1 Thermal transport due to liquid jet impingement on
2 superhydrophobic surfaces with isotropic slip

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6 **Abstract**

7 This paper presents an analytical investigation of thermal transport due
8 to a steady, laminar, axisymmetric liquid jet impinging normally on a super-
9 hydrophobic (SHPo) surface maintained at constant surface temperature. At
10 the liquid-surface boundary of the spreading thin film, an isotropic hydro-
11 dynamic slip and temperature jump are imposed to approximate the SHPo
12 surface boundary condition. Applying an integral analysis within the thin
13 film results in a system of differential equations which are solved numerically
14 to obtain local hydrodynamic and thermal boundary layer thicknesses, thin
15 film height, and local and radially averaged heat flux. The classical smooth
16 hydrophobic scenario with no-slip and no-temperature jump showed excel-
17 lent agreement with previous differential analysis of the same problem. The
18 influence of varying temperature jump length on the local Nusselt number
19 was obtained over a range of Reynolds and Prandtl numbers. Increasing
20 temperature jump length results in a dramatic decrease in the local thermal
21 transport near the impingement point. The greatest decrease occurs at small
22 temperature jump lengths. Further, local and average Nusselt numbers are
23 less influenced by the Reynolds and Prandtl numbers as temperature jump
24 length increases. Overall, variations in the temperature jump length exert
25 much more influence than variations in the hydrodynamic slip length.

26
27 *Keywords:*

28 superhydrophobic surface, jet impingement, convective heat transfer, slip
29 length, temperature jump length

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30 Nomenclature

31	a	jet radius
32	c	liquid specific heat
33	h	liquid film height
34	$\hat{h}$	normalized film height $\hat{h} = h/a$
35	$\hat{h}_0$	normalized liquid film height at end of region I
36	$\hat{h}_1$	normalized film height at end of region II
37	H	control volume height
38	k	liquid thermal conductivity
39	$\dot{m}$	mass flow rate leaving top control volume surface
40	Nu	Nusselt number $Nu = q_w'' a \pi / (k(T_w - T_j))$
41	$\overline{Nu}$	radially-averaged Nusselt number
42	$\overline{Nu}_0$	average Nusselt number for no-slip and no temperature jump case
43	Pr	Prandtl number $Pr = \nu/\alpha$
44	Q	jet volumetric flow rate
45	q_w''	wall heat flux
46	r	radial coordinate
47	$\hat{r}$	normalized radial coordinate $\hat{r} = r/a$
48	$\hat{r}_0$	normalized radial coordinate at end of region I
49	$\hat{r}_1$	normalized radial coordinate at end of region II
50	Re	Reynolds number $Re = Q/(a\nu) = \pi a V/\nu$
51	T	local liquid temperature
52	T_{fs}	free surface temperature
53	T_j	initial jet temperature
54	T_w	wall temperature
55	u	radial velocity
56	$\hat{u}$	normalized velocity $\hat{u} = u/V$
57	U	free surface velocity
58	$\hat{U}$	normalized free surface velocity $\hat{U} = U/V$
59	V	jet velocity
60	z	axial coordinate
61	$\hat{z}$	normalized axial coordinate $\hat{z} = z/a$

62

63 **Greek Symbols**

64	α	thermal diffusivity
65	δ	hydrodynamic boundary layer thickness
66	$\hat{\delta}$	normalized hydrodynamic boundary layer thickness $\hat{\delta} = \delta/a$
67	δ_T	thermal boundary layer thickness
68	$\hat{\delta}_T$	normalized thermal boundary layer thickness $\hat{\delta}_T = \delta_T/a$
69	$\hat{\delta}_{T0}$	normalized thermal boundary layer thickness at end of region I
70	Δr	control volume radial thickness
71	ΔT	wall temperature jump $\Delta T = T_w - T(r, z = 0)$
72	$\Delta \hat{T}$	non-dimensional wall temperature jump $\Delta \hat{T} = \Delta T/(T_w - T_j)$
73	θ	non-dimensional temperature $\theta = (T - T_w)/(T_j - T_w)$
74	θ_{fs}	non-dimensional free surface temperature $\theta_{fs} = (T_{fs} - T_w)/(T_j - T_w)$
75	λ	hydrodynamic slip length
76	$\hat{\lambda}$	normalized hydrodynamic slip length $\hat{\lambda} = \lambda/a$
77	λ_T	temperature jump length
78	$\hat{\lambda}_T$	normalized temperature jump length $\hat{\lambda}_T = \lambda_T/a$
79	ν	kinematic viscosity
80	ρ	fluid density

81

82 **1. Introduction**

83 Numerous studies have demonstrated drag reduction and self-cleaning
84 behavior on superhydrophobic (SHPo) surfaces [1–7]. More recently, thermal
85 transport physics have been explored as well. For a liquid flowing over a SHPo
86 surface, thermal transport is inhibited due to insulating air-filled cavities at
87 the wall. This has been observed in forced internal flow [8–11], natural
88 convection in microchannels [12, 13], and Marangoni convection in droplets
89 [14]. Boiling on SHPo surfaces is inhibited primarily due to an early transition

90 from nucleate to film boiling resulting from the higher liquid-solid contact
91 angle and altered surface geometry [15, 16]. Conversely, condensation heat
92 transfer is enhanced due to increased vapor-surface contact area [17, 18].
93 The focus of this paper is to explore the thermal transport for a liquid flow
94 scenario which has not received attention: a laminar, liquid, axisymmetric
95 jet impinging normally on a SHPo surface with isotropic slip.

96 The hydrodynamics of a jet impinging on a no-slip surface were modeled
97 analytically by Watson [19]. These results specifically capture the develop-
98 ment of the hydrodynamic boundary layer within the spreading thin film,
99 the radial variation of thin film height, and estimate the hydrodynamic jump
100 location. The model compares well with experimental data and has been
101 improved to include surface tension in subsequent studies [20–22].

102 Thermal transport on a surface due to the spreading thin film of an im-
103 pinging jet has also received significant prior attention. This interest has
104 been motivated by industrial applications including cooling of electronic sys-
105 tems and quenching of metals and molded plastics [23, 24]. Analytical models
106 for scenarios of constant wall temperature [25], constant wall heat flux [26],
107 and varying wall temperature or heat flux [27, 28] have been accomplished.
108 Additionally, numerical [29–33] and experimental studies [24, 34, 35] have
109 been performed. In all studies, it was observed that the greatest thermal
110 transport occurs near the impingement point and decreases asymptotically
111 with increasing radial position. Additionally, the local and average Nusselt
112 numbers increase with increasing Prandtl number and jet Reynolds number.

113 SHPo surfaces differ significantly from smooth hydrophobic or hydrophilic
114 surfaces. They are created by combining micro/nano-scale roughness with

115 hydrophobic surface chemistry. Due to the surface hydrophobicity, water
 116 does not wet the cavities between roughness features if the Laplace pressure
 117 is not exceeded. Random or structured grid microfeatures are commonly
 118 employed. Important geometric parameters of SHPo surfaces with repeating
 119 features are the pitch, w , (the distance between microscale features) and
 120 cavity fraction, F_c (ratio of the cavity area projected onto the interface to
 121 the total interface area).

122 If the air-filled cavities are on the micron scale, the hydrodynamic and
 123 thermal boundary conditions at a surface can be significantly altered [17, 36].
 124 Liquid near the wall encounters a no-slip boundary condition at the liquid-
 125 solid microfeature interface and a nearly shear-free boundary condition at
 126 the liquid-air interface between microfeatures. For macroscopic flows, it is
 127 beneficial to define an aggregate slip accounting for the alternating slip and
 128 no-slip boundary conditions to obtain a uniform boundary condition. This
 129 condition allows definition of a local slip velocity, u_s , which is proportional
 130 to the wall shear stress, τ_w , and is given by, $u_s = \lambda\tau_w/\mu$, where μ is liquid
 131 dynamic viscosity, and λ is the hydrodynamic slip length [36]. λ is a property
 132 of the wall microstructure and can be defined in the Stokes flow regime by
 133 the surface parameters, cavity fraction and pitch [36].

134 The jet impingement problem on SHPo surfaces with isotropic [37] and
 135 anisotropic slip lengths [38] has previously been modeled analytically and
 136 validated experimentally [39]. Increasing λ at fixed Reynolds number was
 137 observed to have a similar effect on the hydrodynamics as increasing the
 138 Reynolds number. The hydrodynamic boundary layer developed more slowly,
 139 the film thickness decreased, and the hydrodynamic jump radius increased.

140 The composite liquid-solid and liquid-gas interface also alters the thermal
 141 boundary conditions. The liquid-solid interface yields a conventional convec-
 142 tion boundary condition, while at the liquid-gas interface the heat flux is
 143 greatly reduced. Assuming metallic microfeatures (high thermal conductiv-
 144 ity), the thermal conductivity of the gas filling each cavity is several orders
 145 of magnitude less than that of the microfeatures, rendering the liquid-gas in-
 146 terface nominally adiabatic. Again for macroscopic scale flows, an aggregate
 147 boundary condition can be defined in the form of a wall temperature jump,
 148 ΔT_w . This temperature jump is proportional to the wall heat flux, q_w'' , as
 149 given by the relationship, $\Delta T_w = \lambda_T q_w'' / k$ [11], where k is the liquid thermal
 150 conductivity and λ_T the temperature jump length. The temperature jump
 151 length is the thermal analog to the hydrodynamic slip length [9, 11, 40] and
 152 is a function of microscale feature geometry.

153 The relationship between λ_T and λ is an area of ongoing research. For
 154 Stokes flow between parallel plates with ribs aligned with the flow, λ_T is
 155 very nearly equal to λ [11]. A study of thermal transport in a channel with
 156 ribs oriented perpendicular to the flow observed a more complex relationship
 157 between λ and λ_T [8]. In this study, the ratio of λ_T to λ was shown to depend
 158 on Peclet number, relative module width (ratio of pitch to channel hydraulic
 159 diameter), and cavity fraction. At low Peclet number and small relative
 160 module width, the ratio of λ_T to λ is nominally 2. However, increasing the
 161 Peclet number to the order of 1,000 and setting the relative module width to
 162 1 causes the ratio to decrease.

163 The effect of λ_T on heat transfer in an impinging jet has not been previ-
 164 ously explained, but is of interest as it represents a common cooling scenario

for self-cleaning surfaces. Section 2 of this paper presents an integral analysis to quantify the effect of isotropic slip and temperature jump (λ and λ_T) on the local and average thermal transport. Results from the model are presented and discussed in Section 3 and conclusions are given in Section 4.

2. Analysis

2.1. Model Description

Shown in Fig. 1 is a schematic illustration of a vertical liquid jet impinging on a horizontal SHPo surface. A non-submerged, steady jet is assumed with jet velocity, V ; radius, a ; and flow rate, Q . Temperatures for the incoming jet and wall are T_j and T_w , respectively, both of which are assumed to be uniform and $T_w > T_j$. The wall temperature jump is $\Delta T = T_w - T(r, z = 0)$. A spatially uniform hydrodynamic slip length, λ , and temperature jump length, λ_T , characterize the superhydrophobic wall boundary condition.

For this analysis, $\hat{\lambda}$ and $\hat{\lambda}_T$ are assumed to be equal except in an exploratory case where they are varied independently. This assumption may be reasonable for typical SHPo surfaces but is an area for further research. As noted in the introduction, a recent study found $\hat{\lambda}$ and $\hat{\lambda}_T$ were approximately equal for a parallel plate channel with streamwise oriented ribs and cavities [11]. The text will discuss the influence of varying $\hat{\lambda}_T$ but, since $\hat{\lambda} = \hat{\lambda}_T$, the variation of $\hat{\lambda}$ is implied.

In Fig. 1, the development of the hydrodynamic (δ) and thermal (δ_T) boundary layers are illustrated. A coordinate system is fixed at the intersection of the jet centerline and the horizontal surface, where the z-axis is in the direction of the jet centerline, oriented normal to the surface and opposite

189 the direction of flow. r is the radial coordinate.

Figure 1: Schematic of a liquid jet impinging normally on a flat surface. The thin film is magnified along the z -coordinate to show the film dynamics. The film height (denoted by a solid line), the hydrodynamic boundary layer thickness (denoted by a dashed line), the thermal boundary layer thickness (denoted by a dotted line), and the control volumes utilized (denoted by dash-dotted lines) are shown.

190 Three regions exist moving radially outward from the impingement point
 191 prior to the location of a hydraulic jump. Region I spans r between the
 192 impingement point and the location, r_0 , where the hydrodynamic boundary
 193 layer, δ , merges with the film height, h . Region II extends from r_0 up to the
 194 location where the thermal boundary layer, δ_T , merges with the film height
 195 (r_1). We note that $\delta_T < \delta$, since the jet is liquid and $Pr > 1$, where the
 196 Prandtl number takes the classical form, $Pr = \nu/\alpha$. α is the liquid thermal
 197 diffusivity and ν is the liquid kinematic viscosity. Finally, region III spans r
 198 between r_1 and the location where a hydraulic jump forms.

199 Natural convection, radial conduction, viscous dissipation, and Marangoni
 200 convection are neglected in the analysis and constant liquid properties are
 201 assumed. Forced convection dominates natural convection due to the rel-
 202 atively high jet Reynolds numbers considered. Likewise, radial advection
 203 dominates radial conduction. Since the liquid has a moderate viscosity and
 204 relatively slow velocity, viscous dissipation is negligible. Due to the high jet
 205 Reynolds number, the influence of Marangoni convection is small since it is
 206 driven by variations in surface tension due to temperature variation across
 207 the meniscus between microfeatures. Heat transfer through the top liquid
 208 surface and shear stress between the water and air at this surface are also
 209 neglected and, following prior work, the initial thickness of the boundary
 210 layer was considered to be negligibly small [26].

211 *2.2. Application of Conservation Laws*

212 Conservation laws of mass and energy are applied to an annular control
 213 volume of thickness, Δr , and height, H . Control volume heights are selected
 214 such that known boundary conditions may be applied to the top control
 215 surface. Specifically, control volumes of height δ_T (regions I and II) and h
 216 (region III) were used as shown in Fig. 1.

217 Applying conservation of mass and energy to a general control volume as
 218 shown in Fig. 1 yields

$$\dot{m} = -2\pi\rho\frac{\partial}{\partial r}\left(r\int_0^H u(r,z)\,dz\right)\Delta r \tag{1}$$

219 and

$$-\dot{m}cT(r, H) + q_w''(2\pi r)\Delta r - 2\pi\rho c\frac{\partial}{\partial r}\left(r\int_0^H u(r, z)T(r, z) dz\right)\Delta r = 0 \quad (2)$$

220 where $\dot{m}$ is the mass flow rate through the top surface of the control volume,
 221 ρ is the liquid density, and c is the liquid specific heat. As mentioned, the
 222 hydrodynamic problem was solved previously and is included here only as
 223 needed [37].

224 2.3. Region I

225 In region I, the hydrodynamic boundary layer develops until it merges
 226 with the film height. Throughout this region, the thermal boundary layer is
 227 developing as well, but with a thickness that is less than that of the hydro-
 228 dynamic boundary layer. The control volume height is set to $H = \delta_T$. At
 229 this height, $T(r, \delta_T) = T_j$. Using these boundary conditions and substituting
 230 Eq. 1 into 2 yields

$$q_w'' = \frac{\rho c}{r} \frac{\partial}{\partial r} \left(r \int_0^{\delta_T} u(T(r, z) - T_j) dz \right) \quad (3)$$

231 A cubic velocity profile is assumed for region I, with a slip velocity boundary
 232 condition at $z = 0$ and a free shear condition at $z = \delta$ and is expressed as

$$u = \frac{V}{\frac{2}{3}\delta + \lambda} \left[\lambda + z - \frac{z^3}{3\delta^2} \right] \quad (4)$$

233 A cubic temperature profile is also assumed of the form

$$T = a + bz + cz^2 + dz^3 \quad (5)$$

234 Coefficients for this region I profile are found using the following boundary
235 conditions

$$T(r, z = 0) = T_w - \Delta T_w \quad (6)$$

236

$$T(r, z = \delta_T) = T_j \quad (7)$$

237

$$\left. \frac{\partial T}{\partial z} \right|_{z=\delta_T} = 0 \quad (8)$$

238

$$\left. \frac{\partial^2 T}{\partial z^2} \right|_{z=0} = 0 \quad (9)$$

239 Applying these boundary conditions to Eq. 5 yields

$$T = T_w - \Delta T_w - \frac{3(T_w - \Delta T_w - T_j)}{2\delta_T} z + \frac{T_s - \Delta T_w - T_j}{2\delta_T^3} z^3 \quad (10)$$

240 Using Fourier's law, the heat flux may be obtained from Eq. 5 as

$$q_w'' = -k \left. \frac{dT}{dz} \right|_{z=0} = \frac{3k(T_w - \Delta T_w - T_j)}{2\delta_T} \quad (11)$$

241 The temperature jump length boundary condition [11] $q_w'' = (\Delta T_w k)/(\lambda_T)$ is
242 substituted into the assumed profile (Eq. 11) and solving for the temperature
243 jump gives

$$\Delta T_w = \frac{3\lambda_T(T_w - T_j)}{2\delta_T + 3\lambda_T} \quad (12)$$

244 Substituting Eq. 12 into Eq. 10 gives the following expression for the local
245 liquid temperature

$$T = T_w - \frac{3\lambda_T(T_w - T_j)}{2\delta_T + 3\lambda_T} - \frac{3(T_w - T_j)}{(2\delta_T + 3\lambda_T)} z + \frac{T_w - T_j}{\delta_T^2(2\delta_T + 3\lambda_T)} z^3 \quad (13)$$

246 The wall heat flux is determined by substituting the new temperature profile
 247 (Eq. 13) into Fourier's law and evaluating the derivative

$$q_w'' = \frac{3k(T_w - T_j)}{2\delta_T + 3\lambda_T} \quad (14)$$

248 The velocity profile (Eq. 4), temperature profile (Eq. 13), and heat flux
 249 equation (Eq. 14) are all substituted into the energy equation (Eq. 3), and
 250 non-dimensionalized. Velocities are normalized by the jet velocity, V , and all
 251 lengths by the jet radius, a , with all non-dimensionalized variables denoted
 252 with a hat. The following nonlinear ordinary differential equation (ODE)
 253 describing the relationship between $\hat{\delta}_T$, $\hat{\delta}$, and $\hat{r}$ is obtained

$$\frac{\pi \hat{r}}{RePr} = (2\hat{\delta}_T + 3\hat{\lambda}_T) \frac{d}{d\hat{r}} \left[\hat{r} \frac{\hat{\delta}_T^2 (7\hat{\delta}^2 (4\hat{\delta}_T + 15\hat{\lambda}) - 2\hat{\delta}_T^3)}{140\hat{\delta}^2 (2\hat{\delta} + 3\hat{\lambda}) (2\hat{\delta}_T + 3\hat{\lambda}_T)} \right] \quad (15)$$

254 where the Reynolds number is defined as $Re = Q/(a\nu) = \pi aV/\nu$.

255 In a similar manner, the following ODE was derived for the hydrodynamic
 256 variables in region I by substituting the assumed cubic velocity profile (Eq.
 257 4) into the resulting momentum integral for a similar control volume to yield
 258 [37]

$$\frac{\pi \hat{r}}{Re} = \left(\frac{2}{3}\hat{\delta} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[\hat{r} \hat{\delta}^2 \left(\frac{13}{210}\hat{\delta} + \frac{1}{4}\hat{\lambda} \right) \left(\frac{2}{3}\hat{\delta} + \hat{\lambda} \right)^{-2} \right] \quad (16)$$

259 Applying conservation of mass (Eq. 1), the local film height, $\hat{h}$, as a function
 260 of $\hat{\lambda}$ and $\hat{\delta}$, can be expressed

$$\hat{h} = \frac{1}{2\hat{r}} + \frac{\hat{\delta}^2}{4(\frac{2}{3}\hat{\delta} + \hat{\lambda})} \quad (17)$$

261 A numerical solution for $\hat{h}$, $\hat{\delta}$ and $\hat{\delta}_T$ as a function of $\hat{r}$ was obtained
 262 by solving the system of equations formed by Eqs. 15, 16, and 17 with
 263 boundary conditions $\hat{\delta}(\hat{r} = 0) = 0$, and $\hat{\delta}_T(\hat{r} = 0) = 0$. The extent of region
 264 I was determined by numerically solving for the value of $\hat{r}$ where $\hat{\delta}$ reaches
 265 $\hat{h}$. Normalized values of the thermal boundary layer thickness and local film
 266 height at that point are labeled $\hat{\delta}_{T0}$ and $\hat{h}_0$.

267 Nusselt number, $Nu = q_w'' a \pi / (k(T_w - T_j))$, and the non-dimensional tem-
 268 perature jump, $\Delta\hat{T} = \Delta T / (T_w - T_j)$, as functions of $\hat{r}$ may be obtained
 269 from the wall heat flux (Eq. 14) and the wall temperature jump (Eq. 12),
 270 respectively. These are given as Equations 18 and 19.

$$Nu = \frac{3\pi}{2\hat{\delta}_T + 3\hat{\lambda}_T} \quad (18)$$

$$\Delta\hat{T} = \frac{3\hat{\lambda}_T}{2\hat{\delta}_T + 3\hat{\lambda}_T} \quad (19)$$

271 *Region II*

272 In region II, the hydrodynamic boundary layer has merged with the film
 273 height and the thermal boundary layer continues to grow until it also merges.
 274 The energy equation (Eq. 3), assumed temperature profile (Eq. 13), and the
 275 heat flux equation (Eq. 14) are the same as region I. However, boundary
 276 conditions on the velocity profile are altered in regions II and III as the free
 277 surface velocity is now variable. The resultant cubic velocity profile with a
 278 free shear boundary condition at the film thickness yields

$$\hat{u} = \frac{\hat{U}_{fs}}{\frac{2}{3}\hat{h} + \hat{\lambda}} \left[\hat{\lambda} + \hat{z} - \frac{\hat{z}^3}{3\hat{h}^2} \right] \quad (20)$$

279 where $\hat{U}_{fs}$ is the normalized velocity at the free surface. $\hat{U}_{fs} = U_{fs}/V$ may
 280 be expressed as

$$\hat{U}_{fs} = \frac{\frac{2}{3}\hat{h} + \hat{\lambda}}{2\hat{h}\hat{r}(\frac{5}{12}\hat{h} + \hat{\lambda})} \quad (21)$$

281 which results from conservation of mass (Eq. 1). Substituting Eq. 21 into
 282 the velocity profile (Eq. 20) and then substituting this result and the tem-
 283 perature profile (Eq. 13) into the energy equation (Eq. 3) gives the following

$$\frac{\pi\hat{r}}{RePr} = (2\hat{\delta}_T + 3\hat{\lambda}_T) \frac{d}{d\hat{r}} \left[\frac{\hat{\delta}_T^2(7\hat{h}^2(4\hat{\delta}_T + 15\hat{\lambda}) - 2\hat{\delta}_T^3)}{70\hat{h}^3(5\hat{h} + 12\hat{\lambda})(2\hat{\delta}_T + 3\hat{\lambda}_T)} \right] \quad (22)$$

284 The film thickness height can be determined throughout regions II and III
 285 by applying the integral momentum equation that is detailed in [37] and
 286 expanded as

$$-\frac{2\pi}{Re} = \hat{h}(\frac{5}{12}\hat{h} + \hat{\lambda}) \frac{d}{d\hat{r}} \left[\hat{r}^{-1}\hat{h}^{-1} \left(\frac{68}{315}\hat{h}^2 + \frac{5}{6}\hat{h}\hat{\lambda} + \hat{\lambda}^2 \right) \left(\frac{5}{12}\hat{h} + \hat{\lambda} \right)^{-2} \right] \quad (23)$$

287 The system of equations formed by the hydrodynamic ODE (Eq. 23)
 288 and the energy ODE (Eq. 22) was solved numerically to obtain $\hat{h}$ and $\hat{\delta}_T$ as
 289 functions of $\hat{r}$ throughout region II. The initial conditions were $\hat{h}(\hat{r} = \hat{r}_0) = \hat{h}_0$
 290 and $\hat{\delta}_T(\hat{r} = \hat{r}_0) = \hat{\delta}_{T0}$. The end of region II was determined by numerically
 291 solving for the radial coordinate where $\hat{\delta}_T$ equaled $\hat{h}$. This radial coordinate
 292 was labeled $\hat{r}_1$ and the corresponding film height was labeled $\hat{h}_1$. The Nusselt
 293 number and $\Delta\hat{T}$ as functions of $\hat{r}$ are defined in the same manner as in region
 294 1 (Eq. 18 and Eq. 19).

295 *Region III*

296 In region III, the hydrodynamic and thermal boundary layers have both
 297 merged with the film height. Here the control volume height is $H = h$ such
 298 that $\dot{m} = 0$. Applying this simplification to the energy equation (Eq. 2) and
 299 rearranging yields

$$q_w'' = \frac{\rho c}{r} \frac{\partial}{\partial r} \left(r \int_0^h u T \, dz \right) \quad (24)$$

300 In this region, the assumed velocity profile is the same as in region II (Eq.
 301 20), but the boundary conditions for the temperature profile are now

$$T(r, z = h) = T_{fs}(r) \quad (25)$$

$$\left. \frac{\partial T}{\partial z} \right|_{z=h} = 0 \quad (26)$$

303 The assumed temperature profile is the same as region I and region II (Eq.
 304 13) with the exception that T_j is replaced by $T_{fs}(r)$ and δ_T by h

$$T = T_w - \frac{3\lambda_T(T_w - T_{fs})}{2h + 3\lambda_T} - \frac{3(T_w - T_{fs})}{(2h + 3\lambda_T)}z + \frac{T_w - T_{fs}}{h^2(2h + 3\lambda_T)}z^3 \quad (27)$$

305 where T_{fs} is the temperature at the free surface. Further, the heat flux
 306 equation (Eq. 14) and temperature jump equation (Eq. 12) are altered with
 307 the same substitutions

$$q_w'' = \frac{3k(T_w - T_{fs})}{2h + 3\lambda_T} \quad (28)$$

$$\Delta T_w = \frac{3\lambda_T(T_w - T_{fs})}{2h + 3\lambda_T} \quad (29)$$

Using the assumed velocity (Eq. 20) and temperature (Eq. 27) profiles, as well as the expression for the free surface velocity (Eq. 21) in the governing equation (Eq. 24) results in the following form of the energy equation

$$\frac{\pi \hat{r}}{RePr} = \frac{2\hat{h} + 3\hat{\lambda}}{\theta_{fs}} \frac{d}{d\hat{r}} \left[\frac{\hat{h}\theta_{fs}(26\hat{h} + 105\hat{\lambda})}{70(5\hat{h} + 12\hat{\lambda})(2\hat{h} + 3\hat{\lambda}_T)} - \frac{\theta_{fs}}{6} \right] \quad (30)$$

The non-dimensional temperature is defined as $\theta = (T - T_w)/(T_j - T_w)$ and the non-dimensional free surface temperature is $\theta_{fs} = (T_{fs} - T_w)/(T_j - T_w)$. As defined, the non-dimensional free surface temperature is unity when the free surface temperature is equal to the jet temperature and decreases to zero as the free surface temperature approaches the wall temperature.

The system of equations formed by the hydrodynamic ODE (Eq. 23) and energy ODE (Eq. 30) was solved numerically to obtain $\hat{h}$ and θ_{fs} as functions of $\hat{r}$ throughout all of region III. In this region, the initial conditions were $\hat{h}(\hat{r} = \hat{r}_1) = \hat{h}_1$ and $\theta_{fs}(\hat{r} = \hat{r}_1) = 1$.

Non-dimensionalizing Eq. 28 and Eq. 29 results in expressions for Nu and $\Delta\hat{T}$ as functions of $\hat{r}$ in this final region.

$$Nu = \frac{3\pi\theta_{fs}}{2\hat{h} + 3\hat{\lambda}_T} \quad (31)$$

$$\Delta\hat{T} = \frac{3\hat{\lambda}_T\theta_{fs}}{2\hat{h} + 3\hat{\lambda}_T} \quad (32)$$

The systems of equations for regions I, II, and III were solved using LSODA (a version of the Livermore Solver for Ordinary Differential equations) as implemented by the Mathematica® solver software. The routine switches automatically between a multi-order (1-12) Adams method for non-

stiff problems and a backward-difference method for stiff problems. The numerical error in determining the local Nusselt number is $\pm 1 \times 10^{-4}$.

3. Results

3.1. Comparison with previous results

First, we compare our solution approach to previous work of Chaudhury for jet impingement on a classical smooth no-slip surface [25]. Chaudhury showed that the Reynolds number dependence could be accounted for using the scaling shown in Fig. 2, where the product $\hat{r}^2 \overline{Nu} Re^{-1}$ is shown as a function of $\hat{r} Re^{1/3}$. The results of Fig. 2 correspond to the no-slip and matching temperature boundary conditions for Prandtl numbers of 2, 5, and 8. Excellent agreement exists between the present results and those of Chaudhury [25]. Small discrepancies between the models arise because the present work approximates the velocity and temperature profiles with third-order polynomials, whereas, Chaudhury used fourth-order polynomial profiles in regions I and II and a similarity solution within region III.

A similarity solution is not possible when a uniform slip velocity exists [41], which is the influence we are investigating. Additionally, Chaudhury selected fourth-order polynomials so that the thermal boundary layer thickness would match the hydrodynamic boundary layer thickness when $Pr = 1$. The present work builds upon that of a previous hydrodynamic model [37], which utilized third-order polynomials and obtained excellent agreement with the prior no-slip solution [19]. The approach is similar to that taken by Chaudhury in that the same order polynomials as the hydrodynamic model are selected so that the hydrodynamic and thermal boundary layers are equal

when $Pr = 1$.

3.2. Local behavior: Influence of slip velocity and temperature jump, Reynolds number, and Prandtl number.

This section of the paper explores the influence of slip velocity and temperature jump on the local thermal boundary layer growth and temperature profiles. The thermal boundary layer thickness and film height, $\hat{\delta}_T$ and $\hat{h}$, are shown as functions of $\hat{r}$ for varying $\hat{\lambda}$, $\hat{\lambda}_T$, Re , and Pr in Fig. 3. For all scenarios considered, $\hat{\delta}_T$ increases (starting at 0) with increasing $\hat{r}$ until it eventually merges with $\hat{h}$. The $\hat{\delta}_T$ curves are qualitatively similar across the entire range of temperature jump lengths: the thermal boundary layer

Figure 2: $\hat{r}^2 \overline{Nu} Re^{-1}$ as a function of $\hat{r} Re^{-1/3}$ for the classical no-slip and matching wall temperature scenario for Pr ranging from 2 to 8. Present solution results (dashed lines) are compared to corresponding solutions presented by Chaudhury (solid lines) [25].

grows similar to the classical behavior with the thickness initially increasing with the radial coordinate. The growth rate in $\hat{\delta}_T$ decreases with increasing

radial coordinate up until the existence of an inflection point in the $\hat{\delta}_T$ vs. $\hat{r}$ plot, which marks the transition from region I to region II. This inflection occurs because the hydrodynamic boundary layer has merged with the film thickness in region II. The magnitude of the inflection decreases with increasing temperature jump length, increasing Re , and decreasing Pr . At $\hat{r}$ values beyond the inflection point, the growth rate of $\hat{\delta}_T$ increases because $\hat{h}$ is decreasing less rapidly as the jet spreads into a thin film [37].

Panel (a) of Fig. 3 shows the thin film height and thermal boundary layer thickness as a function of $\hat{r}$ and for varying values of $\hat{\lambda}_T$ ranging from $\hat{\lambda}_T = 0$ (the no-slip condition) to $\hat{\lambda}_T = 0.2$. The Reynolds number and Prandtl number for these cases are constant at $Re = 9,000$ and $Pr = 5$. The local values of both $\hat{h}$ and $\hat{\delta}_T$ decrease as $\hat{\lambda}$ increases. $\hat{h}$ decreases since the velocity near the wall is higher and the hydrodynamic boundary layer develops more slowly as a result of lower wall shear. The thermal boundary layer thickness develops more slowly due to increased thermal resistance at the wall caused by the temperature jump, which results in a decreased heating rate.

Panel (b) of Fig. 3 highlights the influence of varying Re while the temperature jump length is kept constant at $\hat{\lambda}_T = 0.1$. The Reynolds number is varied from 3,000 to 15,000 and, for all cases, $Pr = 5$. Increasing Re results in an increase in the momentum of the thin film, thus decreasing the thickness of $\hat{\delta}$, $\hat{\delta}_T$, and $\hat{h}$. Similar to previous work regarding slip length [37], increasing $\hat{\lambda}$ has a similar effect as increasing Re which is obvious when comparing the results of panel (a) and panel (b).

(a)

(b)

(c)

Figure 3: Film thickness and thermal boundary layer thickness as a function of radial position. Panel (a): $Pr = 5$, $Re = 9,000$, and $\hat{\lambda}$ and $\hat{\lambda}_T$ varying from 0 to 0.2. Panel (b): $Pr = 5$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and Re varying from 3,000 to 15,000. Panel (c): $Re = 9,000$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and Pr varying from 2 to 11.

387 The influence of variations in Pr is illustrated in panel (c) for $\hat{\lambda}_T = 0.1$
 388 and $Re = 9,000$. Of course, the hydrodynamic solution is not affected by
 389 varying Pr and increasing Pr expectedly causes the thermal boundary layer
 390 to grow more slowly with $\hat{r}$ as the thermal energy diffuses more slowly.

391 Non-dimensional temperature profiles ($\theta(\hat{z})$) at two different normalized
 392 radial locations, $\hat{r} = 10$ and $\hat{r} = 30$, are shown in Fig. 4 for various combi-
 393 nations of Re , Pr , and $\hat{\lambda}_T$. The left column (panels (a), (c), and (e)) show
 394 profiles evaluated at $\hat{r} = 10$ and the right column (panels (b), (d), and (f))
 395 show profiles evaluated at $\hat{r} = 30$. The vertical extent of the profiles vary
 396 because all profiles terminate at the height of the thin film, $\hat{h}$, which varies
 397 with $\hat{r}$, Re , and $\hat{\lambda}$, as shown in Fig. 3. At $\hat{h}$, the temperature gradient is zero
 398 as required by the adiabatic boundary condition at the free surface. Also,
 399 due to the non-dimensionalization ($\theta = (T - T_w)/(T_j - T_w)$), $\theta = 1$ when
 400 $T = T_j$ and $\theta = 0$ when $T = T_w$. For all scenarios, as $\hat{z}$ increases (moving
 401 from the surface), θ increases, concomitant with a greater difference between
 402 the local temperature and the wall temperature.

403 In the left column of panels (a, c, e) at $\hat{r} = 10$, many of the profiles are not
 404 fully developed and plateau at $\theta = 1$ where $T = T_j$. This occurs because this
 405 portion of the thin film has not yet been influenced by the conditions at the
 406 wall. For the temperature profiles evaluated at $\hat{r} = 30$, the profile is located
 407 within region III where the thermal boundary layer is fully developed. Here
 408 a plateau does not exist and the profile stretches across the entire thin film.

Figure 4: Non-dimensional temperature profiles, as functions of wall-normal coordinate, $\hat{z}$, at two $\hat{r}$ locations. Panels (a), (c), and (e): $\hat{r} = 10$. Panels (b), (d), and (f): $\hat{r} = 30$. Panels (a) and (b): $Pr = 5$, $Re = 9,000$, and $\hat{\lambda}_T$ ranging from 0 to 0.2. Panels (c) and (d): $Pr = 5$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and Re ranging from 3,000 to 15,000. Panels (e) and (f): $Re = 9,000$, $\hat{\lambda}_T = 0.1$, and Pr ranging from 2 to 11.

409 For all profiles shown, the temperature profiles that do not pass through
 410 the origin exhibit an apparent temperature jump at the wall due to the
 411 superhydrophobic temperature jump condition. This temperature jump may
 412 be as large as 90% of the temperature difference between the wall and the jet
 413 (see panel (a)). In panels (a) and (b) Re and Pr are held constant at 9,000
 414 and 5 respectively and $\hat{\lambda}_T$ is varied through 0, 0.05, 0.1, and 0.2.

415 The results of panels (a) and (b) highlight the difference in thickness of
 416 the thermal boundary layer for the no-slip and no temperature jump case
 417 ($\hat{\lambda} = \hat{\lambda}_T$) relative to the slip and temperature jump cases $\hat{\lambda} = \hat{\lambda}_T > 0$. As
 418 $\hat{\lambda}_T$ increases, there is an increased resistance to heat flux at the surface and
 419 consequently a smaller $\hat{\delta}_T$.

420 The results of panels (c) and (d) correspond to a fixed value of $Pr = 5$
 421 and $\hat{\lambda} = \hat{\lambda}_T = 0.1$ while Re is varied through the values: 3,000, 9,000, and
 422 15,000. As Re increases, the temperature jump at the wall increases at both
 423 $\hat{r} = 10$ and $\hat{r} = 30$, and this is more pronounced at lower $\hat{r}$. Increasing Re
 424 has a similar effect as increasing $\hat{\lambda}_T$ on the temperature jump at the wall,
 425 although larger Re yields larger wall-normal gradients in θ while increasing
 426 $\hat{\lambda}_T$ leads to smaller gradients. The results for panels (e) and (f) correspond
 427 to $Re = 9,000$ and $\hat{\lambda} = \hat{\lambda}_T = 0.1$ while Pr is varied through the values of
 428 2, 5, and 11. As Pr increases, the thermal energy diffuses more slowly from
 429 the wall into the liquid, as is typical, and θ approaches zero more slowly
 430 with increasing $\hat{r}$. Further, the results reveal that the wall non-dimensional
 431 temperature jump also increases with increasing Prandtl number.

432 A local Nu may be obtained from the local values of $\hat{\delta}_T$ and θ_{fs} using
 433 Eq. 18 and 31. Shown in Figs. 5 and 6 is the local Nusselt number as a

function of $\hat{r}$ and the results demonstrate the influence exerted by variations in slip length and temperature jump length. For all scenarios (all values of $\hat{\lambda}_T$), Nu decreases with increasing $\hat{r}$ as expected. The transition from region I to II for all scenarios is indicated by a plus sign + and the region II to III transition is indicated by an X.

Results are shown for $Re = 3,000$ (panel(a)) and $15,000$ (panel (b)) at constant $Pr = 5$ in Fig. 5 for $\hat{\lambda}_T = 0, 0.05, 0.1$, and 0.2 . Results are shown for the same values of $\hat{\lambda}_T$ in Fig. 6, but for $Pr = 2, 5$, and 11 and at a constant $Re = 9,000$. Nu increases (at fixed $\hat{r}$) with increasing Re or Pr for the no-slip and matching temperature cases as expected. The influence of Re or Pr decreases in cases with a non-zero temperature jump length. The negligible change in Nu with Re or Pr at the stagnation point ($\hat{r} = 0$), for cases with a non-zero temperature jump length, illustrates this effect. In general, these surfaces have smaller and more uniform Nu and these effects increase as the temperature jump length increases.

We would like to further emphasize the departure from the smooth surface behavior ($\hat{\lambda}_T = 0$) to what prevails for the superhydrophobic scenarios ($\hat{\lambda}_T \neq 0$). We first consider the transport near the jet centerline and stagnation region ($\hat{r} < 15$), where the influence of temperature jump exerts the greatest influence and large variations in Nu exist for the range of $\hat{\lambda}_T$ values considered. For all scenarios, increases in $\hat{\lambda}_T$ yield notably smaller values of Nu , compared to the smooth surface case. For example, for the $Re = 15,000$, $Pr = 5$ scenario and at $\hat{r} = 4$, Nu decreases by nominally 60%, 77%, and 90% for the $\hat{\lambda}_T = 0.05, 0.1$, and 0.2 scenarios, respectively. The dependence on $\hat{\lambda}_T$ is greatest at the jet centerline and decreases moving radially outward.

Figure 5: Nu as function of $\hat{r}$ for $\hat{\lambda}_T$ ranging from 0 to 0.2. $Pr = 5$. Panel (a): $Re = 3,000$. Panel (b): $Re = 15,000$. A $Re = 9,000$ and $Pr = 5$ case is not shown here but is included in Fig. 10. Plus signs + indicate the region I to region II transition and Xs indicate the region II to region III transition.

Figure 6: Nu as function of $\hat{r}$ for $\hat{\lambda}_T$ ranging from 0 to 0.2. $Re = 9,000$. Pr is varied through 2, 5, and 11 in panels (a), (b), and (c), respectively. As in Fig. 5, plus signs + indicate the region I to region II transition and Xs indicate the region II to region III transition.

459 The relative (compared to the smooth surface case) decrease in magnitude
 460 of Nu with increasing $\hat{\lambda}_T$ is a strong function of Re and a moderate function
 461 of Pr . In general, the relative decrease in Nu (with increasing $\hat{\lambda}_T$) is larger
 462 as either Re or Pr are increased, as illustrated by the data in both figures.

463 Two other points deserve discussion relative to the data of Figs. 5 and
 464 6. First, as $\hat{\lambda}_T$ increases to 0.2, the overall variation with Nu varies from
 465 approximately 18 at the stagnation point to approximately 14 at $\hat{r} = 30$, in
 466 stark contrast to the variation observed in the $\hat{\lambda}_T = 0$ scenario. The implica-
 467 tion of this behavior is that the local heat flux becomes much more uniform
 468 for the larger temperature jump length scenarios. Indeed, at the larger Re ,
 469 Pr , and $\hat{\lambda}_T$ explored, Nu is approaching a uniform value for all $\hat{r}$, an inter-
 470 esting and unexpected result. The second point is related to the first. At
 471 sufficiently large values of $\hat{r}$ (the exact point is dependent on Re and Pr but
 472 is generally between 15 and 25 radii from the stagnation point), the magni-
 473 tude of Nu for the $\hat{\lambda} \neq 0$ scenarios exceeds the value for the $\hat{\lambda} = 0$ surface.
 474 The implication here is that the heat transfer is actually increased on the
 475 superhydrophobic surfaces at large radial locations. This behavior prevails
 476 because of the dramatic decrease in heat transfer in the stagnation region,
 477 which results in a smaller temperature increase in the liquid temperature in
 478 the liquid film. Consequently, at larger radii, the driving temperature dif-
 479 ference between the wall and the mean liquid temperature is greater relative
 480 to the smooth surface case and results in modestly higher local heat transfer
 481 rates. Further, it is important to point out that even small values of $\hat{\lambda}_T$ (see
 482 $\hat{\lambda}_T = 0.05$ data) result in significant departures in the local Nu magnitude
 483 (relative to the smooth surface) and may be important when considering

484 data derived from classical jet impingement experiments on reported smooth
 485 surfaces.

486 For small temperature jump length, the effect of increasing either Re or
 487 Pr , causes the local Nusselt number to increase (seen both in Figs. 5 and 6).
 488 However, as the slip length and the jump length increase, changing Re or Pr
 489 has less effect. Consider the $\hat{\lambda}_T = 0.2$ case in Figs. 5 and 6 for which there
 490 is little apparent change. Here heat transfer is determined by the thermal
 491 resistance at the wall instead of the flow conditions.

492 Shown in Fig. 7 is the average Nusselt number, $\overline{Nu}$, for smooth surfaces
 493 ($\hat{\lambda}_T = 0$) as a function of $\hat{r}$ and the results demonstrate the influence ex-
 494 erted by variations in Re and Pr . Figure 7 provides a point of comparison
 495 for superhydrophobic scenarios ($\hat{\lambda}_T \neq 0$) which are discussed further below.
 496 Results are shown for $Re = 3,000, 9,000$, and $15,000$ at a constant $Pr = 5$
 497 (panel (a)) and for $Pr = 2, 5$, and 11 at a constant $Re = 9,000$ (panel (b)).
 498 In general, $\overline{Nu}$ decreases with increasing $\hat{r}$, Re , and Pr in agreement with
 499 well-known behavior.

500 A fractional reduction in the average thermal transport, $1 - \overline{Nu}/\overline{Nu}_0$, is
 501 used to quantify the reduction in $\overline{Nu}$ for the SHPo cases, where $\overline{Nu}_0$ is $\overline{Nu}$ at
 502 $\hat{\lambda}_T = 0$. A fractional reduction of 1 indicates that the thermal transport has
 503 decreased by 100%. Shown in Fig. 8 is $1 - \overline{Nu}/\overline{Nu}_0$ as a function of $\hat{r}$ and the
 504 results demonstrate the influence exerted by variations in the temperature
 505 jump length. Results are shown for $Re = 3,000$ and $15,000$ (panel (a)) at
 506 constant $Pr = 5$ with $\hat{\lambda}_T = 0.1$ and 0.2 and for $Pr = 2, 5$, and 11 (panel(b))
 507 at constant $Re = 9,000$ with $\hat{\lambda}_T = 0.1$ and 0.2 . In general, the fractional
 508 reduction decreases with increasing $\hat{r}$. This is expected in agreement with

(a)

(b)

Figure 7: $\overline{Nu}$ for no-slip and matching temperature surface as a function of the radius, $\hat{r}$, of the disk over which Nu is averaged. Panel (a): $Pr = 5$. Re is varied from 3,000 to 15,000. Panel (b): $Re = 9,000$. Pr is varied from 2 to 11.

(a)

(b)

Figure 8: Fractional reduction in Nusselt number, $1 - \overline{Nu}/\overline{Nu}_0$, as a function of $\hat{r}$ over which Nu is averaged. $\overline{Nu}_0$ is the average Nusselt number for the no-slip and matching temperature condition. $\hat{\lambda}_T = 0.1$ is indicated by a solid line, while $\hat{\lambda}_T = 0.2$ is indicated by a dashed line. Panel (a): $Pr = 5$ and Re equals 3,000 and 15,000. Panel (b): $Re = 9,000$. Pr equals 2, 5, and 11.

the same reasoning provided previously for the influence of $\hat{r}$ on $\overline{Nu}$. Increasing $\hat{\lambda}_T$ yields increased reduction in $\overline{Nu}$, as expected from the results for Nu

Consider the disk radius $\hat{r} = 10$ position in panels (a) and (b). For the $\hat{\lambda}_T = 0.1$ case, the fractional reduction increases from nominally 0.45 to 0.65 when Re increases from 3,000 to 15,000. Further, the fractional reduction increases through nominally 0.55, 0.65, and 0.7 as Pr increases through 2, 5, and 11. We emphasize this unexpected result that increasing Re or Pr on a SHPo surface increases the fractional reduction in $\overline{Nu}$ at a fixed $\hat{\lambda}_T$ value. This behavior demonstrates coupled interactions between $\hat{\lambda}_T$ and Re and Pr on the magnitude of $\overline{Nu}$.

The actual Nusselt numbers may be obtained by using the value of the fractional reduction displayed in this figure in conjunction with $\overline{Nu}$ for the smooth surface plotted in Fig. 7 or, for select radii, from Figs. 9 and 10.

To further quantify the influence of $\hat{\lambda}_T$ on $\overline{Nu}$, the average Nusselt number is shown as a function of $\hat{\lambda}_T$ at specific $\hat{r}$ values (10, 20, 30, 40) in Fig. 9. For all scenarios (all values of $\hat{r}$ explored), $\overline{Nu}$ decreases with increasing $\hat{\lambda}_T$ as expected. Results are shown at $Pr = 5$ and $Re = 3,000$ (panel(a)) and for the same values of Pr but at $Re = 15,000$ (panel (b)). Increasing $\hat{\lambda}_T$ reduces thermal transport dramatically when Nu is averaged over the region near the stagnation point. A 66% reduction is observed in panel (b) at $\hat{r} = 10$ when $\hat{\lambda}_T$ increases from 0 to 0.1. Less reduction is observed for larger $\hat{r}$ values. Consider a disk of radius $\hat{r} = 40$ where a 25% decrease is observed as $\hat{\lambda}_T$ increases from 0 to 0.2. Similar behavior was observed across the entire range of Re and Pr considered.

(a)

(b)

Figure 9: $\overline{Nu}$ as a function of $\hat{\lambda}_T$ for range of $\hat{r}$, from 10 to 40. $Pr = 5$. Panel (a): $Re = 3,000$. Panel (b): $Re = 15,000$.

534 Shown in Fig. 10 is the average Nusselt number at a fixed $\hat{r}$ value of
535 15 as a function of $\hat{\lambda}_T$. These results demonstrate the influence exerted by
536 variations in Re (panel a) and Pr (panel b) for SHPo ($\hat{\lambda}_T \neq 0$) surfaces. For
537 all scenarios (all values of Re and Pr explored), $\overline{Nu}$ decreases as expected
538 with increasing $\hat{\lambda}_T$. Results are shown for $Re = 3,000, 7,000, 11,000$, and
539 $15,000$ at constant $Pr = 5$ (panel (a)) and for $Pr = 2, 5, 8$, and 11 at constant
540 $Re = 7,000$ (panel (b)). For all cases, $\hat{r} = 15$. Results are shown for the same
541 range of $\hat{\lambda}_T$ (0 to 0.2) in both panels. For smooth surfaces, it is well known
542 that $\overline{Nu}$ increases with Re and Pr . We observe this trend for all $\hat{\lambda}_T$ in panels
543 (a) and (b), respectively.

544 We emphasize the surprising interaction of varying Re and Pr with vary-
545 ing $\hat{\lambda}_T$. Consider the case of $\hat{\lambda}_T = 0.01$ in panels (a) and (b). In panel (a)
546 between $Re = 3,000$ and $Re = 15,000$, $\overline{Nu}$ increases by 164%. In panel (b)
547 between $Pr = 2$ and $Pr = 11$, $\overline{Nu}$ increases by 75%. In contrast, compare
548 the increase at $\hat{\lambda}_T = 0.2$. There the increase is nominally 20% for both Re
549 (increasing from 3,000 to 15,000) and Pr (increasing from 2 to 11). At small
550 $\hat{\lambda}_T$, varying Re or Pr alters $\overline{Nu}$ dramatically while at large $\hat{\lambda}_T$ variations in
551 Re and Pr exercise little influence on $\overline{Nu}$ and the $\overline{Nu}$ vs. $\hat{\lambda}_T$ curves collapse
552 together. This occurs because thermal resistance due to the surface condi-
553 tion becomes more significant than thermal resistance due to the convection
554 boundary condition as $\hat{\lambda}_T$ increases.

Figure 10: $\overline{Nu}$ as a function of $\hat{\lambda}_T$. $\hat{r} = 15$. Panel (a): $Pr = 5$. Re varies from 3,000 to 15,000. Panel (b): $Re = 7,000$. Pr varies from 2 through 11.

555 *3.3. Relative importance of slip length and temperature jump length on the*
556 *solution*

557 This section of the paper explores the relative importance of slip velocity
558 compared to temperature jump on the overall transport. Shown in Fig. 11
559 is the average Nusselt number as a function of $\hat{\lambda}$ and $\hat{\lambda}_T$ at $\hat{r} = 15$.

560 Results are shown at constant $Pr = 5$ and $Re = 7,000$ for three cases.
561 First, $\hat{\lambda}$ is varied from 0 to 0.2 while $\hat{\lambda}_T$ is held constant at $\hat{\lambda}_T = 0$. These
562 results are indicated by the dash-dotted line. Second, $\hat{\lambda}_T$ is varied from 0
563 to 0.2, at a constant value of $\hat{\lambda} = 0$ and these results are indicated by the
564 dashed line. Third, $\hat{\lambda} = \hat{\lambda}_T$ and these results are indicated by the solid line.

565 For case 1, increasing $\hat{\lambda}$ ($\hat{\lambda}_T = 0$) yields an increase in $\overline{Nu}$ because the wall
566 slip velocity increases with $\hat{\lambda}$. This enhanced near-wall advection thus results
567 in a larger $\overline{Nu}$. As $\hat{\lambda}$ increases from 0 to 0.2, $\overline{Nu}$ increases by nominally
568 100%. For case 2, increasing $\hat{\lambda}_T$ ($\hat{\lambda} = 0$) yields an increase in the wall
569 thermal resistance and yields a 67% decrease in $\overline{Nu}$ as $\hat{\lambda}_T$ increases from 0
570 to 0.2. For case 3, $\overline{Nu}$ deviates only modestly from the solution where $\hat{\lambda}_T$
571 was varied at a constant $\hat{\lambda} = 0$ (dashed curve).

572 These results indicate that $\hat{\lambda}_T$ has a much greater influence on $\overline{Nu}$ than a
573 simultaneous change in $\hat{\lambda}$ and that the increased thermal resistance caused by
574 $\hat{\lambda}_T$ has a larger impact on the thermal transport than the enhanced near-wall
575 advection due to the increased wall slip velocity.

Figure 11: $\overline{Nu}$ as a function of $\hat{\lambda}_T$ with $\hat{\lambda} = 0$, as a function of $\hat{\lambda}$ with $\hat{\lambda}_T = 0$, and as a function of $\hat{\lambda} = \hat{\lambda}_T$. $Re = 7,000$ and $Pr = 5$. Nu averaged over 15 jet radii.

576 4. Conclusions

577 SHPo surfaces exhibit an apparent wall slip velocity and temperature
 578 jump. Although the effects of wall slip have been quantified for classical
 579 flows, interest in the influence of SHPo surfaces on thermal transport has
 580 recently emerged. A model has been developed to predict thermal transport
 581 for an axisymmetric, laminar jet impinging on a SHPo surface with isotropic
 582 slip length and temperature jump length. An integral approach was applied
 583 to solve the mass, momentum, and energy equations within the thin film of a
 584 spreading impinged jet. Third-order velocity and temperature profiles were
 585 assumed.

586 The results quantify local and average Nusselt numbers on heated super-
 587 hydrophobic surfaces over a range of temperature jump lengths ranging from

588 0 to 0.2, jet Reynolds numbers varying from 3,000 to 15,000, and Prandtl
589 numbers ranging from 2 to 11.

590 The introduction of an isotropic SHPo surface has the following effects
591 on thermal transport:

- 592 • The local thermal boundary layer thickness decreases relative to the
593 classical scenario.
- 594 • Thermal transport is reduced dramatically in the vicinity of the stag-
595 nation point and this decrease becomes more significant as the level of
596 superhydrophobicity increases.
- 597 • The Nusselt number is sensitive to the introduction of even a small
598 temperature jump length and decreases dramatically.
- 599 • The local and average Nusselt numbers become less dependent on Re
600 and Pr with increasing temperature jump length.
- 601 • Average Nusselt numbers are observed to decrease by up to 60-80% for
602 realizable SHPo surfaces and flow regimes, relative to smooth surfaces.

603 Additionally,

- 604 • The overall thermal transport is a much more dependent on the tem-
605 perature jump length than the hydrodynamic slip length.

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