Pressure Drop of Water Flow Across a Micro-Pin-Fin Array

Part 1: Isothermal Liquid Single-Phase Flow

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Abstract

This is part one of a two-part study that investigates water single-phase and two-phase pressure drop across a staggered array of circular micro-pin-fins. This paper presents results of isothermal liquid single-phase flow, and part two adiabatic liquid-vapor two-phase flow. The micro-pin-fins are 180 *microns* in diameter, 683 *microns* in height, and 399 *microns* in both longitudinal and transverse pitches. Seven temperature levels from 23 to 80 °C, and seventeen maximum mass velocities for each temperature, ranging from 133 to 1538 $\text{kg/m}^2\text{s}$, were tested. The resulting Reynolds number ranged from 26 to 776. Pressure drop across the micro-pin-fin array was measured, and friction factor calculated. The existing friction factor correlations for micro-pin-fin arrays, conventional-size pin-fin arrays, and tube banks were examined and found unable to predict the data. A numerical simulation was performed, and a good agreement was found between the experimental data and numerical results. A new friction factor correlation was developed that is used in part two of the study on calculating liquid-vapor two-phase pressure drop.

Keywords: micro-pin-fins, heat sink, liquid cooling, single-phase, pressure drop, friction factor

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Nomenclature

A_{min}	Minimum transverse flow area
A_{p1}, A_{p2}	Flow area in plenums
A_{ts}	Flow area in test section inlet
C	Constant in a power function
d_h	Hydraulic diameter of flow passage
D	Diameter of a circular pin-fin
f	Friction factor
G_{max}	Maximum mass velocity
G_{p1}, G_{p2}	Mass velocity in plenums
G_{ts}	Mass velocity in test section inlet and outlet
H	Height of a pin-fin
K_{c1}, K_{c2}	Contraction loss coefficient
K_{e1}, K_{e2}	Expansion recovery coefficient
L	Length of test section top platform area
$\dot{m}$	Mass flow rate
M	Total number of data points
MAE	Mean absolute error
n	Power index in a power function
N_L	Total number of pin-fin rows in longitudinal direction
N_{fd}	Number of streamwise pin-fin rows in fully developed region
$N_{T,O}$	Number of pin-fins in an odd row
N_{total}	Total number of pins in the array
P	Pressure
$\Delta P_{c1}, \Delta P_{c2}$	Contraction pressure loss
ΔP_d	Pressure drop across inlet developing rows
ΔP_{exit}	Pressure drop across last row
$\Delta P_{e1}, \Delta P_{e2}$	Expansion pressure recovery
ΔP_{fd}	Averaged per-row pressure drop across fully developed rows
ΔP_{sp}	Pressure drop across test section
$\Delta P_{sp,fin}$	Pressure drop across micro-pin-fin array

Re	Reynolds number
S_L	Longitudinal pitch
S_T	Transverse pitch
S_w	Minimum wall edge to pin center spacing
T	Temperature
v	Specific volume
W	Width of test section top platform area
W_{wall}	Thickness of thin side walls along test section edges

Greek

ϕ	Weighting factor in the Gaddis and Gnielinski correlation
μ	Dynamic viscosity

Subscripts

exp	Experimental
f	Liquid
fin	Pin-fin array
in	Inlet
lam	Laminar
num	Numerical
out	Outlet
$p1$	Deep plenum
$p2$	Shallow plenum
$pred$	Predicted
sp	Single-phase
ts	Test section
$turb$	Turbulent
$wall$	Side wall

1. Introduction

Micro-size pin-fin arrays have recently emerged as a promising alternative to micro-plate-fins as internal heat transfer enhancement structures for liquid-cooled miniature heat sinks intended for high-heat-flux cooling applications [1]. Compared to their plate-fin counterparts, micro-pin-fin arrays offer more design possibilities so as to achieve desired heat sink performance: pin-fins can have different cross-sectional shapes (circular, square, triangle, diamond shape, elliptical, cone shape, airfoil, etc.), and be arranged staggered or aligned in space; characteristic size including diameter, height, and longitudinal and transverse pitches can be varied; and a clearance can be incorporated between fin tips and end wall.

Both single-phase convection and flow boiling can be employed as a means to remove heat from micro-pin-fin arrays. Accordingly, the heat sinks can be classified as single-phase or two-phase. A growing number of studies have been carried out in recent years on thermofluid characteristics of single-phase flow [2-14] and flow boiling [15-21] in micro-pin-fin arrays with the goal of providing fundamental knowledge and accurate predictive tools that are essential to the design and implementation of this novel type of heat sink.

Aiming at further expanding the knowledge in the subject area, the present study concentrates on water flow pressure drop across a staggered circular micro-pin-fin array with height-to-diameter ratio H/D in the intermediate range of 0.5 to 8, and no clearance over the pin-fin tips. Part 1 of the study, this paper, presents results of isothermal liquid single-phase flow. Part 2, documented in the companion paper [22], reports adiabatic liquid-vapor two-phase flow results.

A number of studies are available in the literature on single-phase pressure drop across micro-pin-fin arrays having similar geometrical features [2-6, 10]. Koşar et al. [2] studied water flow pressure drop across two arrays of staggered circular micro-pin-fins having diameter D of 50 and 100 μm , H/D of 2 and 1, respectively, and longitudinal pitch-to-diameter ratio S_L/D and transverse pitch-to-diameter ratio S_T/D of 1.5. Flow Reynolds number ranged from 5 to 128. They found that the existing conventional-size pin-fin array and tube bank correlations were unable to predict their micro-pin-fin data. New friction factor correlations were proposed.

Koşar and Peles [4] investigated water flow pressure drop across a staggered circular micro-pin-fin array with D of 99.5 μm , H/D of 2.43, and S_L/D and S_T/D of 1.5. Flow Reynolds number ranged from 14 to 112. They found that the end wall effect was minor for this particular

H/D value, and the previous tube bank correlations predicted the data well. The finding was later confirmed by Peles et al. [3] and Koşar and Peles [5] using water and refrigerant R-123 as the testing liquid, respectively, in the same array.

Prasher *et al.* [6] studied water flow pressure drop across three arrays of staggered circular micro-pin-fins having D from 55 to 153 μm , H/D from 1.3 to 2.8, S_L/D from 2.4 to 4, and S_T/D from 2 to 3.6. Flow Reynolds number ranged from 40 to 1,000. The friction factor data were plotted against Reynolds number on a log-log scale, and a change in slope in the data was observed to occur at a Re of about 100. They found that the existing micro-pin-fin, conventional-size pin-fin, and tube bank correlations could not predict the data. Two distinct correlations were proposed for the two flow regimes of low ($Re < 100$) and high Reynolds number ($Re > 100$).

Koşar et al. [10] studied water and air flow pressure drop across three arrays of staggered circular micro-pin-fins having D of 100 μm , H/D of 1 and 2.43, S_L/D of 1.5 and 2.5, and S_T/D of 1.5 and 2.5. Flow Reynolds number ranges were 2-310 for water and 2-1,800 for air. They found that the existing conventional-size pin-fin and tube bank correlations were unable to predict the data.

Other relevant studies include those on single-phase pressure drop of low Reynolds number flow ($Re < 1,000$) across conventional-size (millimeter or larger) pin-fin arrays bearing the same geometrical features [23,24]. Short et al. [23] investigated air flow pressure drop across 44 arrays of staggered circular pin-fins having D from 1.57 to 3.18 mm , H/D from 1.88 to 7.25, S_L/D from 1.83 to 3.21, and S_T/D from 2.0 to 6.41. A new correlation was proposed for $175 < Re < 1,000$. Moores and Joshi [24] investigated water flow pressure drop in three arrays of staggered circular pin-fins with and without tip clearance. Pin-fin diameter D ranged from 3.67 to 3.84 mm , H/D from 0.52 to 1.09, S_L/D from 1.13 to 1.18, and S_T/D from 1.3 to 1.36. They also proposed a new correlation for $200 < Re < 1,000$.

Single-phase pressure drop across tube banks ($H/D > 8$) has been studied extensively in the past [25-30]. Unlike that in pin-fin arrays, crossflow in tube banks is dominated by tube size and spatial arrangement with negligible endwall effects. The study by Žukauskas and co-workers led to an empirical $f-Re$ relationship presented in graphical format [28,29]. The relationship was based on a large experimental database, and has been adopted in a heat exchanger design

handbook [30] as well as a popular heat transfer textbook [31]. Another friction factor correlation developed by Gaddis and Gnielinski [27] was also based on a sizable dataset.

Although the aforementioned studies [2-6, 10] provided valuable insight into the characteristics of single-phase pressure drop across staggered circular micro-pin-fin arrays, findings remain inconclusive, especially when comparing experimental data obtained by one party with predictions of the correlations developed by other parties. The objectives of this part 1 of the present study thus are: (1) to provide new data for pressure drop of single-phase flow across a staggered circular micro-pin-fin array over a broad range of Reynolds numbers, (2) to assess the applicability of the existing empirical correlations at predicting the present data, and (3) to perform a three-dimensional numerical analysis, and further assess the effectiveness of numerical methods as a predictive tool.

2. Experimental Apparatus and Procedure

2.1 Flow loop

A flow loop as shown in Fig. 1 was used to supply the working fluid, de-ionized water, to a micro-pin-fin test module at desired conditions. The water was initially stored in a water reservoir, and was deaerated through vigorous boiling prior to testing runs using an immersion heater. A variable speed gear pump was utilized to circulate the water. The water was cooled down to about room temperature before entering the pump using a compact heat exchanger (Heat Exchanger 1). The water was then forced through a 15-micron filter in an effort to reduce particulate fouling of the micro-pin-fin array. After the filter, the water passed through one of two rotameters for flow rate measurements. Additionally, two digital flowmeters were used to provide redundant flow rate measurements. A thermocouple was installed just upstream of the digital flowmeters to take water temperature readings for density calculations. The rotameters and digital flow meters were calibrated using the standard weighting method, and the accuracy of flow rate measurements was better than $\pm 2\%$ of the readings. A second heat exchanger (Heat Exchanger 2) connected to a constant temperature bath was used to bring the water temperature to desired levels. The water then passed through the test module where pressure drop measurements were performed. Upon leaving the test module, the water was cooled down again utilizing another heat exchanger (Heat Exchanger 3) before it returned to the reservoir.

2.2 Test module

Figure 2 shows the test module construction. The test module was composed of three main pieces: a 110-oxygen free copper micro-pin-fin test section, a G-7 fiberglass housing, and a clear polycarbonate plastic (Lexan) cover plate. The test section had a projected top platform area of 3.38 *cm* (length) by 1 *cm* (width). An array of staggered circular micro-pin-fins were micro-end milled out of the top surface. The array contained a total of 1845 pin-fins: 82 rows in the longitudinal (stream-wise) direction, 23 pin-fins in every odd row, and 22 pin-fins in every even row. Due to the staggered arrangement, the odd rows had one more pin-fin than the even rows. The micro-pin-fins were 180 μm in diameter D , 683 μm in height H , and 399 μm in both longitudinal pitch S_L and transverse pitch S_T . The resulting H/D ratio was 3.8, and S_L/D and S_T/D ratios 2.2. Figure 3 shows a top view and key dimensions of the array.

The test section was inserted into the housing, and cemented in place using room temperature vulcanizing (RTV) silicone rubber. A pourable low-viscosity silicone elastomer was then poured into the remaining housing cavity to create a watertight seal around the test section. The cover plate was then bolted down on top of the housing to form a closed flow passage for the water. A leak-proof seal was created by applying a thin layer of RTV along the interface between the housing and cover plate. The cover plate was transparent, which provided direct visual access to the flow. Once assembled, the module was wrapped with multiple layers of fiberglass insulation material in an effort to minimize heat loss to the surroundings.

The housing contained inlet and outlet plenums, consisting of a shallow and deep section to promote even flow distribution across the micro-pin-fin array. The inlet and outlet deep plenums had a width of 19 *mm* and a height of 15.9 *mm*, while the shallow plenums had a width of 19 *mm* and a height of 670 μm . Within the inlet and outlet deep plenums, two type-K thermocouples were installed to measure the inlet and outlet temperatures T_{in} and T_{out} , respectively. The thermocouples had an estimated uncertainty of ± 0.3 $^{\circ}\text{C}$. An absolute pressure transducer was connected to the inlet deep plenum via a pressure tap to measure the test section inlet pressure P_{in} . A differential pressure transducer was connected to the inlet and outlet deep plenums to measure the pressure drop across the test section ΔP_{sp} . The uncertainty in both P_{in} and ΔP_{sp} measurements was estimated to be less than 0.25% of the readings. Readings from the digital flow meters, thermocouples, and pressure transducers were recorded using an Agilent data acquisition and control unit that was interfaced to a PC running a LABVIEW program.

2.3 Experimental procedure

In order to obtain a desired water temperature level at a specific flow rate, the constant temperature bath was adjusted along with the gear pump. Seven water inlet temperatures T_{in} were tested, ranging from 23 °C to 80 °C. For each T_{in} , seventeen different maximum mass velocities G_{max} , ranging from 133 to 1538 kg/m²s, were tested. G_{max} was calculated based on the minimum transverse flow area A_{min} in the micro-pin-fin array, and is given by

$$G_{max} = \frac{\dot{m}}{A_{min}}. \quad (1)$$

A_{min} is calculated using the equation

$$A_{min} = H[(N_{T,O} - 1)(S_T - D) + (2S_w - D)], \quad (2)$$

where $N_{T,O}$ is the number of pin-fins in an odd row ($N_{T,O} = 23$), and S_w the wall to pin distance for the odd row ($S_w = 315 \mu m$). The resulting flow Reynolds number Re_{sp} , defined in Eq. (3), ranged from 26 to 776.

$$Re_{sp} = \frac{G_{max}D}{\mu_f}. \quad (3)$$

Uncertainty in Re_{sp} was estimated to be 2%.

During testing runs, the flow loop was first allowed to reach steady state where readings of flow rate, temperature, and pressure were constant. Once at steady state, the readings were taken by the data acquisition system at 0.5-second intervals for a total of 3 minutes, and outputted to an Excel spreadsheet. The data reported in the paper were the averages of the readings.

Figure 4 shows the recorded T_{out} versus G_{max} for each prescribed T_{in} . It can be observed that T_{out} is slightly lower than T_{in} , indicative of small heat loss from the test module. The temperature drops were generally small with a maximum value of 2.44 °C that occurred at the 80 °C inlet temperature and the lowest G_{max} . The working fluid, i.e., water, was thus assumed to be isothermal as it flowed across the micro-pin-fin array, and the average of T_{in} and T_{out} was used to determine all water properties.

$$T_f = \frac{T_{in} + T_{out}}{2}. \quad (4)$$

3. Results and Discussion

3.1 Pressure drop and friction factor

The measured pressure drop ΔP_{sp} versus G_{max} is presented in Fig. 5. For any given T_{in} , ΔP_{sp} increases with increasing G_{max} as is to be expected. For any given G_{max} , ΔP_{sp} decreases

with increasing T_{in} . This is also the expected trend as the viscosity of water μ_f decreases with increasing T_{in} .

Due to the location of the pressure ports in the test module, ΔP_{sp} represents pressure drop between the deep inlet and outlet plenums, which is composed of pressure drops across the inlet deep and shallow plenums, micro-pin-fin array, outlet shallow and deep plenums, as well as pressure losses and recoveries associated with the consecutive sections. Neglecting the pressure drops within the plenums due to their miniscule contribution, the pressure drop across the micro-pin-fin array $\Delta P_{sp,fin}$ can be calculated from

$$\Delta P_{sp,fin} = \Delta P_{sp} - (\Delta P_{c1} + \Delta P_{c2} + \Delta P_{e2} + \Delta P_{e1}). \quad (5)$$

ΔP_{c1} and ΔP_{c2} in Eq. (5) represent the inlet contraction losses from the deep to shallow plenum and from the shallow plenum to the test section inlet, respectively. The contraction losses are calculated from [8, 32],

$$\Delta P_{c1} = \frac{v_f}{2} (G_{p2}^2 - G_{p1}^2) + \frac{K_{c1} v_f}{2} G_{p2}^2, \quad (6)$$

and

$$\Delta P_{c2} = \frac{v_f}{2} (G_{ts}^2 - G_{p2}^2) + \frac{K_{c2} v_f}{2} G_{ts}^2, \quad (7)$$

where the subscript $p1$ refers to the deep plenum and $p2$ the shallow plenum. G_{p1} , G_{p2} , and G_{ts} are the mass velocities in the deep plenum, shallow plenum, and test section inlet, respectively.

$$G_{p1} = \frac{\dot{m}}{A_{p1}} = \frac{\dot{m}}{H_{p1} W_{p1}}, \quad (8)$$

$$G_{p2} = \frac{\dot{m}}{A_{p2}} = \frac{\dot{m}}{H_{p2} W_{p2}}, \quad (9)$$

and

$$G_{ts} = \frac{\dot{m}}{A_{ts}} = \frac{\dot{m}}{H(W-2W_{wall})}. \quad (10)$$

K_{c1} and K_{c2} are the loss coefficients for the abrupt contractions, and are evaluated from [32]

$$K_{c1} = 0.6740 + 1.2501 \left(\frac{H_{p2}}{W_{p2}} \right) + 0.3417 \left(\frac{H_{p2}}{W_{p2}} \right)^2 - 0.8358 \left(\frac{H_{p2}}{W_{p2}} \right)^3, \quad (11)$$

and

$$K_{c2} = 0.6740 + 1.2501 \left(\frac{H}{W-2W_{wall}} \right) + 0.3417 \left(\frac{H}{W-2W_{wall}} \right)^2 - 0.8358 \left(\frac{H}{W-2W_{wall}} \right)^3. \quad (12)$$

ΔP_{e2} and ΔP_{e1} are the outlet expansion recoveries from the test section outlet to the shallow plenum and from the shallow plenum to the deep plenum, respectively, and are calculated from [8, 32]

$$\Delta P_{e2} = \frac{v_f}{2} (G_{p2}^2 - G_{ts}^2) + \frac{K_{e2} v_f}{2} G_{ts}^2, \quad (13)$$

and

$$\Delta P_{e1} = \frac{v_f}{2} (G_{p1}^2 - G_{p2}^2) + \frac{K_{e1} v_f}{2} G_{p2}^2, \quad (14)$$

where K_{e2} and K_{e1} are the recovery coefficients for the abrupt expansions. They are evaluated from

$$K_{e2} = \left(1 - \frac{A_{ts}}{A_{p2}}\right)^2, \quad (15)$$

and

$$K_{e1} = \left(1 - \frac{A_{p2}}{A_{p1}}\right)^2. \quad (16)$$

The total contraction pressure loss $\Delta P_{c1} + \Delta P_{c2}$ accounts for an average of 3.3% of ΔP_{sp} , while the overall expansion recovery $|\Delta P_{e2} + \Delta P_{e1}|$ accounts for an average of 1.2% of ΔP_{sp} .

Once $\Delta P_{sp,fin}$ is calculated from Eq. (5), friction factor f_{fin} can be determined from

$$f_{fin} = \frac{\Delta P_{sp,fin}}{N_L \left(\frac{v_f G_{max}^2}{2} \right)}, \quad (17)$$

where N_L is the total number of pin-fin rows in the longitudinal direction ($N_L = 82$). Uncertainty in f_{fin} was calculated to be 4.1 %.

Figure 6 shows the variation of f_{fin} versus Re_{sp} on a log-log scale. It can be seen that f_{fin} decreases with increasing Re_{sp} as expected. Of note is the curvature of the data variation, i.e., the slope of the data variation gradually decreases with increasing Re_{sp} over the entire Re_{sp} range. A number of existing pin-fin correlations adopted the power function form $f_{fin} = C Re_{sp}^n$ with an empirical power index n that was a constant [23,24]. Such power functions would appear as straight lines on a log-log scale. The slope change in the data variation as shown in Fig. 6 suggests that a power function might not be suitable for a broad range of Re_{sp} . Interestingly, a similar trend in slope change could also be observed for low Reynolds number flow ($Re < 1,000$) across staggered tube banks in the graphical correlations developed by Žukauskas where f versus Re was plotted on a log-log scale [28-31].

3.2. Correlation method

A select group of single-phase friction factor correlations were assessed by comparing their predictions with the experimental data. The correlations were chosen based on their suitability for the present flow conditions ($Re_{sp} < 1,000$) and geometrical features. The correlations are summarized in Table 1. The mean absolute error (MAE) used here to assess the correlations is given by

$$MAE = \frac{1}{M} \sum \frac{|\Delta P_{sp,exp} - \Delta P_{sp,pred}|}{\Delta P_{sp,exp}} \times 100\%, \quad (18)$$

where M is the total number of data points.

Correlations 1 and 2 are micro-pin-fin correlations. The Koşar *et al.* correlation (correlation 1) used a superposition form by adding two components together: the first component accounted for the effect of pin-fins, and the second one that of top and bottom endwalls [2]. The Prasher *et al.* correlation (correlation 2) was composed of two power functions for different Reynolds number ranges [6]. Figure 7 presents the data along with the predictions of the two correlations. It can be seen that the correlations are unable to predict the present data. The discrepancies suggest that caution must be exercised before a correlation can be extended to a micro-pin-fin configuration that differs from the one for which the correlation was originally developed.

Correlation 3 is a conventional-size pin-fin correlation developed by Short *et al.* [23]. The correlation used a power function form. Figure 8 compares the correlation predictions with the experimental data, and shows a substantial deviation. Furthermore, the power function adopted by the correlation was unable describe the overall trend in the slope change in the data.

Correlations 4 and 5 are the tube bank correlations. As mentioned in the introduction section, the Zukauskas and Ulinskas correlation (correlation 4) was developed based on a large database of flow conditions and geometrical parameters [30]. The $f-Re$ relationship was originally presented in a graphical format, where individual lines were plotted for tube banks with different transverse pitch-to-diameter ratios S_T/D [28-31]. Effects of longitudinal pitch-to-diameter ratio S_L/D was accounted for by using a correction factor. Zukauskas and Ulinskas later used a power series to best fit each line [30]. The present array has a S_T/D ratio of 2.2, and is thus compared against the correlations for $S_T/D = 2.0$ and 2.5. The Gaddis and Gnielinski correlation (correlation 5) used a superposition form: addition of a laminar component and a turbulent component with a weighting factor that was a function of Re . Figure 9 compares the

correlation predictions with the data. For the Zukauskas and Ulinskas correlations, instead being bound by the $S_T/D = 2.0$ and 2.5 lines, the results show the experimental data agree very well with the $S_T/D = 2.5$ correlation predictions over the correlation's valid Reynolds number range, but are overpredicted by the $S_T/D = 2.0$ correlation. The Gaddis & Gnielinski correlation was unable to predict the data well. The inability of tube bank correlations to accurately predict the data might be attributed to the fact that the endwall effects, while absent in tube banks, are significant in the present micro-pin-fin array.

The relatively poor agreement between the correlation predictions and the present data reinforces the notion that for micro-pin-fin arrays, a correlation is accurate only within the parametric range for which it was developed [2,6,10]. For a correlation to be effective over a broad application range, an inordinate number of micro-pin-fin configurations and flow conditions would need to be tested and analyzed, which is likely why it has yet to be done.

A new $f_{fin}-Re_{sp}$ correlation was developed based on the present data. The correlation used the same power series form as that of the Zukauskas and Ulinskas correlation. A non-linear least squares regression was used to obtain the parameters in the correlation.

$$f_{fin} = 4.0947 \times 10^{-1} + \frac{3.9808 \times 10^1}{Re_{sp}} + \frac{7.2713 \times 10^2}{Re_{sp}^2} - \frac{2.5465 \times 10^4}{Re_{sp}^3} + \frac{1.2565 \times 10^5}{Re_{sp}^4}. \quad (19)$$

Figure 10 shows the predictions from Eq. (19) plotted against the data. It should be noted that the correlation is geometry specific, and is intended for use in part 2 of this study to calculate water two-phase pressure drop across the same micro-pin-fin array.

3.3. Numerical simulation

A three-dimensional numerical simulation was performed to assess its effectiveness as a predictive tool for pressure drop across the micro-pin-fin array. Taking advantage of symmetry, a computational domain containing 20 stream-wise half-pins was modeled as shown in Fig. 11. This particular number of rows, i.e., twenty, was chosen as it was found in preliminary numerical simulations that the flow stabilized after 5 or 6 entrance rows, and variations in per-row pressure drop beyond these entrance rows became minuscule.

Four types of flow boundary conditions were applied to the computational domain: velocity inlet, pressure outlet, no-slip condition, and symmetry condition. The inlet consisted of a unidirectional velocity field of specifiabale magnitude. The pressure outlet was set to a constant ambient pressure. The no-slip boundary condition was applied to the top and bottom surfaces of

the computational domain as well as the pin-fin (solid) surfaces. The symmetry boundary condition was applied to the fluid surfaces in between the micro-pin-fins. Additionally, an inlet section with a length of $15D$ and an outlet section with a length of $20D$ were added to avoid a large pressure gradient at the inlet, and to mitigate the effect of backflow at the outlet, respectively.

The governing continuity and momentum equations were solved using commercial CFD software package STAR-CCM+ to obtain the velocity and pressure fields in the computational domain. An upwind differencing scheme was used to discretize and solve the governing equations. Seeing as $Re_{sp} < 776$, a laminar flow model was used in the simulations. The assumption was made due to Zukauskas' studies, which indicated that the flow in tube banks were predominantly laminar for $Re < 1,000$ [28, 29]. Other assumptions made in the simulation include isothermal flow, incompressible fluid, steady state, and constant material properties. The convergence criterion was set to residuals of less than 10^{-5} for the continuity and momentum equations. All computation runs met the convergence criteria.

The computational meshes used in the simulations were created using STAR-CCM+'s built in polyhedral mesher with surface remeshing and prism layer refinement. Figure 12 shows the top view of the numerical mesh for a portion of the computational domain containing three half-pins. Three different meshes were tested to determine mesh independence: a 487,673 cell mesh (coarse), a 720,270 cell mesh (medium), and a 947,847 cell mesh (fine). The differences in the calculated pressure drops between the medium and fine meshes were seen to be less than 1%, and thus the medium mesh was chosen for use.

Once the pressure field in the computational domain was obtained, the pressure drop across each pin-fin row was calculated. The upstream entrance rows and downstream fully-developed rows were identified based on the variation in the values of the per-row pressure drop in the stream-wise direction. The numerical results for the 20-row computational domain were then extended to determine the numerical pressure drop across the entire 82-row micro-pin-fin array by using the following equation,

$$\Delta P_{num} = \Delta P_d + N_{fd} \times \Delta P_{fd} + \Delta P_{exit}. \quad (20)$$

In Eq. (20), ΔP_d is the total pressure drop across the five or six upstream entrance rows, ΔP_{fd} the average per-row pressure drop across the downstream fully-developed rows, and ΔP_{exit} the pressure drop across the last row. N_{fd} represents the number of fully-developed rows in the

micro-pin-fin array, and was obtained by subtracting the number of developing rows and one exit row from N_L ($N_L = 82$).

Three inlet temperatures (23, 50, 80 °C) and seventeen flow rates at each temperature were simulated for comparison with the experimental data. Figure 13 compares the experimental data and numerical results. A good agreement can be found with an overall *MAE* of 6.41%. The result validates the present numerical simulation, and establishes numerical methods based on a laminar flow model as an accurate predictive tool to describe liquid single-phase pressure drop across the micro-pin-fin array for the Reynolds number range tested.

4. Conclusions

Pressure drop characteristics of water isothermal liquid single-phase flow across a staggered circular micro-pin-fin array were investigated both experimentally and numerically. Also presented was an assessment of existing empirical correlations at predicting the experimental data. Key finding from the study are as follows:

1. A gradual decrease in slope was observed in the variation of friction factor with Reynolds number on a log-log scale. The result suggests that power functions with constant index would not be suitable to correlate f and Re over a broad range of Re .
2. Existing micro-pin-fin, conventional-size pin fin, and tube bank correlations were unable to accurately predict the experimental data. This indicates that a pin-fin correlation should not be extended beyond the specific parametric range for which it was developed without a careful validation.
3. A good agreement was observed between the experimental data and numerical results. This proves that numerical methods could serve as an accurate predictive tool for single-phase, low Reynolds number flow pressure drop across micro-pin-fin arrays.

Acknowledgement

The authors are grateful for the support of the National Science Foundation (NSF) (Award no. CBET10-34242). The micro-pin-fin test section was fabricated at the Laser-Assisted Multi-Scale Manufacturing Laboratory, University of Wisconsin-Madison. Professor Frank Pfefferkorn's assistance in fabricating the test section was greatly appreciated.

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Table 1: Single-phase friction factor correlations.

Correlation	Reference	Remarks	Single-phase friction factor, f_{in}	MAE (%)
1	Kosar <i>et al.</i> [2] (2005)	Micro-pin-fins Circular, staggered $H/D = 1, 2$ $S_T/D = S_L/D = 1.5, 5$ $5 < Re_p < 128$	$A = \pi D \cdot H \cdot N_{total} + 2 \left([W - 2W_{wall}]L_{fs} - \frac{\pi D^3}{4} N_{total} \right)$ $d_h = \frac{4DL}{A} \quad Re_d = \frac{G_{max} d_h}{\mu} \quad A_c = \frac{\pi D^2}{4}$ $f_{fin} = \frac{1739}{Re_{sp}^{1.7}} \left(\frac{H/D}{H/D+1} \right)^{1.1} \left(\frac{S_L S_T}{A_c} \right)^{-0.3} + \frac{345}{Re_d} \left(\frac{1}{H/D+1} \right)^2 \left(\frac{S_L S_T}{A_c} \right)^{-0.3}$	42.55
2	Prasher <i>et al.</i> [6] (2007)	Micro-pin-fins Circular, staggered $1.3 < H/D < 2.8$ $2 < S_T/D < 3.6$ $2.4 < S_L/D < 4$ $40 < Re_p < 1,000$	<p>For $Re_p < 100$,</p> $f_{fin} = 679.28 \left(\frac{H}{D} \right)^{-0.64} \left(\frac{S_L - D}{D} \right)^{-0.258} \left(\frac{S_T - D}{D} \right)^{0.283} Re_{sp}^{-1.35}$ <p>For $Re_p > 100$,</p> $f_{fin} = 1.18 \left(\frac{H}{D} \right)^{1.249} \left(\frac{S_L - D}{D} \right)^{-0.7} \left(\frac{S_T - D}{D} \right)^{-0.36} Re_{sp}^{-0.1}$	<div>22.68</div> <div>Overall 291.73</div> <div>394.97</div>
3	Short <i>et al.</i> [23] (2002)	Conventional-size pin-fins Circular, staggered $1.88 < H/D < 7.25$ $2.0 < S_T/D < 6.41$ $1.83 < S_L/D < 3.21$ $175 < Re_p < 4,500$	<p>For $Re_p < 10^3$,</p> $f_{fin} = 140.4 \left(\frac{S_L}{D} \right)^{-1.3} \left(\frac{S_T}{D} \right)^{-0.78} \left(\frac{H}{D} \right)^{-0.55} Re_{sp}^{-0.65} \frac{L}{D \cdot N_L}$	47.22
4	Zukauskas & Ulinskas [30] (1983)	Tube banks Circular, staggered	<p>For $7 < Re_p < 100$,</p> $f_{fin} = 0.713 + \frac{0.448 \times 10^2}{Re_{sp}} - \frac{0.126 \times 10^3}{Re_{sp}^2} - \frac{0.582 \times 10^3}{Re_{sp}^3}$ <p>For $100 < Re_p < 10^4$,</p> $f_{fin} = 0.343 + \frac{0.303 \times 10^3}{Re_{sp}} - \frac{0.717 \times 10^5}{Re_{sp}^2} + \frac{0.88 \times 10^7}{Re_{sp}^3} - \frac{0.38 \times 10^9}{Re_{sp}^4}$ <p>For $100 < Re_p < 5 \times 10^3$,</p> $f_{fin} = 0.33 + \frac{0.989 \times 10^2}{Re_{sp}} - \frac{Re_{sp}^2}{Re_{sp}^3} + \frac{0.148 \times 10^5}{Re_{sp}^2} - \frac{0.192 \times 10^7}{Re_{sp}^3} - \frac{0.862 \times 10^8}{Re_{sp}^4}$	<div>41.62</div> <div>3.10</div>
5	Gaddis & Gnielinski [27] (1985)	Tube banks Circular, staggered	$f_{fin} = f_{lam} + \phi \cdot f_{turb} \quad S_D = \sqrt{\left(\frac{S_T}{z} \right)^2 + S_L^2}$ $f_{lam} = \frac{280\pi \{ [(S_L/D)^{0.5} - 0.6]^2 + 0.75 \}}{[4(S_T/D)(S_L/D) - \pi](S_D/D)^{1.6}} Re_{sp}^{-1}, \quad \phi = 1 - \exp\left(-\frac{Re_{sp} + 200}{1000}\right)$ $f_{turb} = \left\{ 2.5 + \frac{1.2}{[(S_T/D) - 0.85]^{1.06}} + 0.4 \left[\frac{S_L}{S_T} - 1 \right]^3 - 0.01 \left[\frac{S_T}{S_L} - 1 \right]^3 \right\} Re_{sp}^{-0.25}$	35.30

Figure Captions

- Fig. 1: Schematic of flow loop.
- Fig. 2: Test module construction.
- Fig. 3: Top view and key dimensions of the micro-pin-fin array.
- Fig. 4: Outlet temperature versus maximum mass velocity.
- Fig. 5: Measured pressure drop versus maximum mass velocity.
- Fig. 6: Friction factor versus Reynolds number.
- Fig. 7: Comparison of friction factor data with predictions from micro-pin-fin correlations
- Fig. 8: Comparison of friction factor data with predictions from conventional-size pin-fin correlation
- Fig. 9: Comparison of friction factor data with predictions from tube bank correlations
- Fig. 10: Comparison of friction factor data with predictions from the new correlation
- Fig. 11: Computational domain for the micro-pin-fin array.
- Fig. 12: Section of the numerical mesh.
- Fig. 13: Comparison of pressure drop data with numerical results

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13