Model and algorithm for 4PLRP with uncertain delivery time

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Abstract

To address the challenge of logistics routing decision under uncertain environment, this paper studies a fourth party logistics routing problem (4PLRP) with uncertain delivery time (4PLRPU). A novel 4PLRPU model based on uncertainty theory is proposed by describing the delivery time of a third party logistics (3PL) provider as an uncertain variable. After that, the model is transformed into an equivalent deterministic model, and several improved genetic algorithms are designed to get solutions. To handle the problem of infeasible solutions in the proposed 4PLRPU, an improved node-based genetic algorithm (INGA) and an improved distance-based genetic algorithm (IDGA) are developed to reduce the computing time required to repair infeasible solutions, and an improved genetic algorithm based on the simple graph and Dijkstra algorithm (SDGA) is proposed to avoid the generation of infeasible solutions. Numerical experiments are conducted to investigate the performance of the proposed algorithms and verify the effectiveness of the proposed 4PLRPU model. The results show that INGA and SDGA are more effective than the standard genetic algorithm and IDGA at solving large-scale problems. Additionally, compared with the expected value model, the 4PLRPU model is more robust.

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1. Introduction

In supply chain environment, it is accepted that customers' expectations of their suppliers will increase continually[3]. To meet customers' needs, companies turn to third party logistics (3PL) providers. Currently, 3PL has been widely accepted. However, in addition to the professional strength and skills in traditional logistics fields, such as transportation, warehousing, customs clearance and distribution, 3PL providers realize that they must strengthen information technology, knowledge management and strategic alliances to satisfy the requirements of the modern logistics.

There is increasing attention on the development of the fourth party logistics (4PL), which strives to the integration and management of the logistics supply chain. The concept of 4PL was put forward by Andersen Consulting [18]: "It is an integrator that assembles the resources, capability, and technology of its own organization and other organizations to design, build and run comprehensive supply chain solutions." Compared with 3PL, the inevitabilities and advantages of 4PL are gradually and widely becoming recognized [46]. 3PL providers are turning to 4PL to build closer relationships amongst the participants along the supply chain, to support cost cutting initiatives, to develop the flexibility to deal with supply and demand uncertainties and, ultimately, to have a positive impact on the bottom-line [45].

Since 4PL was introduced, many scholars have devoted research time and effort. First, researchers discussed the concepts of 4PL and revealed that 4PL is emerging as a breakthrough solution to modern supply chain challenges [3, 13]. Second, some literature has illustrated the relationship between 3PL and 4PL. Some have studied performance evaluation problems between 4PL and 3PL providers [6, 8, 31], and others have studied the contract design to benefit them both [27, 32]. Third, researchers have focused on the 4PL routing problem (4PLRP). Huang et al. [24] studied the basic 4PLRP. Zhang et al. [50] researched the 3PL selection problem in the 4PLRP. Huang et al. [25] discussed optimization of the 4PLRP with soft time windows. Cui et al. [10] and Huang et al. [26] studied the 4PLRP with fuzzy duration time. Li et al. [33] worked on the multiple transportation modes selection problem in the 4PLRP.

The 4PLRP is a fundamental and unavoidable problem in 4PL's actual operation. Its core content is to manage and optimize the distribution process from the source to the destination; this directly relates to the operational efficiency [28]. The 4PLRP includes two main issues: 3PL providers' selection and path selection. Some scholars [26] deal with these two issues integrally, and use the graph theory to simplify the 4PLRP and clarify the problem's description. Then, the complexity of a graph structure directly relates to the difficulty of the problem. A 4PLRP case can be described as a complex multi-graph network including many alternative 3PL providers, routes and transit cities. Note that in this multi-graph network, more than one edge may exist between two nodes, which dramatically increases the complexity of the problem.

Most of the previous literature on 4PL is worked under deterministic environments [24], but decisions in reality are usually made in the state of indeterminacy [37]. For modeling indeterminacy, there are two types of situations. One is to estimate the probability distribution with enough samples. In this situation, the indeterminacy can be described as randomness based on probability theory [30]. The other is that no samples or not enough samples can be collected to estimate a probability distribution. In this situation, people have to invite some domain experts to evaluate the belief degree that events will occur. Perhaps some people think that the belief degree should be modeled by subjective probability or fuzzy set theory. Tversky and Kahneman [44] claimed that, for both positive and negative prospects, people overweight low probabilities and underweight moderate and high probabilities. Liu [37] claimed that human beings usually overweight unlikely events. Then, the belief degree function will have a much larger variance than the long-run cumulative frequency. Hence, the belief degree cannot be treated as a probability distribution. At the same time, fuzzy set theory is not self-consistent in mathematics and may lead to wrong results in practice [37]. Therefore, the belief degree can neither be described as a stochastic number nor a fuzzy number.

Thus, a new methodology based on belief degree, called uncertainty theory [37], is incorporated into our model to describe the delivery time of 3PL providers. Uncertainty theory, proposed in 2007 and refined in 2010 by Liu [34, 36], is another efficient tool for handling nondeterministic information with expert data [17]. Currently, uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty and has been applied to many areas, including economics [14, 49], finance [9, 20, 29] and

management [17, 47, 48]. For example, Mou et al. [41] researched on an emergency scheduling problem in a transportation network in which the truck times and transportation costs were assumed as uncertain variables. Han et al. [21] studied the maximum flow problem in an uncertain network in which the edge capacities were assumed as uncertain variables. Gao [16] studied the α -shortest path problem and the most shortest path problem in a simple graph based on uncertainty theory.

In this paper, the uncertain programming model of 4PLRP with uncertain delivery time (4PLRPU) is established, where the delivery time is described as an uncertain variable. Because the uncertain measure is included, the 4PLRPU model is difficult to solve. Therefore, the 4PLRPU model is equivalently transformed into a deterministic model, which is regarded as a constrained shortest path problem (CSPP) in a multi-graph. To solve the CSPP, the basic and popular early approaches are the k-shortest path algorithm [12] and dynamic programming [4, 22]. However, these approaches are mainly designed for simple graphs. Huang et al. [26] showed that these approaches are not effective enough to solve CSPP in a multi-graph. Another approach widely used in recent years is the evolutionary algorithm (EA) [5]. There are different types of EAs [2, 19, 42], in which the genetic algorithm [23] is one of the most classic EAs with good performance in solving CSP-P. Therefore, to solve the proposed model after equivalent transformation, an improved enumeration algorithm (ENUM), genetic algorithm (GA) and three improved GAs are designed from different perspectives. The infeasible solution that is not a connected route from the source to the destination is one of the key factors impacting the search efficiency for the proposed problem. Therefore, an improved node-based genetic algorithm (INGA) and an improved distance-based genetic algorithm (IDGA) are proposed to reduce the running time and computing cost of repairing this type of infeasible solutions, and an improved genetic algorithm based on the simple graph and Dijkstra algorithm (SDGA) is proposed to avoid this type of infeasible solution. Numerical experiments are conducted to investigate the performance of the proposed algorithms. Comparison and analysis show the effectiveness of the proposed model and algorithms.

The remainder of this paper is organized as follows: Section 2 introduces basic concepts and properties of uncertainty theory used throughout this paper. Section 3 presents the problem description and formulation of the proposed 4PLRPU. Section 4 introduces the proposed algorithms in detail. In Section 5, numerical examples are used to investigate the performance of

the proposed algorithms and the robustness of the 4PLRPU model. Finally,
Section 6 provides a brief summary of this paper.

2. Preliminary

In this section, basic concepts of the uncertainty theory are presented briefly.

Let Γ be a nonempty set, and $\mathcal{L}$ a σ -algebra over Γ . Each element $\Lambda \in \mathcal{L}$ is assigned a number $\mathcal{M}\{\Lambda\}$. In order to ensure that the number $\mathcal{M}\{\Lambda\}$ has certain mathematical properties, Liu [34] presented the following three axioms:

Axiom 1. (*Normality Axiom*) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ .

Axiom 2. (*Duality Axiom*) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ .

Axiom 3. (*Subadditivity Axiom*) For every countable sequence of events $\Lambda_1, \Lambda_2, \dots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Definition 1 (Liu [34]). The set function $\mathcal{M}$ is called an uncertain measure if it satisfies the normality, self-duality, and countable subadditivity axioms.

Definition 2 (Liu [34]). An uncertain variable is a measurable function ξ from an uncertain space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real number, i.e., $\{\xi \in \mathcal{B}\}$ is an event for any Borel set $\mathcal{B}$.

Definition 3 (Liu [34]). Let $\xi_1, \xi_2, \dots, \xi_n$ be uncertain variables, and f a real-valued measurable function. Then $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$ is an uncertain variable defined by

$$\xi(\gamma) = f(\xi_1(\gamma), \xi_2(\gamma), \dots, \xi_n(\gamma)), \forall \gamma \in \Gamma.$$

Definition 4 (Liu [34]). The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}$$

for any real number x .

134 **Definition 5** (Liu [34]). *An uncertain variable ξ is called normal if it has a*
 135 *normal uncertainty distribution*

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, x \in R$$

136 *denoted by $\mathcal{N}(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.*

137 **Definition 6** (Liu [34]). *Let ξ be an uncertain variable. Then the expected*
 138 *value of ξ is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\}dr - \int_{-\infty}^0 \mathcal{M}\{\xi \leq r\}dr$$

139 *provided that at least one of the two integrals is finite.*

140 **Definition 7** (Liu [34]). *Let ξ be an uncertain variable with finite expected*
 141 *value e . Then the variance of ξ is*

$$V[\xi] = E[(\xi - e)^2].$$

142 **Theorem 1** (Liu [35]). *Assume that the constraint function $g(x, \xi_1, \xi_2, \dots, \xi_n)$*
 143 *is strictly increasing with respect to $\xi_1, \xi_2, \dots, \xi_k$ and strictly decreasing with*
 144 *respect to $\xi_{k+1}, \xi_{k+2}, \dots, \xi_n$. If $g(x, \xi_1, \xi_2, \dots, \xi_n)$ are independent uncertain*
 145 *variables with uncertainty distribution $\Phi_1, \Phi_2, \dots, \Phi_n$, respectively, then the*
 146 *chance constraint*

$$\mathcal{M}\{g(x, \xi_1, \xi_2, \dots, \xi_n) \leq 0\} \geq \alpha$$

147 *holds if and only if*

$$g(x, \Phi_1^{-1}(\alpha), \dots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)) \leq 0,$$

148 *where $\Phi_k^{-1}(\alpha)$ is the inverse function of Φ_k under confidence level α , and*
 149 *called inverse uncertainty distribution of ξ_k .*

150 **Theorem 2** (Liu [36]). *Let ξ_1 and ξ_2 be independent normal uncertain vari-*
 151 *ables $\mathcal{N}(e_1, \sigma_1)$ and $\mathcal{N}(e_2, \sigma_2)$, respectively. Then the sum $\xi_1 + \xi_2$ is also a*
 152 *normal uncertain variable $\mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2)$, i.e.,*

$$\mathcal{N}(e_1, \sigma_1) + \mathcal{N}(e_2, \sigma_2) = \mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2).$$

153 3. Problem description and formulation

154 The 4PLRP can be defined as selecting logistics companies for a 4PL to
 155 optimize the supply chain [26]. One key problem of 4PLRPU in this paper is
 156 **choosing** 3PL providers to finish the transportation task from the source to
 157 the destination to ensure cost minimization and the satisfaction of customer's
 158 time constraint.

159 Assume that a 4PL enterprise undertakes a task from the source to the
 160 destination. The transportation network and candidate 3PL providers are
 161 determined. To more concisely and clearly describe the problem, an undi-
 162 rected multi-graph is used to describe the transportation network as is shown
 163 in Fig. 1. V is the set of nodes, which represent depots, material warehouse,
 164 sale location, etc. $|V| = n$ is the number of nodes. Node v_s represents
 165 the source and node v_t represents the destination. Other nodes are transit
 166 points. E is the set of edges, which represent different 3PL providers that
 167 are available to provide the transportation service between connected nodes.
 168 In such a multi-graph, there is more than one edge (3PL provider) between
 169 each pair of nodes. Nodes and edges have properties of cost and time. Under
 170 uncertain environment, the 3PL provider's delivery time is obtained from
 171 some domain experts; therefore, an uncertain variable is used to address it.
 172 The 4PL needs to select a feasible route which includes interconnected nodes
 173 and edges from the source to the destination, as red lines show in Fig. 1.

To formulate the problem, parameters are defined as follows.

Parameters

r_{ij}	The number of edges (3PL providers) between node i and node j .
e_{ijk}	The k^{th} edge between node i and node j , where $i, j \in (1, 2, \dots, n)$, and k is the index number of edges. For example, e_{122} is the second edge between node v_1 and node v_2 .
C_{ijk}	The cost of the 3PL provider represented by e_{ijk} .
$T_{ijk} + t_{ijk}$	The delivery time of the 3PL provider represented by e_{ijk} . T_{ijk} is the basic time. t_{ijk} is an uncertain variable, which represents time perturbation.
C'_j	The cost of node j , including the cost of processing, inventory, loading and unloading.
T'_j	The time of node j .
T_0	The customer's required due date.
α	The confidence level.

Figure 1: Multi-graph of the 4PLRP.

175 To generalize more, we assume that t_{ijk} are independent uncertain vari-
 176 ables with uncertainty distribution of $\Phi_{ijk}(t)$ and subject to normal uncer-
 177 tainty distribution of $\mathcal{N}(\mu_{ijk}, \sigma_{ijk})$, where μ_{ijk} and σ_{ijk} are the expected val-
 178 ue and the standard variance, and σ_{ijk}^2 is the variance. The confidence level
 179 $\alpha \in (0.5, 1)$.

180 With the definition of parameters, the mathematical model for the pro-
 181 posed 4PLRPU can be built. Note that 3PL provider's delivery time for
 182 each edge consists of two parts T_{ijk} and t_{ijk} , where T_{ijk} is a constant, and
 183 t_{ijk} is an uncertain variable. The goal of this problem is to find a route from
 184 the source to the destination at minimum cost, and is also subject to time
 185 constraint, which requests the belief degree of completing the task before the
 186 due date be larger than the required confidence level. Then, the uncertain
 187 programming model of 4PLRPU is presented as follows:

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{r_{ij}} C_{ijk} x_{ijk}(R) + \sum_{j=1}^n C'_j y_j(R) \quad (1)$$

188 s.t.

$$\mathcal{M} \left\{ \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{r_{ij}} (T_{ijk} + t_{ijk}) x_{ijk}(R) + \sum_{j=1}^n T'_j y_j(R) \leq T_0 \right\} \geq \alpha \quad (2)$$

$$x_{ijk}(R) = \begin{cases} 1, & \text{if } e_{ijk} \in R \\ 0, & \text{else} \end{cases} \quad (3)$$

$$y_j(R) = \begin{cases} 1, & \text{if } v_j \in R \\ 0, & \text{else} \end{cases} \quad (4)$$

$$R = (v_s, \dots, v_i, k, v_j, \dots, v_t) \in G \quad (5)$$

192 In the formulation, Eq. (1) is the objective function, i.e. minimizes the
 193 total cost, including node costs and edge costs. Eq. (2) means that the total

time needed on the route should not be more than the due date T_0 with at least the given confidence level α . $x_{ijk}(R)$ and $y_j(R)$ in Eq. (3) and Eq. (4) are 0-1 variables. Eq. (3) represents whether edge e_{ijk} is included in route R , and Eq. (4) represents whether node v_j is included in route R . Eq. (5) is used to ensure the selected route is legal, which means that nodes and edges of the route R must be interconnected one by one, and must start from the source and end at the destination. Furthermore, there is no loop in the route R .

The 4PLRPU model with uncertain measure $\mathcal{M}$ is difficult to solve, therefore, based on uncertainty theory (Theorem 1), the Eq. (2) can be equivalently written as Eq. (6) shows, where the chance constraint is transformed into a deterministic one. $\Phi_{ijk}^{-1}(\alpha)$ is the inverse uncertainty distribution of $\Phi_{ijk}(t)$.

$$\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{r_{ij}} \Phi_{ijk}^{-1}(\alpha) x_{ijk}(R) + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{r_{ij}} T_{ijk} x_{ijk}(R) + \sum_{j=1}^n T'_j y_j(R) \leq T_0 \quad (6)$$

Since $t_{ijk} \sim \mathcal{N}(\mu_{ijk}, \sigma_{ijk})$, we have

$$\Phi_{ijk}^{-1}(\alpha) = \mu_{ijk} + \frac{\sqrt{3}\sigma_{ijk}}{\pi} \ln \frac{\alpha}{1-\alpha}.$$

4. Algorithm design

Because the 4PLRPU model with uncertain measure and uncertain variables is transformed into a deterministic one, the model can be considered as a CSPP in a multi-graph. Liu et al. [38] stated that the CSPP is an NP-hard problem. Furthermore, for the deterministic equivalent problem of the 4PLRPU, there is more than one 3PL provider between two nodes; therefore, the number of alternative solutions grows exponentially, which increases the difficulty in solving the problem. Thus, EA is introduced to solve this deterministic equivalent problem. GA [23] is one of the most widely used EAs in recent decades to solve constrained optimization problems [43]. Because of the strong search capability, controllable search process and easy improvability, many scholars [38, 40] use GA to solve the CSPP. In this research, several improved GAs are designed to solve our problem.

When solving the 4PLRPU model using GA, the infeasible solution that is not a route from the source to the destination is one of the key factors

223 impacting the search efficiency. Repairing these infeasible solutions is com-
 224 plicated and time-consuming. Therefore, IDGA and INGA are proposed to
 225 reduce the running time and computing cost of repairing this type of infeasible
 226 solution and SDGA is proposed to avoid this type of infeasible solution.
 227 Furthermore, we try to transform the complex problem into multiple simple
 228 problems and solve these sub-problems using classic methods. INGA is de-
 229 signed from the perspective of the k -th shortest path, and SDGA is designed
 230 from the perspective of the constrained Dijkstra algorithm [11].

231 To evaluate the efficiency of the proposed algorithms, ENUM is designed
 232 to search for the optimal solutions of small-scale problems, and is treated as
 233 a benchmark for the other algorithms.

234 4.1. ENUM

235 Because the multi-graph has a huge solution space, the general enumer-
 236 ative algorithm may not be effective. To obtain the optimal solution of the
 237 proposed 4PLRPU, ENUM is designed in two steps, which are briefly de-
 238 scribed as follows:

- 239 (1) Simple graphs are enumerated based on the multi-graph instead of enu-
 240 merating routes directly. The simple graph should contain all of the
 241 nodes on the multi-graph. For each pair of nodes, if there are edges
 242 between them in the multi-graph, only one should be chosen into the
 243 simple graph.
- 244 (2) The constrained shortest path problem (CSPP) on every simple graph is
 245 solved with the constrained Dijkstra algorithm [11].

246 4.2. GA

247 With the advantages of strong search capability, controllable search pro-
 248 cess, easy provision of near-optimal solutions, easy improvability, etc, GA is
 249 considered as one of the most popular EAs.

First, an upper triangular matrix ($F = (f_{ij})_{n \times n}$) of dimension $n \times n$ is
 used to represent the multi-graph. Each element $f_{ij} \in F$ indicates whether
 node i is linked to node j directly in the multi-graph. If there is no edge
 between node i and node j , f_{ij} is set to zero; otherwise, f_{ij} is set to the
 number of edges between node i and node j . For example, the corresponding
 matrix of the multi-graph in Fig. 1 is shown in Eq. (7). In matrix F_1
 ($F_1 = (f_{ij})_{7 \times 7}$), the element value indicates the number of edges between two
 nodes. Therefore, the multi-graph can be simply represented by a nonzero

digital string (342223432332), which is constructed according to the nonzero elements in the matrix from left to right and top to bottom.

$$F_1 \begin{matrix} & v_s & 1 & 2 & 3 & 4 & 5 & v_t \\ \begin{matrix} v_s \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ v_t \end{matrix} & \begin{bmatrix} 0 & 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad (7)$$

Then, when a route is selected, it can also be represented by an upper triangular matrix. Each element f_{ij} represents **the edge between node i and node j that is selected by the task** or which edge exists on the route. If there is no edge selected between node i and node j , f_{ij} is set to zero; otherwise, f_{ij} is set to the marked number of the selected edge. Only one edge can be selected between two adjacent nodes. Therefore, the route can also be simply represented by a digital string. This string is the code of a solution (Chromosome). For example, the red line route in Fig. 1 can be represented as (020001000200); the corresponding matrix F_2 is shown in Eq. (8). Each number of the code (Gene) denotes the selected edge between two nodes at the corresponding position. $f_{13} = 2$ means the second edge between v_s and v_2 is selected.

$$F_2 \begin{matrix} & v_s & 1 & 2 & 3 & 4 & 5 & v_t \\ \begin{matrix} v_s \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ v_t \end{matrix} & \begin{bmatrix} 0 & \underline{0} & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{0} & \underline{0} & \underline{0} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \underline{0} & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{0} & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{0} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad (8)$$

250 Because it is a basic GA, the detailed process will not be described here.

251 4.3. IDGA

252 There are two crucial issues affecting the search efficiency of GA to solve
253 the proposed 4PLRPU. One is how to address infeasible solutions (illegal

254 routes). There are two common ways: discard these infeasible solutions
 255 directly and reselect randomly among the breakpoints. However, neither
 256 approach makes good use of the original individuals. The proposed IDGA
 257 is designed to reduce the running time and computing cost of repairing this
 258 type of infeasible solution; The other is how to reduce the time spent on
 259 backtracking in the complex multi-graph network. Researchers are always
 260 exploring how to search faster and broader. Therefore, IDGA is designed to
 261 improve the standard GA from the following aspects:

262 (1) Repair strategy.

- 263 a) To repair non-connected nodes in an illegal route, the first node v_i
 264 whose following node is not connected and the last node v_j whose
 265 prior node is not connected are determined. In this process, repeated
 266 nodes are avoided.
- b) The distance matrix ($F = (f_{ij})_{n \times n}$) of the multi-graph is determined.
 Each element f_{ij} represents the relative distance between node i and
 node j . If node i and node j are connected directly, f_{ij} is set to 1;
 otherwise, f_{ij} is set to the minimum number of edges needed to reach
 node j from node i . For example, the distance matrix F_3 is shown
 in Eq. (9) for the multi-graph of Fig. 1. $f_{12} = 1$ means that v_s and
 v_1 are adjacent nodes. $f_{14} = 2$ means that the relative distance of v_s
 and v_3 is 2, and at least two edges are needed to reach v_3 from v_s .

$$F_3 \begin{matrix} & v_s & 1 & 2 & 3 & 4 & 5 & v_t \\ \begin{matrix} v_s \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ v_t \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 0 & 1 & 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 0 & 1 & 2 & 1 & 2 \\ 3 & 2 & 1 & 1 & 0 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 & 1 & 0 & 2 & 1 \\ 5 & 2 & 2 & 1 & 1 & 2 & 0 & 1 \\ v_t & 3 & 2 & 2 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix} \quad (9)$$

- 267 c) Beginning from v_i , instead of choosing the following edge (multiple
 268 edges for every two nodes), we choose the following node v_k first based
 269 on the distance matrix (as Eq. (9) shows) of the multi-graph. The dis-
 270 tance matrix is used to calculate the probability of each possible node
 271 according to the reciprocal and the roulette wheel selection scheme.
 272 For example, in the multi-graph of Fig. 1, suppose that $v_i = v_s$ and
 273 $v_j = v_4$; then, the following node v_k may be chosen between v_1 and

274 v_2 . From Eq. (9), we know that the distance from v_1 to v_4 (v_j) is
 275 1 and the distance from v_2 to v_4 is 2. Therefore, the probabilities of
 276 selection for v_1 and v_2 are $2/3$ and $1/3$, respectively. Then, v_k can be
 277 decided. The edge between v_i and v_k is selected randomly.

278 (2) Immigration strategy. We calculate the standard deviation (msd) of the
 279 current population. If $msd < c$ (c is a constant), we generate N new
 280 individuals based on the distance matrix ($v_j = v_t$) and then insert them
 281 into the current population according to the reciprocal and the roulette
 282 wheel selection scheme.

283 (3) Backtracking. Backtracking greatly delays the running time of the al-
 284 gorithm for large-scale problems, so the initialization or repair of an
 285 individual will start again if the number of times of backtracking $bt > b$
 286 (b is a constant).

287 4.4. INGA

288 The multi-graph increases the difficulty in **searching for feasible routes**
 289 **and solving the model**. Therefore, INGA is designed from the perspective of
 290 the simple route and **the** k -th shortest path algorithm [12]. INGA is divided
 291 into two stages. The first stage chooses nodes, and the second stage adds
 292 edges between them.

293 4.4.1. The first stage

294 **First**, the multi-graph is simplified into a simple graph. The simple graph
 295 has all nodes of the multi-graph. If two nodes are connected directly in the
 296 multi-graph, one edge without weight will be added between the two nodes
 297 in the simple graph.

298 Then, we choose routes in the simple graph based on GA without consid-
 299 ering **the** constraints of the proposed model. Because weight is not considered
 300 in this stage, the distance matrix (introduced in IDGA) is used to generate
 301 the initial population.

302 For example, Fig. 2 is the corresponding simple graph of the multi-graph
 303 in Fig. 1, and $\{v_s, v_2, v_3, v_t\}$ is a route of the first stage. In fact, what we
 304 choose in the first stage is a set of nodes that can be connected into a route
 305 from the source to the destination because the edges on the route of this
 306 stage are virtual and have no weight. The edges between those nodes will be
 307 chosen in the second stage.

308 GA is used to search for individuals in the simple graph of the first stage
 309 in which each individual represents a set of nodes. Because the constraints

Figure 2: Simple graph of the first stage for INGA.

are not considered, this process is simpler than the GA introduced in Section 4.2. Here, we briefly introduce the initialization and repair process of the first stage.

Initialization is done to find routes from the source to the destination in the simple graph. We begin from node v_s , and then, the following node will be selected based on the distance between each possible node and the destination v_t . Probabilities are calculated with the reciprocal of their distance. For example, starting from node v_s , v_1 and v_2 are possible nodes, and both of their distances to the destination are 2. Their probabilities can be calculated, and they are 0.5 and 0.5, respectively. Then, the roulette wheel selection scheme is used.

After the crossover operation and mutation operation in the first stage, we need to check the feasibility of each individual. If the individual is infeasible (disconnected), we try to repair it according to the following process. If there are repeated nodes on the individual, we keep the first one and delete others. We then check from the first node v_s until arriving at the breakpoint v_i . We examine whether v_i can be connected directly with other nodes behind v_i in the individual. If that happens, we connect them and repeat this process. Otherwise, we check from the last node v_t until arriving at the breakpoint v_j . We then generate a feasible route to connect v_i and v_j for the individual according to the distance matrix.

4.4.2. The second stage

Each individual from the first stage is a set of nodes that can be connected into a route from the source to the destination. At the beginning, the corresponding edges on the multi-graph between the nodes of each individual are found. The edges and nodes of each individual can form a simplified graph. Then, the constrained shortest path problem will be solved on the graph. For example, Fig. 3 is the graph of the individual $\{v_s, v_2, v_3, v_t\}$. The k-th shortest path algorithm is implemented according to the idea of

339 DA (deletion algorithm) [1]. Traditional methods of the k -th shortest path
 340 problem can only be applied in a simple graph; Azevedo et al. [1] extended
 341 it to the multi-graphs. Detailed steps of the second stage are described as
 follows.

Figure 3: Graph of the second stage for INGA.

342

- 343 Step 1. Compute the current shortest path p_k ($k \in \{1, 2, \dots, K\}$). Let $p_k =$
 344 p_1 , that is, $k = 1$.
 345 Step 2. If $k = K$ or no candidate path is left, the second stage terminates.
 346 Otherwise, advance to Step 3.
 347 Step 3. Determine the first node v_h^m on p_k such that the in-degree is larger
 348 than 1. If v_h^m does not have an extended node, advance to Step 4.
 349 Otherwise, determine the first node v_i^l on p_k without extended nodes
 350 and advance to Step 5. v_h^m represents the m -th extension of v_h .
 351 Step 4. Add an extended node v_h^{m+1} for v_h^m in graph $\overline{G}$ of the second stage.
 352 If v_h^m is not the last node of p_k , mark the next node of v_h^m on p_k as v_i^l .
 353 (v_{h-1}^l, v_h^m, s_j) represents an **edge** that begins at node v_{h-1}^l and ends
 354 at node v_h^m , and its number is s_j . (v_{h-1}^l, v_h^m, s_j) and $(v_{h-1}^l, v_h^{m+1}, s'_j)$
 355 have equal weights. The set of the incoming **edges** of v_h^{m+1} are:

$$I(v_h^{m+1}) \leftarrow \{(v_{h-1}^l, v_h^{m+1}, s'_j) | (v_{h-1}^l, v_h^m, s_j) \in I(v_h^m), (v_{h-1}^l, v_h^m, s_j) \notin p_k\}.$$

- 356 Step 5. Add an extended node for each node on p_k from v_i^l to the end (v_j^m ,
 357 $j \in \{1, 2, \dots, n\}$). If v_{j-1}^l (the last node before v_j^m on p_k) has no
 358 extended nodes, the sets of the incoming **edges** are:

$$I(v_j^m) \leftarrow \{(v_{j-1}^l, v_j^m, s'_q) | (v_{j-1}^l, v_j^{m-1}, s_q) \in I(v_j^{m-1}), (v_{j-1}^l, v_j^{m-1}, s_q) \notin p_k\}.$$

- 359 If v_{j-1}^l has extended node v_{j-1}^{t+1} , the sets of the incoming **edges** are:

$$I(v_j^m) \leftarrow \{(v_{j-1}^l, v_j^m, s'_q) | (v_{j-1}^l, v_j^{m-1}, s_q) \in I(v_j^{m-1}), (v_{j-1}^l, v_j^{m-1}, s_q) \notin p_k\} \\ \cup \{(v_{j-1}^{t+1}, v_j^m, s'_q) | (v_{j-1}^{t+1}, v_j, s_q) \in \overline{G}\}.$$

- 360 Step 6. Update the current shortest path p_k . $k = k + 1$. Return to Step 2.

Table 1: Detailed information of nodes and edges

Type	Number	Weight	Time	Type	Number	Weight	Time
Node	v_s	10	6	Edge	$v_2 \xrightarrow{2} v_3$	12	12
Edge	$v_s \xrightarrow{1} v_2$	18	16	Edge	$v_2 \xrightarrow{3} v_3$	11	13
Edge	$v_s \xrightarrow{2} v_2$	17	17	Node	v_3	9	7
Edge	$v_s \xrightarrow{3} v_2$	19	14	Edge	$v_3 \xrightarrow{1} v_t$	12	9
Edge	$v_s \xrightarrow{4} v_2$	15	20	Edge	$v_3 \xrightarrow{2} v_t$	13	8
Node	v_2	6	5	Edge	$v_3 \xrightarrow{3} v_t$	11	10
Edge	$v_2 \xrightarrow{1} v_3$	10	14	Node	v_t	7	5

361 Taking Fig. 3 as an example, assume that $K = 3$, $T_0 = 65$; detailed
 362 weight information (cost in the proposed model) of the nodes and edges is
 363 shown in Table 1 (randomly generated).

364 In Table 1, rows of “Node” represent the information of nodes. For ex-
 365 ample, the weight of v_s is 10. Rows of “Edge” represent the information of
 366 corresponding edges. For example, $v_s \xrightarrow{1} v_2$ represents the first edge from v_s
 367 to v_2 , and its weight is 18. Detailed steps of the second stage are as follows.

- 368 (1) First, **it is** easy to find the current shortest path p_k , as red lines show in
 369 Fig. 4 when $k=1$. The total weight of each node from v_s is also obtained.
 Now, $W_1 = 68$ (weight of the 1st shortest route), $T_1 = 67$, and $T_1 > T_0$.

Figure 4: The graph when $k = 1$.

- 370
 371 (2) According to p_k , the extended nodes and their incoming **edges** are added
 372 based on Fig. 4, which is shown in Fig. 5. The weights of these extended
 373 nodes are computed and the current shortest path p_k is found, as the red
 lines in Fig. 5 **show**, $k = 2$. Now, $W_2 = 69$, $T_2 = 66$, and $T_2 > T_0$.

Figure 5: The graph when $k = 2$.

375 (3) Similar steps are performed to continue. $k = 3$ and $k = K$. $W_3 = 69$,
376 $T_3 = 66$, and $T_3 > T_0$. The current shortest path p_3 is shown in Fig. 6.
377 Therefore, when $K = 3$, the individual $\{v_s, v_2, v_3, v_t\}$ of the first stage is
378 interpreted as an infeasible route. The second stage of this individual is
finished.

Figure 6: The graph when $k = 3$.

379

380 4.5. SDGA

381 According to the three previously proposed algorithms, feasible routes are
382 selected directly or indirectly from the multi-graph. The three algorithms
383 inevitably encounter the problem of multiple edges between two nodes. In
384 this situation, one idea is to select a simple graph, and to solve the problem
385 in the simple graph with classic methods. According to this idea, SDGA,
386 which uses GA to select simple graphs and uses the constrained Dijkstra
387 algorithm [11] to solve the proposed model in the simple graphs, is designed.
388 Both the problems of infeasible routes and multiple edges in GA are avoided
389 in SDGA.

Figure 7: Multi-graph of 4PLRP for SDGA.

390 In SDGA, every individual is a simple graph and the initial population
391 is generated randomly. A legal individual is a simple graph that contains all

nodes of the multi-graph and one edge between every two adjacent nodes. **First**, a linkage matrix (F_1 in Eq. (7)) of dimension $n \times n$ is also used to represent the multi-graph, as shown in Fig. 7, whose corresponding code is (342223432332). In Fig. 7, F_4 (Eq. (10)) represents an individual (a simple graph) that is constructed by the red lines. The number “1” in row one and column two **indicates** that the first edge between v_s and v_1 is chosen to construct the simple graph and that the upper bound of the number is 3 (only 3 edges in the corresponding position of F_1). The corresponding code of F_4 is (122111322232).

The advantage of this coding method is that it is easy to avoid illegal codes both in the crossover operation and mutation operation in GA. In the crossover operation, any two elements that may exchange position begin in the same node and end at the same node also. Therefore, new codes are also legal codes (i.e., they can form a simple graph). For example, consider parent codes 122|111322|232 and 242|121321|231 and assume that the crossover points are between 3 and 9. Then, the new codes are 122|121321|232 and 242|111322|231. In the mutation operation, the upper bound of each code is treated as the module; **thus**, a legal code will be ensured.

$$F_4 \begin{matrix} & v_s & 1 & 2 & 3 & 4 & 5 & v_t \\ \begin{matrix} v_s \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ v_t \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 2 & 3 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \end{matrix} \quad (10)$$

Then, the problem of each individual can be considered as a constrained shortest path problem, and the constrained Dijkstra algorithm can be used to solve this problem. Due to limited space, **details of the** process will not be introduced here.

5. Numerical experiments

To investigate the performance of the proposed algorithms and verify the effectiveness of the proposed 4PLRPU model, three scales of examples are given in this section. Then, the performances of the proposed algorithms and

the effect of some key parameters on the performance of the algorithms are analyzed. Furthermore, the robustness of the 4PLRPU model is illustrated by comparing with the expected value model of 4PLRP.

5.1. The 4PLRPU examples

As is introduced previously, the 4PLRPU can be represented by an undirected multi-graph. The multi-graphs are generated randomly in a rectangle to represent examples [7]. Detailed steps are shown as follows:

- Step 1. Generate $n-2$ nodes (a total of n nodes on the multi-graph including the source and the destination) randomly in a $d \times d$ square area. $(0, 0)$ and (d, d) represent the source and destination respectively.
- Step 2. If the Euclidean distance between two nodes is less than or equal to D_s , edges exist between them.
- Step 3. Generate a random number $r_{ij} \in [a_1, b_1]$ which represents the number of edges between the two nodes.
- Step 4. Randomly generate cost C_{ijk} and time T_{ijk} for each edge, where $C_{ijk} \in [a_2, b_2]$ and $T_{ijk} \in [a_3, b_3]$.
- Step 5. Randomly generate cost C'_j and time T'_j for each node, where $C'_j \in [a_4, b_4]$ and $T'_j \in [a_5, b_5]$.

To illustrate the ideas discussed above and compare more clearly, three examples with 7 nodes, 15 nodes and 30 nodes ($n = 7, n = 15, n = 30$) are generated. $d = 1$, and $D_s \in [0.5, 0.75]$. Cui et al. [26] consider two test examples in which one is a real logistics network and the other is generated randomly. Based on these, other parameters are set as follows:

$$a_1 = 2, b_1 = 4, a_2 = 6, b_2 = 30, a_3 = 2, b_3 = 22, a_4 = 5, b_4 = 15, a_5 = 4, b_5 = 9.$$

The investigated algorithms are coded with Microsoft Visual Studio 2008 and run on a Intel Core-2 Duo 3.0GHz PC.

5.2. Performance analysis of algorithms

ENUM serves as a basic standard to **evaluate** the other algorithms' performances. After setting an appropriate μ , σ , T_0 and α , three examples with different scales (7 nodes, 15 nodes and 30 nodes) are tested.

447 *5.2.1. Numerical results of algorithms*

448 Without loss of generality, supposing that $\mu = 0, \sigma = 1.20, \alpha = 0.95$, the
 449 experimental results of the 7 nodes example are shown in Table 2. “Algo-
 450 rithm” represents the proposed five algorithms ENUM, GA, IDGA, INGA
 451 and SDGA. “Node” represents the number of nodes and “LCode” represents
 452 the length of code. These two parameters reflect the scale of the problem.
 453 The algorithms are run 100 times, and the parameters considered in the algo-
 454 rithms are population size “NP”, the number of generation “NG”, crossover
 455 rate “Pc”, mutation rate “Pm”, the best solution “Best”, the worst solution
 456 “Bad”, the average value “Avg”, standard deviation “Msd” in Eq. (11) and
 457 algorithm running time “RTime”(once). “-” indicates that the algorithm
 458 does not have this parameter.

$$\text{Msd}(S) = \sqrt{\frac{\sum_{i=1}^n (S_i - \bar{S})^2}{n - 1}} \quad (11)$$

459 In Eq. (11), $\bar{S}$ is the mean, S_i is the best solution in the i -th running,
 460 and $n = 100$.

Table 2: Experimental results of the algorithms for the 7 nodes example

Algorithm	Node	LCode	NP	NG	Pc	Pm	Best	Bad	Avg	Msd	RTime
ENUM	7	12	-	-	-	-	68	-	-	-	24.78s
GA	7	12	50	12	0.90	0.05	68	68	68	0.00	0.98s
IDGA	7	12	50	11	0.90	0.05	68	68	68	0.00	0.27s
INGA	7	12	50	6	0.90	0.05	68	68	68	0.00	0.24s
SDGA	7	12	50	5	0.90	0.05	68	68	68	0.00	0.05s

461 Here, the due date T_0 is set to 73, which can be changed based on the
 462 customer’s requirement. In INGA, the parameter k (i.e., the second stage,
 463 k -th shortest path algorithm) is set to 8. It can be observed from Table 2 that
 464 both GA and the three improved GAs can find the optimal solution quickly
 465 for small-scale problems (they obtain the same solution as the enumeration
 466 algorithm). For small-scale problems, the three improved GAs need less
 467 running time than GA.

468 Expanding the scale of the problem, a 15 nodes example is tested, and the
 469 experimental results are presented in Table 3. In spite of using the improved
 470 strategy, ENUM cannot determine the optimal solution within an acceptable
 471 time (more than 12 hours). In this case, four GAs are compared with each
 472 other.

Table 3: Experimental results of the algorithms for the 15 nodes example

Algorithm	Node	LCode	NP	NG	Pc	Pm	Best	Bad	Avg	Msd	RTime
ENUM	15	36	-	-	-	-	-	-	-	-	-
GA	15	36	200	30	0.90	0.05	107	107	107	0.00	5.62s
IDGA	15	36	50	15	0.90	0.05	107	107	107	0.00	0.99s
INGA	15	36	50	14	0.90	0.05	107	107	107	0.00	1.37s
SDGA	15	36	50	20	0.90	0.05	107	107	107	0.00	2.41s

473 We can see from Table 3 that all of the proposed intelligent algorithm-
474 s obtain solutions with the same value (they may choose different routes,
475 but the cost is the same) within an acceptable running time. For medium-
476 scale problems, the average running time of IDGA is the least under the
477 premise of guaranteeing solution quality and the standard deviation, which
478 illustrates the effectiveness of strategies in IDGA. Compared with GA, the
479 three improved algorithms converge to the solution faster and need smaller
480 population size and generation.

481 Then, the large-scale problem with 30 nodes and 198 edges is tested to
482 investigate the performances of the proposed algorithms. In this case, each
483 code contains 71 genes, which poses a challenge for general algorithms. The
484 experimental results are shown in Table 4.

Table 4: Experimental results of the algorithms for the 30 nodes example

Algorithm	Node	LCode	NP	NG	Pc	Pm	Best	Bad	Avg	Msd	RTime
ENUM	30	71	-	-	-	-	-	-	-	-	-
GA	30	71	2000	35	0.90	0.05	182	183	182.02	0.14	35mins22s
IDGA	30	71	300	40	0.90	0.05	182	182	182	0.00	182.70s
INGA	30	71	200	20	0.95	0.05	182	182	182	0.00	15.31s
SDGA	30	71	100	25	0.90	0.05	182	182	182	0.00	3.97s

485 GA can get the same best solution as the other improved algorithms but
486 needs more running time. The proposed IDGA greatly reduces the running
487 time, and its standard deviation is smaller than the GA's. For large-scale
488 problems, the improvement is obvious. However, the running time of IDGA
489 is also more than 3 minutes, which is not efficient enough. From Table 4,
490 we can see that SDGA and INGA search faster than GA and IDGA and
491 that SDGA's running time is smaller than INGA's. In addition to reducing
492 the running time of repairing infeasible solutions, INGA and SDGA also
493 simplify the complexity of the multi-graph, which improves the efficiency of

the algorithms.

5.2.2. Analysis of algorithm convergence

To see the algorithms' search processes more clearly, the mean value and good rate of algorithms are tested along with the parameter of population generation (NG). Then, how the algorithms converge to the best solution can be shown.

$$\text{Mean}(c) = \frac{1}{n} \sum_{i=1}^n c_i \quad (12)$$

$$\text{Rate}(c) = \frac{m}{n} \quad (13)$$

c_i is the cost of the i -th individual of the current generation, n is the population size, and m is the number of individuals whose cost is equal to the best solution.

Figure 8: Variation diagram of average cost for the 15 nodes example.

Fig. 8 shows the variation diagram of average cost changes with NG for the 15 nodes example. To make a clearer comparison, the population size (NP) is set to 200 for all algorithms. It can be seen that the convergence processes of INGA and SDGA are quite similar and that they converge faster than IDGA and GA. Because the Dijkstra algorithm and k-th shortest path algorithm are used, respectively, SDGA and INGA have obvious advantages in the initial solutions. It is obvious that SDGA and INGA converge faster than IDGA, and they all converge faster than GA.

Fig. 9 shows the variation diagram of the 30 nodes example. “1” is the detailed figure of part “2”. Because of difficulties in solving large-scale

Figure 9: Variation diagram of average cost for the 30 nodes example.

Figure 10: Variation diagram of good rates for the 15 nodes example.

514 problems, especially for GA and IDGA, the population size (NP) is no longer
515 equal. The NP of GA, IDGA, INGA and SDGA are set to 2000, 300, 200
516 and 100, respectively. **Generally**, the three improvement strategies of IDGA,
517 INGA and SDGA are efficient, and INGA and SDGA are more obvious.
518 INGA reduces the complexity of multiple edges between neighboring nodes,
519 and SDGA solves the problem of illegal coding that often appears in GA and
520 its improved algorithms. It can be observed in Fig. 9 that the improved
521 algorithms significantly accelerate the search process. The three improved
522 GAs' NP are much smaller than that of GA; however, they can also converge
523 to the best solution with less generations.

524 Fig. 10 and Fig. 11 are good rate variation diagrams, along with NG,
525 for the 15 nodes example and the 30 nodes example. As introduced above,
526 NP is set to 200 in Fig. 10, and the NP of GA, IDGA, INGA and SDGA
527 are set to 2000, 300, 200 and 100, respectively, in Fig. 11. We can also see

Figure 11: Variation diagram of good rates for the 30 nodes example.

that the three improved algorithms show higher efficiency than GA. With the increase in the problem scale from 7 nodes to 30 nodes, INGA and SDGA show advantages gradually. Generally, SDGA always needs a smaller NP, and its initial solutions are better than those of INGA. However, INGA needs fewer generations to converge to the best solution. SDGA and INGA perform better than the other proposed algorithms when solving large-scale problems for the proposed 4PLRPU.

5.2.3. The effects of some key problem parameters on the algorithms

Some key parameters also impact the algorithms' efficiency, such as the due date T_0 from the customer, the problem scale represented by the number of nodes in the proposed model, the transportation environment represented by μ and σ , and the confidence level α asked by the customer. In this section, the effect of T_0 is tested for the 15 nodes example first, and then, the four GAs are compared to solve a larger scale problem with 100 nodes. The experimental results are shown in Fig. 12 and Table 5.

It can be observed from Fig. 12 that T_0 obviously impacts SDGA. As T_0 increases, the running time of SDGA decreases quickly. The constrained Dijkstra algorithm is easily affected by the tightness of the constraints. However, once T_0 increases, the running time of INGA and IDGA will also decrease, but not very clearly. In particular, the influence on INGA is even more obvious because the idea of the k-th shortest path is used. It is similar to SDGA to some extent, and a more strict constraint often leads to a larger K.

Table 5 shows the experimental results of the 100 nodes example, where $T_0 = 1000$ and $dev = 1$. It can be seen that the length of code "LCode" for the four algorithms is a very large number and that it is very hard to search

Figure 12: The effect of parameter T_0 .

Table 5: Experimental results of the algorithms for the 100 nodes example

Algorithm	Node	LCode	NP	NG	Pc	Pm	Best	Avg	RTime
GA	100	11020	5000	500	0.95	0.15	609	612.4532	75mins36s
IDGA	100	11020	3000	500	0.95	0.15	584	584.7463	43mins52s
INGA	100	11020	3000	250	0.95	0.10	578	578.2156	136.37s
SDGA	100	11020	3000	100	0.95	0.15	575	575.4770	31.43s

for the optimal solution. Here, the average value of the objective function is used to show the performances of the algorithms. In this condition, GA and IDGA are not very effective. INGA can help us solve large-scale problems but is less effective than SDGA. SDGA's solution has the smallest "Avg" and uses the least running time.

Overall, the proposed GA and the three improved GAs can find optimal solutions accurately and quickly and are all effective for solving the small-scale problems of 4PLRPU. When the problem scale increases, IDGA's improvement strategies play a role and IDGA has a better performance than GA. INGA and SDGA have obvious advantages in the initial population, especially for large-scale problems. The running times of INGA and SDGA are always smaller than those of the other algorithms, and at the same time, they ensure the solution quality. INGA simplifies the complexity of multiple edges in multi-graph and limits the problem of illegal routes, and SDGA avoids the problem of illegal routes. The improvements are significant.

5.3. Robustness analysis of the 4PLRPU model

Many existing studies on 4PL established deterministic models considering the delivery time as a deterministic value [24, 25, 39]. These models can

571 be regarded as expected value models under an uncertain environment. To
 572 illustrate the effectiveness of the proposed 4PLRPU model under an uncertain
 573 environment, the corresponding expected value model of 4PLRP, called
 574 4PLRPE, is **developed** by replacing Eq. (2) with Eq. (14). Furthermore,
 575 the three examples with different scales are tested, and the results of both
 576 models are compared in this section.

$$E \left[\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{r_{ij}} (T_{ijk} + t_{ijk}) x_{ijk}(R) + \sum_{j=1}^n T'_j y_j(R) \right] \leq T_0 \quad (14)$$

577 To more clearly analyze the effectiveness of the 4PLRPU model under an
 578 uncertain environment, the robustness of solution “Robustness” is defined as
 579 shown in Eq. (15).

$$Robustness = \frac{N_f}{N_s} \quad (15)$$

580 Given a solution (a route R) of the problem, its distribution function of
 581 the total delivery time can be easily obtained, and the robustness is calculated
 582 **via** Monte Carlo Simulation according to the distribution function. In Eq.
 583 (15), N_s is the number of samples generated according to the distribution
 584 function of route R . N_f is the number of feasible samples that satisfy the
 585 due date T_0 required by the customer.

586 The experimental results are presented in Table 6. It can be seen that the
 587 4PLRPU model considers the tardiness risk under an uncertain environment
 588 and improves the robustness of solutions. With the same T_0 , the 4PLRPU
 589 model chooses routes with higher robustness and higher cost to guarantee the
 590 predetermined confidence level α . However, although the 4PLRPE chooses
 591 routes with **lower costs**, its robustness is far lower than that of the 4PLRPU
 592 model. The 4PLRPE only considers expectations and has a higher tardiness
 593 risk. From Table 6, we can also see that, with the increase in σ , the robust-
 594 ness is reduced and the 4PLRPE is influenced obviously by σ , whereas the
 595 4PLRPU model is not. Taking 7 nodes as an example, when T_0 is set to 66
 596 and $\sigma = 1.2$, the 4PLRPE chooses the route $R_1 = (v_s \xrightarrow{4} v_2 \xrightarrow{3} v_3 \xrightarrow{3} v_t)$. R_1
 597 has a lower total cost; however, the robustness is only 0.5007. If the confi-
 598 dence level $\alpha = 0.95$ is considered, R_1 cannot meet the customer’s demand.
 599 The route that the 4PLRPU model selects is $R_2 = (v_s \xrightarrow{3} v_2 \xrightarrow{3} v_3 \xrightarrow{3} v_t)$. Al-
 600 though the cost of R_2 is slightly higher than that of R_1 , the robustness of R_2
 601 is 0.9538. Overall, compared with the expected value model, the 4PLRPU
 602 model can obtain higher robustness with little cost increase.

Table 6: Comparison of the 4PLRPE and 4PLRPU model

Model	Algorithm	Node	T_0	α	σ	Best	Robustness
4PLRPE	SDGA	7	66	-	1.2	69	0.5007
4PLRPU	SDGA	7	66	0.95	1.2	73	0.9538
4PLRPE	SDGA	7	66	-	2.0	69	0.4955
4PLRPU	SDGA	7	66	0.95	2.0	80	0.9514
4PLRPE	SDGA	7	70	-	1.2	68	0.8199
4PLRPU	SDGA	7	70	0.95	1.2	70	0.9546
4PLRPE	SDGA	7	70	-	2.0	68	0.7160
4PLRPU	SDGA	7	70	0.95	2.0	73	0.9519
4PLRPE	SDGA	15	75	-	1.2	103	0.6013
4PLRPU	SDGA	15	75	0.95	1.2	110	0.9550
4PLRPE	SDGA	15	75	-	2.0	103	0.5586
4PLRPU	SDGA	15	75	0.95	2.0	114	0.9520
4PLRPE	SDGA	15	80	-	1.2	100	0.6784
4PLRPU	SDGA	15	80	0.95	1.2	107	0.9787
4PLRPE	SDGA	15	80	-	2.0	100	0.6112
4PLRPU	SDGA	15	80	0.95	2.0	110	0.9510
4PLRPE	SDGA	30	140	-	1.2	177	0.6895
4PLRPU	SDGA	30	140	0.95	1.2	186	0.9548
4PLRPE	SDGA	30	140	-	2.0	177	0.6176
4PLRPU	SDGA	30	140	0.95	2.0	196	0.9537
4PLRPE	SDGA	30	143	-	1.2	174	0.6404
4PLRPU	SDGA	30	143	0.95	1.2	183	0.9553
4PLRPE	SDGA	30	143	-	2.0	174	0.5886
4PLRPU	SDGA	30	143	0.95	2.0	191	0.9492

6. Conclusion

Because of the lack of historical data regarding delivery time in uncertain 4PL environments, routing decisions are usually made in the state of indeterminacy. To address this type of uncertainty, the delivery time is described as an uncertain variable, and a novel 4PLRPU uncertain programming model is developed based on uncertainty theory.

To solve the proposed 4PLRPU model, it is first transformed into an equivalent deterministic model, and then, IDGA, INGA and SDGA are designed according to the characteristics of the problem. Numerical analysis shows the effectiveness of the algorithms and the model. For small-scale problems, GA, IDGA, INGA and SDGA obtain the optimal solutions quick-

ly, which are calculated by the ENUM. For medium-scale problems, IDGA, INGA and SDGA show the advantages in searching speed compared with GA, and all of them can obtain the same best solutions. Further expanding the scale of the problem, compared with GA and IDGA, INGA gradually shows its obvious advantages in convergence and efficiency, because INGA's computing time required to repair infeasible solutions is greatly reduced by decreasing the complexity of multiple edges. Additionally, compared with GA and IDGA, SDGA shows obvious advantages in effectiveness and efficiency due to avoiding infeasible solutions. Finally, compared with the expected value model of 4PLRP, the results indicate that the 4PLRPU model is more robust. In general, the proposed 4PLRPU model, as well as INGA and SDGA, provides a useful routing decision methodology for 4PL under uncertain environments.

In the future, the work in this paper can be extended by considering more uncertain factors or more tasks.

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