High velocity compressive response of metallic corrugated core sandwich columns

Jae-Yong Lim^{a*} and Hilary Bart-Smith^b

^a New Transportation Systems Research Center, Korea Railroad Research Institute, 176 Cheoldo bangmulgwan-ro, Uiwang, Gyeonggi, 437-757, Republic of Korea

^b Mechanical and Aerospace Engineering Department, University of Virginia, Charlottesville, Virginia, 22904, U.S.A.

* Corresponding author.

E-mail address: jyylim@krri.re.kr (Jae-Yong Lim)

Abstract

The high velocity compressive response of metallic corrugated core sandwich columns was characterized. One end of a column was compressed, either perpendicular-to-corrugations or parallel-to-corrugations, at a uniform velocity up to 100m/s whereas the other end was held fixed. Numerical simulations revealed that the response differs from the low velocity response dominated by inertial stabilization; the perpendicular-to-corrugations compressive response was characterized by repetitive densification in a unit cell while the parallel-to-corrugations one occurred during one-way trip of stress wave. The effects of geometric imperfections, column length, compression velocity were investigated through the analysis on rate independent Al6061-T6 corrugated core sandwich columns. On the other hand, the influence of material strain-rate sensitivity was numerically assessed for SS304 corrugated core sandwich columns. As a result, the front end reaction forces increased with imposed velocity while the peak reaction forces on the back end remained invariant. No significant effect of geometric imperfections was observed, and the column length effect was associated with the time scale: the arrival time of stress wave and reaction force variation. Consequently, the analytical expressions for the response were given in terms of sandwich geometric dimensions and

loading intensity.

Keywords: Corrugated core sandwich columns, stress wave propagation, dynamic buckling, in-plane compression, impact, blast.

Nomenclature

Superscript

(0), (1), ..., (k)	material regions
(back)	back end
(front)	front end
(SW)	corrugated core sandwich column
(perp)	perpendicular-to-corrugations
(para)	parallel-to-corrugations

Subscript

(el)	elastic
(pl)	plastic
(fw)	face wrinkling mode
(g)	global buckling mode

ν	Poisson's ratio
ν_P	plastic Poisson's ratio (here, $\nu_P = 0.5$)
ω	corrugation angle
$\bar{\rho}$	core relative density
ρ	parent material density
σ_Y^0	quasi-static yield strength
$\sigma_Y(\dot{p})$	dynamic yield strength
σ_{eq}	equivalent stress
ε_Y'	yield strain in plane strain condition

ξ	the imperfection amplitude of global curvature
c	core height
$c_{(\cdot)}^{(\cdot)}$	elastic or elastic-plastic wave speed of columns such as corrugated core sandwich columns compressed perpendicular-to-corrugations or parallel-to-corrugations
D	material constant
E	Young's modulus
E_P	plastic modulus
E_T	tangent modulus
E'_T	the tangent modulus in plane strain condition
E^*	effective in-plane core stiffness
h	face sheet thickness
L	column span length
l	core ligament length
m	material constant
P_Y	in-plane load associated with yield stress
P	equivalent plastic strain
$\dot{p}$	equivalent plastic strain rate
R	the dynamic yield strength enhancement ratio
t	core ligament thickness, or time
$\Delta t_{(\cdot)}^{(\cdot)}$	the arrival time of elastic or plastic stress wave from the front end to the back end of sandwich column
V	compression velocity, rate-of-compression

1. Introduction

Metallic core sandwich structures have been recognized as promising structural elements protecting from high intensity dynamic loading. The benefits come from the distinctive attributes in sandwich constructions; distributed mass in the sandwich design considerably reduces momentum transferred from external environments, and proper selection of cores or incorporation of energy dissipating media in empty core sections absorbs the kinetic energy of flying objects [1,2]. Recent investigations on the mechanical response of the sandwich structures when exposed to various types of intense loading extend their potential applicability toward ship hull design against underwater blast [3-5], crashworthiness design [6,7], vehicular armor systems against soil blast [8,9], and ballistic impact mitigation structures [10-13].

The metallic core sandwich structures can also be subject to dynamic in-plane loading. For example, if employed for decks or bulkheads in a ship design, the structures may possibly be exposed to in-plane or oblique impacts. Recently, the authors discussed the response and dynamic effects of corrugated core sandwich columns under in-plane loading [14-16]. In the loading rate considered in those studies, the response is mainly dominated by lateral inertia and material strain-rate dependence, and quite sensitive to geometric curvature imperfections. However, there is a lack of information on the dynamic in-plane response of the metallic core sandwich structures if higher-intensity loading greater than 20m/s is applied.

There are several investigations on the response of metallic core sandwich structures at the compression rate of loading rate ranging up to a few of hundred m/s, which front faces of typical sandwich panels can attain during an underwater blast [17]. However, their main focuses are on the out-of-plane compression of core. St-Pierre et al.[18] investigated the dynamic indentation response of panels with a corrugated or Y-frame

1 core. From their numerical study, the response has been explored by increasing the
2 imposed velocity up to 100m/s for consideration of the structural elements into hull
3 structures or land vehicles against blast loading or collision. In the basic study done by
4 Ferri et al. [19], fundamental understanding on the high velocity dynamic response was
5 gained through investigating the dynamic crushing response of I-core. In addition,
6 Tilbrook [20] and Lee [21] independently investigated the dynamic out-of-plane
7 response of corrugated core and of pyramidal core, respectively. On the other hand,
8 collapse response of constituent inclined core plates was analyzed depending on the
9 compression velocities: (1) inertial-stabilization dominating velocity regime; (2) buckle
10 wave dominating velocity regime; (3) axial-plastic-shock-wave dominating velocity
11 regime [22]. Vaughn et al. [23] presented the buckling wave phenomena when the
12 constituent elements of pyramidal cores are subject to imposed compression velocities
13 on the order of 100m/s. However, there is little known about the dynamic in-plane
14 response of sandwich columns at this range of loading rates.

15 In this study, the dynamic response of corrugated core sandwich columns under high
16 velocities is numerically explored via FEM to gain insight on the dynamic in-plane
17 response. In particular, the effects of applied velocity and sandwich column geometric
18 dimensions on reaction force are noted. The investigation is mainly focused on the
19 slender sandwich columns made of a rate-independent material, Al6061-T6, whose
20 material strain-rate dependence can be neglected. Also, the effect of material strain-rate
21 dependence on the dynamic response is analyzed through FE simulations on SS304
22 corrugated core sandwich columns by comparing two kinds of FE simulations, with and
23 without consideration of the strain-rate sensitivity of the parent material.

24 The paper is organized as follows: In Section 2, details of the FE model are described.
25 In Sections 3.1 and 3.2, the high velocity responses for corrugated core sandwich

columns compressed perpendicular-to-corrugations and parallel-to-corrugations are investigated, respectively. Finally, simple analytic equations describing the observations are suggested in Section 3.3.

2. Numerical Methods

2.1. Specimens and loadings

Consider a corrugated core sandwich column subject to dynamic in-plane loading as shown in Fig. 1. One end of sandwich column (front end) is compressed by a rigid plate at a constant rate of the order of tens of m/s (up to 100m/s) while the other end (back end) is completely fixed. To find the distinction with respect to the loading directions, two in-plane loading directions, *perpendicular-to-corrugations* and *parallel-to-corrugations*, were considered independently as described in Fig. 1(a)(b).

Two kinds of sandwich columns were considered depending on their parent materials: Al6061-T6 columns and SS304 columns. The geometry of sandwich columns is characterized by face sheet thickness, h , the length of corrugation leg, l , core height, c , core angle, ω , column length, L , as indicated in Fig. 1(c). Core relative density, $\bar{\rho}$, which defines the volume fraction occupied by materials in the core section, is given by

$$\bar{\rho}_{Al} = \frac{t/\sin \omega}{\ell \cos \omega + t/\sin \omega} \quad (1)$$

$$\bar{\rho}_{SS304} = \frac{2t}{\ell \sin 2\omega} \quad (2)$$

Taking the general manufacturing range into consideration, the core relative densities in this study were selected from the previous investigations, 25% and 12% for Al6061T6 and SS304 corrugated core sandwich columns, respectively [13, 24].

However, the responses are explored for various geometries by increasing L and h with core relative densities fixed. Our main focuses are on the slender columns whose non-dimensional ratio, L/c, is greater than 40. Column geometries and loading conditions for this investigation are summarized in Table 1.

2.2 FEA

A commercial FE package, ABAQUS/Explicit, is employed for the FE simulations. In-plane compression velocities, V, in the range of 20~100m/s are imposed at one end of sandwich columns such that

$$V / \left(c_{el}^{(SW)} \varepsilon_Y \right) > 1 \quad (3)$$

with a longitudinal wave speed of a sandwich column, $c_{el}^{(SW)}$, and yield strain, $\varepsilon_Y = \sigma_Y / E$. Depending on the in-plane loading orientations, $c_{el}^{(SW)}$, can be given in terms of sandwich geometric dimensions and material properties [14].

$$c_{el}^{(SW)} = \begin{cases} c_{el}^{(perp)} = \sqrt{\frac{E}{(1-\nu^2)(2h + \bar{\rho}c)\rho}} \frac{2h}{2h + \bar{\rho}c}, & \text{perpendicular-to-corrugations} \\ c_{el}^{(para)} = \sqrt{\frac{2hE/(1-\nu^2) + \bar{\rho}cE}{(2h + \bar{\rho}c)\rho}}, & \text{parallel-to-corrugations} \end{cases} \quad (4)$$

where E, ν denote Young's modulus and Poisson's ratio, respectively. The condition written in Eqn. (3) implies that elastic-plastic stress waves are generated at the loaded end due to the high in-plane compressive velocities.

Conventional shell elements (S4R) and 3D continuum elements (C3D8R) or continuum shell elements (SC8R) were meshed for the construction SS304 corrugated core sandwich columns and Al6061-T6 columns, respectively. The parent materials,

Al6061-T6 and SS304, were modeled as a rate-independent elastic-plastic material with linear strain-hardening and a rate-dependent elastic-plastic model with a bilinear strain-hardening as shown in Fig. 2. The aluminum alloy is characterized by Young's modulus $E=75150(\text{MPa})$, the yield stress, $\sigma_Y=293.9(\text{MPa})$ and the plastic hardening modulus, $E_p = 534.2(\text{MPa})$. On the other hand, the quasi-static elastic-plastic behavior of SS304 is described by Young's modulus, $E=230769.2(\text{MPa})$, and the two hardening moduli for the two plastic regions, $E_p^{(1)} = 8117.65(\text{MPa})$ and $E_p^{(2)} = 2460.66(\text{MPa})$, $\sigma_Y^{(0)} = 271.8(\text{MPa})$, $\sigma_Y^{(1)} = 450.0(\text{MPa})$. The rate dependence of SS304 is taken into account by employing the Cowper-Symonds model, in which the dynamic yield strength enhancement ratio, $R = \frac{\sigma_Y(\dot{p})}{\sigma_Y^0}$, (i.e. the ratio of the dynamic yield strength, $\sigma_Y(\dot{p})$, and the quasi-static one, σ_Y^0) is given by a function of equivalent plastic strain rate, $\dot{p}$. The following material parameters $D=4920$ and $m=0.154$ are used for the rate dependence of the stainless steel alloy [19, 20].

$$R = \frac{\sigma_Y(\dot{p})}{\sigma_Y^0} = 1 + \left(\frac{\dot{p}}{D} \right)^m \quad (5)$$

Consequently, the dynamic flow stress can be expressed by a product of the dynamic yield strength enhancement ratio, R , and quasi-static stress, σ^{qs} , which are in terms of equivalent plastic strain rate, $\dot{p}$, and equivalent plastic strain, p , respectively.

$$\sigma(p, \dot{p}) = R(\dot{p}) \cdot \sigma^{qs}(p) \quad (6)$$

The densities of Al6061-T6 and SS304 are used as 2700kg/m^3 and 7920kg/m^3 , respectively.

To incorporate geometric imperfections in the dynamic analyses, static buckling modes were obtained with the mesh for the dynamic simulation. Interesting modes were

selected from the eigenanalysis and inserted into the dynamic simulation. And thus, columns having geometric imperfections can be investigated in the dynamic simulations by relocating the nodal positions of initially perfect column geometry in the pre-processing step.

For the contact between constituent members, a general contact algorithm is defined for core-to-core, face-to-face and face-to-core. In addition, symmetry conditions on the lateral sides are imposed to describe large width dimension of sandwich panel columns. However, fracture due to excessive strain has not been considered in this model. The compression is imposed by a rigid plate, displaced at a constant velocity, V until the characteristic structural response is observed. For example, when there is a loss of axial stability due to global buckling or progressive densification of unit cells observed in the perpendicular-to-corrugations models.

3. Results and Discussion

3.1. High Velocity Response of Sandwich Columns Compressed Perpendicular-to Corrugations

In this section, the high velocity response of sandwich columns compressed perpendicular-to-corrugations is investigated. FE simulations are performed to obtain the response of the sandwich columns of varied geometric dimensions subject to compression velocities up to 100m/s. First, the rate-independent (Al6061-T6) corrugated core sandwich columns are analyzed and subsequently the influence of material strain-rate dependence is assessed numerically for columns manufactured from SS304.

3.1.1 Selective column response

First, we present a typical dynamic column response perpendicular-to-corrugations. The representative column shown in this subsection is an Al6061-T6 sandwich column of $h=2.2\text{mm}$, $L=469\text{mm}$, $\bar{\rho} = 25\%$ ($t=3.175\text{mm}$, $l=22\text{mm}$, $w=60^\circ$). The curves in Fig. 3(a) represents the reaction force curves measured at the front and back ends of the Al6061-T6 sandwich column compressed at $V=60\text{m/s}$. The front end reaction force reveals an instantaneous rise after imposing the compressive velocity, followed by an immediate decay over $100\mu\text{s}$. While this observation at the front end is in progress, no response is detected at the back end. However, the back end reaction force rises to achieve a peak reaction force around $t=150\mu\text{s}$, remain at the peak for a period of time, and declines eventually. Also, additional increase in the front end reaction force is observed at $t=400\mu\text{s}$.

A graphical illustration of corresponding deformation shapes is shown in a time sequential order in Fig. 3(c). The contour plot shows the equivalent plastic strain. The decay in the front end reaction force within $100\mu\text{s}$ can be accounted for by the local buckling of the face ligament closest to the rigid plate as shown in the deformation at $t=70\mu\text{s}$. On the other hand, the rise and decay of the back end reaction force are inferred from the arrival of an elastic stress wave followed by face wrinkling of the face ligament at the back end, which is confirmed by the contour at $t=140\mu\text{s}$ and the magnified view of the deformation near the back end at $t=420\mu\text{s}$. The increase in the front end reaction force around $t=400\sim 600\mu\text{s}$ is explained by the densification of a cell, as shown in the zoom-in view of deformation shape near the front end at $t=420\mu\text{s}$. The densifications, which make the subsequent rise and decay of reaction forces, have been repeated several times until the column collapses due to the appreciable face wrinkling

deformations of back face ligaments (See Fig. 3b)¹. The high-velocity dynamic event happens during the propagation of one elastic wave propagation along the column length, and the dynamic response curves can be characterized through the following parametric analysis.

3.1.2 Reaction Force Characteristics

As demonstrated in Section 3.1.1, the high velocity response of corrugated core sandwich columns loaded perpendicular-to-corrugations is dominated by highly localized buckling deformation. In this section, in-depth analysis on the effects of applied velocity, overall column geometry, and imperfections are conducted to gain insight on the reaction force characteristics.

Effect of Compression Velocity: For applied compression velocities of 20, 60, 100m/s, the reaction forces at the back and front ends are examined in Fig. 4, respectively. The column has the geometric dimensions as the one presented in Section 3.1.1. As shown in Fig. 4(a), all of the back end reaction forces rise after a time period of 100 μ s and maintain their peaks at $P_{\text{back}} = 1500\text{N/mm}$, invariable to the applied compression velocities. In other words, the shapes of the back end reaction forces around their peaks ($0 < t < 300\mu\text{s}$) are independent of loading intensities. This can be interpreted as follows: an elastic stress wave front, whose magnitude and propagation speed are independent of applied velocities, arrives and is reflected at the back end. Subsequently, a face ligament

¹ The cell densification response is more appreciable in Al columns whose face ligament length is short. On the other hand, SS304 columns described in the subsequent section 3.1.3 do not show such a clear response in the front end reaction force.

near the back end buckles by the plastic wave created from the reflection of the elastic wave.

In contrast to the back end reaction forces, the front end reaction forces are dependent on the applied velocities as shown in Fig. 4(b). They drop quickly within approximately 100 microseconds. However, the increase in the front end reaction forces for $V=60\text{m/s}$ and 100m/s are observed again at 420 and 220 μs , respectively, which result from cell densification that causes contact between face and core ligaments.

Effect of Column Length: The effect of column length on the dynamic response is investigated in Fig. 5. Through this investigation, the response accounted for by wave propagation will be corroborated. The column lengths of sandwich columns are varied as $L=469, 938, 1877\text{mm}$ while the same face sheet thickness and core dimensions retained. The column lengths are the same, twice and four times as long as that of that representative column in Section 3.1.1, respectively. The front end reaction forces are invariable to the column lengths, thus confirming highly localized buckling.

On the other hand, each back end reaction force curve is translated along the time axis by $\Delta t_{\text{el}}^{(\text{Perp})}$ such that the arrival time of elastic stress wave to the back end is set to zero. The shape of the back end reaction curves until the load drops ($0 < t - \Delta t_{\text{el}} < 200\mu\text{s}$) are insensitive to column length. Therefore, the high velocity responses are dominated by highly localized buckling deformation irrespective of column lengths. However, the longer columns ($L=938, 1877\text{mm}$) show slightly longer duration at peak loads than the shorter column ($L=469\text{mm}$) does.

Effect of Geometric Imperfection: The effects of imperfections on the dynamic response, specifically the choice of imperfection modes and their magnitudes, are shown in Figs.

6 and 7. First, the dynamic response of an Al6061-T6 sandwich column having a global curvature is investigated. To perform the dynamic FEA, a globally deformed eigenmode from static buckling analysis, (here, mode #7 as shown in Fig. 6(a)) is superposed on the mesh for a perfect geometry. (Mode #7 is the lowest globally-buckled mode whereas mode #1 through #20 except for the mode #7 correspond to face-wrinkled modes.) Figs. 6(b) and(c) show the reaction forces at the front and back ends by varying the imperfection magnitude between $0 < \xi_g < 0.1c$, respectively. However, it is demonstrated that no significant difference is found between them.

Secondly, FEA is performed on a sandwich column having geometrically imperfect face ligaments. To represent such a geometrically imperfect column, an eigenmode of Mode #6 from an additional static buckling analysis is incorporated into the dynamic FE simulations. (Mode #6 is one of face-wrinkled modes among the twenty static buckling modes obtained from the static buckling analysis. And, the maximum wrinkled faces in the selected mode are located in both back and front ends.) Although this type of imperfection is identical to the deformation characteristics of sandwich columns under high velocity compression perpendicular-to-corrugations, there is no significant difference in the front end reaction forces at the imperfection range of $0 < \xi_{fw} < 0.1h$ as shown in Fig. 7(a). It is noted that the imperfection only affects a duration of the back end reaction force at $P=1500\text{N/mm}$ slightly as shown in Fig. 7(b).

To summarize the observations in Figs. 6 and 7, the considered range of imperfections does not cause appreciable differences to the front end reaction force curves due to intense local buckling deformation accompanying substantial plastic deformation at the front end. However, the duration at the peak of back end reaction force is slightly affected by the imperfection in the considered imperfection range. Because a mild intensity of back end reaction force, corresponding to yield stress, is

maintained until the face ligament buckles, the duration will be affected by the imperfections: Put in another way, the face wrinkling near the back end is analogous to the dynamic buckling problem of a plastic column under a constant load associated with σ_Y .

3.1.3 Material Strain-rate Dependence

So far in this section, the FE observations and analytic characterization have been focused on corrugated core sandwich columns fabricated from rate independent materials (Al6061-T6). In this subsection, how the material strain-rate dependence affects the high velocity dynamic response is discussed through FE analysis of SS304 corrugated core sandwich columns.

FE simulations are carried out for SS304 corrugated core sandwich columns of $h=2.5\text{mm}$, $L=707\text{mm}$ subject to $V=40\text{m/s}$ with and without considering material strain-rate dependence. The FE simulation without material strain-rate dependence is computed by suppressing the dynamic strengthening enhancement ratio, $R(\dot{p})$, in the parent material description. The reaction forces from the two FE computations are compared in Fig. 8. It is evident that the higher magnitude in the rate-dependent simulation attributes to the elevation in dynamic yield strength, i.e. the effects of material strain-rate dependence.

To examine whether the rate-dependent result can be inferred from the rate-independent computation, the rate-independent back and front end reaction curves are scaled with $R(\dot{p})$, and then the scaled curves are compared to those of the rate-dependent simulation in Fig. 9. As a result, the reaction force curves from the rate-dependent FE simulation and the R-scaled curves from the rate-independent simulations,

which are created by multiplying the curves by $R(\dot{p})$, show similarity in magnitude and shape.

From the comparative observations of SS304 corrugated core sandwich columns with and without consideration of the material strain-rate dependence, it is confirmed that the response of a rate-dependent corrugated core sandwich column is successfully approximated by the R-scaled response from the rate-independent analysis.

3.2. High Velocity Response of Sandwich Columns Compressed Parallel-to Corrugations

In this section, the high velocity response of corrugated core sandwich columns compressed parallel-to-corrugations is investigated by utilizing FEA. The initial investigation focuses on the response of the rate-independent (Al6061-T6) sandwich columns of varied geometric dimensions subject to compression velocities up to 100m/s with material strain-rate effects excluded. The dynamic response is explored with respect to various geometric dimensions, loading rate, imperfections. And then, to account for the influence of material strain rate-dependence, FE analysis on a rate-dependent (SS304) sandwich column is performed.

3.2.1. Selective response

Representative reaction forces numerically recorded at the front and back ends are plotted in Fig. 10(a). The demonstrated curves are on an Al6061-T6 corrugated core sandwich column compressed parallel-to-corrugations at $V=60\text{m/s}$, having the same geometric dimensions of $h=2.2\text{mm}$, $L=469\text{mm}$, $\bar{\rho} = 0.25$ as that presented in Section 3.1.1.

1 Upon compression at the front end, the front end reaction force rises instantaneously
2
3 to approximately 3500N/mm, and remains at this magnitude for several hundreds of
4
5 microseconds, which differs from the front end reaction characteristics of the
6
7 perpendicular-to-corrugations results in Section 3.1.1. On the other hand, the back end
8
9 reaction force rises after a delay of 100 μ s, and then, a constant reaction force is
10
11 maintained for several hundreds of microseconds.
12
13

14 The dynamic event in the corrugated core sandwich column is described in a time
15
16 sequential order in Fig. 10(b). Equivalent plastic strain is identified by the colored
17
18 contour plot on the deformed column. Upon compression at the front end (left), two
19
20 distinct wave fronts, plastic wave front (red) and elastic wave front (not indicated in the
21
22 contour plot), appear simultaneously as shown at $t=100\mu$ s. At this point, the elastic wave
23
24 propagation is not shown in the contour because the contour represents equivalent
25
26 plastic strain, however, the elastic wave front travelling at the higher wave speed must
27
28 have arrived at the back end before $t=100\mu$ s. The elastic wave front is reflected just
29
30 before $t=100\mu$ s and transformed into a plastic stress wave proceeding toward the front
31
32 end as demonstrated in the contours of $t=200$ and 300μ s. The two plastic wave fronts,
33
34 which are the incident and reflected plastic waves, meet at about mid-point of the
35
36 column length as shown in the contours of $t=400$ and 500μ s. Subsequently, the
37
38 superposition of the two plastic wave leads to local and global buckling growth, which
39
40 is coincident with the drops of the reaction force curves.
41
42
43
44
45
46
47

48 Therefore, a considerable amount of information in the reaction force characteristics
49
50 can be explained by elastic-plastic wave propagation.
51
52
53
54
55
56
57

58 3.2.2. Reaction Force Characteristics

59
60
61
62
63
64
65

1 A parametric analysis is carried out with emphasis on the effects of compression
2 velocity, column length, and imperfections.
3
4
5
6
7

8 *Effect of Compression Velocity:* The front and back end reaction forces of the sandwich
9 column, as a function of applied compression velocities are investigated in Figs. 11(a)
10 and (b), respectively. The column has the same geometric dimensions as the one
11 presented in Section 3.2.1. The magnitude of the front end reaction forces increases in
12 proportion to compression velocity from 40 to 100m/s while the back end reaction
13 forces are invariant to the magnitude of compression velocity.
14
15
16
17
18
19
20
21
22
23

24 *Effect of Column Length:* The investigation of the dynamic response of sandwich
25 columns having different column lengths is illustrated in Fig. 12. This column length
26 investigation will identify the wave propagation phenomenon within this structure and
27 provide an insight on the corresponding reaction force characteristics. The sandwich
28 columns have column lengths twice ($L=938\text{mm}$) and three times ($L=1407\text{mm}$) as long
29 as that ($L=469\text{mm}$) presented in Fig. 12, with the same face sheet thickness ($h=2.2\text{mm}$)
30 and core dimensions ($\bar{\rho} = 0.25$) retained.
31
32
33
34
35
36
37
38
39
40

41 As shown in Fig. 12(a), the durations at the constant level of front end reaction forces
42 are clearly dependent on the column lengths. With regard to back end reaction forces,
43 each back end reaction force curve is translated along the time axis by $\Delta t_{el}^{(Para)}$ such
44 that the arrival time of each elastic stress wave to the back end coincides. The
45 transformed back end reaction curves are shown in Fig. 12(b) with the wave
46 propagation of a longer sandwich column of $L=938\text{mm}$ in Fig. 13. From the illustrated
47 contours in Figs. 10(b) and 13, the excessive deformation initiated near the
48 superposition of the two plastic waves leads to the termination of maintaining a constant
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

level of reaction forces.

Effect of Geometric Imperfections: To show the effects of imperfections, FE analysis on an Al6061-T6 corrugated core sandwich column with a global initial curvature are performed by varying its magnitude up to $\xi_g = 0.1c$. The column has the same geometric dimensions as that presented in Section 3.2.1. The varied magnitudes of the assigned global curvature influence the duration at the constant level of reaction force curves with minimal impact on the magnitude as shown in Fig. 14.

3.2.3. Material Strain-rate Dependence

In this subsection, how the material strain-rate dependence is incorporated into the high velocity dynamic response is discussed through the FE analysis of SS304 corrugated core sandwich columns.

FE simulations are carried out for SS304 corrugated core sandwich columns of $h=2.5\text{mm}$, $L=707\text{mm}$ subject to $V=40\text{m/s}$ with and without considering material strain-rate dependence. The FE simulation without material strain-rate dependence is also computed by suppressing the dynamic strengthening enhancement ratio, $R(\dot{p})$, in the parent material description. The reaction forces from the two FE computations are compared in Fig. 15. It is evident that the higher magnitude in the rate-dependent result attributes to the elevation in dynamic yield strength, i.e., the effects of material strain-rate dependence.

It is important to note that the SS304 back end reaction force curves show significant hardening around $t=700\mu\text{s}$ compared to the Al6061-T6 columns. That is because the plastic part of SS304 is modeled by two plastic hardening moduli in the FE simulations,

$E_p^{(1)} = 8117.65(\text{MPa})$ for small strain region ($0 < p < 0.0164$), and
 $E_p^{(2)} = 2460.66(\text{MPa})$ for large strain region ($p > 0.0164$) as described in Fig. 16. In
other words, the hardening response that emerged in the back end reaction force curves
in Fig. 15(b) is explained by an arrival of the plastic wave associated with the first
strain-hardening region whose corresponding stiffness and wave speed are $E_p^{(1)}$ and
 $c_{pl,1}^{(SW)}$ as depicted in Fig. 16.

To examine if the rate-dependent result can be inferred from the rate-independent
analysis, the rate-independent back and front end reaction curves scaled with R are
compared to the rate-dependent results in Fig. 17. The two results, the rate-dependent
FE reaction force curves and the R -scaled curves from the rate-independent FE curves,
are similar. The only difference is that the hardening in the R -scaled curves appears
stepwise while the rate-dependent FE results shows smooth increase in the hardening
region. This phenomenon can be explained by the findings from the two independent
studies, Ferri [19] and Garnet [26] work: The shape of wave fronts travelling along a
rate-dependent column is indistinct. As a result, associated plastic strain distribution is
quite dispersed [19].

Nevertheless, the rate-dependent response is reasonably approximated from scaling
the rate-independent results. Therefore, it can be concluded that the response of a rate-
dependent material under high velocity compression is approximately predicted from a
rate-independent analysis of treating the rate-dependent material as a rate-independent
material having R -scaled yield stress and tangent modulus.

3.3. Analytic simplification using one-dimensional stress wave theory

3.3.1 Perpendicular-to-corrugations

The high velocity response of corrugated core sandwich columns compressed perpendicular-to-corrugations is sensitive to face wrinkling, in which a face ligament near the front end fails while an incident plastic wave passes one unit cell, and the cell near the back end buckles due to the plastic wave generated by the reflection of an incident elastic stress wave.

The reaction forces of corrugated core sandwich columns compressed perpendicular-to-corrugations can be modeled as shown in Fig. 18. Correspondingly, the peak loads can be deduced from the theory of rate-independent elastic-plastic wave propagations using an analogy between sandwich columns and monolithic columns. The peak of front end reaction force curve is given by [19, 20, 22]

$$\frac{P^{(front)}}{P_Y^{(perp)}} = f_{correction} \cdot \left[1 + \left(\sqrt{\frac{E'_T}{E/(1-\nu^2)}} \right) \left(\frac{V}{c_{el}^{(perp)} \cdot \epsilon'_Y} - 1 \right) \right] \quad (7)$$

where E'_T , ϵ'_Y denote the tangent modulus in plane strain condition, yield strain in

plane strain condition, $\epsilon'_Y \approx \left(\frac{\sigma_Y}{\sqrt{1-\nu_P+\nu_P^2}} \right) / \left(\frac{E}{1-\nu^2} \right)$, and P_Y is the in-plane load associated with yield stress, $P_Y^{(perp)} = \frac{2h\sigma_Y}{\sqrt{1-\nu_P+\nu_P^2}}$. The correction factor, $f_{correction}$, is

the ratio of the in-plane stiffness of a sandwich column with and without the core's in-plane contribution, $f_{correction} = \left(\frac{2hE/(1-\nu^2) + cE_{core}^*}{2hE/(1-\nu^2)} \right)$ and the effective in-plane core stiffness,

E_{core}^* can be derived by including the effect of core axial deformation.

$$E_{core}^* = \frac{(\cos \omega)^3}{\sin \omega} \frac{tE}{l(1-\nu^2)} \quad (8)$$

As seen in Fig. 19(a), the major core deformation mechanism by a low-velocity compression is folding at nodes between face sheets and a core so that the contribution of a core to the in-plane stiffness of sandwich columns is minimal. However, the

deformation of a core strut positioned closest to the front end under high velocity compression differs from the deformation mechanism of the other cores or the deformation mechanism observed under a low velocity compression. The deformation of the core strut closest to the front end is instantaneous so that considerable axial compression and flexural deformation is involved.

On the other hand, the upper and lower bounds of back end reaction force can be obtained assuming the buckling of a face ligament near the back end either by a propagating elastic wave or by its reflected wave.

$$\frac{P_{lower}^{(back)}}{P_Y} \approx 1 \quad (9a)$$

$$\frac{P_{upper}^{(back)}}{P_Y} \approx 1 + \sqrt{\frac{E'_T}{E/(1-\nu^2)}} \quad (9b)$$

In Fig. 20, the analytic expressions given in Eqns. (7) and (9) are compared with FE results of sandwich columns having several face sheet thicknesses of $h=1.31, 2.2,$ and 4.4 mm subjected to various applied velocities. The peaks in back end reaction force computed from FEM are bounded by analytical expressions of Eqn. (9).

For the high velocity response of SS304 sandwich columns, most of deformation is plastic so that the rate-dependent reaction force curves of corrugated core sandwich columns compressed perpendicular-to-corrugations can be approximated by multiplying $R(\dot{p})$, a function of plastic strain rate, from the rate independent simulation. Accordingly, the peak forces considering the material strain-rate dependence of a parent material whose plastic part is modeled by two plastic hardening moduli can be obtained by simply multiplying the rate-independent expressions by $R(\dot{p})$. If V is sufficiently larger than V_{min} , which is approximately 30m/s for the SS304 columns in this study,

$$\left. \frac{P_Y^{(front)}}{P_Y^{(0)}} \right| \approx R(\dot{p}) \cdot f_{correction} \cdot \left[\frac{P_Y^{(1)}}{P_Y^{(0)}} + \sqrt{\frac{E_T^{(2)}}{E/(1-\nu^2)}} \left(\frac{V}{c_{el}^{(0)(perp)} \mathcal{E}_Y^{(0)}} - 1 \right) - \sqrt{\frac{E_T^{(2)}}{E_T^{(1)}}} \frac{(P_Y^{(1)} - P_Y^{(0)})}{P_Y^{(0)}} \right] \quad (10)$$

$$\frac{P_{upper}^{(back)}}{P_Y^{(0)}} \approx R(\dot{p}) \cdot \left[\frac{P_Y^{(1)}}{P_Y^{(0)}} + \sqrt{\frac{E_T^{(2)}}{E/(1-\nu^2)}} - \sqrt{\frac{E_T^{(2)}}{E_T^{(1)}}} \cdot \frac{P_Y^{(1)} - P_Y^{(0)}}{P_Y^{(0)}} \right] \quad (11a)$$

$$\frac{P_{lower}^{(back)}}{P_Y^{(0)}} \approx R(\dot{p}) \quad (11b)$$

$$V_{min} = c_{el}^{(perp)} \mathcal{E}_Y^{(0)} + c_{pl}^{(1)(perp)} \left(\mathcal{E}_Y^{(1)} - \mathcal{E}_Y^{(0)} \right) \quad (12)$$

In the above equations, the third term in the RHS of Eqn. (10) and the second term in the RHS of Eqn. (11a) result from the bilinear modeling of the plastic part in SS304 (See Appendix A.) The superscripts (1) and (2) represent the respective plastic regions while (0) represents the elastic region, as denoted in the σ_{11} - ε_{11} diagram of Fig. 21. Although the relationship between the axial stress versus and the axial strain in the presence of constraint in lateral expansion (i.e., the plane-strain condition) does not generally show a strict discontinuity point around the region of the elastic-plastic transition [20], the relationship between σ_{11} and ε_{11} is assumed as shown in Fig. 21, for convenience. Hence, $E_T^{(k)}$ and $P_Y^{(k)}$ are computed from the relation of $P_Y^{(k)} = 2h \cdot$

$$\sigma_Y^{(k)} \approx 2h \cdot \frac{\sigma_Y^{(k)}}{\sqrt{1-\nu_P+\nu_P^2}} \text{ and, } E_T^{(k)'} = \frac{1}{1/E-\nu/(2E)+3/(4E_P^{(k)})} \text{ respectively [15].}$$

In Fig. 22, the analytical equations (10) and (11) are plotted with the FE calculations for the high velocity response of SS304 corrugated core sandwich columns having $h=1.25$ and 2.5mm with the same L and $\bar{p}$ as the column.

3.3.2 Parallel-to-corrugations

For the sandwich columns compressed parallel-to-corrugations, the front end reaction

force reveals an initial rise followed by a constant peak maintained for some period, and similarly, back end reaction force shows similar response to the front end reaction force after a delay, which is taken for the arrival of an elastic wave, $L/c_{el}^{(Para)}$. Accordingly, the reaction force curve can be simplified as shown in Fig. 23.

Much information about the reaction force characteristics can be explained by elastic-plastic wave propagation. For a rate independent material, peak reaction forces of the front and back end reaction characteristics, $P^{(front)}$ and $P^{(back)}$, can be given by

$$\frac{P^{(front)}}{P_Y^{(para)}} \approx 1 + \left(\frac{\sqrt{2hE'_T + \bar{\rho}cE_T}}{\sqrt{2hE/(1-\nu^2) + \bar{\rho}cE}} \right) \left(\frac{V}{c_{el}^{(para)} \cdot \varepsilon'_Y} - 1 \right) \quad (13)$$

$$\frac{P^{(back)}}{P_Y^{(para)}} \approx 1 + \frac{\sqrt{2hE'_T + \bar{\rho}cE_T}}{\sqrt{2hE/(1-\nu^2) + \bar{\rho}cE}} \quad (14)$$

where $P_Y^{(para)} = \frac{2h\sigma_Y}{\sqrt{1-\nu_P + \nu_P^2}} + \bar{\rho}c\sigma_Y$, $\varepsilon'_Y = \frac{P_Y^{(para)}}{2hE/(1-\nu^2) + \bar{\rho}cE}$. The duration of front end

reaction force curve at $P^{(front)}$ is approximated as $\Delta t^{(para)} \approx L/2c_{pl}^{(Para)}$.

In Fig. 24, the analytic expressions for $P^{(front)}$ and $P^{(back)}$ described in Eqns.(13) and (14) are compared with FE results on the sandwich columns of varied face sheet thickness, h , with retaining the same core dimensions ($\bar{\rho} = 0.25$) and column length ($L=469\text{mm}$). The analytic expressions based on the theory of rate-independent wave propagation agree well with the FE results.

As in Section 3.2.3, the peak forces considering the material strain-rate dependence of a parent material whose plastic part is modeled by two plastic hardening moduli can be obtained just by simply multiplying the rate-independent expressions by $R(\dot{p})$. If V is sufficiently larger than V_{min} , which is approximately 40m/s for the SS304 columns compressed parallel-to-corrugations in this study,

$$\left. \frac{P^{(front)}}{P_Y^{(0)}} \right| \approx R(\dot{p}) \cdot \left[\frac{P_Y^{(1)}}{P_Y^{(0)}} + \sqrt{\frac{2hE_T^{(2)} + \bar{\rho}cE_T^{(2)}}{2hE/(1-v^2) + \bar{\rho}cE}} \left(\frac{V}{c_{el}^{(0)(para)} \mathcal{E}_Y^{(0)}} - 1 \right) - \sqrt{\frac{2hE_T^{(2)} + \bar{\rho}cE_T^{(2)}}{2hE_T^{(1)} + \bar{\rho}cE_T^{(1)}}} \frac{(P_Y^{(1)} - P_Y^{(0)})}{P_Y^{(0)}} \right] \quad (15)$$

$$\frac{P_{upper}^{(back)}}{P_Y^{(0)}} \approx R(\dot{p}) \cdot \left[\frac{P_Y^{(1)}}{P_Y^{(0)}} + \sqrt{\frac{2hE_T^{(2)} + \bar{\rho}cE_T^{(2)}}{2hE/(1-v^2) + \bar{\rho}cE}} - \sqrt{\frac{2hE_T^{(2)} + \bar{\rho}cE_T^{(2)}}{2hE_T^{(1)} + \bar{\rho}cE_T^{(1)}}} \cdot \frac{P_Y^{(1)} - P_Y^{(0)}}{P_Y^{(0)}} \right] \quad (16a)$$

$$\frac{P_{lower}^{(back)}}{P_Y^{(0)}} \approx R(\dot{p}) \quad (16b)$$

$$V_{min} = c_{el}^{(para)} \mathcal{E}_Y^{(0)} + c_{pl}^{(1)(para)} (\mathcal{E}_Y^{(1)} - \mathcal{E}_Y^{(0)}) \quad (17)$$

$$\text{where } \mathcal{E}_Y^{(0)} = \frac{P_Y^{(0)}}{2hE/(1-v^2) + \bar{\rho}cE}, \quad P_Y^{(k)} = 2h \frac{\sigma_Y^{(k)}}{\sqrt{1-v_p + v_p^2}} + \bar{\rho}c\sigma_Y^{(k)}, \quad c_{el}^{(0)} \approx \sqrt{\frac{2hE/(1-v^2) + \bar{\rho}cE}{(2h + \bar{\rho}c)\rho}}.$$

In Fig. 25, the analytic expressions corrected for the inclusion of the material strain-rate effects are compared with FE calculations of SS304 corrugated core sandwich columns of $h=1.25, 2.5\text{mm}$, respectively.

4. Conclusion

This study investigates the dynamic response of corrugated core sandwich columns subject to a velocity in the high velocity region via FE simulations. The peak and duration in reaction forces are parameterized with respect to compression velocities, material properties, and sandwich geometric parameters utilizing the theory of rate-independent elastic-plastic wave propagation. The responses in the high velocity range differs from the low-velocity response in that the response time scale is of the order of a one-way trip time of an plastic stress wave along the column length.

In the perpendicular-to-corrugation loading orientation, face wrinkling dominates the structural response. The face sheet ligaments closest to the front end exhibit face wrinkling before an incident plastic wave passes one unit cell under the intense loading situation. Macroscopically, contact forces between face and core members from progressive densification of unit cells are repeatedly observed. The back end reaction forces are determined by the face wrinkling failure of the face ligament closest to the back end due to the incident elastic wave and its reflected plastic wave. The parameters for the front and back end reaction forces in the in-plane loading orientation are well predicted by the modified analytic expressions.

In the parallel-to-corrugation loading orientation, the failure leading to load drop is the result of global buckling motion when the two plastic waves meet and superpose in the mid-section of the column. The high velocity response of corrugated core sandwich columns compressed parallel-to-corrugations is also well-characterized by the analytic expressions based on the theory of wave propagations.

Acknowledgements

The authors gratefully acknowledge the support of the Office of Naval Research (Dr. David Shifler, Program Officer). And, this research was partially supported by a grant from the R&D program of the Korea Railroad Research Institute, Republic of Korea. The authors gratefully acknowledge Dr. H.N.G. Wadley for the many insightful discussions during this work.

Appendix Analytic Expression for Peak Reaction Forces in a Rate-Independent Elastic-Plastic Material with Two Hardening Moduli.

According to the rate-independent theory of wave propagation [26-28] in an one-dimensional rod,

$$V = \int_0^{\varepsilon} c(\varepsilon) \cdot d\varepsilon \quad (\text{A1})$$

Rewriting Eqn. (A1) for a rate-independent elastic-plastic material with two plastic hardening moduli with the assumption of axial stress-strain behavior as shown in Fig. 13,

$$V = c_{el}'^{(0)} \varepsilon_Y'^{(0)} + c_{pl}'^{(1)} \left(\varepsilon_Y'^{(1)} - \varepsilon_Y'^{(0)} \right) + c_{pl}'^{(2)} \left(\varepsilon - \varepsilon_Y'^{(1)} \right) \quad (\text{A2})$$

$$\varepsilon_Y'^{(0)} = \frac{P_Y'^{(0)}}{(AE)'_{eff}^{(0)}} \quad (\text{A3a})$$

$$\varepsilon_Y'^{(1)} - \varepsilon_Y'^{(0)} = \frac{P_Y'^{(1)} - P_Y'^{(0)}}{(AE)'^{(1)}} \quad (\text{A3b})$$

$$\varepsilon - \varepsilon_Y'^{(1)} = \frac{P^{(front)} - P_Y'^{(1)}}{(AE)'^{(2)}} \quad (\text{A3c})$$

Substituting Eqns. (A3a),(A3b),(A3c) into Eqn. (A2),

$$\frac{P^{(front)}}{P_Y'^{(0)}} = \frac{P_Y'^{(1)}}{P_Y'^{(0)}} - \sqrt{\frac{(AE)_{eff}'^{(2)}}{(AE)_{eff}'^{(1)}}} \left(\frac{P_Y'^{(1)} - P_Y'^{(0)}}{P_Y'^{(0)}} \right) + \sqrt{\frac{(AE)_{eff}'^{(2)}}{(AE)_{eff}'^{(0)}}} \left(\frac{V}{c_{el}'^{(0)} \varepsilon_Y'^{(0)}} - 1 \right) \quad (\text{A4})$$

Upper bound of back end reaction force can be obtained considering wave reflection of elastic stress wave and Eqn. (A1).

$$c_{el}'^{(0)} \varepsilon_Y'^{(0)} = c_{pl}'^{(1)} \cdot (\varepsilon_Y'^{(1)} - \varepsilon_Y'^{(0)}) + c_{pl}'^{(2)} \cdot (\varepsilon - \varepsilon_Y'^{(1)}) \quad (\text{A5})$$

$$\frac{P_{upper}^{(back)}}{P_Y'^{(0)}} \approx \frac{P_Y'^{(1)}}{P_Y'^{(0)}} + \sqrt{\frac{(AE)_{eff}'^{(2)}}{(AE)_{eff}'^{(0)}}} - \sqrt{\frac{(AE)_{eff}'^{(2)}}{(AE)_{eff}'^{(1)}}} \cdot \frac{P_Y'^{(1)} - P_Y'^{(0)}}{P_Y'^{(0)}} \quad (\text{A6})$$

$$\frac{P_{lower}^{(back)}}{P_Y'^{(0)}} \approx 1 \quad (\text{A7})$$

Reference

- [1] Wei Z, Dharmasena KP, Wadley HNG, Evans AG. Analysis and interpretation of a test for characterizing the response of sandwich panels to water blast. *Int J Impact Engng* 2007; 34(10): 1602-1618.
- [2] Xue Z, Hutchinson JW. A comparative study of impulse-resistant metal sandwich plates. *Int J Impact Engng* 2004; 30: 1283-1305.
- [3] Liang Y, Spuskanyuk A, Flores SE, Hayhurst DR, Hutchinson JW, McMeeking RM, Evans AG. The response of metallic sandwich panels to water blast. *J Appl Mech* 2007; 74(1): 81-99.
- [4] McShane GJ, Deshpande VS, Fleck NA. The underwater blast resistance of metallic sandwich beams with prismatic lattice cores. *J Appl Mech* 2007; 74(2): 352-364.
- [5] St-Pierre L, Deshpande VS, Fleck NA. The low velocity impact response of sandwich beams with a corrugated core or a Y-frame core. *Int J Mech Sci* 2015; 91:71-80.
- [6] Hou S, Zhao S, Ren L, Han X. Li Qing. Crashworthiness optimization of corrugated sandwich panels. *Mater Des* 2013; 51: 1071-1084.
- [7] Ehlers S, Tabri K, Romanoff J, Varsta P. Numerical and experimental investigation on the collision resistance of the X-core structure. *Ships Offshore Struct* 2010; 7(1) : 21–29.
- [8] Liu T, Fleck NA, Wadley HNG, Deshpande VS. The impact of sand slugs against beams and plates : Coupled discrete particle/finite element simulations. *J Mech Phys of Solids* 2013; 61(8): 1798-1821.
- [9] Rimoli JJ, Talamini B, Wetzel JJ, Dharmasena KP, Radovitzky R, Wadley HNG. Wet-sand impulse loading of metallic plates and corrugated core sandwich panels. *Int J Impact Engng* 2011; 38(10): 837-848.

- [10] Yungwirth CJ, Wadley HNG, O'Connor JH, Zakraysek AJ, Deshpande VS. Impact response of sandwich plates with a pyramidal lattice core. *Int J Impact Engng* 2008; 35 (8): 920-936.
- [11] Yungwirth CJ, O'Connor JH, Zakraysek AJ, Deshpande VS, Wadley HNG. Explorations of hybrid sandwich panel concepts for projectile impact mitigation. *J. Am Ceram Soc* 2011; 94: S62-S75.
- [12] Wadley HNG, O'Masta MR, Dharmasena KP, Compton BG, Gamble EA, Zok FW, Effect of core topology on projectile penetration in hybrid aluminum/alumina sandwich structures. *Int J Impact Engng* 2013; 62: 99-113.
- [13] Wadley HNG, Dharmasena KP, O'Masta MR, Wetzel JJ. Impact response of aluminum corrugated core sandwich panels. *Int J Impact Engng* 2013; 62: 114-128.
- [14] Lim JY, Bart-Smith H. Theoretical approach on the dynamic global buckling response of metallic corrugated core sandwich columns. *Int J Nonlinear Mech* 2014; 65:14-31.
- [15] Lim JY, Bart-Smith H. An analytical model for the face wrinkling failure prediction of metallic corrugated core sandwich columns in dynamic compression. *Int J Mech Sci* 2015; 92:290-303.
- [16] Lim JY, Bart-Smith H. Dynamic buckling response of long plates for the prediction of local plate buckling of corrugated core sandwich columns. *J Appl Mech* 2015; 82(11): 111008.
- [17] Vaughn DG, Hutchinson JW. Bucklewaves. *Eur J Mech A-Solids* 2006; 25: 1-12.
- [18] St-Pierre L, Fleck NA, Deshpande VS. The dynamic indentation response of sandwich panels with a corrugated or Y-frame core. *Int J Mech Sci* 2015; 92:279-289.
- [19] Ferri E, Antinucci E, He MY, Hutchinson JW, Zok FW, Evans AG. Dynamic buckling of impulsively loaded prismatic cores. *J Mech Mater Struct* 2006; 1(8): 1345-

1365.

[20] Tilbrook MT, Radford DD, Deshpande VS, Fleck NA. Dynamic crushing of sandwich panels with prismatic lattice cores. *Int J Solids Struct* 2007; 44(18-19): 6101-6123.

[21] Lee S, Barthelat F, Hutchinson JW, Espinosa HD. Dynamic failure of metallic pyramidal truss core materials. *Int J Plasticity* 2006; 22: 2118-2145.

[22] McShane GJ, Pingle SM, Deshpande, VS, Fleck NA. Dynamic buckling of an inclined strut. *Int J Solids Struct* 2012; 49(19-20): 2830-2838.

[23] Vaughn DG, Canning JM, Hutchinson JW, Coupled plastic wave propagation and column buckling. *J Appl Mech* 2005; 72(1): 139-146.

[24] Biagi R, Bart-Smith H. In-plane column response of metallic corrugated core sandwich panels. *Int J Solids Struct* 2012; 49(26): 3901-3914.

[25] Khan AS, Huang S. *Continuum Theory of Plasticity*. John Wiley, New York, 1995.

[26] Garnet H, Amen H. One dimensional elasto-plastic wave interaction and boundary reflections. *Comput Struct* 1975; 5(5-6): 327-334.

[27] von Karman T, Duwez P. The propagation of plastic deformation in solids. *J Appl Phys* 1950; 21(10):987-994.

[28] Taylor GI. The plastic wave in a wire extended by an impact load. In: Batchelor GK, editor. *The Scientific Papers of G. I. Taylor Vol. 1 Mechanics of Solids*. Cambridge: University Press (1958) 467-479.

Table Captions

Table 1 Summary of FE simulations.

Figure Captions

Figure 1: In-plane loading orientations: (a) Perpendicular-to-corrugations; (b) Parallel-to-corrugations; (c) geometric parameters.

Figure 2: Quasi-static stress-strain material behavior compared with the constitutive models of (a) Al6061-T6 and (b) SS304.

Figure 3: Typical dynamic response of corrugated core sandwich columns compressed perpendicular-to-corrugations: (a)(b) Reaction force curves on wave-traveling time scale and on a long time scale , and (c) Deformation of an Al6061-T6 sandwich column of $h=2.2\text{mm}$, $L=469\text{mm}$, $\bar{\rho} = 25\%$ ($t=3.175\text{mm}$, $l=22\text{mm}$, $w=60^\circ$) under $V=60\text{m/s}$.

Figure 4: Reaction forces depending on applied velocities of $V=20, 60, 100\text{m/s}$: (a) Back end; (b) Front end.

Figure 5: Length dependence, (a) Front end reaction force, (b) Back end reaction force.

Figure 6: Global buckling mode (mode 7) imperfection sensitivity, (a) imposed eigenmode for the imperfection, (b) front end reaction force, (c) back end reaction force.

Figure 7: Face wrinkling mode (mode 6) imperfection sensitivity, (a) front end reaction force, (b) back end reaction force.

Figure 8: The comparison of reaction forces between rate-dependent FEA and rate-independent FEA: (a) front reaction force, (b) back end reaction force.

Figure 9: Comparison between R-scaled rate-independent analysis and the rate-dependent analysis, (a) front end reaction force, (b) back end reaction force.

Figure 10: Typical dynamic response of corrugated core sandwich columns compressed parallel-to-corrugations: (a) Reaction forces, and (b) Wave propagation of an Al6061-T6

sandwich column of $h=2.2\text{mm}$, $L=469\text{mm}$, $\bar{\rho} = 25\%$ ($t=3.175\text{mm}$, $l=22\text{mm}$, $w=60^\circ$) under $V=60\text{m/s}$.

Figure 11: Velocity-dependent reaction forces at (a) the front end, (b) the back end.

Figure 12: Effects of column length at $V=60\text{m/s}$; (a) front end reaction force, (b) back end reaction force.

Figure 13. Sandwich column length dependence ($L=938\text{mm}$).

Figure 14: Imperfection-sensitive reaction force curves, (a) Front end reaction force, (b) Back end reaction force curves.

Figure 15: Material strain-rate dependence of SS304 corrugated core sandwich columns compressed parallel-to-corrugations; (a) Front end reaction force, (b) Back end reaction force.

Figure 16: Three wave fronts propagated in a rate-independent elastic- plastic material with bilinear hardening moduli.

Figure 17: Comparison between the rate-dependent analysis and the scaled curves from the rate-independent analysis; (a) Front end reaction forces, (b) Back end reaction forces.

Figure 18: Simplified model of the front and back end reaction force curves for corrugated core sandwich columns under high-velocity compression perpendicular-to-corrugations.

Figure 19 : Core contribution to in-plane stiffness depending on the magnitude of compression velocity : (a) low velocity, (b) high velocity.

Figure 20: Analytic prediction of peak reaction forces of corrugated core sandwich columns under high velocity compression perpendicular-to-corrugations, (a) $h=1.31\text{mm}$, (b) $h=2.2\text{mm}$, (c) $h=4.4\text{mm}$.

Figure 21: Approximated σ_{11} - ϵ_{11} diagram of an elastic-bilinear plastic material (SS304) in the presence of lateral expansion constraint.

Figure 22: Analytic prediction of peak reaction forces of rate-dependent (SS304) corrugated core sandwich columns under high velocity compression perpendicular-to-corrugations, (a) $h=1.25\text{mm}$, (b) $h=2.5\text{mm}$.

Figure 23: Simplified model of the front and back end reaction force curves for corrugated core sandwich columns under high-velocity compression parallel-to-corrugations.

Figure 24: Comparison between analytic expressions and FE simulations of compression-velocity dependent peak reaction forces, $P^{(\text{front})}$ and $P^{(\text{back})}$, of Al6061-T6 sandwich columns having various face sheet thicknesses. (a) $h=0.457\text{mm}$, (b) $h=2.2\text{mm}$, (c) $h=4.4\text{mm}$.

Figure 25: Analytic prediction of peak reaction forces of rate-dependent (SS304) corrugated core sandwich columns under high velocity compression parallel-to-corrugations, (a) $h=1.25\text{mm}$, (b) $h=2.5\text{mm}$.

Table 1 Summary of FE simulations.

Type of columns	h (mm)	t (mm)	l (mm)	w (°)	$\bar{\rho}$ (%)	L (mm)	ξ	V (m/s)
Al6061-T6 (perpendicular-to-corrugation)	2.2	3.175	22	60	25	469 938 1877	-	60
	2.2	3.175	22	60	25	469	$\xi^{(g)}: 0.05c, 0.1c$	60
	2.2	3.175	22	60	25	469	$\xi^{(f)}: 0.05h, 0.1h$	60
	1.31	3.175	22	60	25	469	-	20, 40, 60, 80, 100
	2.2	3.175	22	60	25	469	-	20, 40, 60, 80, 100
	4.4	3.175	22	60	25	469	-	20, 40, 60, 80, 100
SS304 (perpendicular-to-corrugation)	2.5	1.5	25	45	12	707	-	20, 40, 60, 80, 100
	2.5	1.5	25	45	12	707	-	40 (Rate independent)
	1.25	1.5	25	45	12	707	-	20, 40, 60, 80, 100
Al6061-T6 (parallel-to-corrugations)	2.2	3.175	22	60	25	469 938 1407	-	60
	2.2	3.175	22	60	25	469	$\xi^{(g)}: 0.05c, 0.1c$	60
	0.457	3.175	22	60	25	469	-	40, 60, 80, 100
	2.2	3.175	22	60	25	469	-	40, 60, 80, 100
	4.4	3.175	22	60	25	469	-	40, 60, 80, 100
SS304 (parallel-to-corrugation)	2.5	1.5	25	45	12	707	-	40, 60, 80, 100
	2.5	1.5	25	45	12	707	-	40 (Rate independent)
	1.25	1.5	25	45	12	707	-	40, 60, 80, 100

Figure 1: In-plane loading orientations: (a) *Perpendicular-to-corrugations*; (b) *Parallel-to-corrugations*; (c) geometric parameters.

(a)

(b)

Figure 2: Quasi-static stress-strain material behavior compared with the constitutive models of (a) Al6061-T6 and (b) SS304.

Figure 3: Typical dynamic response of corrugated core sandwich columns compressed perpendicular-to-corrugations: (a)(b) Reaction force curves on wave-traveling time scale and on a long time scale, and (c) Deformation of an Al6061-T6 sandwich column of $h=2.2mm$, $L=469mm$, $\bar{\rho}=25\%$ ($t=3.175mm$, $l=22mm$, $w=60^\circ$) under $V=60m/s$.

(a)

(b)

Figure 4: Reaction forces depending on applied velocities of $V=20, 60, 100\text{m/s}$: (a) Back end; (b) Front end.

(a)

(b)

Figure 5: Length dependence, (a) Front end reaction force, (b) Back end reaction force.

(a)

(b)

(c)

Figure 6: Global buckling mode (mode 7) imperfection sensitivity, (a) imposed eigenmode for the imperfection, (b) front end reaction force, (c) back end reaction force.

(a)

(b)

(c)

Figure 7: Face wrinkling mode (mode 6) imperfection sensitivity, (a) front end reaction force, (b) back end reaction force.

(a)

(b)

Figure 8: The comparison of reaction forces between rate-dependent FEA and rate-independent FEA: (a) front reaction force, (b) back end reaction force.

(a)

(b)

Figure 9: Comparison between R-scaled rate-independent analysis and the rate-dependent analysis, (a) front end reaction force, (b) back end reaction force.

(a)

(b)

Figure 10: Typical dynamic response of corrugated core sandwich columns compressed parallel-to-corrugations: (a) Reaction forces, and (b) Wave propagation of an Al6061-T6 sandwich column of $h=2.2\text{mm}$, $L=469\text{mm}$, $\bar{\rho} = 25\%$ ($t=3.175\text{mm}$, $l=22\text{mm}$, $w=60^\circ$) under $V=60\text{m/s}$.

(a)

(b)

Figure 11: Velocity-dependent reaction forces at (a) the front end, (b) the back end.

(a)

(b)

Figure 12: Effects of column length at $V=60\text{m/s}$; (a) front end reaction force, (b) back end reaction force.

Figure 13. Sandwich column length dependence (L=938mm).

(a)

(b)

Figure 14: Imperfection-sensitive reaction force curves, (a) Front end reaction force, (b) Back end reaction force curves.

(a)

(b)

Figure 15: Material strain-rate dependence of SS304 corrugated core sandwich columns compressed parallel-to-corrugations; (a) Front end reaction force, (b) Back end reaction force.

Figure 16: Three wave fronts propagated in a rate-independent elastic- plastic material with bilinear hardening moduli.

(a)

(b)

Figure 17: Comparison between the rate-dependent analysis and the scaled curves from the rate-independent analysis; (a) Front end reaction forces, (b) Back end reaction forces.

Figure 18: Simplified model of the front and back end reaction force curves for corrugated core sandwich columns under high-velocity compression perpendicular-to-corrugations.

Figure 19 : Core contribution to in-plane stiffness depending on the magnitude of compression velocity :
(a) low velocity, (b) high velocity.

(a)

(b)

(c)

Figure 20: Analytic prediction of peak reaction forces of corrugated core sandwich columns under high velocity compression perpendicular-to-corrugations, (a) h=1.31mm, (b) h=2.2mm, (c) h=4.4mm.

Figure 21: Approximated σ_{11} - ϵ_{11} diagram of an elastic-bilinear plastic material (SS304) in the presence of lateral expansion constraint.

(a)

(b)

Figure 22: Analytic prediction of peak reaction forces of rate-dependent (SS304) corrugated core sandwich columns under high velocity compression perpendicular-to-corrugations, (a) h=1.25mm, (b) h=2.5mm.

Figure 23: Simplified model of the front and back end reaction force curves for corrugated core sandwich columns under high-velocity compression parallel-to-corrugations.

(a)

(b)

(c)

Figure 24: Comparison between analytic expressions and FE simulations of compression-velocity dependent peak reaction forces, $P^{(\text{front})}$ and $P^{(\text{back})}$, of Al6061-T6 sandwich columns having various face sheet thicknesses. (a) $h=0.457\text{mm}$, (b) $h=2.2\text{mm}$, (c) $h=4.4\text{mm}$.

(a)

(b)

Figure 25: Analytic prediction of peak reaction forces of rate-dependent (SS304) corrugated core sandwich columns under high velocity compression parallel-to-corrugations, (a) $h=1.25\text{mm}$, (b) $h=2.5\text{mm}$.