac{3}{4} \psi_3 A^2 \quad (30)$$

Then, solving the updated Eq. (29) gives one second-order approximated solution of

$\eta_{1,1}(\chi)$ as

$$\begin{aligned} \eta_{1,1}(\chi) = & \frac{1}{480\omega^2} \left[(96\psi_4 A^4 + 160\psi_2 A^2 - 15\psi_3 A^3 + 480\psi_0) \cos(\omega \chi) \right. \\ & + (80\psi_4 A^4 + 80\psi_2 A^2) \cos(2\omega \chi) + 15\psi_3 A^3 \cos(3\omega \chi) \\ & \left. + 4\psi_4 A^4 \cos(4\omega \chi) - (480\psi_0 + 180\psi_4 A^4 + 240\psi_2 A^2) \right] \end{aligned} \quad (31)$$

In addition, Eq. (26) with two terms approximation of power series when $p \rightarrow 1$,

$$\omega_2 = \omega^2 - \psi_1 - \omega_1 \quad (32)$$

Substituting Eq. (32) into Eq. (27c) for $\eta_{1,2}(\chi)$ and eliminating the secular terms, the

frequency-amplitude equation is expressed as

$$\begin{aligned} \omega^4 A - \psi_1 \omega^2 A - \frac{3}{4} \psi_3 \omega^2 A^3 + 2\psi_0 \psi_2 A + \frac{5}{6} \psi_2^2 A^3 - \frac{3}{2} \psi_0 \psi_3 A^2 - \frac{1}{2} \psi_2 \psi_3 A^4 \\ + \frac{3}{128} \psi_3^2 A^5 + 3\psi_0 \psi_4 A^3 + \frac{7}{4} \psi_2 \psi_4 A^5 - \frac{3}{10} \psi_3 \psi_4 A^6 + \frac{63}{80} \psi_4^2 A^7 = 0 \end{aligned} \quad (33)$$

Finally, by solving Eq. (33) respect to the fundamental frequency ω , the second-order frequency-amplitude relationship function can be easily determined.

3.3 Bifurcation and dynamic stability

To analyze the bifurcation characteristics of autonomous system, letting $\dot{\eta}_1(\chi) = \dot{q}_1(\chi) = q_2$ and substituting them into Eq. (23) lead to

$$\begin{cases} \dot{q}_1 = q_2 \\ \dot{q}_2 = -f(q_1, q_2) \end{cases} \quad (34)$$

where $f(q_1, q_2) = \psi_1 + \psi_2 q_1 + \psi_3 q_1^2 + \psi_4 q_1^3 + \psi_5 q_1^4$. At the equilibrium points, the dynamic system is at rest, and the equilibrium points are computed by $q_2 = 0$ or $f(q_1, q_2) = 0$. To check the stability of equilibrium points, the corresponding Jacobian matrix is expressed as

$$J = \begin{bmatrix} 0 & 1 \\ \psi_2 + 2\psi_3 q_1 + 3\psi_4 q_1^2 + 4\psi_5 q_1^3 & 0 \end{bmatrix} \quad (35)$$

The corresponding eigenvalue of Jacobian matrix at $q_1 = q_{1,i}$ satisfies $\lambda^2 - \psi_2 - 2\psi_3 q_1 - 3\psi_4 q_1^2 - 4\psi_5 q_1^3 = 0$. For $\lambda^2 < 0$, the system has two pure imaginary roots and has the equilibrium point $(q_{1,i}, K_2)$ as the center point. For the other equilibrium points $(q_{1,2}, K_2)$, its eigenvalue satisfies $\lambda^2 > 0$ and the system has two real eigenvalues, where one is positive and the other one is negative. This means that this equilibrium point is an unstable saddle point.

Note that the fixed points or physically equilibrium positions of governing equation (23) exist in $\eta_1 \in [0, 1]$, but some points as solution of the governing equation (23) might exist outside the domain $[0, 1]$. Actually, the number and positions fixed points

and phase diagrams of dynamic system can be obtained by numerical analysis for the following equation:

$$f(q_1, q_2) = \psi_1 + \psi_2 q_1 + \psi_3 q_1^2 + \psi_4 q_1^3 + \psi_5 q_1^4 = 0 \quad (36)$$

The positions of fixed points depending on $\{\psi_m\}, m \in [1, 5]$ are considered as a function of applied electrostatic voltage, dispersion forces and physical parameters of electrically actuated circular nanoplate. When dispersion forces and physical parameters are given, the dynamic stability of circular nanoplate is dependent on electrostatic voltage.

4. Results and discussion

4.1 Validation of HPM with numerical method

In order to investigate the convergence and accuracy of HPM used to solve the dynamic pull-in instability considering the influence of Casimir force, the comparison of results from HPM and Runge-Kutta numerical method is performed. **Table 1** lists the deflection of circular nanoplate with the influence of Casimir force ($\alpha_4 = 1$) under different initial amplitudes and electrostatic voltages when the non-dimensional time is taken as $\chi = 0.1$. It is observed that the deflections of circular nanoplate computed by second-order HPM well agree with those computed by Runge-Kutta numerical method. Even if the errors of results between HPM and Runge-Kutta numerical method slightly increase with the increment of initial amplitude, the largest error of deflection at initial amplitude $A = 0.20$ and electrostatic voltage $K_2 = 5.0$ is only 3.79%.

To further prove that the HPM in this paper is effective employed to investigate the dynamic pull-in of circular plate, the dynamic pull-in voltages of micro-plate obtained by the present HPM is compared to those from other methods in Refs.[50-52]. **Table 2** gives the geometrical and material parameters of micro-plate for validation. The comparisons of the dynamic pull-in voltage obtained by HPM with those of experiment [61], finite element method (FEM) simulation [62], and step-by-step

linearization method (SSLM) with 20 nodes[63] are list in **Table 3**. The results show that the result from HPM has good agreement with those from experiment and other numerical methods in literatures.

4.2 Dynamic frequency and pull-in voltage

From the nonlinear dynamic equation (34), the relation of dynamic frequency and initial amplitude of circular nanoplate can be easily determined. This section focused on the coupling influences of various physical parameters on the dynamic pull-in voltage of circular nanoplate. The relevant material and geometrical parameters of circular nanoplate in examples calculation are given in **Table 4**.

Fig. 2 illustrates the influences of the surface elasticity on the dynamic pull-in voltage and non-dimensional fundamental frequency of circular nanoplate. Due to the lack of existing experimental data, four different values of the surface elasticity in the range of 0-10 nN/nm are considered to investigate the pull-in voltage. It is observed that the frequency gradually decreases as the applied voltage increases; when the voltage continue to increase, the frequency would suddenly drop to zero, namely, the pull-in instability occurs. Furthermore, under the identical voltage, the frequency increases as the value of surface elasticity increases. The surface elasticity enhances the structural stiffness of circular nanoplate, thus the pull-in voltage with large surface elasticity is higher than that with small surface elasticity.

Fig. 3 shows the effect of surface residual stress on the pull-in voltage and non-dimensional frequency of circular nanoplate. The values of surface residual stress are considered in the range 0-1.0 nN/nm. As the surface residual stress increases, the pull-in voltage of circular nanoplate reduces. The surface residual stress also decreases the fundamental frequency of circular nanoplate under the same applied voltage.

Fig. 4 and **Fig. 5** depict that influences of the radial mechanical loading and environment temperature change on the dynamic pull-in voltage and fundamental frequency, respectively. From **Fig. 4**, the negative values of radial loading represent tensile stress while the positive values of radial loading are compressive stress. This

means the radial compressive loading reduces the dynamic pull-in voltage and frequency of circular nanoplate. It is seen from **Fig. 5** that as the temperature change increases, both of the pull-in voltage and frequency reduce, which means the increase of temperature change shows the compressive stress in radial direction of circular nanoplate.

Fig. 6 illustrates the effect of nonlocal parameter on the dynamic pull-in voltage and frequency of circular nanoplate. According to the MD simulation results, the values of nonlocal parameter $(e_0 a)^2$ are in the range 0-3 nm². It is observed that the pull-in voltage reduces with the increment of nonlocal parameter, and under the same voltage, the frequency declines as the nonlocal parameter increase. This means that the nonlocal parameter induces the decrease of frequency and pull-in voltage of circular nanoplate.

To study the coupling influence of surface elasticity and surface residual stress in conjunction with the nonlocal parameter on the dynamic behaviors of circular nanoplate, the non-dimensional fundamental frequency versus nonlocal parameters for different surface elasticity and surface residual stress is plotted in **Fig. 7**. It is seen that the frequency declines as the nonlocal parameter for all selection of surface elasticity and residual stress, which means the effect of nonlocal parameter is significant on the frequency of circular nanoplate. For the identical nonlocal parameter, the surface elasticity enhances the structural stiffness whereas the increment of surface residual stress reduces the frequency of circular nanoplate.

Fig. 8 depicts the frequency versus nonlocal parameter for different radial mechanical loadings and temperature changes. Herein, the positive temperature change and radial compressive loading are considered. The results show that the frequency of circular nanoplate with the clamped boundary decreases as the temperature changes and mechanical compression loading increase.

The influences of ratio of radius-to-thickness on non-dimensional frequency and voltage of electrostatically actuated circular nanoplate are studied, shown as in **Fig. 9**. The non-dimensional frequency decreases with the increment of ratio of

radius-to-thickness at the identical applied voltage. Moreover, the effects of nonlocal parameter on non-dimensional frequency for different ratios of radius-to-thickness are investigated in **Fig. 10**. As the nonlocal parameters and ratio of radius-to-thickness increase, the non-dimensional frequency declines. Based on Eringen's nonlocal elasticity, the nonlocal parameters are determined by the ratios of internal characteristic length to external characteristic length, and thus the nonlocal parameter $\mu = e_0 a / R$ decreases when radius of circular nanoplate gradually increase. Therefore, the influence of nonlocal parameters become weakening as the size of device system increases.

4.3 Dynamic instability and bifurcation

The coupling effects of the applied voltage and Casimir force on the transition history of equilibrium points are summarized in phase diagrams of circular nanoplate for different initial conditions. **Fig. 11(a~d)** illustrate the motional trajectories of circular nanoplate under applied voltages. It should be pointed that those solutions beyond the non-dimensional domain $[0, 1]$ of dynamic governing equation have not practically physical meaning, thus can be canceled. To emphasizes the effect of applied voltage on the fundamental frequency, the Casimir force is not considered here; whereas other physical parameters are taken as $E^s = 10 \text{ nN/nm}$, $\tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$ and $(e_0 a)^2 = 1 \text{ nm}^2$.

Fig. 11(a) shows the phase diagram of circular nanoplate when no voltage is applied ($K_2 = 0$). There is one physically stable center C , where the dynamic system is in a rest state at this moment. When a small actuation voltage is applied on the circular nanoplate ($K_2 = 3.0$), the phase diagram of system undergoes change, depicted in **Fig. 11(b)**. There exist one stable focus F , and there are periodic orbits around the focus F . Homoclinic orbits originate from the other equilibrium point as one unstable saddle node S and goes back to it. When the applied voltage continue to increase ($K_2 = 17.8$), the two nodes approach to each other, shown as in **Fig. 11(c)**.

Finally, when the applied voltage equals to the pull-in voltage ($K_2 = 19.23$), two nodes meet together, the stable focus F eliminates and the dynamic pull-in instability of system occurs, depicted in **Fig. 11(d)**. At this moment, the circular nanoplate loses the structural balance between elastic storing energy and electrical energy, thus collapses downwards onto the substrate.

Furthermore, **Fig. 12** and **Fig. 13** illustrate the pull-in instability procedure of circular nanoplate subjected to electrostatic and Casimir forces for the identical initial conditions (the initial deflection and velocity are zero), where the value of non-dimensional Casimir force is taken as $K_4 = 1$. It is observed from **Fig. 12** that when no actuation voltage is applied ($K_2 = 0$), the small deflection of circular plate occurs because the Casimir force has an attracting effect on the circular nanoplate within the nanoscale separation distance. As the applied voltage increases, the deflection gradually becomes larger; as the voltage approaches to the pull-in value ($K_2^{PI} = 9.90$), the deflection sharply increases. This means the system loses the dynamic stability and the pull-in instability occurs. At the pull-in voltage, the motion trajectories in the phase plane approach to the unstable saddle node, shown as in **Fig. 13**. As the voltage continues to increase, the circular nanoplate becomes dynamically unstable and sticks onto the substrate.

To elucidate the effect of nonlocal parameter on the dynamic pull-in instability of circular nanoplate, the time history and instable phase diagram at the pull-in voltage for different nonlocal parameters are, respectively, plotted in **Fig. 14** and **Fig. 15**, where the surface modulus and surface residual stress are not considered ($E^s = \tau^s = 0$), and the corresponding dynamic pull-in voltage is shown as $K_2^{PI} = 16.38$. **Fig. 15** illustrates that the effect of nonlocal parameter on the instable phase diagram when the values of applied voltage increase to the pull-in voltage. It is seen from Fig.15 that the non-dimensional velocity of dynamic system slightly decreases with the increase of nonlocal parameter; nevertheless, the pull-in voltage reduce as the nonlocal parameter increases. At the dynamic instability state, the influences of nonlocal

parameter on velocity are not significant on the whole. The present results are consistent with those in Ref. [40], where the dynamic pull-in behaviors of graphene sheets are studied. The effect of nonlocal parameter can be explained to actually enlarge the equivalent mass, but soften the structural stiffness of nanoplate.

5. Conclusion

The present analytical model of 2-D circular nanoplate actuator can be harnessed in applications of nano-actuators and nano-switches in MEMS/NEMS. The switch on/off state can be precisely determined by the critical conditions of pull-in instability. The size-dependent coupling influences of elasticity deformation, electrical energy and Casimir effect are accounted for determining the dynamic behaviors and structural stability of nano-actuator system. The results indicate that the Casimir force has a softening effect on circular nanoplate, thus decreases the fundamental frequency of nano-actuator. On the other hand, the small scale parameter weakens the structural stiffness of nanoplate, so that the frequencies of the nanostructure are declined. Therefore, the values of pull-in voltage of disk-type nano-actuator are over-estimated; consequently, the excessive voltage could lead the devices to fail or damage. New results in this work may be helpful in design of electrostatically actuated nano-devices. This work aims to study the influence of small scale on the dynamic pull-in instability of circular nanoplate under the electrostatic and Casimir forces. Here, the bifurcation results around equilibrium points in Figs.11(a-d)~Fig.15 have practically physical meaning for nano-devices, while other mathematical solutions beyond the non-dimensional physical range $[0, 1]$ have been eliminated. The main conclusions are given by

1) Dynamic frequency and the pull-in voltage increase with the increment of surface modulus due to that surface modulus enhances the structural stiffness of circular nanoplate.

2) The pull-in voltage of circular nanoplate reduces as the surface residual stress increases. The surface residual stress decreases the frequency of circular nanoplate

subjected to the same applied voltage.

3) Both of the pull-in voltage and frequency reduce when radial compressive mechanical loading and temperature change increase.

4) The pull-in voltage decreases as the nonlocal parameter increases and this means the nonlocal parameter has a softening effect on the rigidity of circular nanoplate.

5) At the pull-in voltage, the motion trajectories in phase diagram become closer to the unstable saddle node; by continuing to increase the voltage, the circular nanoplate loses the stable center point and becomes dynamically unstable.

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Appendix A

The associated coefficients of governing equation of electrically actuated nanoplate are defined as follows

$$\begin{aligned}\Psi_0 &= \mu^4 \int_0^1 \rho^5 \phi_m^{IV}(\rho) \phi_k(\rho) d\rho + 6\mu^4 \int_0^1 \rho^4 \phi_m^{III}(\rho) \phi_k(\rho) d\rho \\ &\quad - \mu^2 \int_0^1 (2\rho^5 - 3\mu^2 \rho^3) \phi_m^{II}(\rho) \phi_k(\rho) d\rho \\ &\quad - 3\mu^2 \int_0^1 (2\rho^4 + \mu^2 \rho^2) \phi_m^I(\rho) \phi_k(\rho) d\rho \\ &\quad + \int_0^1 \rho^5 \phi_m(\rho) \phi_k(\rho) d\rho \\ \Psi_1 &= -(K_2 + K_4) \int_0^1 \rho^5 \phi_m(\rho) \phi_k(\rho) d\rho\end{aligned}$$

$$\begin{aligned}
\Psi_2 = & k^s \int_0^1 \left(\rho^5 \phi_m^{IV}(\rho) + 2\rho^4 \phi_m^{III}(\rho) - \rho^3 \phi_m^{II}(\rho) + \rho^2 \phi_m^I(\rho) \right) \phi_k(\rho) d\rho \\
& - (N_0 + 2\tau_0^s) \int_0^1 \left(\rho^5 \phi_m^{II}(\rho) + \rho^4 \phi_m^I(\rho) \right) \phi_k(\rho) d\rho \\
& - \mu^2 \left[k^s + \mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^5 \phi_m^{VI}(\rho) \phi_k(\rho) d\rho \\
& - 7\mu^2 \left[k^s + \mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^4 \phi_m^V(\rho) \phi_k(\rho) d\rho \\
& + 2\mu^2 (N_0 + 2\tau_0^s) \int_0^1 \rho^5 \phi_m^{IV}(\rho) \phi_k(\rho) d\rho \\
& - \mu^2 \left[k^s (3 + 2\nu) + 5\mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^3 \phi_m^{IV}(\rho) \phi_k(\rho) d\rho \\
& + 8\mu^2 (N_0 + 2\tau_0^s) \int_0^1 \rho^4 \phi_m^{III}(\rho) \phi_k(\rho) d\rho \\
& + 2\mu^2 \left[k^s (1 + 2\nu) + 3\mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^2 \phi_m^{III}(\rho) \phi_k(\rho) d\rho \\
& + 2\mu^2 (N_0 + 2\tau_0^s) \int_0^1 \rho^3 \phi_m^{II}(\rho) \phi_k(\rho) d\rho \\
& + 3\mu^2 \left[k^s (1 - 2\nu) - \mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^2 \phi_m^{II}(\rho) \phi_k(\rho) d\rho \\
& - 2\mu^2 (N_0 + 2\tau_0^s) \int_0^1 \rho^2 \phi_m^I(\rho) \phi_k(\rho) d\rho \\
& - 3\mu^2 \left[k^s (1 - 2\nu) - \mu^2 (N_0 + 2\tau_0^s) \right] \int_0^1 \rho^2 \phi_m^I(\rho) \phi_k(\rho) d\rho \\
& - (2K_2 + 4K_4) \int_0^1 \left[\rho^5 - 3\mu^2 (2\rho^2 + \mu^2) \rho^2 \phi_m^I(\rho) \right. \\
& \left. + \mu^2 (2\rho^2 - 3\mu^2) \rho^3 \phi_m^{II}(\rho) + 6\mu^4 \rho^4 \phi_m^{III}(\rho) + \mu^4 \rho^5 \phi_m^{IV}(\rho) \right] \phi_k(\rho) d\rho \\
\Psi_3 = & -(3K_2 + 10K_4) \int_0^1 \left[\rho^5 \phi_m(\rho) \phi_k(\rho) - 6\mu^2 (2\rho^2 + \mu^2) \rho^2 \phi_m^I(\rho) \phi_k(\rho) \right. \\
& + \mu^2 (2\rho^2 - 3\mu^2) \rho^3 \left(2\phi_m^I(\rho) \phi_k^I(\rho) + 2\phi_m(\rho) \phi_k^{II}(\rho) \right) \\
& + 6\mu^4 \rho^4 \left(6\phi_m^I(\rho) \phi_k^{II}(\rho) + 2\phi_m(\rho) \phi_k^{III}(\rho) \right) \\
& \left. + \mu^4 \rho^5 \left(6\phi_m^{II}(\rho) \phi_k^{II}(\rho) + 8\phi_m(\rho) \phi_k^{III}(\rho) + 2\phi_m(\rho) \phi_k^{IV}(\rho) \right) \right] \phi_l(\rho) d\rho \\
\Psi_4 = & -(4K_2 + 20K_4) \int_0^1 \left[\rho^5 \phi_m(\rho) \phi_k(\rho) \phi_l(\rho) - 9\mu^2 (2\rho^2 + \mu^2) \rho^2 \phi_m(\rho) \phi_k(\rho) \phi_l^I(\rho) \right. \\
& + \mu^2 (2\rho^2 - 3\mu^2) \rho^3 \left(6\phi_m(\rho) \phi_k^I(\rho) \phi_l^I(\rho) + 3\phi_m(\rho) \phi_k(\rho) \phi_l^{II}(\rho) \right) \\
& + 6\mu^4 \rho^4 \left(6\phi_m^I(\rho) \phi_k^I(\rho) \phi_l^I(\rho) + 18\phi_m(\rho) \phi_k^I(\rho) \phi_l^{II}(\rho) + 3\phi_m(\rho) \phi_k(\rho) \phi_l^{III}(\rho) \right) \\
& + \mu^4 \rho^5 \left(36\phi_m^I(\rho) \phi_k^I(\rho) \phi_l^{II}(\rho) + 18\phi_m(\rho) \phi_k^{II}(\rho) \phi_l^{II}(\rho) + 24\phi_m(\rho) \phi_k^I(\rho) \phi_l^{III}(\rho) \right. \\
& \left. + 3\phi_m(\rho) \phi_k(\rho) \phi_l^{IV}(\rho) \right) \left. \right] \phi_n(\rho) d\rho
\end{aligned}$$

$$\begin{aligned}
\Psi_5 = & -(5K_2 + 35K_4) \int_0^1 \left[\rho^5 \phi_m(\rho) \phi_k(\rho) \phi_l(\rho) \phi_n(\rho) \right. \\
& - 12\mu^2 (2\rho^2 + \mu^2) \rho^2 \phi_m(\rho) \phi_k(\rho) \phi_l(\rho) \phi_n^I(\rho) \\
& + \mu^2 (2\rho^2 - 3\mu^2) \rho^3 (12\phi_m(\rho) \phi_k(\rho) \phi_l^I(\rho) \phi_n^I(\rho) \\
& + 4\phi_m(\rho) \phi_k(\rho) \phi_l(\rho) \phi_n^{II}(\rho)) + 6\mu^4 \rho^4 (24\phi_m^I(\rho) \phi_k^I(\rho) \phi_l^I(\rho) \phi_n(\rho) \\
& + 36\phi_m(\rho) \phi_k(\rho) \phi_l^I(\rho) \phi_n^{II}(\rho) + 4\phi_m(\rho) \phi_k(\rho) \phi_l(\rho) \phi_n^{III}(\rho)) \\
& + \mu^4 \rho^5 (24\phi_m^I(\rho) \phi_k^I(\rho) \phi_l^I(\rho) \phi_n^I(\rho) + 144\phi_m(\rho) \phi_k^I(\rho) \phi_l^I(\rho) \phi_n^{II}(\rho) \\
& + 36\phi_m(\rho) \phi_k(\rho) \phi_l^{II}(\rho) \phi_n^{II}(\rho) + 48\phi_m(\rho) \phi_k(\rho) \phi_l^I(\rho) \phi_n^{III}(\rho) \\
& \left. + 4\phi_m(\rho) \phi_k(\rho) \phi_l(\rho) \phi_n^{IV}(\rho) \right] \phi_j(\rho) d\rho
\end{aligned}$$

Appendix B

The flowchart of algorithm realization

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Caption of Figures

Fig. 1 Schematic of electrically actuated circular nanoplate with surface layer for (a) top view and (b) side view.

Fig. 2. The influences of surface elasticity modulus on non-dimensional frequency and voltage ($\tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$, $A = 0.2$).

Fig. 3. The influences of surface residual stress on non-dimensional frequency and voltage ($E^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$, $A = 0.2$).

Fig. 4. The influences of axial mechanical loading on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$, $A = 0.2$).

Fig. 5. The influences of temperature change on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$, $A = 0.2$).

Fig. 6. The influences of nonlocal parameter on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $K_4 = 1$, $A = 0.2$).

Fig. 7. The coupling effect of surface elasticity and residual stress on dynamic frequency ($K_2 = 5$, $N_{me} = \Delta T = 0$, $K_4 = 1$, $A = 0.2$).

Fig. 8. The coupling effect of axial mechanical loading and temperature change on dynamic frequency ($K_2 = 5$, $E^s = \tau^s = 0$, $K_4 = 1$, $A = 0.2$).

Fig. 9. The influences of ratio of radius-to-thickness on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $\Delta T = 100 \text{ K}$, $K_4 = 1$, $A = 0.2$).

Fig. 10. The effect of nonlocal parameter on non-dimensional frequency for different ratios of radius-to-thickness ($K_2 = 0$, $E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$,

$\Delta T = 100 \text{ K}$, $K_4 = 1$, $A = 0.2$).

Fig. 11. The transition history of equilibrium points in phase diagram of circular nanoplate when (a) without applied voltage ($K_2 = 0$) and Casimir force, (b) small voltage ($K_2 = 3.0$), (c) near pull-in voltage ($K_2 = 17.8$), and (d) at the pull-in state ($K_2 = 19.23$).

Fig. 12. The time history responses of circular nanoplate under applied voltage and Casimir force ($E^s = 10.605 \text{ nN/nm}$, $\tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$,

$(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$).

Fig. 13. The phase diagram of circular nanoplate under applied voltage and Casimir force ($E^s = 10.605 \text{ nN/nm}$, $\tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$,

$(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$).

Fig. 14. The small scale effect on time history response of circular nanoplate at the dynamic pull-in state ($E^s = \tau^s = 0$, $N_{me} = \Delta T = 0$, $K_4 = 1$).

Fig. 15. The small scale effect on phase diagram of circular nanoplate at the dynamic pull-in state ($E^s = \tau^s = 0$, $N_{me} = \Delta T = 0$, $K_4 = 1$).

Table 1. Influence of Casimir force on deflection by HPM and numerical method.

Table 2. Geometrical and material parameters of micro-plate for validation.

Table 3. Comparison of dynamic pull-in voltages from various methods.

Table 4. The material and geometrical parameters of circular nanoplate.

(a) Top view

(b) Side view

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Fig. 5. The influences of temperature change on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$, $A = 0.2$).

Fig. 6. The influences of nonlocal parameter on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $K_4 = 1$, $A = 0.2$).

Fig. 7. The coupling effect of surface elasticity and residual stress on dynamic frequency ($K_2 = 5$, $N_{me} = \Delta T = 0$, $K_4 = 1$, $A = 0.2$).

Fig. 8. The coupling effect of axial mechanical loading and temperature change on dynamic frequency ($K_2 = 5$, $E^s = \tau^s = 0$, $K_4 = 1$, $A = 0.2$).

Fig. 9. The influences of ratio of radius-to-thickness on non-dimensional frequency and voltage ($E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $\Delta T = 100 \text{ K}$, $K_4 = 1$, $A = 0.2$).

Fig. 10. The effect of nonlocal parameter on non-dimensional frequency for different ratios of radius-to-thickness ($K_2 = 0$, $E^s = \tau^s = 1.0 \text{ nN/nm}$, $N_{me} = 1 \text{ nN/nm}$,

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Fig. 13. The phase diagram of circular nanoplate under applied voltage and Casimir force ($E^s = 10.605 \text{ nN/nm}$, $\tau^s = 1.0 \text{ nN/nm}$, $\Delta T = 100 \text{ K}$, $N_{me} = 1 \text{ nN/nm}$, $(e_0 a)^2 = 1 \text{ nm}^2$, $K_4 = 1$).

Fig. 14. The small scale effect on time history response of circular nanoplate at the dynamic pull-in state ($E^s = \tau^s = 0$, $N_{me} = \Delta T = 0$, $K_4 = 1$).

Fig. 15. The small scale effect on the instable phase diagram of circular nanoplate at the dynamic pull-in state ($E^s = \tau^s = 0$, $N_{me} = \Delta T = 0$, $K_4 = 1$).

Table 1. Influence of Casimir force on deflection by HPM and numerical method.

Initial amplitude: A	Electrostatic voltage: K_2	without Casimir force ($\alpha_4 = 0$)		with Casimir force ($\alpha_4 = 1$)	
		2 nd HPM	Numerical	2 nd HPM	Numerical
0	2.5	0.0129089	0.0129039	0.0180201	0.0179980
	5.0	0.0257259	0.0256876	0.0308019	0.0307126
0.05	2.5	0.0120472	0.0119438	0.0061568	0.0058273
	5.0	0.0020747	0.0020638	0.0081619	0.0080864
0.10	2.5	0.0368892	0.0366446	0.0301626	0.0293288
	5.0	0.022177	0.0212676	0.0149420	0.0140721
0.15	2.5	0.0616543	0.0602191	0.0541304	0.0525407
	5.0	0.0459902	0.0443494	0.0393859	0.0385215
0.20	2.5	0.0863587	0.0856631	0.0768227	0.0754041
	5.0	0.0698275	0.0671754	0.0609557	0.0582059

Table 2. Geometrical and material parameters of micro-plate for validation.

Parameters	Symbol	Value/unit
Radius	R	250 μm
Thickness	h	20 μm
Initial gap	g_0	1 μm
Young's modulus	E	169 GPa
Poisson's ratio	ν	0.30

Table 3. Comparison of dynamic pull-in voltages from various methods.

Method	Step size (V)	Dynamic pull-in voltage (V)	Δ (%)	Reference
Experiment		311.6		Osterberg [61]
Finite element method (FEM)		314	0.77	Raback and Pursula [62]
	$\delta\bar{V}_e = 0.1$	315.1	1.12	
Step-by-step linearization method (SSLM) (N=20)	$\delta\bar{V}_e = 0.05$	315.00	1.09	Rashvand et al. [63]
	$\delta\bar{V}_e = 0.01$	314.89	1.06	
	$\delta\bar{V}_e = 0.001$	314.860	1.05	
Homotopy perturbation method (HPM)		314.878	1.05	Present work

$$\text{-With: } \Delta(\%) = \frac{\text{ABS(Experiment - Num)}}{\text{Experiment}} \times 100$$

Table 4. The material and geometrical parameters of circular nanoplate.

Parameters	Symbol	Value/unit
Thickness	h	5 nm
Radius	R	150 nm
Initial gap	g_0	6 nm
Young's modulus	E	210 GPa
Poisson's ratio	ν	0.27
Mass density	ρ	2370 kg/m ³
CTE	α	2.3×10 ⁻⁶ K ⁻¹
Permittivity of vacuum	ϵ_0	8.854×10 ⁻¹² F/ m

Graphical abstract

Fig.1. The influences of temperature change on curves of non-dimensional frequency and voltage

Fig. 2. The phase diagram of circular nanoplate with applied voltage at the pull-in voltage ($K_2 = 19.23$).