Role of Fluid Injection in the Evolution of Fractured Reservoirs

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Abstract

A survey is provided of some of the better known examples of quantitative results during fluid injection on number, quality, and weakening effects for fractures in earth reservoirs along with some comparisons to either well-known or better-known theories of both fracture arrival and/or new growth of existing fractures through both fluid injection and stress application. The detailed analyses presented focus on reservoirs having (at worst) orthotropic symmetry.

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1. Introduction

The following discussion is intended to highlight various aspects of fracture analysis as it may be applied to problems in the earth sciences, and particularly in those cases where fluids (water, gas, oil, brine, CO₂, etc.) may be involved. Certainly the importance of understanding how fluids effect the overall mechanical properties when they may be present in existing fractures in such systems is well-known, and of considerable practical importance. The additional role that fluids may play in affecting the overall behavior of these systems while being injected into existing fractures, or their role in either expanding or contracting such existing fractures, or creating new fractures is our main focus throughout.

2. Some measures of scientific and/or engineering significance

Many papers will be presented briefly in the following sections. It therefore seems both necessary and useful to comment briefly on how we might choose to grade or order the many papers discussed here. One popular method of doing so is to consider the number of times each paper has been cited in the literature by others. To account for this measure, the author has researched the citation histories of all the published papers being cited in this paper. However, it is still not clear how these data might be best presented. Should we show the raw numbers for the citations as of the writing of the present paper? Or should there be some normalization by the number of years since the publication of each paper? And how should very recent papers be treated, since they may have either no citations or very few citations as of the time of writing of the present work? The author has chosen NOT to normalize the paper citations by age, but rather to simply quote the numbers in cases where the papers have been in print long enough to accumulate some significant number of citations. In the case of the various newer papers, it was decided that all papers having at least 20 citations or more would have these numbers listed after the paper reference in the following bibliography. For papers with fewer than 20 citations as of the writing of the present paper, the resulting typically rather small number of citations is not being listed here.

3. Early Theories and Analyses of Fractured Systems

Perhaps the earliest attempt to analyze the mechanics of fracture in solids was due to Griffith (1924). His work provides a wide-ranging discussion of fracture of various materials. His main example was glass rods, but he also discusses cracked plates, fibers, along with a brief treatment of applications to liquids. Sack (1946) (on page 730) corrects some mathematical errors in Griffith's paper, especially for materials containing circular cracks, and considers materials containing circular cracks. Sack's results may depend on Poisson's ratio of the material, whereas Griffith's results generally do not. Elliott (1947) generalized Griffith's approach for applications to metals, and provides a discussion of cracks in both 2D and 3D. This work concentrates especially on penny-shaped cracks and Griffith cracks. Results are in agreement with the experimental results of Griffith, but several differing versions of the formulas are also examined. Orowan (1949) presents an extensive review of the Griffith theory, a rederivation of those results based on atomic considerations, and also a detailed discussion about brittle strength of polycrystalline aggregates. However, the work of Brace (1960), which generalizes Griffith's approach and applies it to rocks, is one good example of work more pertinent to our present goals. Rice (1968) points out that the Griffith theory of elastic brittle fracture is also mathematically identical to the theory of fracture based on atomic cohesive forces as presented by Barenblatt (1962).

The works of Dugdale (1960) and Barenblatt (1962) are not themselves so directly pertinent to our targeted earth sciences problems, but they are nevertheless mentioned frequently in the references in later works by Rice (1968), Broberg (1971) and others, and therefore provide useful background for various related fracture analyses and applications.

Another early paper, and one that will play a major role in the later analyses of fluid effects here, is due to Athy (1930). This paper is the earliest one known to the author that introduces a nonlinear (or exponential) dependence into the formulas for mechanical behavior (*i.e.*, elastic deformation) of soils. In particular, Athy shows that

$$D = B + A(1 - e^{-bx}), \tag{1}$$

where D is the density of the soil, B is 1.4 – which is the density of a surface clay, A is 1.3 – which is the maximum density of increase possible, b is a constant, and x is the depth of burial.

Brace, W. F. (1960) provides useful information about early applications of Griffith’s ideas (originally emphasizing applications to glass – but Griffith’s discussion is more wide ranging than that) when applied to rocks. There is also a related preprint available online by Brace on the same topic, having the title: “Dependence of fracture strength of rocks on grain size.”

Warren, J. E., and Root, P. J. (1963) is one of the classic references on double porosity modeling of reservoirs. Porosity is then assumed to be of two types: The first type is called *primary porosity*, being intergranular and controlled by deposition and lithification. It is highly interconnected and typically can be correlated with the permeability. The second type is called *secondary porosity* and is controlled by fracturing, jointing, or solution in circulating water. Secondary porosity is not highly interconnected, and therefore usually not correlated very well with the system permeability. They introduce the now well-known “sugar-cube model” with the matrix (being the primary regions containing porosity) composed of the cubes, and thin rough regions between these cubes being the locations of the fractures.

Broberg, K. B. (1971) gives an extensive discussion of stable and unstable crack growth in several areas of application, but mostly treats sheets of PVC, and metals. He quotes the Dugdale crack model extensively, and also the J integral for crack applications due to Rice. His work compares and contrasts those results with the results of the Griffith energy approach to crack modeling.

Nilsson, F. (1973) emphasizes the fact that Rice’s J-integral formulas are limited to static cases. So the work of Rice (1971), Broberg (1971), and others involving the J-integral cannot be applied directly in cases that involve dynamics, only statics and quasi-statics. However, he also shows how to modify these earlier analyses in order to include inertia effects in order to make the problems more realistic for a wider range of applications.

White, J. W. (1973) provides an efficient method of defining the failure surface in a numerical modeling code. He develops a systematic way of classifying and presenting material failure data for earth media that are approximately isotropic on the average. His functional is defined by

$$I_{2D}^{1/2}(f) = A_1(I_1) + A_2(I_1)I_{3D}^{1/3} + A_3(I_1)I_{3D}^{2/3}, \quad (2)$$

where

$$I_1 = -(\sigma_1 + \sigma_2 + \sigma_3), \quad (3)$$

which is three times the pressure.

$$S_i = \sigma_i - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3), \quad (4)$$

for $i = 1, 2, 3$.

$$I_{2D}^{1/2} = +\{(1/2)[(S_1)^2 + (S_2)^2 + (S_3)^2]\}^{1/2}, \quad (5)$$

$$I_{3D}^{1/2} = (S_1 \cdot S_2 \cdot S_3)^{1/3}. \quad (6)$$

The more recent work by Rubin (1991) is based in part on White’s paper.

Another class of approximations that has repeatedly appeared in the earth sciences related literature is well-represented by the early paper of Budiansky and O’Connell (1976). This paper may well be the first to appear in the geophysics-related literature to reference the work of Bristow (1960) concerning the definition and make use of the “crack density parameters,” which since then have become a recurring theme in many of the pertinent papers published on this topic. This paper introduced what are now standard ideas about how cracks influence the values of elastic constants and electrical conductivities in metals, and later works have applied the ideas to other materials including rocks. The work by Bristow has been used heavily in later works by Sayers and Kachanov (1991), Grechka and Kachanov (2006), Berryman and Grechka (2006), and many other followers of this line of work.

Tullis, J., and Yund, R. A. (1980) also discuss effects of temperature, strain, and strain rate on crack related problems.

Cheng, A. H.-D., and Liggett, J. A. (1984a) is the first of two papers by these authors on an efficient and accurate method for linear consolidation problems. They state their method is always preferred over other numerical methods — including finite element and finite difference methods — for such problems, whenever feasible.

Similarly, Cheng, A. H.-D., and Liggett, J. A. (1984b), present a second paper in their series, which uses the boundary integral equation method (based on a reciprocal principle) to formulate governing equations of Biot’s linear porous-elasticity. Resulting fracture propagation criteria are then based on both elastic and plastic constitutive relations. Applications include failure of overconsolidated clay, earthquake prediction, and underground hydraulic fracturing for oil and gas exploration.

Tullis, J., and Yund, R. A. (1989) present their main result that trace amounts of water at high pressure can strongly enhance the rate of dislocation climb in deformed quartz

aggregates. In general, the concentration of important water-related defects increases with pressure.

Rubin, M. R. (1991) presents a failure surface that is both simple (from the point of view of the algebra required) and isotropic. The paper gives examples of the use of the failure surface for both limestone and Monterey sand. The method is based in part on the earlier work of White (1973), which is quoted often. White's method is much simpler than (but also not as general as) Rubin's approach.

Sayers, C. M., and Kachanov, M. (1991) do not actually reference the work of Bristow (1960) in this paper, but it nevertheless depends heavily on ideas like those concerning crack density introduced earlier by Bristow (1960). These authors do give Bristow appropriate credit in their later works.

Johnson, P. A., and Rasolofosaon, P. N. J. (1996) actually has *no equations*, but includes many useful plots of laboratory data showing examples of nonlinear rock behavior.

Tuncay, K., and Corapcioglu, M. Y. (1996a) discuss some double-porosity models (such as those of Warren and Root) including the possibility of more than one fluid being present in the pores. The theory is strictly limited to linear pressure dependent behaviors however.

Tuncay, K., and Corapcioglu, M. Y. (1996b) continue the analysis of their previous paper by including some air-filled pores, and thus two-fluid modeling. Fast and slow compressional waves are included (as in Biot's theory), a second compressional wave arises due to the fractures, and a fourth compressional wave arises due to the pressure difference between the fluid phases in different porous blocks of their sugar-cube type model as also seen in Warren and Root (1963).

Tsvankin, I. (1997) does not treat nonlinear pressure effects, but does provide a uniform description of anisotropic media with either orthorhombic or transversely isotropic symmetry.

Wang, H. F. (1997) discusses various aspects of undrained compression in saturated sedimentary rock. In addition to Skempton's coefficients, the analysis also emphasizes Henkel's (1960) generalization of Skempton's coefficients, which is especially important for analysis of fully three-dimensional and polyaxial generalizations of Skempton's coefficients.

Vasco, D. W., and Karasaki, K. (2001) set up a numerical inversion scheme with equations that mostly involve linear integral equations. There can be some nonlinear interactions resulting from this formulation, but they are mostly covered within the complex formulation

of the resulting integrals.

Berryman, J. G., Pride, S. R., and Wang, H. F. (2002) present a detailed analysis of a differential effective medium (DEM) scheme applied to rocks with either dry (gas-filled) or liquid-saturated fractures. Results show that as porosity tends to unity, the overall Poisson's ratio of these systems tends towards small positive values for the dry-fracture porous media, and thus tends towards one-half for the liquid-saturated cases. Examples treated include granites and sandstones.

Vasco, D. W. (2004) relates pressure and saturation changes to permeability variations within a reservoir. Theoretical estimates are shown to agree well with observed changes due to water and CO₂ saturation changes.

Grechka, V., and Kachanov, M. (2006a,b) published two papers on rocks with closely spaced and intersecting cracks. The first paper performed a series of 3D finite element simulations on effective elasticity for systems that contain several sets of fractures embedded in otherwise isotropic host rock. Cracks might be close together, so that strong interactions among their stress fields may occur and the cracks may also overlap in some cases. They observed that the Noninteraction Approximation (NIA) nevertheless performed well in all the cases studied. In the second paper, they find that, while the theories of Schoenberg and Kachanov both fit finite-element computational results with comparable accuracy, the Kachanov approach appears to be the more useful one for fracture characterization. In particular, it correctly predicts the crack-induced anisotropy up to orthotropy. Other theories considered in the paper do not appear to predict this behavior quite as well.

Berryman, J. G., and Grechka, V. (2006) treats isotropic cracked systems as ensembles of cracked grains by making use of analogs to isotropic polycrystalline elastic media. They also make use of the numerical results from earlier work of Grechka and Kachanov (2006a,b) which included computer studies on sets of fractures embedded in otherwise isotropic host rock. Berryman and Grechka also found that the spread in values obtained was very comparable to those expected based on the Voigt (1928) and Reuss (1929) bounds. But the Hashin-Shtrikman(1961) bounds were generally found to be much too tight. On the other hand, the self-consistent (Berryman, 1992) estimates obtained at the same time were very good estimators of the average behavior of these systems.

Berryman, J. G., and Nakagawa, S. (2010) show how to deduce drained constants (for connected pores) from undrained (*i.e.*, fluid-saturated pores, but the system of pores may not

be very well interconnected) constants for general anisotropic systems having symmetries ranging from isotropic to orthotropic. Laboratory ultrasound data were inverted for the drained constants in three samples: one of glass beads, and two for angular sand grain packings. Experiments were performed under uniaxial stress, which produced hexagonal (*i.e.*, transversely isotropic) symmetry for the poroelastic response. When the porosity, grain bulk modulus, and pore-fluid bulk modulus were known, the drained constants could then be uniquely determined.

Tsvankin, I., Gaiser, J., Grechka, V., van der Baan, M., and Thomsen, L. (2010) presents little detailed analysis here, but instead gives a general discussion of various issues involving data processing, and fracture characterization. This paper references other work mentioned here of Kachanov (1980) and Schoenberg and Sayers (1995).

Saevik, P. N., Berre, I., Jakobsen, M., and Lien, M. (2013) reconsider the asymmetric self-consistent, symmetric self-consistent, and differential schemes for permeability upscaling in fractured media. For open fractures, only the asymmetric method is seen to have correct asymptotic behavior. The asymmetric method is also found to predict percolation thresholds quite accurately.

Johnson, L. R. (2014) shows how seismic events are characterized in terms of first-degree moment tensors and interpreted in terms of a model combining shear cracks and wing cracks. Results can be used in either static or dynamic moment tensors.

As we shall see in the following discussion, such analyses have been broadly expanded upon in recent times by many researchers, especially when fluid injection plays an important role in the evolving fracture system behavior.

4. Nonlinear Theories of Fluid-Enhanced Fracturing of Earth Materials

After Athy (1930), the next well-known work on nonlinear response of earth materials via fluid injection is apparently that due to Wyble (1958). This author's main result is

$$100\frac{W_P}{W_0} - B = (100 - B)10^{-\alpha P}, \quad (7)$$

where W_0 is the value of the property of interest at zero pressure, W_P is the value at pressure P , and B is the per cent to which the curve is asymptotic. Symbol α is here the coefficient of the exponential decrease (determined from the slope of a semi-logarithmic plot).

This result of Athy (1930) is similar to forms used later by Bandis, Lumsden, and Barton (1983) and also by Barton, Bandis, and Bakhtar (1985):

$$V_m = A + B(JRC) + C(JCS/a_1)^D, \quad (8)$$

where the maximum fracture closure is V_m , JRC is the joint closure strength, JCS is a measure of the joint roughness, D is the exponent of the nonlinear term. Typical experimental values found through multiple regressions of data sets were $A \simeq -0.10$ to -0.30 , $B \simeq -0.007$ to -0.006 , $C \simeq -1.01$ to -2.24 , and $D \simeq -0.23$ to -0.25 . Samples considered in the studies included slate, dolerite, limestone, siltstone, and sandstone, as well as samples from fresh, to moderately weathered, to weathered.

The result of Athy (1930) is also very similar to forms used later by Zimmerman *et al.* (1986), Schoenberg (2002), Daley *et al.* (2006), and Liu *et al.* (2009). For Zimmerman *et al.* (1986), the strain ϵ is given by:

$$\epsilon = AP - Be^{-P/P^*} + D, \quad (9)$$

where A , B , D , and P^* are constants found by least-squares regression. For compressibility C , we have:

$$C = A + (B/P^*)e^{-P/P^*}, \quad (10)$$

which is obtained by differentiating the previous equation with respect to pressure P . This result is commonly known and fairly widely used within the earth sciences community.

Han, D., Nur, A., and Morgan, D. (1986) introduces nonlinear behavior into the equations via

$$V = B_0 + B_1\phi + B_2\sqrt{c} + B_3[p - e^{-Dp}], \quad (11)$$

where V is velocity, p is pressure, c is clay content, ϕ is porosity, and coefficients B_0 , B_1 , B_2 , B_3 , and D are all curve-fitting parameters.

Zimmerman, R. W., Somerton, W., and King, M. S. (1986) find that the compressibility for three quarried, consolidated sandstones (Boise, Berea, and Bandera) is given generally by

$$C = A + (B/P^*)e^{P/P^*}. \quad (12)$$

Constants A and B were found by least-squares regression. The parameter P^* is found via a further nonlinear regression.

FIG. 1: Plot of pore strain versus confining stress using data from Zimmerman *et al.* (1986) for Boise Sandstone. Open circles are for $P_p = 1.75$ MPa, black x's are for $P_p = 8.55$ MPa, and black squares are for $P_p = 15.45$ MPa.

Figure 1 shows some plots of data on pressure-dependent fracture closure due to applied stress in Boise Sandstone as measured and discussed in this work by Zimmerman *et al.* (1986).

Brown, E. T., Bray, J. W., and Santarelli, J. T. (1989) also introduce nonlinear stress-dependence into the formulas for the elastic constants related to boreholes and fractured earth systems. The pertinent equations for Young's modulus E are

$$E = (\alpha\sigma_3 + b)^m \quad (13)$$

and

$$E = B - Ce^{-A\sigma_3}, \quad (14)$$

where $B = E_\infty$ and $C = E_\infty - E_0$. E_0 and E_∞ are respectively the values of E when $\sigma_3 = 0$ and when $\sigma_3 \rightarrow \infty$.

Eberhart-Phillips, D., Han, D.-H., and Zoback, M. D. (1989) established empirical relationships including two equations of the following form which were found for the compressional and shear wave velocities in sandstones:

$$V = B_0 + B_1\phi + B_2\sqrt{C} + B_3[P_e - e^{-DP_e}], \quad (15)$$

where ϕ is porosity, C is clay content, and while D and the B 's are fitting constants, and P_e is effective pressure.

Zimmerman, R. W. (1991) also treats the nonlinear case

$$C_{bc} = C_{bc}^\infty + (C_{bc}^i - C_{bc}^\infty)e^{-P/P^\infty}, \quad (16)$$

where C_{bc} is the zero pressure compressibility, C_{bc}^∞ is the asymptotic high pressure compressibility when cracks are closed. P^∞ is a pressure scaling factor determining the limiting value of the compressibility. This equation is based in part on Wyble (1958) and in part on Zimmerman *et al.* (1986).

Matsuki, K., *et al* (2001), study the nonlinear effects of crack closure in granite under normal stress conditions. They show that the elastic closure Δ is approximately linear in the logarithm of normal stress σ_n . So we have

$$\Delta = A + B \ln \sigma_n \quad (17)$$

with A and B being fitting constants. Time dependent closure is then given by

$$\Delta(\infty) - \Delta(0) = B \ln \frac{Y_E(0)}{Y_E(\infty)}. \quad (18)$$

Figure 2 shows some plots of data on time-dependent fracture closure due to applied stress in Inada Granite as discussed in the work by Matsuki *et al.* (2001).

From Schoenberg (2002), we have:

$$Z_N(\theta) = Z_{N_\infty}(\theta) + [Z_{N_0} - Z_{N_\infty}(\theta)]e^{-\tau_\perp(\theta)/\tau_N(\theta)} \quad (19)$$

FIG. 2: Plot of crack closure versus applied stress for data from Matsuki *et al.* (2001) for Inada Granite (IG). Red squares are for sample 10, black *’s are for sample 15, blue x’s are for sample 20, and green triangles are for sample 25.

and

$$Z_T(\theta) = Z_{T_\infty}(\theta) + [Z_{T_0} - Z_{T_\infty}(\theta)]e^{-\tau_\perp(\theta)/\tau_T(\theta)}. \quad (20)$$

where Z_N and Z_T are constants for excess normal stress and excess tangential stress, respectively.

Malama, B., and Kulatilake, P. H. S. M. (2003) show that the rate of decrease of the jointed rock normal compliance, C_j , with increasing normal compressive stress σ , is expressed in formulas as: $\partial C_j / \partial \sigma = -f(\sigma)$, where $f(\sigma) = ae^{-b\sigma}$ is a function determined in the presented analysis. The relation between the total deformation v and the normal stress σ is

FIG. 3: Plot of crack closure versus applied stress for data from Malama and Kulatilake (2003) for Diorite and Granodiorite rock. Blue x's are for sample Dr-1, red squares are for Grd-1, blue *'s are for Grd-2, and black circles are for Grd-3.

given by

$$v = \frac{a}{b^2}[1 - e^{-b\sigma}] + C_i\sigma, \quad (21)$$

where C_i is the limiting value as $\sigma \rightarrow \infty$. And

$$\Delta v = \Delta v_m[1 - e^{-(\sigma/\sigma_{1/2}) \ln 2}], \quad (22)$$

where Δv_m is the maximum joint closure in the limit as $\sigma \rightarrow \infty$, and given by $\Delta v_m = a/b^2$, and $b = (\ln 2)/\sigma_{1/2}$, where $\Delta v_m/2$ determines the value of $\sigma_{1/2} = \sigma(\Delta v_m/2)$.

Figure 3 shows the types of behavior observed by Malama and Kultatilake (2003).

Neuzil, C. E. (2003), makes good use of the earlier work of Athy (1930). See (1).

Shapiro, S. A. (2003) shows that, for compressibility, we have

$$C_{dr}(P) = C_{drs}[1 - \theta_s(C_{drs} - C_{gr})P + \theta_c\phi_{c0}\exp(-\theta_cPC_{drs})]. \quad (23)$$

And for bulk modulus:

$$K_{dr}(P) = K_{drs}[1 + \theta_s(C_{drs} - C_{gr})P - \theta_c\phi_{c0}\exp(-\theta_cPC_{drs})]. \quad (24)$$

Here K_{drs} and C_{drs} are related by

$$K_{drs} = 1/C_{drs}. \quad (25)$$

The skeleton shear modulus is given by

$$\mu_{dr}(P) = \mu_{drs}[1 + \theta_{s\mu}(C_{drs} - C_{gr})P - \theta_{c\mu}\phi_{c0}\exp(-\theta_cPC_{drs})]. \quad (26)$$

The shear modulus of the dry rock is μ_{dr} in the case of closed compliant porosity, while the stiff porosity is equal to ϕ_{s0} , and

$$\theta_{s\mu} = -\frac{1}{\mu_{drs}} \frac{\partial \mu_{dr}}{\partial \phi_s}, \quad \theta_{c\mu} = -\frac{1}{\mu_{drs}} \frac{\partial \mu_{dr}}{\partial \phi_c}. \quad (27)$$

The quantity θ_s can also be estimated using the Hashin-Shtrikman upper bound for μ_{dr} , which is given approximately by:

$$\theta_{s\mu} \simeq 1 + \frac{6K_{gr} + 2\mu_{gr}}{9K_{gr} + 8\mu_{gr}}. \quad (28)$$

Similarly, Daley *et al.* (2006) show that:

$$Z_T = Z_{T\infty} + [Z_{T0} - Z_{T\infty}]e^{-\sigma_{11}/\tau_T}, \quad (29)$$

$$Z_N = Z_{N\infty} + [Z_{N0} - Z_{N\infty}]e^{-\sigma_{11}/\tau_N}, \quad (30)$$

where Z_N and Z_T again are constants for excess normal stress and excess tangential stress, respectively. Then, the fracture permeability k_T is also determined by

$$k_T = k_{T\infty} + [k_{T0} - k_{T\infty}]e^{-\sigma_{11}/\tau_k}. \quad (31)$$

The decay constants for Z_T , Z_N , and k_T are τ_T , τ_N , and τ_k , respectively. This paper also references earlier work by Bandis, Lumsden, and Barton (1983), and by Barton, Bandis, and Bakhtar (1985).

Chen, C. Y., and Horne, R. N. (2006) treat rough-walled fractures and two-phase (specifically, water and gas) flow problems. They found that their tortuous channel model (or TCM) could be expressed via relative permeabilities given by

$$k_{rw} = 0.74S_w^{*2} + 0.26S_w^* \quad (32)$$

and

$$k_{rg} = 0.43S_g^3 + 0.38S_g^2 + 0.19S_g, \quad (33)$$

where subscripts w and g stand for water and gas respectively, with S_w and S_g being their respective saturations.

Einav, I. (2007a) introduces the concept of “breakage” in the context of granular materials. This work distinguishes between the older “damage mechanics” that had been developed by others for describing decreases in stiffness and strength of “solid-like” materials (unlike granular materials that are assumed to have many distinct grains and gaps between and among the grains’ surfaces when we begin our analysis). Relative breakage B_r is defined by

$$B_r = B_t/B_p, \quad (34)$$

where B_p is breakage potential, and B_t is total breakage. Thus, $0 \leq B_r \leq 1$. The paper gives detailed comparisons to earlier work on both damage mechanics and breakage analysis. He calls this approach “continuum breakage mechanics” or CBM.

Einav, I. (2007b) continues the work of the previous paper by discussing both pure elastic-breakage theory and elastic-plastic breakage. There are examples of comparisons between the proposed theory and experiments on well-graded sands, among several others. Expressions for elastic compressional free energy e_v^e are given by

$$e_v^e - e_{v0} = \frac{(p/p_r)^{1-m} - 1}{K(1-m)}, \quad (35)$$

where e_{v0} is an initial volumetric strain, while m and K are material constants. m typically ranges from 1/3 to 1/2, while taking $m \rightarrow 1$ results in a logarithmic form of this equation.

Zhang, Y., Sayers, C. M., and Adachi, J. I. (2009) derived the following equations which were based in part on the work of Schoenberg (2002):

$$B_N = B_{N_\infty} + [B_{N_0} - B_{N_\infty}]e^{-\sigma_{neff}/\tau_N} \quad (36)$$

and

$$B_T = B_{T_\infty} + [B_{T_0} - B_{T_\infty}]e^{-\sigma_{neff}/\tau_T}. \quad (37)$$

where B_N and B_T are the normal and shear compliances of the fractures. σ_{neff} is the normal effective stress acting normal to the fracture. τ_N and τ_T are characteristic stresses that determine the rate of B_N and B_T with increasing effective stress σ_{neff} .

Salimi, H., and Bruining, H. (2010), continue the analysis of Warren and Root (1963) to include some air-filled pores, and thus two-fluid modeling. Fast and slow compressional waves are included (as in Biot's theory), a second compressional wave arises due to the fractures, and a fourth compressional wave arises due to the pressure difference between the fluid phases in different porous blocks of their sugar-cube type model.

From Liu *et al.* (2009), we have bulk rock compressibility $C_{bc} = \frac{1}{K}$ determined by:

$$C_{bc} = C_{bc}^{\infty} + (C_{bc}^i - C_{bc}^{\infty}) \exp(-\sigma/P), \quad (38)$$

where P is some characteristic (or reference) pressure or stress, i indicates the initial value of the bulk rock compressibility C_{bc} which is then defined by (38), where σ is the applied stress. This work was based in part on earlier work by Wyble (1958), as well as that of Zimmerman *et al.* (1986), Zimmerman (1991), and a few others as quoted in the paper.

This work was then later extended by Liu and Rutqvist (2013), who generalized the work of Liu *et al.* (2009) to multiphase flow in a dual-continuum system, where the second phase (t) contains cracks or fractures, and the first phase (e) involves harder (perhaps rounder and therefore stiffer) pores. Liu, Wei, and Rutqvist (2013) also show that, if b is the averaged fracture aperture, then

$$b = b_{0,e} \left(1 - \frac{\sigma_n}{K_{F,e}}\right) + b_{0,t} \exp\left(-\frac{\sigma_n}{K_{F,t}}\right), \quad (39)$$

where label F is fracture, label 0 is for the unstressed state, label e is for the hard part, and label t is for the soft part. They also found that computed results using these formulas agree well with several previously published data sets.

Das, A., Nguyen, G. D., and Einav, I. (2011) show that the Helmholtz free energy is:

$$H = (1 - \theta B)(1/2)[K(\epsilon_v^e)^2 + 3G(\epsilon_s^e)^2], \quad (40)$$

where B is the internal variable for continuum breakage, K is bulk modulus, G is shear modulus, and

$$p = (1 - \theta B)K\epsilon_v^e, \quad q = 3(1 - \theta B)G\epsilon_s^e. \quad (41)$$

Here p is mean effective stress, q is shear stress, ϵ_v is volumetric strain, elastic volumetric strain is ϵ_v^e , and elastic shear strain is ϵ_s^e . For shear related dissipation, they adopt either

the Hoek-Brown failure criterion, or the Mohr-Coulomb failure criterion. Then, breakage energy is given by

$$E_B = [\theta/2(1 - \theta B)^2](p^2/K + q^2/3G). \quad (42)$$

Rubin, M. B., and Einav, I. (2011) show that the effective bulk and shear moduli are given by

$$K_{eff} \simeq k(1 - \phi)^{3/2}[(1 - \theta B)p_1/k]^{1/2}, \quad (43)$$

and

$$G_{eff} \simeq (g/k)K_{eff}, \quad (44)$$

where ϕ is porosity, B is the breakage coefficient, θ is a grading index, and p_1 is pressure. The scaling constants for shear and bulk modulus are g and k .

Lian, P. Q., Cheng, L. S., and Liu, L. F. (2011) finds that

$$k = k_0 e^{-\alpha_0(p_0 - p)}, \quad (45)$$

where k_0 is initial permeability, k is permeability under stress, p_0 is initial pressure, α_0 is the deformation coefficient, and p is applied pressure. This analysis treats the region outside the wellbore as having a closed zone near the wellbore, and a fractured zone outside the closed zone.

David, E. C., and Zimmerman, R. W. (2012) discuss the gradual closure of compliant pores under stress, and the remaining influence of stiff non-closable pores at high-pressure. They also incorporate results from the differential scheme for the resulting changes in values for the elastic moduli.

Sayers, C. M., and den Boer, L. D. (2012) show that normal and shear compliances are given by

$$B_N(\Delta\sigma'_n) = B_N^0 e^{-\Delta\sigma'_n/\sigma_c} \quad (46)$$

and

$$B_T(\Delta\sigma'_n) = B_T^0 e^{-\Delta\sigma'_n/\sigma_c}. \quad (47)$$

The characteristic stress determining the rate of decrease of fracture compliance with increasing normal compressive stress is given by σ_c . These equations were also derived in part based on the work of Schoenberg (2002).

Bunger, A. P., Gordeliy, E., and Detournay, E. (2013) present data and simulations on hydraulic fractures that tend to grow at shallow depths towards the surface, or in more general situations they curve towards any free surface that happens to be near the fracture.

Katsuki, D., Gutierrez, M., and Almrabat, A. (2014) incorporates into its main equations the same ones as those of presented earlier by Zimmerman *et al.* (1986), and Han *et al.* (1986).

5. Fluid Injection into Orthotropic Fractured Reservoirs

Berryman (2012) treats the poroelastic behavior of orthotropic fractured reservoirs. Our focus here is on fluid injection and how such fluid injection might lead to new or growing fractures. But normal fluids cannot support the storage of any shear energy. Recall [see Mavko *et al.*, (1998)] that an orthorhombic system under hydrostatic stress, or under uniaxial stress applied parallel to the axis of symmetry, or under triaxial stress applied parallel to be axis of symmetry will in all these cases still result in a system also having orthorhombic symmetry. (There remain a few cases in which the resulting symmetry could be either monoclinic or triclinic, but we shall exclude these from consideration here.) So the resulting poroelastic equations we need to use for such applications to orthotropy can all be simplified to:

$$\begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ -\zeta \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & -\beta_1 \\ S_{12} & S_{22} & S_{23} & -\beta_2 \\ S_{13} & S_{23} & S_{33} & -\beta_3 \\ -\beta_1 & -\beta_2 & -\beta_3 & \gamma \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -p_f \end{pmatrix}, \quad (48)$$

where the e_{ii} (no summation over repeated indices) are strains in the $i = 1, 2, 3$ directions. The σ_{ii} are the corresponding stresses. The fluid pressure is p_f . The increment of fluid content is ζ , which (like the strains) is dimensionless. The drained compliances are $S_{ij}^d \equiv S_{ij}$. The drained Reuss average bulk modulus is then defined via

$$\frac{1}{K_R^d} \equiv \sum_{ij=1,2,3} S_{ij}^d, \quad (49)$$

a quantity which is the one commonly taken to be the definition of the bulk modulus of such a simple (non-heterogeneous) anisotropic system. Undrained compliances will be symbolized by S_{ij}^u . For the Reuss average undrained bulk modulus K_R^u , we have drained compliances

replaced by undrained compliances in a formula otherwise identical in form to (49). Off-diagonal coefficients $\beta_i = S_{i1} + S_{i2} + S_{i3} - 1/3K_R^g$, where K_R^g is again the Reuss average modulus of the grains – simply replace d ’s with g ’s in (49) to determine K_R^g .

One good choice of the resulting changes in the fracture aperture is then given by (39). Both the hard and soft porosities are then taken into account along with their appropriate weighting factors, as well as sensitivities to the known distinct sources of pressure effects.

6. Fluid Injection and Fracture Extension

Zhou, X., and Burbey, T. J. (2014) provide a recent review and extensive survey of ways in which fluid injection may influence both fracture initiation and fracture propagation. These authors provide a listing and discussion of 78 pertinent references. We will emphasize a smaller subset of these works here, including only those that are most pertinent to our present goals.

Howard, G. C., and Fast, C. R. (1957) discuss optimal fluid characteristics for successful fracture extension. They emphasize a coefficient C called the fracturing-fluid coefficient. This coefficient is defined in relationship to three types of linear flow mechanisms, which are: (a) viscosity and relative-permeability effects, (b) reservoir-fluid viscosity-compressibility effects, and (c) wall-building effects. They show results for the effect of fracturing-fluid viscosity on area of fracture, for the effect of reservoir fluid on area of fracture, and for the effect of fracturing-fluid coefficient and pump rate on fracture extent, among a total of eight figures.

Rice, J. R., and Cleary, M. P. (1976) study fluid-saturated porous media with compressible constituents while emphasizing stress-induced diffusion effects. This work is so well-known and also sufficiently complicated that we will not attempt to summarize their main results here.

Zoback, M. D., *et al.* (1977) present results of hydraulic fracturing experiments on both intact and prefractured rocks in a laboratory setting. These authors compared results of controlled pressurization rate experiments on Ruhr sandstone and gabbro to those of controlled flow rate experiments on Ruhr sandstone. For pressurization rate of $3.0 \text{ MNm}^{-2}/\text{s}$, the apparent tensile strength was 29.2 MNm^{-2} . For controlled flow rate of 24.3 MNm^{-2} , the apparent tensile strength was 24.3 MNm^{-2} . For further details of these very extensive

experimental results, the original paper should be consulted.

Spence, D. A., and Turcotte, D. L. (1985) study crack propagation when the forcing fluid is magma. They found that the stress intensity factor γ was given approximately by

$$\gamma = k^{1/2}(A_0 - \frac{B}{2}), \quad (50)$$

where $A_0 = 0.5503$, $B = 1.1006$, and $k = 0.9315$. These results were also in reasonable agreement with those from earlier work by Geertsma and Haafkens (1979), where $A_0 = 0.55$, $B = 1.32$, and $k = 0.95$.

Boone, T. J., and Ingraffea, A. R. (1990) present numerical methods for studying hydraulically forced fracture propagation in poroelastic media. They use an equilibrium fracture model based mostly on work of Dugdale (1960) and Barenblatt (1962) to determine the fracture dimensions. The governing equations used are largely based on specialized cases from the more general work of Rice and Cleary (1976).

Sousa, J., *et al.* (1993) present a numerical simulation approach for hydraulic fracturing induced by Newtonian and power-law fluids. Their code is called FRANSYS (FRacture ANalysis SYstem) and it operates on a workstation with 3D graphical display. They present six viable examples of applications for their code. These include penny-shaped cracks with and without leakoff, penny-shaped crack pressure distributions, an inclined elliptical crack, perforated wellbore with Newtonian fluid, and elliptical fractures with power-law fluid.

Durham, W. B., and Bonner, B. P. (1994) treat the interactions of fluid flow and self-propping at high effective pressure and in slightly offset joints.

Figure 4 shows some plots of data on time-dependent fracture closure due to applied stress in Westerly Granite as discussed in the work by Durham and Bonner (1994).

Renshaw, C. E., and Harvey, C. F. (1994) study fracture propagation velocity in a poroelastic medium. They emphasize the distinction between natural and induced fractures. The rate at which fluid can flow from saturated rock into void space created by fracture growth limits the propagation rate of a natural hydraulic fracture. They show that the governing equations for such natural fractures are controlled mostly by three factors: (a) the hydraulic conductivity, (b) the storage coefficient, and (c) the initial flaw length.

Falk, M. A., *et al.* (2001) treat dynamic fracture simulations involving cohesive surfaces. They conclude that simulations of crack branching with cohesive surfaces between elements are often not well-posed in the sense that results continually change with needed mesh

FIG. 4: Plot of crack closure versus applied stress using data from Durham and Bonner (1994) for Westerly Granite. Open circles are for Sample 1 offset, black x's are for Sample 1 mated, and blue squares are for Sample 2 mated.

refinements.

Shet, C., and Chandra, N. (2002), treat the energy balance of cohesive zone models when being used to simulate creation of new fractures. They show that the plastic energy in the material surrounding a crack is not accounted for correctly in the cohesive energy balance. So cohesive zone energy includes all the inelastic energy – such as energy required for cavitation, grain bridging, internal sliding, and surface energy, but excludes all inelastic strain energy in the bounding material.

Detournay, E. (2004) discusses propagation characteristics of fluid-driven fractures in

otherwise impermeable rocks. The analysis focuses on the important distinction that the energy expended to create new fracture surfaces in rock is small compared to the energy dissipated in viscous flow, whereas viscous dissipation is small in comparison to the energy dissipated at the crack tip. The work also discusses significant differences between two- and three-dimensional cracks.

Bunger, A. P, and Detournay, E. (2008), provide experimental results validating the predicted tip asymptotics for a fluid-driven crack. This work brings together theory, computations, and experimental results, and shows graphical comparisons of experimental results to boundary layer solutions. Results show that the tip region of a fluid-driven crack is dominated by the viscous dissipation asymptote rather than by the classical linear elastic fracture mechanics asymptote, as had been previously predicted by this groups' earlier work.

Peirce, A., and Detournay, E. (2008a) apply level set methods to studies of hydraulically driven fractures, while Peirce, A., and Detournay, E. (2008b) apply an Eulerian moving front algorithm for modeling hydraulically driven fractures.

Tagirova, V. R. (2009) treats hydraulic fractures when the injected fluid is non-Newtonian via an exact self-similar solution constructed when the fluid flow rate is known at the fracture inlet. Results show that use of a non-Newtonian hydraulic fracturing fluid may increase the resulting hydraulic fracture's propagation speed. For simple shear flows a complete solution of the problem can be obtained.

Carrier, B., and Granet, S. (2012) use the cohesive zone model to construct a numerical method for various hydraulic fracture problems in already permeable media. Four propagation regimes are considered, including: toughness-fracture storage, toughness-leak-off, viscosity fracture storage, and cases dominated by viscosity-leak-off.

Saevik, P. N., *et al.* (2013) consider permeability upscaling methods in fractured media. Main results presented are for the differential scheme, and modified versions of both the symmetric and asymmetric self-consistent schemes. Advantages of the methods developed include avoidance of singularities found in some earlier versions, and also fewer parameters are needed to evaluate their formulas.

7. Summary and Conclusions

Nonlinear pressure effects are observed in loosely consolidated sands, as was observed early on by Athy (1930). Other nonlinear effects occur in highly fractured materials, both near the surface of the earth and at greater depths. Recent work by Einav (2007a,b) provides a rather different point of view on such behavior, while the earlier work of Zimmerman, Somerton, and King (1986), Sayers and Kachanov (1995), Schoenberg and Sayers (1995), and Schoenberg (2002) – among many others discussed here – have all contributed very useful insights into the nonlinear behavior of many of these systems. Work on these systems will no doubt continue as more and different types of examples of such systems appear in data sets that need to be analyzed and understood for various practical reasons.

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