Fault detection filter design for data reconstruction-based continuous-time networked control systems

Yu-Long Wang^a, Tian-Bao Wang^{b,*}, Qing-Long Han^c

^a*School of Electronics and Information, Jiangsu University of Science and Technology, Zhenjiang Jiangsu 212003, China*

^b*School of Electrical and Information Engineering, Jiangsu University, Zhenjiang Jiangsu 212013, China*

^c*Griffith School of Engineering, Griffith University, Gold Coast Campus, QLD 4222, Australia*

Abstract

This paper is concerned with observer-based fault detection filter design for a continuous-time networked control system (NCS) by taking packet dropouts and network-induced delays into account. An observer-based fault detection filter and a reference residual model are introduced to construct a novel model for the continuous-time NCS. To reduce the time for fault detection, a new data reconstruction scheme is proposed and the corresponding closed-loop model is established. Based on the established models, new fault detection filter design criteria are derived to asymptotically stabilize the residual system. The mutually exclusive distribution characteristic of interval time-varying delays is made full use to deal with integral inequalities for products of vectors. The designed fault detection filters can guarantee the sensitivity of the residual signal to faults. Simulation results illustrate the effectiveness of the proposed observer-based fault detection filter design.

Keywords: Networked control systems, fault detection, data reconstruction

1. Introduction

Networked control systems (NCSs) are spatially distributed systems in which sensors, actuators, and controllers are connected through a shared

*Corresponding author. Email: wtbcn@163.com

Email addresses: feixiangwyl@163.com (Yu-Long Wang), wtbcn@163.com (Tian-Bao Wang), q.han@griffith.edu.au (Qing-Long Han)

band-limited digital communication network. Introducing the communication network into control systems has several advantages, such as flexibility in communication architecture, plug and play devices, and ease of system diagnosis and maintenance, and so on. NCSs are the subject of intensive research and many results are available about stability analysis and/or stabilization [1, 16, 17, 27, 34, 36, 42], sampled-data control [4, 5], sensors/actuators assignment [7, 8, 10], quantization [9, 29], packet dropouts and network-induced delays [26, 31, 38, 41, 43, 44, 45], and the non-uniformly distributed interval time-varying delay [39, 40].

For NCSs, the occurrence of the sensor fault or the actuator fault is usually unavoidable. Then, it is of paramount importance to study how to detect the occurrence of faults in time. The fault detection for traditional control systems has been paid much attention and some interesting results have been reported, see [14, 18, 20, 30, 46] and the references therein. On the other hand, some nice results dealing with fault detection for NCSs have been obtained. For example, Dong *et al.* [2] was concerned with the network-based robust fault detection problem for discrete-time Takagi-Sugeno fuzzy systems. He *et al.* [12] studied the problems of fault detection, isolation, and estimation for discrete time-varying networked sensing systems. Huang and Nguang [13] investigated robust fault estimation for NCSs with network-induced delays. Liu *et al.* [21] was devoted to the fault detection of linear systems over networks with bounded packet dropouts. Long and Yang [22, 23] investigated the problem of fault detection in finite frequency domain for NCSs. Mao *et al.* [24] proposed a method for the fault detection of a nonlinear NCS. Wang *et al.* [32] was devoted to fault detection of NCSs subject to uncertain time-varying network-induced delays. Wang *et al.* [33] proposed a so-called packet-based periodic communication strategy to design optimal observer-based residual generators. It should be mentioned that discrete-time systems are considered in [2, 12, 22, 24, 33], while discretized NCSs are considered in [21, 23, 32]. Moreover, packet dropouts and/or network-induced delays are neglected in some of the above mentioned work. *For continuous-time NCSs considering packet dropouts and network-induced delays simultaneously, how to propose an appropriate observer-based fault detection filter design scheme is interesting and practically valuable, which is the first motivation of this paper.*

For NCSs with faults, the feasible fault signals are included in the measurement outputs $y(t_{k-1}), y(t_k), \dots$, where k denotes a positive integer, and $y(t_{k-1}), y(t_k), \dots$ are adopted by the fault detection filter (FDF) to detect

the occurrence of faults. For a specific time interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$, where t_k and τ_k denote the sampling instant and the length of the network-induced delays, respectively, if we combine the recently received measurement output $y(t_k)$ and the formerly received measurement output $y(t_{k-1})$ together, which is named as data reconstruction in this paper, and transmit the data reconstruction-based measurement outputs to the FDF, enlarged fault signals will be received by the FDF and the fault detection time will be reduced correspondingly. However, such a data reconstruction scheme has not been considered in the literature. *To improve the fault detection performance of NCSs, one should take the data reconstruction scheme into account, which is the second and main motivation of this paper.*

For continuous-time NCSs with $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, the artificial delay $\tau(t) = t - t_k$, which is named as an interval time-varying delay, is usually introduced in the literature to study stability/stabilization of considered systems. Suppose that $\tau(t) \in [\tau_m, \eta)$, where τ_m and η denote known constants. Divide $[\tau_m, \eta)$ into ρ equidistant time intervals with ρ denoting a given positive integer. One can conclude that for a specific instant t , $\tau(t)$ can not be included simultaneously in two different time intervals derived above, in what follows, we refer to such a phenomenon as mutually exclusive distribution. When dealing with integral inequalities for products of vectors, *how to make full use of the mutually exclusive distribution characteristic of interval time-varying delays to derive less conservative results is of paramount importance and still unsolved.*

This paper is concerned with observer-based fault detection filter design for a continuous-time NCS under consideration of packet dropouts and network-induced delays. By introducing an observer-based fault detection filter, a reference residual model, and a new data reconstruction scheme, new closed-loop models for the considered NCS are established. Based on the established models, fault detection filter design criteria are proposed to asymptotically stabilize the residual systems.

The main contributions of this paper are summarized as follows:

- By taking packet dropouts and network-induced delays into consideration, and proposing a new data reconstruction scheme, closed-loop models for a continuous-time NCS with actuator faults are established.
- The mutually exclusive distribution characteristic, which introduces less conservatism, of interval time-varying delays is made full use to deal with integral inequalities for products of vectors.
- The observer-based fault detection filter design criteria are proposed.

The designed fault detection filters can guarantee the sensitivity of the residual signal to faults.

The remainder of this paper is organized as follows. By introducing the data reconstruction scheme, Section 2 establishes new models for a continuous-time NCS considering faults, packet dropouts, and network-induced delays. Based on the established models, Section 3 is concerned with observer-based fault detection filter design. Section 4 is concerned with data reconstruction-based fault detection filter design. The results of numerical simulation are presented in Section 5. Conclusions are drawn in Section 6.

Notation. $\mathbb{R}^n$ denotes n -dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. I and 0 represent an identity matrix and a zero matrix with appropriate dimensions, respectively. $*$ denotes the entries of a matrix implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

2. Modelling for an NCS with faults and data reconstruction

Consider a continuous-time NCS, whose faults are to be detected, described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D_1\omega(t) + E_1f(t) \\ y(t) = Cx(t) \\ x(t_0) = x_0 \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^s$, $\omega(t) \in \mathbb{R}^p$, $f(t) \in \mathbb{R}^q$ are the state vector, control input vector, measurement output, disturbance, and actuator fault signal, respectively; $\omega(t)$ is assumed to belong to $L_2[t_0, \infty)$; $x_0 \in \mathbb{R}^n$ denotes the initial condition; A , B , D_1 , E_1 , and C are known constant matrices of appropriate dimensions. C^T is assumed to be full column rank and (A, C) is detectable.

Throughout this paper, we introduce a buffer at the fault detection filter to store the outputs of the FDF and the recently received measurement outputs. Let $t_k, t_k + h, t_k + 2h, t_k + \dots, t_{k+1}, t_{k+1} + h, t_{k+1} + 2h, t_{k+1} + \dots$ ($k = 0, 1, 2, \dots$) denote the sampling instants, where h is the length of the sampling period. Suppose that the measurement outputs which are sampled at the instants $t_k, t_{k+1}, \dots$ are transmitted to the FDF successfully, while the measurement outputs sampled between the instants t_k and t_{k+1} ($k = 0, 1, 2, \dots$) are dropped, and δ denotes the upper bound of consecutive packet dropouts. Define τ_k as the time from the instant t_k when the sensor

samples data from the plant to the instant when the FDF receives the data. Suppose that $\tau_m \leq \tau_k < \tau_M$, where τ_m and τ_M are given scalars satisfying $\tau_m > 0, \tau_M > 0$.

In this paper, the following fault detection filter is adopted to generate the residual signal

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = S(y(t) - \hat{y}(t)) \end{cases} \quad (2)$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{y}(t) \in \mathbb{R}^s$ and $r(t) \in \mathbb{R}^{q_r}$ are the state of the FDF, the output of the FDF and the residual signal, respectively; L and S are to be determined.

A reference residual model is usually needed to describe the desired behavior of the residual signal $r(t)$. In this paper, we introduce the following reference residual model [32, 46]

$$\begin{cases} \dot{e}_r(t) = A_r e_r(t) + B_r \omega(t) + C_r f(t) \\ r_{ref}(t) = D_r e_r(t) + E_r \omega(t) + F_r f(t) \end{cases} \quad (3)$$

where $e_r(t) \in \mathbb{R}^{n_r}$ and $r_{ref}(t) \in \mathbb{R}^{q_r}$ are the state and the output of the reference residual model, respectively; A_r, B_r, C_r, D_r, E_r and F_r are given constant matrices of appropriate dimensions; A_r is a stable matrix.

Over the time interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$, the available measurement output for the fault detection filter is $y(t_k)$. At the instant t_k , the output of the FDF, i.e., $\hat{y}(t_k)$, is stored in the buffer. Define $e(t) = x(t) - \hat{x}(t)$, $\xi(t) = [x^T(t) \ e^T(t) \ e_r^T(t)]^T$, $\nu(t) = [u^T(t) \ \omega^T(t) \ f^T(t)]^T$, $r_e(t) = r_{ref}(t) - r(t)$, $\tau(t) = t - t_k$. For $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, one has $\tau(t) \in [\tau_k, t_{k+1} - t_k + \tau_{k+1})$. Considering that $\tau_m \leq \tau_k < \tau_M$ and $t_{k+1} - t_k \leq (\delta + 1)h$, one has $\tau(t) \in [\tau_m, \eta)$, where $\eta = (\delta + 1)h + \tau_M$. Based on (1)-(3) and the above given parameters, one can establish the following closed-loop system

$$\begin{cases} \dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{B}\xi(t - \tau(t)) + \tilde{D}\nu(t) \\ r_e(t) = \tilde{C}\xi(t) + \tilde{E}\xi(t - \tau(t)) + \tilde{F}\nu(t) \end{cases} \quad (4)$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A_r \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -LC & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} B & D_1 & E_1 \\ 0 & D_1 & E_1 \\ 0 & B_r & C_r \end{bmatrix}, \\ \tilde{C} &= [0 \ 0 \ D_r], \quad \tilde{E} = [0 \ -SC \ 0], \quad \tilde{F} = [0 \ E_r \ F_r] \end{aligned} \quad (5)$$

Remark 1. For a fault detection system, how to detect the occurrence of faults in time is quite important. Notice that the feasible fault signals are included in the measurement outputs $y(t_{k-1})$, $y(t_k)$, $\dots$. For $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, if we combine the recently received measurement output $y(t_k)$ and the formerly received measurement output $y(t_{k-1})$ together, which method is named as data reconstruction in this paper, and transmit the data reconstruction-based measurement outputs to the FDF, the fault signals generated at the instants t_k and t_{k-1} can affect simultaneously the residual signal $r(t)$ during the time interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$. Compared with the fault detection scheme which adopts only $y(t_k)$ over the time interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$, the data reconstruction-based fault detection scheme helps to reduce the time for fault detection.

In what follows, we turn to modelling for a fault detection system considering data reconstruction. Notice that a buffer is equipped at the FDF to store the recently received measurement outputs. The outputs $\hat{y}(t_{k-1})$, $\hat{y}(t_k)$, $\dots$ of the FDF are also stored in the buffer. It should be mentioned that the storage capacity of the buffer is limited, and only the recent data can be stored in the buffer. Then, for $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, the measurement outputs, which are actually adopted by the FDF, can be described by

$$\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-1})) \quad (6)$$

where $\tilde{a} = a_1 + a_2$ with a_1 and a_2 denoting given parameters.

Remark 2. For the data reconstruction scheme presented in (6), if $a_1 = 1$ and $a_2 = 0$, the actually adopted measurement output at the FDF reduces to $y(t_k)$. Then, the data reconstruction scheme in (6) provides more flexibility for fault detection. On the other hand, to reduce the time for fault detection, fault information, which is included in $y(t_{k-1})$, should be assigned a higher weight than fault information included in $y(t_k)$, that is, one should set $a_1 < a_2$. Moreover, if the storage capacity of the buffer is enough, one can construct the measurement outputs in (6) as $\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-2}))$, or $\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-3}))$, etc. For given a_1 and a_2 , if choosing $\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-\varrho_1}))$ with $\varrho_1 = 1, 2, \dots$ can provide shorter fault detection time than $\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-\varrho_2}))$ with $\varrho_2 = 1, 2, \dots$ and $\varrho_1 \neq \varrho_2$, one should choose $\tilde{y}(t) = \frac{1}{\tilde{a}}(a_1 y(t_k) + a_2 y(t_{k-\varrho_1}))$.

Define $d(t) = t - t_{k-1}$. Considering that $t - t_{k-1} = (t - t_k) + (t_k - t_{k-1})$ and $t_k - t_{k-1} \in \{h, 2h, \dots, (\delta + 1)h\}$, one has $d(t) \in [\tau_m + h, \tilde{\eta})$, where

$\tilde{\eta} = 2(\delta + 1)h + \tau_M$. Based on the data reconstruction scheme in (6) and the closed-loop system (4), and for $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, one can establish the following closed-loop system

$$\begin{cases} \dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{B}_1\xi(t - \tau(t)) + \tilde{B}_2\xi(t - d(t)) + \tilde{D}\nu(t) \\ r_e(t) = \tilde{C}\xi(t) + \tilde{E}_1\xi(t - \tau(t)) + \tilde{E}_2\xi(t - d(t)) + \tilde{F}\nu(t) \end{cases} \quad (7)$$

where $\tilde{A}$, $\tilde{D}$, $\tilde{C}$, and $\tilde{F}$ are the same as the corresponding items in (4), and

$$\begin{aligned} \tilde{B}_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a_1}{a}LC & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a_2}{a}LC & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \tilde{E}_1 &= [0 \quad -\frac{a_1}{a}SC \quad 0], \quad \tilde{E}_2 = [0 \quad -\frac{a_2}{a}SC \quad 0] \end{aligned}$$

To detect the occurrence of faults, one should construct a residual evaluation function $J(t)$. When the value of the residual evaluation function $J(t)$ is larger than a given threshold J_{th} , an alarm of faults will be generated. Define the residual evaluation function as

$$J(t) \triangleq \left(\int_0^t r_e^T(s)r_e(s)ds \right)^{\frac{1}{2}} \quad (8)$$

Choose a residual evaluation function threshold J_{th} as follows (see [3] for the selection criterion of J_{th})

$$J_{th} = \sup_{\nu(t) \in L_2, f(t)=0} J(t) \quad (9)$$

The fault detection logic is

$$\begin{cases} J(t) > J_{th}, & \text{with faults} \\ J(t) \leq J_{th}, & \text{without faults} \end{cases} \quad (10)$$

Based on the systems (4) and (7), and the fault detection logic (10), we study the problem of fault detection filter design for the continuous-time NCS considering packet dropouts and network-induced delays.

It should be pointed out that $\tau(t)$ and $d(t)$, which are named as interval time-varying delays in the literature, are different from the network-induced delays τ_k . In this paper, we take the mutually exclusive distribution characteristic of interval time-varying delays $\tau(t)$ and $d(t)$ into full consideration to

derive less conservative FDF design criteria. Take the interval time-varying delay $\tau(t)$ for example. Considering that $\tau(t) \in [\tau_m, \eta]$, we divide $[\tau_m, \eta]$ into ρ equidistant time intervals with ρ denoting a given positive integer, and define $\sigma_1 = (\eta - \tau_m)/\rho$, $\eta_1 = \tau_m + \sigma_1$, $\eta_2 = \tau_m + 2\sigma_1, \dots, \eta_{\rho-1} = \tau_m + (\rho-1)\sigma_1$. Then, one can conclude that at any instant t with $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, $\tau(t) \in [\tau_m, \eta_1)$ or $\tau(t) \in [\eta_1, \eta_2), \dots$, or $\tau(t) \in [\eta_{\rho-1}, \eta)$. On the other hand, for the specific instant t , $\tau(t) \in [\tau_m, \eta_1)$ or $\tau(t) \in [\eta_1, \eta_2), \dots$, or $\tau(t) \in [\eta_{\rho-1}, \eta)$ can not occur simultaneously. In this paper, we refer to such a phenomenon as mutually exclusive distribution. The mutually exclusive distribution characteristic of $d(t)$ can be achieved similarly. To make full use of the mutually exclusive distribution characteristic of $\tau(t)$ and $d(t)$, define $\sigma_3 = (\tilde{\eta} - \tau_m - h)/\rho$, $\tilde{\eta}_1 = \tau_m + h + \sigma_3$, $\tilde{\eta}_2 = \tau_m + h + 2\sigma_3, \dots, \tilde{\eta}_{\rho-1} = \tau_m + h + (\rho-1)\sigma_3$. Define scalars $\lambda_1, \lambda_2, \dots, \lambda_\rho$ and $\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_\rho$, where

$$\begin{aligned} \lambda_1 &= \begin{cases} 1, & \tau(t) \in [\tau_m, \eta_1) \\ 0, & \text{otherwise} \end{cases} \\ \lambda_2 &= \begin{cases} 1, & \tau(t) \in [\eta_1, \eta_2) \\ 0, & \text{otherwise} \end{cases} \\ &\vdots \\ \lambda_\rho &= \begin{cases} 1, & \tau(t) \in [\eta_{\rho-1}, \eta) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{11}$$

and

$$\begin{aligned} \tilde{\lambda}_1 &= \begin{cases} 1, & d(t) \in [\tau_m + h, \tilde{\eta}_1) \\ 0, & \text{otherwise} \end{cases} \\ \tilde{\lambda}_2 &= \begin{cases} 1, & d(t) \in [\tilde{\eta}_1, \tilde{\eta}_2) \\ 0, & \text{otherwise} \end{cases} \\ &\vdots \\ \tilde{\lambda}_\rho &= \begin{cases} 1, & d(t) \in [\tilde{\eta}_{\rho-1}, \tilde{\eta}) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{12}$$

Remark 3. *The discrete delay decomposition approach was proposed in [11] to study stability of linear retarded and neutral systems in the case of constant time-delays. Notice that the mutually exclusive distribution characteristic of interval time-varying delays is used to deal with integral inequalities for*

products of vectors in this paper, while the delay decomposition approach is used to construct the Lyapunov functional, they are different in essence.

3. Observer-based fault detection filter design for the NCS

In this section, we first present a new observer-based fault detection filter design for the closed-loop system (4). Then, we analyze the merits for considering the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$.

3.1. Fault detection filter design for the NCS

This subsection is concerned with observer-based fault detection filter design for the closed-loop system (4). Construct the following Lyapunov functional

$$V(t, \xi_t) = \sum_{i=1}^4 V_i(t, \xi_t) \quad (13)$$

where

$$\begin{aligned} V_1(t, \xi_t) &= \xi(t)^T P \xi(t) \\ V_2(t, \xi_t) &= (\eta - \tau(t)) \int_{t-\tau(t)}^t \dot{\xi}^T(s) Q \dot{\xi}(s) ds \\ V_3(t, \xi_t) &= \int_{t-\tau_m}^t \xi^T(s) R_1 \xi(s) ds + \int_{t-\eta}^{t-\tau_m} \xi^T(s) R_2 \xi(s) ds \\ V_4(t, \xi_t) &= \int_{-\tau_m}^0 \int_{t+s}^t \dot{\xi}^T(\theta) S_1 \dot{\xi}(\theta) d\theta ds + \int_{-\eta}^{-\tau_m} \int_{t+s}^t \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta ds \end{aligned}$$

P , Q , R_1 , R_2 , S_1 , and S_2 are symmetric positive definite matrices with appropriate dimensions. We state and establish the following result.

Theorem 1. *For given scalars τ_m , τ_M , δ , h , and ρ , the residual system (4) is asymptotically stable with an H_∞ norm bound γ and the fault detection filter gains $L = V_2^T N^{-T}$ and $S = V_1^T N^{-T}$ if there exist symmetric positive definite matrices W_1 , W_2 , W_3 , $\tilde{Q}$, $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{S}_1$, $\tilde{S}_2$, and matrices V_1 , V_2 , N , such that (14) and (15) hold for every feasible value of λ_i ($i = 1, 2, \dots, \rho$)*

$$\begin{bmatrix} \tilde{H}_{11} & \tilde{H}_{12} \\ * & \tilde{H}_{22} \end{bmatrix} < 0 \quad (14)$$

$$W_2 C^T = C^T N \quad (15)$$

where

$$\begin{aligned} \tilde{H}_{11} &= \begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} & 0 & \tilde{D} \\ * & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} & 0 & 0 \\ * & * & \tilde{\Pi}_{33} & \tilde{\Pi}_{34} & 0 \\ * & * & * & \tilde{\Pi}_{44} & 0 \\ * & * & * & * & -\gamma I \end{bmatrix}, \\ \tilde{H}_{12} &= \begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_2 & \tilde{\Phi}_2 \end{bmatrix}, \quad \tilde{H}_{22} = \text{diag}\{-\gamma I, \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3\}, \\ \tilde{\Pi}_{11} &= \tilde{A}W + W\tilde{A}^T + \tilde{R}_1 - \frac{\tilde{S}_1}{\tau_m} - \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q}, \\ \tilde{\Pi}_{12} &= \frac{\tilde{S}_1}{\tau_m}, \quad \tilde{\Pi}_{13} = \tilde{V}_2^T + \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q}, \\ \tilde{\Pi}_{22} &= \tilde{R}_2 - \tilde{R}_1 - \frac{\tilde{S}_1}{\tau_m} - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2, \\ \tilde{\Pi}_{23} &= \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2, \\ \tilde{\Pi}_{33} &= -\left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q} - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2 \\ &\quad - \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \\ \tilde{\Pi}_{34} &= \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \\ \tilde{\Pi}_{44} &= -\tilde{R}_2 - \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \\ \tilde{\Phi}_1 &= \begin{bmatrix} \tilde{C}W & 0 & \tilde{\Pi}_{36}^T & 0 & \tilde{F} \end{bmatrix}^T, \quad \tilde{\Phi}_2 = \begin{bmatrix} \tilde{A}W & 0 & \tilde{\Pi}_{37}^T & 0 & \tilde{D} \end{bmatrix}^T, \\ \mathcal{X}_1 &= \psi^{-1}(\tilde{Q} - 2W), \quad \mathcal{X}_2 = \tau_m^{-1}(\tilde{S}_1 - 2W), \quad \mathcal{X}_3 = (\eta - \tau_m)^{-1}(\tilde{S}_2 - 2W), \\ W &= \text{diag}\{W_1, W_2, W_3\}, \quad \psi = \lambda_1\rho\sigma_1 + \lambda_2(\rho-1)\sigma_1 + \dots + \lambda_\rho\sigma_1, \\ \tilde{\Pi}_{36} &= \tilde{V}_1, \quad \tilde{\Pi}_{37} = \tilde{V}_2, \quad \tilde{V}_1 = \begin{bmatrix} 0 \\ -C^T V_1 \\ 0 \end{bmatrix}, \quad \tilde{V}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -C^T V_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Proof. See Appendix A.

Remark 4. As shown in (11) and (12), λ_i and $\tilde{\lambda}_i$ ($i = 1, 2, \dots, \rho$) are not fixed but variable. The variations of λ_i and $\tilde{\lambda}_i$ are determined by the variations of $\tau(t)$ and $d(t)$, respectively. To guarantee the asymptotic stability of the residual system (4), the fault detection filter design criterion in Theorem 1 should hold for every feasible value of λ_i ($i = 1, 2, \dots, \rho$) with $\lambda_i = 1$ or $\lambda_i = 0$. Moreover, for any $\lambda_i = 1$, one has $\lambda_j = 0$, where $j = 1, 2, \dots, \rho$, and $j \neq i$.

3.2. Analysis of the merits for considering the mutually exclusive distribution characteristic of interval time-varying delays

The mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$ is made full use in Theorem 1 to deal with integral inequalities for products of vectors. If such a mutually exclusive distribution characteristic is not considered, the proposed FDF design method in Theorem 1 is still applicable, and the following FDF design criterion is followed immediately.

Corollary 1. For given scalars τ_m , τ_M , δ , and h , the residual system (4) is asymptotically stable with an H_∞ norm bound γ and the fault detection filter gains $L = V_2^T N^{-T}$ and $S = V_1^T N^{-T}$ if there exist symmetric positive definite matrices W_1 , W_2 , W_3 , $\tilde{Q}$, $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{S}_1$, $\tilde{S}_2$, and matrices V_1 , V_2 , N , such that (16) and (17) hold

$$\begin{bmatrix} \hat{H}_{11} & \tilde{H}_{12} \\ * & \hat{H}_{22} \end{bmatrix} < 0 \quad (16)$$

$$W_2 C^T = C^T N \quad (17)$$

where

$$\hat{H}_{11} = \begin{bmatrix} \hat{\Pi}_{11} & \tilde{\Pi}_{12} & \hat{\Pi}_{13} & 0 & \tilde{D} \\ * & \hat{\Pi}_{22} & \hat{\Pi}_{23} & 0 & 0 \\ * & * & \hat{\Pi}_{33} & \hat{\Pi}_{34} & 0 \\ * & * & * & \hat{\Pi}_{44} & 0 \\ * & * & * & * & -\gamma I \end{bmatrix},$$

$$\hat{H}_{22} = \text{diag}\{-\gamma I, \hat{\mathcal{X}}_1, \mathcal{X}_2, \mathcal{X}_3\}, \quad \hat{\Pi}_{11} = \tilde{A}W + W\tilde{A}^T + \tilde{R}_1 - \frac{\tilde{S}_1}{\tau_m} - \frac{\tilde{Q}}{\eta},$$

$$\hat{\Pi}_{13} = \tilde{V}_2^T + \frac{\tilde{Q}}{\eta}, \quad \hat{\Pi}_{22} = \tilde{R}_2 - \tilde{R}_1 - \frac{\tilde{S}_1}{\tau_m} - \frac{\tilde{S}_2}{\eta - \tau_m}, \quad \hat{\Pi}_{23} = \frac{\tilde{S}_2}{\eta - \tau_m},$$

$$\begin{aligned}\hat{\Pi}_{33} &= -\frac{\tilde{Q}}{\eta} - \frac{2}{\eta - \tau_m} \tilde{S}_2, \quad \hat{\Pi}_{34} = \frac{\tilde{S}_2}{\eta - \tau_m}, \\ \hat{\Pi}_{44} &= -\tilde{R}_2 - \frac{\tilde{S}_2}{\eta - \tau_m}, \quad \hat{\mathcal{X}}_1 = (\eta - \tau_m)^{-1}(\tilde{Q} - 2W),\end{aligned}$$

while $\tilde{H}_{12}$, $\tilde{\Pi}_{12}$, $\mathcal{X}_2$, $\mathcal{X}_3$, $\tilde{V}_2$ and W are the same as the corresponding items in Theorem 1.

Proof. If the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$ is not considered, the bounding inequalities in (A.5), (A.6) and (A.8) are converted into (18), (19) and (20), respectively,

$$(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t) \leq (\eta - \tau_m)\dot{\xi}^T(t)Q\dot{\xi}(t), \quad (18)$$

$$-\int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds \leq -\frac{1}{\eta}\varphi_1^T Q \varphi_1, \quad (19)$$

$$-\int_{t-\eta}^{t-\tau_m} \dot{\xi}^T(\theta)S_2\dot{\xi}(\theta)d\theta \leq -\frac{1}{\eta - \tau_m}(\varphi_3^T S_2 \varphi_3 + \varphi_4^T S_2 \varphi_4) \quad (20)$$

where φ_1 in (19) is the same as the corresponding item in (A.6), φ_3 and φ_4 in (20) are the same as the corresponding items in (A.8).

The rest of the proof is similar to the proof of Theorem 1, here it is omitted. This completes the proof.

The following theorem establishes the relationship between the FDF design criteria in Theorem 1 and Corollary 1.

Theorem 2. *For the residual system (4), if the FDF design criterion in Corollary 1 is satisfied, then the FDF design criterion in Theorem 1 is also satisfied.*

Proof. Notice that $(\eta - \tau_m)\dot{\xi}^T(t)Q\dot{\xi}(t)$ in (18) can be rewritten as $(\eta - \tau_m)\dot{\xi}^T(t)Q\dot{\xi}(t) = \Gamma_1 + \Delta_1$, where Γ_1 denotes the right side of the inequality (A.5), $\Delta_1 = [(\eta - \tau_m) - \psi]\dot{\xi}^T(t)Q\dot{\xi}(t)$, and ψ is the same as the corresponding item in (A.5).

Moreover, $-\frac{1}{\eta}\varphi_1^T Q \varphi_1$ in (19) can be rewritten as $-\frac{1}{\eta}\varphi_1^T Q \varphi_1 = \Gamma_2 + \Delta_2$, where Γ_2 denotes the right side of the inequality (A.6), $\Delta_2 = [-\frac{1}{\eta} + (\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_p}{\eta_p})]\varphi_1^T Q \varphi_1$.

Furthermore, $-\frac{1}{\eta-\tau_m}(\varphi_3^T S_2 \varphi_3 + \varphi_4^T S_2 \varphi_4)$ in (20) can be rewritten as $-\frac{1}{\eta-\tau_m} \times (\varphi_3^T S_2 \varphi_3 + \varphi_4^T S_2 \varphi_4) = \Gamma_3 + \Delta_3 + \Delta_4$, where Γ_3 denotes the right side of the inequality (A.8), $\Delta_3 = [-\frac{1}{\eta-\tau_m} + (\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1})]\varphi_3^T S_2 \varphi_3$, $\Delta_4 = [-\frac{1}{\eta-\tau_m} + (\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1})]\varphi_4^T S_2 \varphi_4$, while $\varphi_1, \varphi_2, \varphi_3$, and φ_4 are the same as the corresponding items in the proof of Theorem 1.

Considering that $\Delta_1 \geq 0$, $\Delta_2 \geq 0$, and Δ_1 and Δ_2 are not equal to zero simultaneously, one has $\Delta_1 + \Delta_2 > 0$. Similarly, one can see that $\Delta_3 + \Delta_4 > 0$. Then the integral inequalities in (A.5), (A.6) and (A.8) are more easier to be satisfied than the ones in (18), (19) and (20), respectively. From the proof of Theorem 1 and Corollary 1, one can conclude that if the FDF design criterion in Corollary 1 is satisfied, the FDF design criterion in Theorem 1 is also satisfied. This completes the proof.

Remark 5. *It has been proved that if the FDF design criterion in Corollary 1 is satisfied, then the FDF design criterion in Theorem 1 is also satisfied, which implies that considering the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$ can introduce less conservatism. If the mutually exclusive distribution characteristic of interval time-varying delays, which is considered in this paper for the first time, is made full use to deal with the problems in [15, 17, 47], less conservative results are expected to be obtained. The corresponding results are omitted here for brevity. Notice that the reciprocally convex approach is proposed in [19, 25] to deal with integral inequalities for products of vectors. The theoretical comparison between the reciprocally convex approach and the mutually exclusive distribution characteristic-based approach is a subject for further research.*

In the proof of Theorem 1, the mutually exclusive distribution characteristic of $\tau(t)$ has been made full use by dividing the time interval $[\tau_m, \eta)$ into ρ equidistant time intervals, and it has been proved in Theorem 2 that considering the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$ can introduce less conservatism. Then, a natural question is whether dividing $[\tau_m, \eta)$ into $\rho + 1$ equidistant time intervals will introduce less conservatism than dividing $[\tau_m, \eta)$ into ρ equidistant time intervals or not.

If the time interval $[\tau_m, \eta)$ is divided into $\rho + 1$ equidistant time intervals, define $\sigma_2 = (\eta - \tau_m)/(\rho + 1)$. Then, the bounding inequality in (A.5) is

converted into

$$\begin{aligned}
& (\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t) \\
&= \hat{\lambda}_1[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] + \hat{\lambda}_2[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] \\
&\quad + \cdots + \hat{\lambda}_\rho[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] + \hat{\lambda}_{\rho+1}[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] \\
&\leq \hat{\psi}\dot{\xi}^T(t)Q\dot{\xi}(t)
\end{aligned} \tag{21}$$

where $\hat{\psi} = \hat{\lambda}_1(\rho + 1)\sigma_2 + \hat{\lambda}_2\rho\sigma_2 + \cdots + 2\hat{\lambda}_\rho\sigma_2 + \hat{\lambda}_{\rho+1}\sigma_2$, and the definition for $\hat{\lambda}_1, \hat{\lambda}_2, \dots$, and $\hat{\lambda}_{\rho+1}$ is similar to the definition for $\lambda_1, \lambda_2, \dots$, and λ_ρ in (11).

Since σ_1 is different from σ_2 , it is difficult to prove theoretically whether $\hat{\psi}\dot{\xi}^T(t)Q\dot{\xi}(t)$ in (21) can introduce less conservatism than $\psi\dot{\xi}^T(t)Q\dot{\xi}(t)$ in (A.5) or not. Similar conclusions are drawn for the inequalities in (A.6) and (A.8).

In conclusion, if the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$ is adopted to deal with integral inequalities for products of vectors, not a theoretical proof shows that dividing $[\tau_m, \eta)$ into $\rho + 1$ equidistant time intervals can introduce less conservatism than dividing $[\tau_m, \eta)$ into ρ equidistant time intervals. However, the simulation results illustrate that dividing $[\tau_m, \eta)$ into $\rho + 1$ equidistant time intervals can introduce less conservatism.

4. Data reconstruction-based fault detection filter design

This section is concerned with the data reconstruction-based fault detection filter design for the closed-loop system (7). We state and establish the following result.

Theorem 3. *For given scalars $\tau_m, \tau_M, \delta, h$, and ρ , the residual system (7) is asymptotically stable with an H_∞ norm bound γ and the fault detection filter gains $L = V_2^T N^{-T}$ and $S = V_1^T N^{-T}$ if there exist symmetric positive definite matrices $W_1, W_2, W_3, \tilde{Q}, \tilde{M}, \tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4$, and matrices V_1, V_2, N , such that (22) and (23) hold for every feasible value of λ_i and $\tilde{\lambda}_i$ ($i = 1, 2, \dots, \rho$)*

$$\begin{bmatrix} \bar{H}_{11} & \bar{H}_{12} \\ * & \bar{H}_{22} \end{bmatrix} < 0 \tag{22}$$

$$W_2 C^T = C^T N \quad (23)$$

where

$$\begin{aligned} \bar{H}_{11} &= \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & 0 & \bar{\Pi}_{15} & \bar{\Pi}_{16} & 0 & \tilde{D} \\ * & \bar{\Pi}_{22} & \bar{\Pi}_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \bar{\Pi}_{33} & \bar{\Pi}_{34} & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Pi}_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Pi}_{55} & \bar{\Pi}_{56} & 0 & 0 \\ * & * & * & * & * & \bar{\Pi}_{66} & \bar{\Pi}_{67} & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{77} & 0 \\ * & * & * & * & * & * & * & -\gamma I \end{bmatrix}, \\ \bar{H}_{12} &= [\bar{\Phi}_1 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2 \quad \bar{\Phi}_2], \\ \bar{H}_{22} &= \text{diag}\{-\gamma I, \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \mathcal{X}_4, \mathcal{X}_5, \mathcal{X}_6\}, \\ \bar{\Pi}_{11} &= \tilde{A}W + W\tilde{A}^T - \frac{\tilde{S}_1}{\tau_m} - \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q} \\ &\quad + \tilde{R}_1 + \tilde{R}_3 - \frac{\tilde{S}_3}{\tau_m + h} - \left(\frac{\tilde{\lambda}_1}{\tilde{\eta}_1} + \frac{\tilde{\lambda}_2}{\tilde{\eta}_2} + \dots + \frac{\tilde{\lambda}_\rho}{\tilde{\eta}}\right)\tilde{M}, \\ \bar{\Pi}_{12} &= \frac{\tilde{S}_1}{\tau_m}, \quad \bar{\Pi}_{13} = \bar{V}_2^T + \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q}, \\ \bar{\Pi}_{15} &= \frac{\tilde{S}_3}{\tau_m + h}, \quad \bar{\Pi}_{16} = \hat{V}_2^T + \left(\frac{\tilde{\lambda}_1}{\tilde{\eta}_1} + \frac{\tilde{\lambda}_2}{\tilde{\eta}_2} + \dots + \frac{\tilde{\lambda}_\rho}{\tilde{\eta}}\right)\tilde{M}, \\ \bar{\Pi}_{22} &= \tilde{R}_2 - \tilde{R}_1 - \frac{\tilde{S}_1}{\tau_m} - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2, \\ \bar{\Pi}_{23} &= \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2, \\ \bar{\Pi}_{33} &= -\left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\tilde{Q} - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1}\right)\tilde{S}_2 \\ &\quad - \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \\ \bar{\Pi}_{34} &= \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \\ \bar{\Pi}_{44} &= -\tilde{R}_2 - \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1}\right)\tilde{S}_2, \end{aligned}$$

$$\begin{aligned}
\bar{\Pi}_{55} &= \tilde{R}_4 - \tilde{R}_3 - \frac{\tilde{S}_3}{\tau_m + h} - \left(\frac{\tilde{\lambda}_1}{\sigma_3} + \frac{\tilde{\lambda}_2}{2\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\rho\sigma_3} \right) \tilde{S}_4, \\
\bar{\Pi}_{56} &= \left(\frac{\tilde{\lambda}_1}{\sigma_3} + \frac{\tilde{\lambda}_2}{2\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\rho\sigma_3} \right) \tilde{S}_4, \\
\bar{\Pi}_{66} &= - \left(\frac{\tilde{\lambda}_1}{\sigma_3} + \frac{\tilde{\lambda}_2}{2\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\rho\sigma_3} \right) \tilde{S}_4 - \left(\frac{\tilde{\lambda}_1}{\rho\sigma_3} + \frac{\tilde{\lambda}_2}{(\rho-1)\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\sigma_3} \right) \tilde{S}_4 \\
&\quad - \left(\frac{\tilde{\lambda}_1}{\tilde{\eta}_1} + \frac{\tilde{\lambda}_2}{\tilde{\eta}_2} + \cdots + \frac{\tilde{\lambda}_\rho}{\tilde{\eta}} \right) \tilde{M}, \\
\bar{\Pi}_{67} &= \left(\frac{\tilde{\lambda}_1}{\rho\sigma_3} + \frac{\tilde{\lambda}_2}{(\rho-1)\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\sigma_3} \right) \tilde{S}_4, \\
\bar{\Pi}_{77} &= - \tilde{R}_4 - \left(\frac{\tilde{\lambda}_1}{\rho\sigma_3} + \frac{\tilde{\lambda}_2}{(\rho-1)\sigma_3} + \cdots + \frac{\tilde{\lambda}_\rho}{\sigma_3} \right) \tilde{S}_4, \\
\bar{\Phi}_1 &= \left[\tilde{C}W \quad 0 \quad \bar{\Pi}_{39}^T \quad 0 \quad 0 \quad \bar{\Pi}_{69}^T \quad 0 \quad \tilde{F} \right]^T, \\
\bar{\Phi}_2 &= \left[\tilde{A}W \quad 0 \quad \bar{\Pi}_{3,10}^T \quad 0 \quad 0 \quad \bar{\Pi}_{6,10}^T \quad 0 \quad \tilde{D} \right]^T, \\
\bar{\mathcal{X}}_1 &= \psi^{-1}(\tilde{Q} - 2W), \quad \bar{\mathcal{X}}_2 = \tau_m^{-1}(\tilde{S}_1 - 2W), \quad \bar{\mathcal{X}}_3 = (\eta - \tau_m)^{-1}(\tilde{S}_2 - 2W), \\
\bar{\mathcal{X}}_4 &= \bar{\psi}^{-1}(\tilde{M} - 2W), \quad \bar{\mathcal{X}}_5 = (\tau_m + h)^{-1}(\tilde{S}_3 - 2W), \\
\bar{\mathcal{X}}_6 &= (\tilde{\eta} - \tau_m - h)^{-1}(\tilde{S}_4 - 2W), \quad W = \text{diag}\{W_1, W_2, W_3\}, \\
\psi &= \lambda_1\rho\sigma_1 + \lambda_2(\rho-1)\sigma_1 + \cdots + \lambda_\rho\sigma_1, \\
\bar{\psi} &= \tilde{\lambda}_1\rho\sigma_3 + \tilde{\lambda}_2(\rho-1)\sigma_3 + \cdots + \tilde{\lambda}_\rho\sigma_3, \\
\bar{\Pi}_{39} &= \bar{V}_1, \quad \bar{\Pi}_{3,10} = \bar{V}_2, \quad \bar{\Pi}_{69} = \hat{V}_1, \quad \bar{\Pi}_{6,10} = \hat{V}_2, \\
\bar{V}_1 &= \begin{bmatrix} 0 \\ -\frac{a_1}{a}C^TV_1 \\ 0 \end{bmatrix}, \quad \bar{V}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a_1}{a}C^TV_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\hat{V}_1 &= \begin{bmatrix} 0 \\ -\frac{a_2}{a}C^TV_1 \\ 0 \end{bmatrix}, \quad \hat{V}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{a_2}{a}C^TV_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

Proof. Construct the following Lyapunov functional

$$V(t, \xi_t) = \sum_{i=1}^7 V_i(t, \xi_t) \quad (24)$$

where $V_l(t, \xi_t)$ ($l = 1, \dots, 4$) are the same as the corresponding items in

(13), and

$$\begin{aligned}
V_5(t, \xi_t) &= (\tilde{\eta} - d(t)) \int_{t-d(t)}^t \dot{\xi}^T(s) M \dot{\xi}(s) ds \\
V_6(t, \xi_t) &= \int_{t-\tau_m-h}^t \xi^T(s) R_3 \xi(s) ds + \int_{t-\tilde{\eta}}^{t-\tau_m-h} \xi^T(s) R_4 \xi(s) ds \\
V_7(t, \xi_t) &= \int_{-\tau_m-h}^0 \int_{t+s}^t \dot{\xi}^T(\theta) S_3 \dot{\xi}(\theta) d\theta ds + \int_{-\tilde{\eta}}^{-\tau_m-h} \int_{t+s}^t \dot{\xi}^T(\theta) S_4 \dot{\xi}(\theta) d\theta ds
\end{aligned}$$

M , R_3 , R_4 , S_3 , and S_4 are symmetric positive definite matrices with appropriate dimensions.

The rest of the proof is similar to the proof of Theorem 1, here it is omitted. This completes the proof.

Remark 6. To guarantee the asymptotic stability of the residual system (7), the fault detection filter design criterion in Theorem 3 should hold for every feasible value of λ_i and $\tilde{\lambda}_i$ ($i = 1, 2, \dots, \rho$) with $\lambda_i = 1$ or $\lambda_i = 0$, and $\tilde{\lambda}_i = 1$ or $\tilde{\lambda}_i = 0$. Moreover, for any $\lambda_i = 1$, one has $\lambda_j = 0$, where $j = 1, 2, \dots, \rho$, and $j \neq i$. Furthermore, for any $\tilde{\lambda}_i = 1$, one has $\tilde{\lambda}_j = 0$, where $j = 1, 2, \dots, \rho$, and $j \neq i$.

For the data reconstruction-based fault detection filter design criterion in Theorem 3, different data reconstruction parameters a_1 and a_2 presented in (6) lead to different fault detection performance. The following algorithm describes an approach how to choose appropriate a_1 and a_2 .

Algorithm 1.

Step 1. For given scalars τ_m , τ_M , δ , h , and ρ , choose the initial values $a_{\epsilon,0} > 0$ ($\epsilon = 1, 2$, and $a_{1,0} < a_{2,0}$) and the final values $a_{\epsilon,ult} > 0$ ($a_{\epsilon,ult} < a_{\epsilon,0}$) for a_{ϵ} , set appropriate step lengths $a_{\epsilon,dec} > 0$; choose a large enough H_{∞} norm bound γ_{opt} and set $a_{\epsilon,opt} = a_{\epsilon,0}$; set $a_1 = a_{1,0}$.

Step 2. Set $a_2 = a_{2,0}$.

Step 3. Solve the fault detection filter design criterion presented in Theorem 3, if $\gamma < \gamma_{opt}$, set $\gamma_{opt} = \gamma$, $a_{\epsilon,opt} = a_{\epsilon}$ and go to step 4; otherwise, go to step 4 directly.

Step 4. Set $a_2 = a_2 - a_{2,dec}$

if $a_2 \geq a_{2,ult}$
go to step 3
else

```

 $a_1 = a_1 - a_{1,dec}$ 
if  $a_1 \geq a_{1,ult}$ 
    go to step 2
else
    go to step 5
endif
endif
endif

```

Step 5. Output the locally optimal $a_{\epsilon,opt}$ and γ_{opt} .

By using Algorithm 1, one can get the locally optimal $a_{\epsilon,opt}$ to guarantee better fault detection performance.

It should be pointed out that the mutually exclusive distribution characteristic of interval time-varying delays $\tau(t)$ and $d(t)$ is made full use in Theorem 3 to deal with integral inequalities for products of vectors. In fact, if such a mutually exclusive distribution characteristic is not considered, the fault detection filter design method in Theorem 3 is still applicable, the corresponding result is omitted here for brevity.

Remark 7. In Theorem 1, Corollary 1, and Theorem 3, the fault detection filter gains are described as $L = V_2^T N^{-T}$ and $S = V_1^T N^{-T}$. For a specific NCS, if the derived matrix N is not invertible, one should add a constraint such as $N + N^T < 5I$ for the fault detection filter design criteria to get an invertible matrix N , and the corresponding fault detection filter design in Theorem 1, Corollary 1, and Theorem 3 is a suboptimal solution. Moreover, by assuming that C^T is full column rank, where C is presented in (1), and adopting the method in Theorem 2 in [35], the equality constraint $W_2 C^T = C^T N$ in Theorem 1, Corollary 1, and Theorem 3 can be removed, and one can derive the fault detection filter gains L and S which are independent on the matrix N . The detailed result is an extension of Theorem 2 in [35], here it is omitted for brevity.

Remark 8. Notice that both the robust design and the sensitivity constraint of an FDF are considered in [21]. From (29) in [21], one can see that when dealing with the robust design of the FDF, the fault signal is not considered. Similarly, from (50) in [21], one can see that the external disturbance is not considered when dealing with the sensitivity constraint of the FDF. For the FDF design in this paper, if the actuator fault signal $f(t)$ in (1) is assumed to be zero, the design criteria in Theorem 1, Corollary 1, and Theorem 3 reduce to criteria concerning the robustness of the residual signal to the disturbance and control input.

5. Simulation results and discussion

In this section, we give an example to illustrate the merits and effectiveness of the newly proposed fault detection scheme.

Example 1. Consider the two-tank benchmark [28]. Suppose that the linearized continuous time model of the two-tank system is (see [28, 33, 37])

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -3 & 3 \\ 3 & -6 \end{bmatrix} x(t) + \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix} u(t) + \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \omega(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t) \\ y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{cases} \quad (25)$$

The reference model is given by

$$\begin{cases} \dot{e}_r(t) = \begin{bmatrix} -5 & 1 \\ 2 & -6 \end{bmatrix} e_r(t) + \begin{bmatrix} 0.2 \\ 1 \end{bmatrix} \omega(t) + \begin{bmatrix} 0.9 \\ -0.5 \end{bmatrix} f(t) \\ r_{ref}(t) = \begin{bmatrix} -5 & -2 \end{bmatrix} e_r(t) + 2\omega(t) - 0.8f(t) \end{cases} \quad (26)$$

Firstly, we show the merits for considering the mutually exclusive distribution characteristic of interval time-varying delays. For Theorem 1 and Corollary 1, suppose that $\tau_m = 0.001$, $\tau_M = 0.05$, $\delta = 3$, $h = 0.05s$. To avoid that some elements of the obtained matrix S are close to zero, we assume that $N + N^T < 5I$ for Theorem 1, Corollary 1, and Theorem 3.

As discussed in Remark 4, the fault detection filter design criterion in Theorem 1 should hold for every feasible value of λ_i ($i = 1, 2, \dots, \rho$) with $\lambda_i = 1$ or $\lambda_i = 0$. By solving the FDF design criterion in Theorem 1 for every feasible value of λ_i ($i = 1, 2, \dots, \rho$), one can obtain that the H_∞ norm bounds corresponding to $\rho = 2$ and $\rho = 3$ are $\gamma_1 = 2.8041$ and $\gamma_2 = 2.7993$, respectively. On the other hand, solving the FDF design criterion in Corollary 1, one can obtain the H_∞ norm bound $\gamma_3 = 2.8401$. From $\gamma_2 < \gamma_1$, one can see that if the mutually exclusive distribution characteristic of interval time-varying delays is adopted to deal with integral inequalities for products of vectors, *the larger the number (that is ρ) of subintervals, the better the system performance*. Moreover, $\gamma_1 < \gamma_3$ and $\gamma_2 < \gamma_3$ verify the fact that the FDF design scheme considering the mutually exclusive distribution characteristic of interval time-varying delays introduces less conservatism than the FDF design scheme without considering such a mutually exclusive distribution characteristic.

In the following, we illustrate the merits and effectiveness of the newly proposed fault detection scheme. For Theorem 1 and Theorem 3, suppose that $\delta = 0$, $\rho = 2$, while τ_m , τ_M and h are the same as the ones presented above. By solving the FDF design criterion in Theorem 1, one can see that the FDF gain matrices derived by Theorem 1 are

$$L = \begin{bmatrix} 2.6508 & 2.0729 \\ 3.3788 & 1.7374 \end{bmatrix}, S = \begin{bmatrix} 0.0283 & 0.0007 \end{bmatrix}$$

For Theorem 3, suppose that $a_{1,0} = 1$, $a_{2,0} = 3$, $a_{1,ult} = 0.2$, $a_{2,ult} = 2$, $a_{1,dec} = 0.4$, $a_{2,dec} = 0.5$. By using Algorithm 1, one can get the locally optimal $a_{1,opt} = 1$, $a_{2,opt} = 3$, and the corresponding FDF gain matrices derived by Theorem 3 are

$$L = \begin{bmatrix} -0.2257 & 3.2462 \\ 2.7005 & -1.9388 \end{bmatrix}, S = \begin{bmatrix} 0.0753 & -0.0393 \end{bmatrix}$$

Figure 1: The curve of $\omega(t)$

Suppose that the initial state of the augmented systems (4) and (7) is $\xi_0 = [0.1 \ -0.1 \ 0.05 \ -0.05 \ 0.2 \ -0.2]^T$. The control input $u(t)$ is assumed to be zero. The disturbance input $\omega(t)$ is a normally (Gaussian) distributed random signal whose mean and variance are 0 and 0.01, respectively, and the curve of $\omega(t)$ is presented in Fig. 1. The fault signal $f(t)$ is described by

$$f(t) = \begin{cases} 1.8, & t \in [0.8s, 1.8s] \\ 0, & otherwise \end{cases} \quad (27)$$

Figure 2: The curve of $\tau(t)$

Figure 3: The curve of $d(t)$

Figure 4: The residual response $r_e(t)$ of the system (4)

Figure 5: The residual evaluation function response $J(t)$ of the system (4)

Figure 6: The residual response $r_e(t)$ of the system (7)

Figure 7: The residual evaluation function response $J(t)$ of the system (7)

Figure 8: The residual evaluation function response $J(t)$ of systems (4) and (7)

The interval time-varying delays $\tau(t)$ and $d(t)$ are given in Fig. 2 and Fig. 3, respectively. Then, Fig. 4 and Fig. 5 show the residual response $r_e(t)$ and the residual evaluation function response $J(t)$, respectively, for the system (4). Fig. 6 and Fig. 7 show the residual response $r_e(t)$ and the residual evaluation function response $J(t)$, respectively, for the system (7) which considers the data reconstruction.

From Fig. 5 and Fig. 7, one can see that the newly proposed fault detection scheme can not only reflect the occurrence of faults in time (that is, the proposed fault detection scheme is sensitive to faults), but also recognize faults without confusing them with the disturbance $\omega(t)$.

To show the merits of the data reconstruction-based fault detection filter design scheme, Fig. 8 presents the residual evaluation function response $J(t)$ for the system (4) without considering data reconstruction and the system (7) considering data reconstruction, where the fault signal is the same as the one in (27). From (9), Fig. 5 and Fig. 7, one can see that J_{th} for the system (4) and the system (7) are 0.3919 and 0.3878, respectively. Then, from Fig. 8, the fault detection logic presented in (10), and J_{th} presented above, one can see that the faults in the systems (4) and (7) are detected at the instants 0.8398s and 0.8357s, respectively. For systems (4) and (7), the faults occur at the instant 0.8s. Then, the fault detection time for the system (4) is 0.0398s, while the fault detection time for the system (7) is 0.0357s. Considering that 0.0357s is only 89.69% of 0.0398s, one can conclude that the data reconstruction-based fault detection scheme provides a shorter fault detection time than the fault detection scheme without considering data reconstruction, which illustrates the merits of the data reconstruction-based

fault detection scheme.

6. Conclusions

The FDF design for a continuous-time NCS considering packet dropouts, network-induced delays, and actuator faults has been studied. An observer-based FDF and a new data reconstruction scheme have been introduced to establish new closed-loop models. Based on the established models, FDF design criteria have been derived to asymptotically stabilize the residual systems. When dealing with integral inequalities for products of vectors, the mutually exclusive distribution characteristic of interval time-varying delays has been made full use to derive less conservative FDF design criteria. The designed FDFs can guarantee the sensitivity of the residual signal to faults, which has been verified by the simulation results.

7. Acknowledgements

This work was supported in part by the National Science Foundation of China (Grant Nos. 61004025, 61210306066, 61374063, and 61403170); the “333 Project” in Jiangsu Province, China; the “Qing Lan Project” in Jiangsu Province, China; the Key Project for International Science and Technology Cooperation and Inviting Experts in Universities of Jiangsu Province, China; and the Overseas Research Project for Excellent Young and Middle-Aged Teachers in Universities of Jiangsu Province, China.

Appendix A. Proof of Theorem 1

Taking the time derivative of the Lyapunov functional $V(t, \xi_t)$ given in (13) along the trajectory of the system (4), one has

$$\dot{V}_1(t, \xi_t) = 2\xi^T(t)P\dot{\xi}(t) \quad (\text{A.1})$$

$$\dot{V}_2(t, \xi_t) = (\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t) - \int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds \quad (\text{A.2})$$

$$\dot{V}_3(t, \xi_t) = \xi^T(t)R_1\dot{\xi}(t) + \xi^T(t - \tau_m)(R_2 - R_1)\dot{\xi}(t - \tau_m) - \xi^T(t - \eta)R_2\dot{\xi}(t - \eta) \quad (\text{A.3})$$

$$\dot{V}_4(t, \xi_t) = \dot{\xi}^T(t)\Theta\dot{\xi}(t) - \int_{t-\tau_m}^t \dot{\xi}^T(\theta)S_1\ddot{\xi}(\theta)d\theta - \int_{t-\eta}^{t-\tau_m} \dot{\xi}^T(\theta)S_2\ddot{\xi}(\theta)d\theta \quad (\text{A.4})$$

where $\Theta = \tau_m S_1 + (\eta - \tau_m)S_2$.

Notice that $\tau(t) \in [\tau_m, \eta]$. To make full use of the mutually exclusive distribution characteristic of the interval time-varying delay $\tau(t)$, we divide $[\tau_m, \eta]$ into ρ equidistant time intervals. Define $\sigma_1 = (\eta - \tau_m)/\rho$, $\eta_1 = \tau_m + \sigma_1$, $\eta_2 = \tau_m + 2\sigma_1$, $\dots$, $\eta_{\rho-1} = \tau_m + (\rho - 1)\sigma_1$. From the definition of σ_1 , one can see that η is equal to $\tau_m + \rho\sigma_1$.

Considering the mutually exclusive distribution characteristic of $\tau(t)$, and adopting the Jensen integral inequality in [6], one obtains

$$\begin{aligned}
& (\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t) \\
&= \lambda_1[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] + \lambda_2[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] \\
&\quad + \dots + \lambda_\rho[(\eta - \tau(t))\dot{\xi}^T(t)Q\dot{\xi}(t)] \\
&\leq \psi\dot{\xi}^T(t)Q\dot{\xi}(t)
\end{aligned} \tag{A.5}$$

where $\psi = \lambda_1\rho\sigma_1 + \lambda_2(\rho - 1)\sigma_1 + \dots + \lambda_\rho\sigma_1$, and

$$\begin{aligned}
& - \int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds \\
&= \lambda_1\left[\frac{1}{\tau(t)}(-\tau(t)) \int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds\right] + \lambda_2\left[\frac{1}{\tau(t)}(-\tau(t)) \int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds\right] \\
&\quad + \dots + \lambda_\rho\left[\frac{1}{\tau(t)}(-\tau(t)) \int_{t-\tau(t)}^t \dot{\xi}^T(s)Q\dot{\xi}(s)ds\right] \\
&\leq -\frac{\lambda_1}{\eta_1}\varphi_1^T Q\varphi_1 - \frac{\lambda_2}{\eta_2}\varphi_1^T Q\varphi_1 - \dots - \frac{\lambda_\rho}{\eta}\varphi_1^T Q\varphi_1 \\
&= -\left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta}\right)\varphi_1^T Q\varphi_1
\end{aligned} \tag{A.6}$$

where $\varphi_1 = [\xi(t) - \xi(t - \tau(t))]$, and

$$- \int_{t-\tau_m}^t \dot{\xi}^T(\theta)S_1\dot{\xi}(\theta)d\theta \leq -\frac{1}{\tau_m}\varphi_2^T S_1\varphi_2 \tag{A.7}$$

where $\varphi_2 = [\xi(t) - \xi(t - \tau_m)]$.

Similarly, one has

$$\begin{aligned}
& - \int_{t-\eta}^{t-\tau_m} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta \\
& = \lambda_1 \left[- \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta - \int_{t-\eta}^{t-\tau(t)} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta \right] \\
& \quad + \lambda_2 \left[- \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta - \int_{t-\eta}^{t-\tau(t)} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta \right] \\
& \quad + \cdots + \lambda_\rho \left[- \int_{t-\tau(t)}^{t-\tau_m} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta - \int_{t-\eta}^{t-\tau(t)} \dot{\xi}^T(\theta) S_2 \dot{\xi}(\theta) d\theta \right] \\
& \leq - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \cdots + \frac{\lambda_\rho}{\rho\sigma_1} \right) \varphi_3^T S_2 \varphi_3 - \left(\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \cdots + \frac{\lambda_\rho}{\sigma_1} \right) \varphi_4^T S_2 \varphi_4
\end{aligned} \tag{A.8}$$

where $\varphi_3 = [\xi(t - \tau_m) - \xi(t - \tau(t))]$, $\varphi_4 = [\xi(t - \tau(t)) - \xi(t - \eta)]$.

Then, considering the $r_e(t)$ in (4), and combining (A.1)-(A.8) together, one has

$$\dot{V}(t, \xi_t) + \gamma^{-1} r_e^T(t) r_e(t) - \gamma \nu^T(t) \nu(t) \leq \tilde{\xi}^T(t) (\Pi + \Xi) \tilde{\xi}(t) \tag{A.9}$$

where $\tilde{\xi}(t) = [\xi^T(t) \quad \xi^T(t - \tau_m) \quad \xi^T(t - \tau(t)) \quad \xi^T(t - \eta) \quad \nu^T(t)]^T$, and

$$\begin{aligned}
\Pi &= \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & P\tilde{D} \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 \\ * & * & \Pi_{33} & \Pi_{34} & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & -\gamma I \end{bmatrix}, \\
\Xi &= \gamma^{-1} \Phi_1 \Phi_1^T + \Phi_2 (\psi Q + \tau_m S_1 + (\eta - \tau_m) S_2) \Phi_2^T, \\
\Pi_{11} &= P\tilde{A} + \tilde{A}^T P + R_1 - \frac{S_1}{\tau_m} - \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \cdots + \frac{\lambda_\rho}{\eta} \right) Q, \\
\Pi_{12} &= \frac{S_1}{\tau_m}, \quad \Pi_{13} = P\tilde{B} + \left(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \cdots + \frac{\lambda_\rho}{\eta} \right) Q, \\
\Pi_{22} &= R_2 - R_1 - \frac{S_1}{\tau_m} - \left(\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \cdots + \frac{\lambda_\rho}{\rho\sigma_1} \right) S_2,
\end{aligned}$$

$$\begin{aligned}
\Pi_{23} &= (\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1})S_2, \\
\Pi_{33} &= -(\frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} + \dots + \frac{\lambda_\rho}{\eta})Q - (\frac{\lambda_1}{\sigma_1} + \frac{\lambda_2}{2\sigma_1} + \dots + \frac{\lambda_\rho}{\rho\sigma_1})S_2 \\
&\quad - (\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1})S_2, \\
\Pi_{34} &= (\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1})S_2, \\
\Pi_{44} &= -R_2 - (\frac{\lambda_1}{\rho\sigma_1} + \frac{\lambda_2}{(\rho-1)\sigma_1} + \dots + \frac{\lambda_\rho}{\sigma_1})S_2, \\
\Phi_1 &= [\tilde{C} \quad 0 \quad \tilde{E} \quad 0 \quad \tilde{F}]^T, \quad \Phi_2 = [\tilde{A} \quad 0 \quad \tilde{B} \quad 0 \quad \tilde{D}]^T
\end{aligned}$$

From the inequality in (A.9), one can see that if $\Pi + \Xi < 0$, one has $\dot{V}(t, \xi_t) + \gamma^{-1}r_e^T(t)r_e(t) - \gamma\nu^T(t)\nu(t) < 0$. By using Schur complement, $\Pi + \Xi < 0$ is equivalent to

$$\begin{bmatrix} \Pi & H_{12} \\ * & H_{22} \end{bmatrix} < 0 \quad (\text{A.10})$$

where Π is the same as the corresponding item presented above, and

$$\begin{aligned}
H_{12} &= [\Phi_1 \quad \Phi_2 \quad \Phi_2 \quad \Phi_2], \\
H_{22} &= \text{diag}\{-\gamma I, \quad -\psi^{-1}Q^{-1}, \quad -\tau_m^{-1}S_1^{-1}, \quad -(\eta - \tau_m)^{-1}S_2^{-1}\}
\end{aligned}$$

Pre- and post-multiplying both sides of (A.10) with $\text{diag}\{\underbrace{P^{-1}, \dots, P^{-1}}_4, \underbrace{I, \dots, I}_5\}$ and its transpose, supposing that there exists a matrix N such that $W_2 C^T = C^T N$, defining $P^{-1} = W$, $W = \text{diag}\{W_1, W_2, W_3\}$, $P^{-1}QP^{-1} = \tilde{Q}$, $P^{-1}R_1P^{-1} = \tilde{R}_1$, $P^{-1}R_2P^{-1} = \tilde{R}_2$, $P^{-1}S_1P^{-1} = \tilde{S}_1$, $P^{-1}S_2P^{-1} = \tilde{S}_2$, $NS^T = V_1$, $NL^T = V_2$, and considering that $-Q^{-1} \leq \tilde{Q} - 2W$, $-S_1^{-1} \leq \tilde{S}_1 - 2W$, $-S_2^{-1} \leq \tilde{S}_2 - 2W$, one can see that if (14) and (15) are satisfied simultaneously, $\Pi + \Xi < 0$ is also satisfied, which implies that $\dot{V}(t, \xi_t) + \gamma^{-1}r_e^T(t)r_e(t) - \gamma\nu^T(t)\nu(t) < 0$.

By using the definition for H_∞ performance, one can prove that if $\nu(t) = 0$, (14) and (15) can ensure the asymptotic stability of the system (4); if $\nu(t) \neq 0$, we have $\|r_e(t)\|_2 < \gamma\|\nu(t)\|_2$. Therefore, if (14)-(15) are satisfied, the system (4) is asymptotically stable with an H_∞ norm bound γ . This completes the proof.

References

- [1] M.G. Cea, G.C. Goodwin, Stabilization of systems over bit-rate-constrained networked control architectures, *IEEE Transactions on Industrial Informatics* 9 (2013) 357-364.
- [2] H. Dong, Z. Wang, J. Lam, H. Gao, Fuzzy-model-based robust fault detection with stochastic mixed time delays and successive packet dropouts, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics* 42 (2012) 365-376.
- [3] P.M. Frank, X. Ding, Survey of robust residual generation and evaluation methods in observer-based fault detection systems, *Journal of Process Control* 7 (1997) 403-424.
- [4] E. Fridman, A refined input delay approach to sampled-data control, *Automatica* 46 (2010) 421-427.
- [5] H. Gao, J. Wu, P. Shi, Robust sampled-data H_∞ control with stochastic sampling, *Automatica* 45 (2009) 1729-1736.
- [6] K. Gu, V.L. Kharitonov, J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, 2003.
- [7] G. Guo, A switching system approach to sensor and actuator assignment for stabilisation via limited multi-packet transmitting channels, *International Journal of Control* 84 (2011) 78-93.
- [8] G. Guo, Linear systems with medium-access constraint and Markov actuator assignment, *IEEE Transactions on Circuits and Systems-I: Regular Papers* 57 (2010) 2999-3010.
- [9] G. Guo, H. Jin, A switching system approach to actuator assignment with limited channels, *International Journal of Robust and Nonlinear Control* 20 (2010) 1407-1426.
- [10] G. Guo, Z. Lu, Q.-L. Han, Control with Markov sensors/actuators assignment, *IEEE Transactions on Automatic Control* 57 (2012) 1799-1804.
- [11] Q.-L. Han, A discrete delay decomposition approach to stability of linear retarded and neutral systems, *Automatica* 45 (2009) 517-524.

- [12] X. He, Z. Wang, Y. Liu, D.H. Zhou, Least-squares fault detection and diagnosis for networked sensing systems using a direct state estimation approach, *IEEE Transactions on Industrial Informatics* 9 (2013) 1670-1679.
- [13] D. Huang, S.K. Nguang, Robust fault estimator design for uncertain networked control systems with random time delays: an ILMI approach, *Information Sciences* 180 (2010) 465-480.
- [14] I. Izadi, T. Chen, Q. Zhao, Norm invariant discretization for sampled-data fault detection, *Automatica* 41 (2005) 1633-1637.
- [15] X. Jia, D. Zhang, X. Hao, N. Zheng, Fuzzy H_∞ tracking control for non-linear networked control systems in T-S fuzzy model, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics* 39 (2009) 1073-1079.
- [16] X. Jiang, Q.-L. Han, New stability criteria for linear systems with interval time-varying delay, *Automatica* 44 (2008) 2680-2685.
- [17] X. Jiang, Q.-L. Han, S. Liu, A. Xue, A new H_∞ stabilization criterion for networked control systems, *IEEE Transactions on Automatic Control* 53 (2008) 1025-1032.
- [18] A. Johansson, M. Bask, T. Norlander, Dynamic threshold generators for robust fault detection in linear systems with parameter uncertainty, *Automatica* 42 (2006) 1095-1106.
- [19] W.I. Lee, P. Park, Second-order reciprocally convex approach to stability of systems with interval time-varying delays, *Applied Mathematics and Computation* 229 (2014) 245-253.
- [20] X.-J. Li, G.-H. Yang, Dynamic observer-based robust control and fault detection for linear systems, *IET Control Theory & Applications* 6 (2012) 2657-2666.
- [21] B. Liu, Y. Xia, Y. Yang, M. Fu, Robust fault detection of linear systems over networks with bounded packet loss, *Journal of the Franklin Institute* 349 (2012) 2480-2499.

- [22] Y. Long, G.-H. Yang, Fault detection for a class of networked control systems with finite-frequency servo inputs and random packet dropouts, *IET Control Theory & Applications* 6 (2012) 2397-2408.
- [23] Y. Long, G.-H. Yang, Fault detection in finite frequency domain for networked control systems with missing measurements, *Journal of the Franklin Institute* 350 (2013) 2605-2626.
- [24] Z. Mao, B. Jiang, P. Shi, Protocol and fault detection design for non-linear networked control systems, *IEEE Transactions on Circuits and Systems-II: Express Briefs* 56 (2009) 255-259.
- [25] P. Park, J.W. Ko, C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, *Automatica* 47 (2011) 235-238.
- [26] C. Peng, M.-R. Fei, E. Tian, Y.-P. Guan, On hold or drop out-of-order packets in networked control systems, *Information Sciences* 268 (2014) 436-446.
- [27] C. Peng, Q.-L. Han, D. Yue, To transmit or not to transmit: a discrete event-triggered communication scheme for networked Takagi-Sugeno fuzzy systems, *IEEE Transactions on Fuzzy Systems*, 21 (2013) 164-170.
- [28] D. Sauter, T. Boukhobza, Robustness against unknown networked induced delays of observer based FDI, in: *Proceedings of the IFAC Safe-process*, Beijing, China, 2006, pp. 331-336.
- [29] H. Shao, Q.-L. Han, Z. Zhang, X. Zhu, Sampling-interval-dependent stability for sampled-data systems with state quantization, *International Journal of Robust and Nonlinear Control* 24 (2014) 2995-3008.
- [30] A.A. Stoorvogel, H.H. Niemann, A. Saberi, P. Sannuti, Optimal fault signal estimation, *International Journal of Robust and Nonlinear Control* 12 (2002) 697-727.
- [31] E. Tian, W.K. Wong, D. Yue, Robust H_∞ control for switched systems with input delays: a sojourn-probability-dependent method, *Information Sciences* 283 (2014) 22-35.
- [32] Y. Wang, S.X. Ding, H. Ye, G. Wang, A new fault detection scheme for networked control systems subject to uncertain time-varying delay, *IEEE Transactions on Signal Processing* 56 (2008) 5258-5268.

- [33] Y.Q. Wang, S.X. Ding, H. Ye, L. Wei, P. Zhang, G.Z. Wang, Fault detection of networked control systems with packet based periodic communication, *International Journal of Adaptive Control and Signal Processing* 23 (2009) 682-698.
- [34] Y.-L. Wang, Q.-L. Han, Modelling and controller design for discrete-time networked control systems with limited channels and data drift, *Information Sciences* 269 (2014) 332-348.
- [35] Y.-L. Wang, Q.-L. Han, Modelling and observer-based H_∞ controller design for networked control systems, *IET Control Theory & Applications* 8 (2014) 1478-1486.
- [36] D. Wang, J. Wang, W. Wang, H_∞ controller design of networked control systems with markov packet dropouts, *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 43 (2013) 689-697.
- [37] Y.Q. Wang, H. Ye, G.Z. Wang, Fault detection of NCS based on eigendecomposition, adaptive evaluation and adaptive threshold, *International Journal of Control* 80 (2007) 1903-1911.
- [38] F. Yang, Q.-L. Han, H_∞ control for networked systems with multiple packet dropouts, *Information Sciences* 252 (2013) 106-117.
- [39] D. Yue, E. Tian, Z. Wang, J. Lam, Stabilization of systems with probabilistic interval input delays and its applications to networked control systems, *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans* 39 (2009) 939-945.
- [40] D. Yue, E. Tian, Y. Zhang, C. Peng, Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics* 39 (2009) 503-516.
- [41] L. Zhang, H. Gao, O. Kaynak, Network-induced constraints in networked control systems—a survey, *IEEE Transactions on Industrial Informatics* 9 (2013) 403-416.
- [42] B.-L. Zhang, Q.-L. Han, Network-based modelling and active control for offshore steel jacket platform with TMD mechanisms, *Journal of Sound and Vibration* 333 (2014) 6796-6814.

- [43] J. Zhang, J. Lam, Y. Xia, Output feedback delay compensation control for networked control systems with random delays, *Information Sciences* 265 (2014) 154-166.
- [44] D. Zhang, Q.-G. Wang, L. Yu, Q.-K. Shao, H_∞ filtering for networked systems with multiple time-varying transmissions and random packet dropouts, *IEEE Transactions on Industrial Informatics* 9 (2013) 1705-1716.
- [45] Y.-B. Zhao, J. Kim, G.-P. Liu, Error bounded sensing for packet-based networked control systems, *IEEE Transactions on Industrial Electronics* 58 (2011) 1980-1989.
- [46] Y. Zhao, J. Lam, H. Gao, Fault detection for fuzzy systems with intermittent measurements, *IEEE Transactions on Fuzzy Systems* 17 (2009) 398-410.
- [47] X.-L. Zhu, G.-H. Yang, Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay, *IET Control Theory & Applications* 2 (2008) 524-534.