Constructal design for convection melting of a phase change body

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Abstract

This paper documents a numerical study of time-dependent melting dominated by natural convection in a cylinder filled with a phase change material. In accord with constructal design, the search is for effective heat-flow architectures. The volume-constrained improvement of the designs for heat flow begins with assuming the simplest structure, where a single line serves as heat source. Next, the heat source is endowed with freedom to change its shape as it grows. The objective of the numerical simulations is to discover the geometric features that lead to the fastest melting process. The results show that the heat transfer rate density increases as the complexity and number of degrees of freedom of the structure are increased. Furthermore, the angles between heat invasion lines have a minor effect on the global performance compared to other degrees of freedom: number of branching levels, stem length, and branch lengths. The effect of natural convection in the melt zone is documented.

Keywords: Constructal; Evolutionary design; Melting; Natural convection; Energy storage; Phase change material; Tree shaped; Dendritic.

Nomenclature

B	volume, m^3
c_p	specific heat at constant pressure, kJ/kg K
D	enclosure diameter (height), m
F	buoyancy force, N
g	gravity acceleration, m/s^2
k	thermal conductivity, W/m K
L_f	latent heat of fusion, J/kg
L_0	stem length, m
L_1	branch length, m
n	number of bifurcation levels
P	pressure, Pa
Ra	Rayleigh number
t	time, s
t_f	duration of melting by conduction, s
T	temperature, K
T_0	melting temperature, K
T_1	invading line temperature, K
u	velocity vector, m/s
V	speed of invading line, m/s

Greek symbols

α	tilt angle, degree
β	thermal expansion coefficient, K^{-1}

β_1 bifurcation angle, degree
 δ radial thickness of the melted domain, m
 δ_f thickness of the melted layer at the end of the conduction mode, m
 ΔT excess temperature
 μ dynamic viscosity, Pa s
 ρ density, kg/m³
 ν kinematic viscosity
 τ temperature interval, K

Subscripts

c end of consolidation phase

Symbols

$\sim$ nondimensional

1. Introduction

The evolution of flow designs is a widespread occurrence in engineering and nature. One design approach that has gained increasing attention in recent years is based on the constructal law [1], which is the thought that geometry (flow architecture) is generated in the pursuit of global performance subject to global constraints, in flow systems that are free to morph. The growing interest in the design approach is due to its implications in the field of heat transfer and fluid flow with the objective of achieving greater heat transfer rate density in a finite volume [2-24].

In this article, we apply the method of constructal design to latent heat storage. Energy storage is essential in designs for the effective utilization of unsteady and random renewable

energy (e.g., solar energy and wind energy) by storing the energy in the form of latent heat during the high-availability period, for later use.

Traditionally, the study of latent heat storage is based on models where the heating entity, wall or embedded pipe, is solid and stationary. In such models, the melting of the phase-change material proceeds in the direction perpendicular to the stationary solid surface. In the present study we show that the model with stationary heating surface corresponds to sufficiently long times, when the flowing heating agent has had time to travel along and occupy the entire extent of the solid surface. This long-times behavior will be defined as the “consolidation” period in the present model, where the study of the heating and melting process begins earlier, when the heating agent begins to invade the surface that immediately begins to heat and melt the surrounding phase-change material. This behavior at short time will be defined as the “invasion” phase.

We showed in an earlier study that this complete model, with invasion followed by consolidation, accounts for the S-shape of the growth of the melt volume over time [25]. The melting rate is initially small, it then increases to a maximum, and finally it decreases monotonically until the melting is complete. The objective of the present study is to determine the relationship between the configuration of the invasion path of the heating effect and the maximum melting rate, or the steepness of the S-shaped history of the volume of melt. We seek to determine three-dimensional heat invasion architecture that offers the shortest time for melting the entire amount of phase-change material.

2. Model

Consider the time-dependent melting of a phase-change material in a cylindrical enclosure heated by a growing line of higher temperature (T_1) placed on the axis, Fig. 1. The line tip speed

is V . Initially, the volume $B = \pi D^3/4$ is filled entirely by the solid phase, the initial temperature T_0 of which is uniform and identical to the melting temperature. The material properties are: $k = 0.2$ W/m K, $\rho = 800$ kg/m³, $c_p = 1.25$ kJ/kg K, $L_f = 125$ kJ/kg, $\mu = 0.008$ Pa s, $T_0 = 303$ K, $\tau = 1$ K, $\beta = 0.002$ K⁻¹, where τ is the temperature range over which melting occurs. The boundary of this volume is modeled as adiabatic. Beginning with the time $t = 0$, heat begins to flow from the heat source to the phase change material and as time passes, a finger-shaped melt volume continues to grow along the advancing line [26].

The Navier-Stokes and the heat transfer equations are coupled to model natural convection. The fluid is modeled as Boussinesq-incompressible, in other words, the variation in density ρ is negligible everywhere except in the body force term of the momentum equation

$$F = \rho g [-1 + \beta(T - T_0)] \quad (1)$$

where β is the coefficient of thermal expansion. With this assumption, the governing equations of conservation of mass, momentum and energy can be written as follows:

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla P + \mu \nabla^2 \mathbf{u} + F \quad (3)$$

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T \quad (4)$$

The heat transfer mechanism through the incipient liquid layer of thickness δ is that of pure conduction. Therefore, the scale analysis of the energy equation shows that the radial thickness of the melted domain in the early stage of the process increases as [27]

$$\delta \sim \left(\frac{kt}{\rho c_p} \right)^{1/2} \quad (5)$$

Here the Prandtl number of the fluid equals to 50 and since its order of magnitude is greater than

1, in the momentum equation, the effects of inertia terms are negligible relative to the driving effect of buoyancy and the retarding effect of viscous shear. The velocity scale obtained from the momentum balance shows that the convective contribution increases with time. The time t_f when the δ layer becomes convective is determined from the balance between convection and conduction in the energy equation

$$t_f \sim \left(\frac{D\nu}{\alpha g \beta \Delta T} \right)^{1/2} \quad (6)$$

$$\delta_f \sim D Ra_D^{-1/4} \quad (7)$$

where δ_f marks the onset of the natural convection regime at the time t_f , and Ra_D is the Rayleigh number based on the diameter (height) of the cylinder, namely $Ra_D = g \beta \rho c_p D^3 \Delta T / k \nu = 2.45 \times 10^5$. In the present study the conduction layer thickness is small relative to the cylinder radius, meaning that the heat transfer is ruled by natural convection. In order to investigate the effect of natural convection strength on the melting rate, we decreased the Rayleigh to $Ra_D = 9.8 \times 10^3$ by lowering the excess temperature, where heat conduction is the dominant process. Figure 2 compares the shape and amount of melt volume during the invasion and consolidation phases at these two different Rayleigh numbers. It is worth mentioning that the invasion time is $t_i = D/V$, which is independent of Rayleigh number. The melting durations of the corresponding models are compared on the logarithmic scale in Fig. 3. Results show that increasing the Rayleigh number accelerates the melting process due to stronger natural convection in the melt region.

The effective heat capacity method is used to account for both the latent heat of fusion (L_f) and the sensible energy required to increase the temperature of the phase change material over the melting temperature range, τ [28]. The modified specific heat is

$$c_p^* = c_p + \frac{L_f}{\tau}, \text{ when } T_0 < T < T_0 + \tau \quad (8)$$

Bellow T_0 and above $T_0 + \tau$, the specific heat has the value $c_p = 1.25 \text{ kJ/kg K}$. The dynamic viscosity μ was implemented in the model as a piecewise, continuous, second derivative function centered around T_0 where its value at temperatures below melting was four orders of magnitude higher than for the liquid phase-change material, $\mu = 0.008 \text{ Pa s}$.

The simulations were performed by using a finite element package [29]. The mesh independence test was conducted for all models presented in this study and the ratio of maximum element size to the geometry diameter was fixed to 0.08 to ensure the accuracy of the numerical results.

3. First construct

We started with the simplest configuration of Fig. 1, in which the source of heat is the single straight line advancing at the constant speed V with the excess temperature of ΔT . This configuration is used as reference for the new performance exhibited by optimized tree structures by varying some geometric features.

We then considered the first construct, Fig. 4, where the advancing line splits into three branches at $L_0/D = 0.5$ which divide the cross-section into three equal areas; in other words, branches connect the centerline to three points positioned equidistantly on the circular perimeter. This construct is labeled $n = 1$, where n is the number of levels where branching occurs. In this section and the following, all the numerical simulations account for the convection-dominated melting process, $Ra_D = 2.45 \times 10^5$. The body of revolution of melt around the invading lines at the end of invasion phase are illustrated in Fig. 5 when $n = 0$ and $n = 1$. The finger-shaped melting fronts during the invasion phase is in agreement with the theoretical findings of Ref. 26.

The morphing of the first construct has two degrees of freedom, namely, the stem length ratio

(L_0/D) and tilt angle (α), where α is the angle between the branch and the cylinder radius in the same plane that contains the cylinder axis. An example for $\alpha = 45^\circ$ is illustrated in Fig. 4. This angle varies from zero in the original configuration, to 90 degrees which corresponds to the straight line invasion. The effect of changing the stem length ratio of the assumed structure, $\alpha = 0^\circ$, on the melting duration is reported in Fig. 6. It shows that the fastest melting occurs when $L_0/D = 0.6$. The ordinate parameter is the melting time of each configuration relative to the complete melting time of $L_0/D = 0.5$ configuration

$$\tilde{t}_1 = t/t_{c,n=1,\alpha=0,L_0/D=0.5} \quad (9)$$

Next, we varied both the tilt angle and the stem length ratio in order to accelerate the melting by further increasing the complexity of the assumed configuration. Figure 7 shows how to select the tilt angle for each L_0/D value so that the total melting duration is minimized. The results indicate that the performance of the postulated configuration is augmented by 16 percent when $L_0/D = 0.45$ and $\alpha = 30.5^\circ$. The improvement made in performance by adding design freedom is in accord with the constructal law.

4. Second construct

We endowed the flow structure with more freedom to morph by allowing the tree geometry to grow to the second level ($n = 2$) where the number of branches doubles. As shown in Fig. 8, there are six fixed and equidistant points on the perimeter in the second construct.

The first branch length (L_1) can also vary, and this adds a new degree of freedom (L_1/D). This geometric feature is represented by the angle of bifurcation β_1 , Fig. 8. The second construct configuration when $\alpha = 45^\circ$ is illustrated in Fig. 8. Figures 9 and 10 show how to combine L_0/D and β_1 in order to achieve the fastest spreading of heat from tree design of the second construct when $\alpha = 0^\circ$ and 25° .

The shortest melting times and their corresponding β_1 for different tilt angles are summarized in Fig. 11. The tilt angle has a relatively small effect on the melting performance and the geometric features of the enhanced tree configuration are $L_0/D = 0.4$, $\alpha = 30^\circ$, and $\beta_1 = 70^\circ$. In Fig. 12 we compared the evolution of the dimensionless melted volume in the simplest structure with those of the optimized first and second constructs. This figure also shows how the increase in complexity leads to better global performance. The ordinate and abscissa parameters of Figs. 9-12 are time ratios defined as

$$\begin{aligned}\tilde{t}_0 &= t/t_{c,n=0} \\ \tilde{t}_2 &= t/t_{c,n=2,\alpha=0,L_0/D=L_1/D=0.5}\end{aligned}\tag{10}$$

Conclusions

In this paper we studied numerically the melting process of the phase change material by natural convection in a cylindrical enclosure where the heating agent is not stationary and steady, as opposed to the traditional designs. The problem is how to arrange the heat-flow path buried in and transferring heat to a phase change volume with the objective of achieving greater heat transfer rate, or shorter melting time. We started the search by assuming a tree-shaped structure with freely varying geometric which is based on the constructal law.

The numerical results showed that the global performance of the storage system can be augmented by endowing its geometry with progressively more degrees of freedom, that is, number of branching levels, stem length, branch length, and tilt angle. We reported the best geometric features and also discovered that the tilt angle has a relatively minor impact on the melting performance. Relative to numerical simulations based on pure conduction, we show that the melting process is accelerated dramatically by the presence of natural convection in the liquid melt. The stronger natural convection effect, higher Rayleigh number, in the fixed enclosure can be achieved by increasing the temperature of the heat source.

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Figure Captions

- Figure 1 Cylindrical enclosure filled with phase change material (PCM) heated by a single invading line.
- Figure 2 The effect of natural convection strength (Rayleigh number) on the evolution of the melting front during invasion and consolidation phases of the melting phenomenon: $t = 100$ s marks the end of the invasion phase, and the start of the consolidation phase.
- Figure 3 Comparison between the melting rate (in the logarithmic scale) of two melting processes with the weak and strong natural convection effects.
- Figure 4 Configuration of the morphing tree structure with one level of branching.
- Figure 5 The evolution of the melt volume around the invading lines at the end of invasion when $n = 0$ and $n = 1$.
- Figure 6 The effect of the position of the first branch in the first construct on the melting duration when $\alpha = 0^\circ$.
- Figure 7 The shortest melting time and its corresponding tilt angle α_{opt} as function of stem length ratio in the first construct.
- Figure 8 Configuration of the morphing tree structure with two levels of branching.
- Figure 9 The shortest melting time and its corresponding bifurcation angle $\beta_{1,\text{opt}}$ as function of stem length ratio in the second construct when $\alpha = 0^\circ$.
- Figure 10 The shortest melting time and its corresponding bifurcation angle $\beta_{1,\text{opt}}$ as function of stem length ratio in the second construct when $\alpha = 25^\circ$.
- Figure 11 The shortest melting time and its corresponding bifurcation angle $\beta_{1,\text{opt}}$ as function of tilt angle α in the second construct.
- Figure 12 The effect of complexity (n) on the evolution of the dimensionless melted volume.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

$n=0$

$n=1$

Figure 6

Figure 9

Figure 10

Figure 11

Figure 12