MIXED CONVECTIVE HEAT TRANSFER FROM A HEATED SPHERE AT AN ARBITRARY INCIDENT FLOW ANGLE IN LAMINAR FLOWS

Samuel Gem Musong

Zhi-Gang Feng¹

Department of Mechanical Engineering

University of Texas at San Antonio, Texas

ABSTRACT

The mixed convection from a heated sphere for an arbitrary flow incident angle (θ) at low to moderate Reynolds numbers ($1 \leq Re \leq 100$) and Richardson numbers ($0 \leq Ri \leq 5$) is studied by an immersed boundary method, thereby investigating the influence of different flow incident angles ($0 \leq \theta \leq 180^\circ$) on the buoyancy flow and heat transfer. The numerical method is validated by comparing the results with the simulation results of pure forced convection as well as those of mixed convection with assisting flow (0° flow incident angle) published in the literature. Extensive simulations for a wide range of different incident flow angles have been performed. New correlations are obtained for the overall Nusselt number (Nu) in terms of θ and Ri , and Re , showing a quadratic decrease in Nu with respect to θ only for $0^\circ \leq \theta \leq 90^\circ$ (aiding and cross flow) and a half bell-shaped decrease in Nu for $90^\circ < \theta \leq 180^\circ$ (opposed flow). The combined treatment of mixed convection for the completely upward flow, cross flow, and completely downward flow (i.e. at incident angle 0° , 90° and 180° , respectively) was achieved showing almost linear relationships between the heat transfer rates and Ri for $Ri \geq 1$.

Keywords: Mixed convection; aiding flow; opposing flow; immersed boundary method; correlation of Nusselt number; heat transfer from a sphere; laminar flows.

¹ Corresponding author.

Email: zhigang.feng@utsa.edu; Tel.: 210-4585737; Fax: 210-4586504

Nomenclature

A	radius of a sphere	u, U	Velocity
Bi	Biot number	x, X	position coordinate
C	specific heat		
D	diameter of a sphere		Greek symbols
f	force density	ρ	Density
F	surface boundary force	λ	energy density function
Gr	Grashof number	Ω	entire domain
H	Grid spacing	β	thermal expansion coefficient
K	thermal conductivity	μ	dynamic viscosity
L, l	length variable	Θ	dimensionless temperature
Nu	overall Nusselt number	θ	flow angle of incidence
p	pressure	Λ	energy flux at boundary
Pe	Peclet number		
Pr	Prandtl number		
Ra	Rayleigh number ($GrPr$)		Subscripts/superscripts
Ri	Richardson number	0	value of ambient fluid
Re	Reynolds number	f	property related to fluid
s	surface coordinate	s	property related to sphere
T	temperature	G	natural convection
t	time variable	R	forced convection

1 Introduction

Heat transfer on a heated sphere immersed in a viscous fluid can be treated as a free convection or a forced convection problem. However, in most heating and cooling applications in real world, the convective heat transfer is of a mixed type. Mixed convection occurs whenever there is a forced fluid flow along with fluid motion driven by a temperature difference between the body and the surrounding fluid. The free convection occurs along the direction of gravity and the forced convection depends on the direction of the forced flow. The two directions are not necessarily the same. The overall heat transfer of a sphere resulting from a mixed flow is significantly influenced by the direction of the forced flow.

The Richardson number (Ri) characterizes the importance of free convection with respect to forced convection. Yuge [1] found that the forced convection is predominant when $Ri < 1$ and the free convective heat contribution becomes negligible when $Ri < 0.01$. This was confirmed by Ziskind et al. [2] who noticed that when $Ri > 1$, the buoyancy induced flow dominates over the forced convective flow. A pure forced convection therefore occurs at $Ri = 0$ while a pure free convection occurs at $Ri = \infty$. $Ri = 1$ is the intermediary that separates strong free convection from strong forced convection.

Most researchers have approached the problem of mixed convection by dividing it into two regimes based on the relationship between the imposed flow and buoyancy flow directions [3,4,5]: aiding and cross flow ($0 \leq \theta \leq 90^\circ$) and opposing flow ($90^\circ < \theta \leq 180^\circ$). Here, θ is the angle between the direction of the forced flow and the direction of the free convection. The first theoretical study on mixed convection for aiding flow past a sphere was done by Acrivos [6] who used a laminar boundary layer approximation for large Re . Hieber and Gebhart [7] used a matched asymptotic expansion to study the cases of small Gr and Re . Yuge [1] experimentally

investigated the heat transfer between a gas and a spherical surface, defining the forced convection Nusselt number (N_R) and natural convection Nusselt numbers (N_G) for $3.5 \leq Re \leq 110$ and $0 \leq Gr \leq 1818$ in the aiding, cross, and opposing mixed flow regimes. Klyachko [8] critically analyzed Yuge's study and refined the correlations for the mixed convection of a sphere in air by identifying a critical Richardson number Ri_{crit} when either the inertial forces or buoyancy forces prevail in the heat transfer process. An alternative correlation for Yuge's experimental results has been provided by Armaly et al. [9] in terms of N_R and N_G . Chen and Mucoglu [10] numerically investigated the assisting and opposing flows past a sphere using a finite difference method and studied the surface heat transfer on a sphere with constant surface temperature and uniform heat flux for large Gr and Re . They found the dynamics of mixed convection in the configuration of opposing flow to be more complex than that of assisting flow.

Buoyancy effects in mixed convection have also been extensively studied. Chen and Mucoglu [5] concluded that the local Nusselt number increases with increasing buoyancy force for aiding flow, and decreases with increasing buoyancy force for opposing flow. This finding was also confirmed by Wong et al. [11] and Gopmandal et al. [12]; the latter studied thermal buoyancy flows around a sphere translating in a quiescent fluid and found that the separation point shifted towards upstream for the buoyancy-opposed flow ($Ri < 0$), resulting in a larger recirculation downstream.

Since the Nusselt numbers for either forced or natural convection can be predicted experimentally, several authors have combined these results to predict the Nu for mixed convection. Tang et al. [3] explored the effective diameter scalar addition method which was first developed by Kirk and Johnson [4] to predict mixed convection in the cross flow and aiding flow regions. The former used a composite power curve to predict opposing flow mixed convection. Kirk and

Johnson [4] conducted experiments on mixed convection heat loss from spheres in air for Re varying from 123 to 1070 and collected data for cross flow, aiding flow, and opposing flow with Grashof number values in the order of 10^5 . Their results indicated that cross and aiding flow ($0 \leq \theta \leq 90^\circ$) must be treated separately from opposing flow ($90^\circ < \theta \leq 180^\circ$). They also found that over a wide range of Re , opposing flows show a maximum and minimum Nu , which confirms the results of Yuge [1].

From previous studies on the subject matter, a generalized correlation equation for mixed convective flow over a sphere for all flow regimes and a wide range of flow numbers has not been achieved. A clear understanding of the heat transfer characteristics for mixed flow at an arbitrary angle is not available. Churchill [13] obtained a correlation equation for mixed convective flow about a sphere which is applicable only to aiding flow; he also stated that attempts to generalize the correlating equation in some simple fashion to include forced convection oblique to or opposed to free convection were unsuccessful. The only existing correlation of the heat transfer rate in terms of the incident flow angle was obtained by Tsubouchi and Sato [14] for $\theta \geq 45$, $3 \times 10^{-2} \leq Gr \leq 40$, $0.3 \leq Re \leq 30$, and $Ri > 3/4$ in air as follows:

$$Nu = 2 + 0.5[Re^2 + (4/3)Gr - (4/\sqrt{3})ReGr^{1/2}\cos(\vartheta)]^{1/4} \quad (1)$$

In the present study, a three dimensional immersed boundary based method has been developed to solve the mixed convection from a heated sphere for an arbitrary flow incident angle at low to moderate Reynolds numbers $1 \leq Re \leq 100$ and Richardson numbers $0 \leq Ri \leq 5$. The influence of different flow incident angles on the buoyancy flow and heat transfer rate is investigated numerically. We first apply the numerical method to study the forced convection as well as the mixed convection with assisting flow. The simulation results are compared to those

published in the literature and good agreements are found. We then extend the method to study the mixed convection at various flow incident angles. The flow structures are analyzed. Based on the present simulation results, a new correlation is derived for the Nusselt number in terms of a wider range of flow numbers and incident angles.

2 Problem Description and the Immersed Boundary Method (IBM)

The immersed boundary method (IBM) [15] combined with a direct forcing scheme [16] has been implemented for solving particle motions and fluid velocity fields of particulate flows by Feng and Michaelides [17]. It has also been extended to solve heat transfer of moving particles in a viscous fluid by Feng and Michaelides [18, 19] and by Feng [20]. For the present problem, the sphere is considered fixed with no rotational or translational velocity. Two types of grids are used to solve the solid interactions: one is a fixed regular background grid or Eulerian grid for the entire flow domain, and the other is a Lagrangian grid for outlining the immersed solid boundary of the particle.

For the momentum interaction between the fluid and the solid boundary, the presence of the solid boundary can be represented by a virtual boundary with a distribution surface force $\vec{F}$ assigned on this boundary or volumetric force density $\vec{f}$ around the boundary region. The virtual boundary behaves the same as the solid boundary: when fluid approaches the virtual boundary, a distributing force arises that repels the fluid and prevents it from penetrating; this distribution force depends on the incoming flows and can be conveniently computed by a direct forcing scheme. The force density function enforces the no-slip boundary condition on the particle surface. Similarly, with respect to thermal interaction between the fluid and the solid boundary, a thermal energy density λ can be introduced to make the virtual boundary satisfy the constant temperature boundary condition at the solid surface. Under such treatment, the whole

computation domain can be regarded as filled with fluid; however, in the region that is occupied by solid, special force density and energy density terms are used to account for the effect of a physical solid boundary. For a particle that has a different temperature than the fluid, the presence of an energy density term will create a temperature gradient within the fluid, which would modify the fluid properties. For relatively small temperature differences between the particle and fluid, the Boussinesq approximation has been often used successfully for the coupling of the energy and momentum equations. Below we briefly describe the equations used to implement a finite-difference based IBM approach.

The concept of the IBM introduced by Peskin [15], is illustrated in a two-dimensional representation shown in Fig. 1. Consider a (circular) rigid particle suspended in an incompressible Newtonian fluid. The entire computational domain Ω is composed of the fluid region Ω_L and the solid/particle region Ω_S . The surface of the particle is denoted by $\partial\Omega_S$. The temperature of the fluid, $T_{f0}(\vec{x}, t)$, is different than that of the particle $T_s(t)$. The IBM enables one to describe the entire domain $\Omega = \Omega_L + \Omega_S$ by the modified Navier-Stokes equations for incompressible viscous flows. It should also be noted that the IBM (a non-boundary fitted method) is well suited in treating the complexities in elastic bodies by coupling the structure deformations between fluid and an elastic boundary. Moreover, the IBM is more efficient than boundary-fitted methods because expensive remeshing techniques are eliminated.

Fig.1. Conceptual model of a circular particle suspended in a fluid.

In the present study, we choose the incoming free stream velocity U_{f0} as the reference velocity U_{ref} , the particle diameter D as the reference length L_{ref} , D/U_{f0} as the reference time t_{ref} , and the reference temperature T_{ref} as the initial dimensional temperature difference between the sphere and the ambient fluid, $T_s - T_{f0}$. The dimensionless temperature is defined as:

$$\Theta = \frac{T_f - T_{f0}}{T_{ref}} \quad (2)$$

where T_f is the dimensional fluid temperature field. We also define the following dimensionless parameters:

$$Re = \frac{\rho_f U_{ref} L_{ref}}{\mu_f}, Pr = \frac{\mu_f c_f}{k_f}, Pe = Pr Re, Gr = \frac{g \beta_f \rho_f^2 L_{ref}^3 (T_s - T_{f0})}{\mu_{ref}^2}, Ri = \frac{Gr}{Re^2}. \quad (3)$$

where ρ_f , μ_f , and β_f are the density, dynamic viscosity, and thermal expansion coefficient of the fluid respectively; c_f and k_f are the specific heat and thermal conductivity of fluid; Re , Pr , Pe , Gr , and Ri are the Reynolds, Prandtl, Peclet, Grashof, and Richardson numbers respectively.

For the velocity field of the entire domain, it follows the modified Navier-Stokes equation [20]:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} = -\vec{\nabla} p + \frac{1}{Re} \nabla^2 \vec{u} + \frac{Gr}{Re^2} \theta \hat{e}_g + \vec{f}, \quad \vec{x} \in \Omega \quad (4)$$

where $\vec{u}$ is the dimensionless flow velocity field developed in the fluid, p is the dimensionless fluid pressure, and $\hat{e}_g$ is the unit vector in the direction of gravitational acceleration; the continuity equation is given as:

$$\vec{\nabla} \cdot \vec{u} = 0, \quad \vec{x} \in \Omega; \quad (5)$$

The force density function $\vec{f}$ is related to the boundary force $\vec{F}$ as follows [15],

$$\vec{f}(\vec{x}, t) = \int_{\partial\Omega} \vec{F}(\vec{X}, t) \delta(\vec{x} - \vec{X}(\vec{s}, t)) dA \quad (6)$$

There are several approaches in computing the force density function, such as the penalty method and adaptive scheme [21]. In the present study, the force density function is computed directly from Eqn. (4) by forcing the current fluid velocity at the boundary point to be equal to the velocity of the particle at the same location while computing the local derivative term, i.e.,

$$\frac{\partial \vec{u}}{\partial t} \approx \frac{\vec{U}_p - \vec{u}_p}{\Delta t}. \quad (7)$$

Here, $\vec{u}_p$ is the fluid velocity at the boundary point P extrapolated from the Eulerian velocity field and $\vec{U}_p$ is the velocity of the particle at the same boundary point P.

For the temperature field, we have

$$\frac{\partial \theta}{\partial t} + \vec{u} \cdot \vec{\nabla} \theta = \frac{1}{Pe} \nabla^2 \theta + \lambda, \quad \vec{x} \in \Omega \quad (7)$$

Similar to the force density function, the energy density function is given as:

$$\lambda(\vec{x}, t) = \int_{\Gamma} \Lambda(\vec{X}, t) \delta(\vec{x} - \vec{X}(\vec{s}, t)) dA \quad (8)$$

Here, Λ is the energy flux at the boundary of the particle. For the thermal density function, we also choose a direct forcing scheme by enforcing the fluid temperature at the boundary node to

be equal to the particle temperature at the same location while computing the local derivative term in Eqn. (8). More details can be found in our previous work [19].

The sphere temperature at its surface is assumed to be unit at any time,

$$\theta(\vec{x}, t) = 1 ; \vec{x} \in \partial\Omega_s \quad (9)$$

The implication of this assumption requires that the particle has a very high thermal conductivity compared to the conductivity of fluid, or the Biot number has to be very small so the heat resistance inside the sphere is less than the resistance between the particle and its surrounding.

The heat transfer rate between the sphere and its surrounding fluid is denoted by the dimensionless average Nusselt number,

$$Nu = -\frac{1}{\pi} \oint_A \vec{\nu} \theta \cdot d\vec{A} \quad (10)$$

where A is the dimensionless surface area of the sphere. Following the immersed boundary concept, the above surface integral can be converted to a volume integral and evaluated as follows:

$$Nu \approx \frac{1}{\pi} \sum_i \lambda_i \delta V_i \quad (11)$$

where λ_i is the value of thermal density function at i -th boundary node and δV_i denotes the volume associated to the i -th boundary node. Ren et al. proposed two methods of evaluating the average Nusselt number [22]: one is from temperature corrections at Eulerian points, and the other is from heat flux at Lagrangian points. The present method falls into their second method. At the limit of zero Reynolds number or steady conduction, $Nu = 2$.

The fluid momentum and heat transfer partial differential equations outlined above are solved simultaneously on a regular three-dimensional staggered grid. The velocity components and the force density components are defined at the center of the faces of each cell. The pressure, temperature and energy density λ are defined at the center of the cell. An explicit projection

method is used in solving the momentum and heat transfer equations. The solution procedures are similar to the ones described for two-dimensional flow problems in our previous work [19].

3 Simulation Results and Discussions

3.1 Mixed convection of a heated isothermal sphere in an oblique fluid flow

Consider an isothermal sphere of radius a immersed in an incompressible viscous Newtonian fluid flow with an undisturbed oncoming free stream of velocity U_{f0} and temperature T_{f0} . The stream has an incident angle θ which is defined as the angle between stream direction and the opposite direction of the gravity, as shown in Fig. 2. The surface of the sphere is maintained at a uniform temperature T_s . The buoyancy flow from the sphere surface depends on the temperature gradient between the sphere and the viscous fluid. When $T_s > T_{f0}$, a thermal buoyancy force arises between the hot isothermal sphere and fluid that produces an upward free convective flow; on the other hand, a downward buoyancy flow occurs when $T_s < T_{f0}$ [12,23]. Based on the incident angle, the ambient stream flow can be defined as aiding and cross flow ($0 \leq \theta \leq 90^\circ$) or opposing flow ($90^\circ < \theta \leq 180^\circ$). Obviously the combined resulting flow depends on the directions and strengths of ambient stream flow as well as the buoyancy flow.

To mimic a sphere in a large ambient uniform flow, we choose a three-dimensional channel in the z -direction as the simulation domain. Flow enters the channel at the inlet and leaves the channel at the outlet. The flow velocity and temperature are given at the inlet; a fully developed (zero-gradient) boundary condition is used at the outlet. To minimize the effect of the boundary from the side surfaces, the periodic boundary conditions are imposed at the surfaces of the simulation domain along both x and y directions.

Fig.2. An isothermal sphere of radius a and temperature T_p immersed in a viscous fluid at ambient temperature T_{f0} in oblique flow.

To identify the optimal simulation parameters, we carry out convergence studies for mixed convection. We consider the convective heat transfer from a heated fixed sphere in air ($Pr=0.72$) at $Re = 100$ and $Ri = 5$. Firstly, convergence tests on the grid spacing h and time step δt are conducted for a completely upward flow ($\theta=0^\circ$). To test the effect of the grid spacing on the simulation results, the computational domain size is fixed to be $8a \times 8a \times 24a$ and the time step $\delta t = 0.001$ is used. Three cases of grid spacing, $D/10$, $D/20$, and $D/25$, respectively, are tested. Fig. 3 shows the overall Nusselt number for these three cases. Clearly, a grid spacing of $D/10$ is too crude such that for a finer grid $D/20$ the value of the steady state Nu is increased by 10%. However, refining the grid from $D/20$ to $D/25$ slightly increases the steady state Nu by approximately 2%. It can therefore be concluded that a minimum grid spacing of $D/20$ is needed in order to achieve a reasonable accurate result.

Fig.3: The effect of the grid spacing h on the overall Nusselt number.

The use of forward time differencing will introduce numerical diffusion and affect simulation results. To test the sensitivity of the time step on the computed Nu , the computational domain size is kept at $8a \times 8a \times 24a$ with a grid spacing of $D/10$, three different time steps, 0.005, 0.0005, and 0.00025 are chosen, respectively. From the results shown in Fig. 4, it is found that the calculated Nu decreases slightly as the time step δt decreases. The difference in the computed Nu is 6% when decreasing the time step from $\delta t = 0.005$ to $\delta t = 0.0005$, and 1% from $\delta t = 0.0005$ to $\delta t = 0.00025$. Therefore, a minimum time step of 0.0005 is sufficient to achieve results that are independent of time step.

Fig.4: The effect of the time step δt on the computed overall Nusselt number

In our study on the heat diffusion from a heated sphere [24, 20], we found that the size of the computational domain could significantly affect the temperature field in the outer region at longer times; however, it has an insignificant effect on the inner field and the overall Nusselt number. Maintaining the grid spacing $h = D/20$ and the time step $\delta t = 0.0005$, the effect of the domain size on the mixed convection for a completely upward flow ($\theta = 0^\circ$) on a heated sphere is further investigated by choosing three domain sizes: $8a \times 8a \times 24a$, $10a \times 10a \times 20a$, and $4a \times 20a \times 10a$, respectively. The simulation results show almost identical overall Nusselt numbers for the domains $8a \times 8a \times 24a$ and $10a \times 10a \times 20a$ in which cases the center of the sphere is positioned at $(4a, 4a, 4a)$ and $(5a, 5a, 5a)$, respectively. For the domain $4a \times 20a \times 10a$, which has a narrower and longer flow inlet, the center of the sphere is located at $(2a, 10a, 5a)$. The simulation results show less than 1% difference in the overall Nusselt number between any two cases among the three cases studied here, indicating that the flow inner field and the overall

Nusselt number are not sensitive to the choice of simulation domain size. However, we must point out that the use of periodic boundary conditions in the x and y directions also has significantly minimized the computational domain size effect.

Fig.5: The effect of the domain on the overall Nusselt number.

3.2 Mixed convection of a sphere in air ($Pr=0.72$): a validation case

To validate the present method, we conduct simulations of mixed convection of a heated sphere for cross and aiding flow in air at various Re and Ri and compare our results with those found in the literature. A domain of $10a \times 10a \times 20a$ is used in these simulations. The overall Nusselt numbers are computed and listed in Table 1. The results of the Nusselt Number for pure forced flow ($\theta = 90^\circ$ and $Ri = 0$) of Wong et al. [11], Armaly et al. [9], and Gopmandal et al. [12] are also listed in Table 1. Armaly's results for mixed flow are in closer agreement with ours as compared to Wong's data in Table 1. Yuge's experimental results for mixed aiding flow were presented by Armaly et al. [9] in terms of N_R and N_G respectively as:

$$(Nu - 2)/N_R = \left[1 + (0.795 Ri^{1/4})^{3.5}\right]^{1/3.5} \quad (12)$$

$$(Nu - 2)/N_G = 1.258 \left[1 + (0.795 Ri^{1/4})^{3.5}\right]^{1/3.5} Ri^{1/4}$$

where $N_R = 0.493 Re^{0.5}$, $N_G = 0.392 Gr^{0.25}$, $3.5 \leq Re \leq 110$, $0 \leq Gr \leq 1818$, and $Pr = 0.71$.

Note that the results of Nu for $Ri = 0$ are true for any arbitrary flow incident angle, $0^\circ \leq \theta \leq 180^\circ$,

Excellent agreements are found between the present simulation results and those found in the literature for all the cases considered here. This shows that the present method is reliable and the simulation parameters used are sufficient to produce accurate results.

Table 1: Nusselt numbers for pure forced and mixed convection over a sphere in air ($Pr=0.72$) at $\theta=0$

Re	Ri	Nu			
		present	Wong [11]	Armaly [9]	Gopmandal[12]
1	0	2.083	-	-	2.061
5	0	2.924	2.808	3.102	-
10	0	3.363	3.282	3.559	3.225
50	0	5.407	-	5.486	5.440
100	0	6.982	7.351	6.930	6.993
60	0.2	5.980	6.021	5.934	-
60	4	7.203	7.183	-	-
100	0.1	7.075	-	7.012	-
100	0.2	7.326	7.509	-	-

3.3 Mixed flow for Aiding and Cross flow in air ($0^\circ \leq \theta \leq 90^\circ$)

For the incident flow angles $0^\circ \leq \theta \leq 90^\circ$, also known as the aiding and cross flow regime, the mixed convection on a sphere in a domain $24a \times 8a \times 24a$ is simulated with a uniform

flow imposed in the xy -plane. To visualize the flow structures, plots of the velocity vector map, contour slices, and isosurface contours of temperature and vorticity using lambda-2 criteria [25] are generated for cases of $\theta = 0^\circ$ and $\theta = 45^\circ$. All contour slices and velocity vector maps presented in this work are colored by temperature. In the case of $\theta = 0^\circ$ shown in Fig. 6 with the sphere's centre positioned at $(12a, 4a, 5a)$, the forced flow aligns the rising fluid upward into a vertical fluid column; this is unlike the case of natural convection whereby the fluid column is created by a weaker realignment of rising fluid currents. A thermal plume results with a cylindrical stem. The regions colored by velocity magnitude in red on the velocity vector map in Fig. 6(a) indicate high velocity levels in the wake of the flow, which appear in the region very close to the surface of the sphere and at the plume cap.

Fig.6: Aiding flow over a sphere for $\theta = 0^\circ$, (completely upward flow): $Re=100$, $Ri=5$, and $t=1$

(a) Velocity vector map (b) temperature contour slice

In the case of $\theta = 45^\circ$ shown both in perspective and side views in Fig. 7(a-d), the sphere's centre is positioned at $(4a, 4a, 5a)$. The free convective thermal plume is tilted from its vertical alignment or buoyancy force direction to an angle due to the forced flow; the degree of tilting

from the vertical alignment increases with the increase of θ . From the isosurface temperature contour shown in Fig. 7(c), the stem of the evolving tilted plume loses its cylindrical shape. At $\theta = 45^\circ$, vortex tubes are formed along the stem of the plume while a doughnut-shaped vortex ring is formed at the plume cap as shown in Fig. 7(d), likely caused by the confined simulation boundary effect. The helical streams of rising fluid shown in the velocity vector map give rise to these vortex tubes.

Fig.7: Visualizations of mixed aiding flow over a sphere at $t=3$ with $\theta = 45^\circ$, $Re=100$, $Ri=5$

- (a) Velocity vector map (b) temperature contour slice (c) isosurface temperature contour , $\theta=0.6$
(d) isosurface vorticity contour

A plot of the time evolution of the Nu at several different assisting flow angles is shown in Fig. 8. A decreasing trend in Nu can be observed as the flow incident angle increases within

the range of $0^\circ \leq \theta \leq 90^\circ$; a correlation in Nu with respect to θ will be obtained subsequently. For a given Reynolds number, the Nu decreases with a decrease in the Richardson number Ri since the buoyancy force contribution equally reduces. We have therefore shown that a smooth decreasing trend in Nu is observable as θ increases for the range $0^\circ \leq \theta \leq 90^\circ$ based on our numerical simulation results. Furthermore, the sensitivity of the Nusselt number to the change in ϑ increases as the flow angle increases from an upward flow ($\theta = 0^\circ$) to a cross flow ($\theta = 90^\circ$). It should be noted that Nu at the completely upward flow ($\theta = 0^\circ$) is always greater than that for cross flow ($\theta = 90^\circ$). This will be further illustrated in the next section.

Fig.8: Heat transfer characteristics of mixed flow over a sphere for incident flow angles ($0^\circ \leq \theta \leq 90^\circ$, $Re = 100$, $Ri=5$): Time evolution of Nu

3.5 Mixed cross flow in air ($\theta = 90^\circ$, $Pr = 0.72$)

Simulation studies and experiments on mixed cross flow ($\theta = 90^\circ$) are very limited. Many studies exist for the case of $Ri = 0$, when the free convection is suppressed ($Gr=0$) and there is no difference between assisting, opposing, and cross flows. The cross flow empirical formulae and graphical procedures of Yuge [1] appear to be cumbersome although they apply to a wide range of Ri . Here, we analyze in detail the mixed convection of a sphere with centre positioned at $(4a, 4a, 5a)$ in cross flow when buoyancy forces are predominant ($Ri > 1$). Fig. 9 shows the flow structures resulting from the mixed convection at cross flow over an isolated heated sphere visualized in side and perspective views. Due to the fact that the forced flow acts along a direction perpendicular to the gravity force, the buoyancy flow is pushed from central to the left, leading to a tilted strong plume rising downstream. As the flow angle increases from completely upward flow to cross flow, the cylindrical tip of the isosurface temperature contour downstream eventually merges and splits up into double horn-shaped contours with pointed tips, as shown in Fig. 9 (c). Notice how the vortex tubes for $\theta = 90^\circ$ in Fig. 9 (d) are curved inwards as compared to those for $\theta = 45^\circ$ shown in Fig. 7 (d).

Further analysis of the mixed cross flow on a heated sphere at $\theta = 90^\circ$ is performed. Fig. 10 shows the Nusselt numbers of such flows at $Gr=10,000$ and several different Reynolds numbers. Results from the completely upward flow and completely downward flow are also plotted in Fig.10. These simulation results are compared with those found in the literature, which include pure forced flow [11, 26, 27, 28], mixed aiding flow [8], and free convection [1]. As the Reynolds number increases, the cross flow Nu ($\theta = 90^\circ$) increases nonlinearly, rising below the aiding flow Nu at $\theta = 0^\circ$, staying above the opposing flow Nu at $\theta = 180^\circ$, and always exceeding the pure free convection Nu . The mixed crossed flow Nu can therefore be said to be an

enhancement of the free convection buoyancy flow Nu , but to a lesser degree compared to that of a mixed completely upward flow at a given flow Reynolds number. As seen in Fig. 10, the Nu for mixed cross flow exceeds that for pure forced flow with increase in the Reynolds number; however, these two Nu -curves approach each other as they both increase with Re , to coincide at $Re = 100$, where $Ri \approx 1$ for mixed cross flow.

Fig.9: Visualizations of mixed cross flow over a sphere at $t=3$ with $\theta = 90^\circ$, $Re=100$, and $Ri=5$:

(a) velocity vector map (b) temperature contour slice (c) Temperature isosurface contour at $\theta=0.6$ (d) isosurface vorticity contour

As the Reynolds number increases, the Nu of pure forced convection overtakes the Nu of free convection at $Re = 50$. A correlation for mixed cross flow at $\theta = 90^\circ$ will be obtained subsequently. The minimum value observed in the opposing flow Nu for changing Reynolds

numbers was also observed in the experiments of Yuge [1]. Klyachko [8], after analyzing Yuge's data for aiding flows, proposed

$$Nu = Nu_R [1 + 0.1(1 + Re_o/Re) Ri^{1/4}] ; \text{ where } Nu_R = 2(1 + 0.276 Re^{1/2} Pr^{1/3}); \quad (13a)$$

$$Re_o = [1/19044 (295.0 Ra^{1/8} - 500.0) Pr^{-2/3}] , \quad 10^{-3} \leq Ra \leq 500$$

$$Re_o = [1/19044 (135.0 Ra^{1/4} - 500.0) Pr^{-2/3}] , \quad 500 \leq Ra \leq 2 \times 10^7 \quad (14b)$$

These correlations show a good agreement with our results.

Fig.10: Comparison of the overall Nusselt number for mixed cross flow over a sphere at $\theta=90^\circ$ for different Re with results in literature for $(1/Ri)=0$ and $Ri=0$, ($Gr=10000$).

3.6 Mixed Opposing flow in air ($90^\circ < \theta \leq 180^\circ$)

A forced flow imposed on a heated sphere would generally diminish or enhance the heat transfer rate. The degree of this change depends on the flow incident angle. For a completely upward imposed flow, the inertial forces enhance the heat transfer rate. For cross flow as

previously discussed, the inertial forces affect the thermal plume formation and push it to the downstream direction; it also increases Nu , but to a lesser degree compared to completely upward flow. For opposing flow angles ($90^\circ < \theta \leq 180^\circ$), the inertial forces greatly counteract the formation of a thermal plume, which may possibly lead to flow recirculation. This can be clearly observed in the case of a completely downward flow at $\theta = 180^\circ$ shown in Fig. 11, with the sphere's centre positioned at $(4a, 4a, 5a)$. At time $t=1$, circulations are formed around the sphere due to the combined effect of forced flow and natural convection. The forced flow prevents the thermal plume from rising upward.

Fig.11: Opposing flow over a sphere at $\theta = 180^\circ$ (completely downward flow): $Re = 100$, $Ri = 5$

(a) Velocity vector map (b) temperature contour slice (c) isosurface temperature contour at $\theta = 0.6$

As time evolves, the inertial forces prevail over the buoyancy force, creating a wider low velocity area above the sphere; the circulation on both sides of the sphere propagates downstream, creating a mushroom cloud vortex turned upside down. This causes the slowdown of heat transfer rate compared to a completely upward flow. In the special case of $\theta = 135^\circ$, a close observation of the flow structures shows a double horn-shaped temperature iso-surface contour that is reduced in length shown in Fig. 12 (b) and (c), as compared to that for $\theta = 90^\circ$ shown in Fig. 9 (b) and (c). Also, the vortex tubes for $\theta = 135^\circ$ in Fig. 12 (d) are further curved inwards than those for $\theta = 90^\circ$ shown in Fig. 9 (d).

Fig.12: Visualizations of mixed flow over a sphere at $t=3$ with $\theta = 135^\circ$, $Re = 100$, and $Ri = 5$
(a) velocity vector map (b) temperature contour slice (c) isosurface temperature contour at $\theta = 0.6$
(d) isosurface vorticity contour

The heat transfer characteristics of mixed flow at an arbitrary angle over a sphere for the two main regimes (aiding and cross flow, and opposing flow) are presented in Fig. 13. In general, a decreasing trend in the steady state Nu as θ increases from 0° to 180° is observed. For aiding and cross flow angles $0^\circ \leq \theta \leq 90^\circ$, a quadratic decrease in Nu is observed. However, at opposing flow incident angles in the range $90^\circ < \theta \leq 180^\circ$, a half bell-shaped characteristic curve is obtained for Nu which could be likened to the right half of a Gaussian curve symmetrically split into two. For a given value of Ri , more than 60% of the difference in the heat transfer rate between completely upward flow ($\theta = 0^\circ$) and completely downward flow ($\theta = 180^\circ$) is taken by the span or “height” of the half bell-shaped characteristic curve.

For $Ri < 1$, the Gaussian-like half curves appear to coincide at a common point of symmetry. The minimum Nu occurs for a completely downward flow ($\theta = 180^\circ$) at $Ri=1$, which is in accordance with the counter flow observations of Tang et al.[3]. For $0 < Ri \leq 0.01$, the simulation results approach the result for $Ri=0$; this would require that the half bell-shaped curves flatten out almost horizontally, such that the effect of the flow incident angle on Nu is insignificant. The shape of the resulting characteristic curves of Nu for $0^\circ \leq \theta \leq 180^\circ$ in Fig. 13 clearly shows the need to isolate and treat cross and aiding flow ($0 \leq \theta \leq 90^\circ$) from opposing flow ($90^\circ < \theta \leq 180^\circ$) separately, as Kirk and Johnson [4] indicated. Like Churchill [13] mentioned, there is no generalized correlation that could represent the behavior of these curves; however, the isolated treatments of Nu for $0 \leq \theta \leq 90^\circ$ and $90^\circ < \theta \leq 180^\circ$ enable better descriptions of the Nu -curves respectively as a quadratic decrease with θ and a half bell-shaped decrease with θ .

Fig.13: Mixed flow Nu for incident angles $0^\circ \leq \theta \leq 180^\circ$, $0.01 \leq Ri \leq 5$, and $Re=100$

3.7 Correlation of Nu in terms of flow incident angle in air

To obtain a correlation of Nu for a sphere immersed in an oblique air flow ($Pr = 0.72$) in terms of flow Re and Ri , and the arbitrary flow incident angle θ , we perform more simulations at Reynolds numbers 10, 20, 40, 50, 60, and 80. Finding a Nu correlation for the half bell-shaped curve in the range $90^\circ < \theta \leq 180^\circ$ would pose a great degree of difficulty to obtain. We have therefore focused on obtaining a correlation for the quadratic curve in the range $0^\circ \leq \theta \leq 90^\circ$, which is less cumbersome.

Based on these simulation results, a correlation for the mixed aiding and cross flow regime with incident angles in the range $0 \leq \theta < \frac{\pi}{2}$, $10 \leq Re \leq 100$, and $1 \leq Ri \leq 5$ was obtained as follows:

$$Nu = aRe^b Ri^{0.5b} \theta^2 + cRe^d Ri^{0.5d} \theta + nRe^f Ri^{0.5f} - gRe^{1+h} Ri^{0.5} - kRe^m \quad (14)$$

where $a = -0.0208$; $b = 0.4851$; $c = 0.0250$; $d = 0.05$; $n = 1.436$; $f = 0.3642$; $g = 0.2534$;

$h = -0.722$; $k = -0.2602$; $m = 0.2412$

Also, for the forced flow with $10 \leq Re \leq 100$ and $Ri = 0$, we found that:

$$Nu = 1.5862Re^{0.3159} \quad (15)$$

The correlation produces a very good agreement with the simulation results with differences below 5% for most of cases. Fig. 14 shows the correlation compared with the simulation results for $Ri = 1$ at different Reynolds numbers and incident angles.

Fig.14: Comparing Results with the correlation of Nu for $0^\circ \leq \theta \leq 180^\circ$, $10 \leq Re \leq 100$, $Ri=1$

An interesting observation on how the Nu changes with Ri for completely upward flow, mixed cross flow, and completely downward flow ($\theta = 0^\circ$, 90° , and 180°) based on our simulation data can be seen in Fig. 15. The difference in Nu on a sphere when the imposed flow is switched from completely upward flow ($\theta = 0^\circ$) to completely downward flow ($\theta = 180^\circ$) for $Re = 100$ and $Ri \leq 5$ increases with increase in Ri . However, for $Ri > 1$, shown in Fig. 15, this difference slowly decreases with increase in Ri . At $Ri = 0.01$, Nu for completely upward, cross, and completely downward flows appear to converge since the free convection buoyancy force is negligible. On the other hand, as Ri increases for $Ri > 1$, keeping the Re fixed at $Re = 100$, the Nu for $\theta = 0^\circ$, and $\theta = 90^\circ$ slowly approach each other and converge at some large value of Ri where the forced flow becomes negligible. The minimum Nu occurs for completely downward flow conditions at $Ri = 1$ (when $Gr = 10000$)—this is confirmed by the counter flow equations of Tang et al.[3] that adequately describe the minimum to occur when forced and natural flows roughly equal each other for large Re flows. We have therefore shown that for moderate Reynolds numbers $10 \leq Re \leq 100$, the maximum difference in Nu between completely upward and completely downward flow occurs at $Ri = 1$.

Fig.15: Linear relationships between Nu and Ri for $\theta = 0^\circ, 90^\circ, 180^\circ$, and $Re = 100$

4 Conclusions

The mixed convection on a heated sphere at various flow incident angles ranging from 0° to 180° has been studied using a three-dimensional numerical simulation method combined with the immersed boundary method. Based on extensive simulation results, we obtained a correlation of Nu for angles in the aiding and cross flow range ($0 \leq \theta \leq \frac{\pi}{2}$) for $1 \leq Ri \leq 5$ and $10 \leq Re \leq 100$, showing a quadratic decrease in Nu with θ , and an increase in Nu with Ri . At opposing flow incident angles in the range $90^\circ < \theta \leq 180^\circ$, the heat transfer characteristic shows a half bell-shaped decrease in Nu as θ increases. Furthermore, we have shown that for low to moderate Reynolds numbers in the range $10 \leq Re \leq 100$, the maximum difference in the overall Nusselt number between completely upward and completely downward flows occurs at $Ri = 1$. We were able to observe a relationship between Nu and Ri for each of the three flow incident angles $\theta = 0^\circ, 90^\circ$, and 180° , treated together simultaneously as completely upward flow, mixed cross flow and completely downward flow, respectively. At $Ri = 0.01$, the Nu for flows at $\theta = 0$

0° , 90° , 180° showed insignificant differences because natural convection becomes negligible; on the other hand, the Nu for flows at $\theta = 0^\circ$, 90° , 180° slowly approach each other for $Ri > 1$ and converge at some large value of Ri where the forced flow becomes negligible.

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References:

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- [1] T. Yuge, Experiments on heat transfer from spheres including combined natural and forced convection, *ASME J. Heat Transfer* **82**, 214–220 (1960).
 - [2] G. Ziskind, B. Zhao, D. Katoshevski, E. Bar-Ziv, Experimental study of the forces associated with mixed convection from a heated sphere at small Reynolds and Grashof numbers. Part I: Cross-flow, *Int. J. Heat and Mass Transfer* **44**, 4381–4389 (2001).
 - [3] L. Tang, A. T. Johnson, R. H. McCuen, Empirical study of mixed convection about a sphere, *J. Agricultural Eng. Research* **50**, 197–208 (1991).
 - [4] G. D. Kirk, A. T. Johnson, Experimental determination of mixed convective heat transfer from a sphere to air, *Int. Communications in Heat and Mass Transfer* **13**, 369–387 (1986).
 - [5] T. S. Chen, A. Mucoglu, Analysis of mixed forced and free convection about a sphere, *Int. J. Heat and Mass Transfer*, **20**, 867–875 (1977).
 - [6] A. Acrivos, On the combined effect of forced and free convection heat transfer in laminar boundary layer flows, *Chem. Eng. Science* **21**, 343–352 (1966).

-
- [7] C. A. Hieber, B. Gebhart, Mixed convection from a sphere at small Reynolds and Grashof numbers, *J. Fluid Mech.* **38**, 137–159 (1969).
- [8] L.S. Klyachko, Heat transfer between a gas and a spherical surface with the combined action of free and forced convection, *ASME J. Heat Transfer* **85**, 355–357 (1963).
- [9] B. F. Armaly, T. S. Chen, N. Ramachandran , Correlations for laminar mixed convection on vertical, inclined and horizontal flat plates with uniform surface heat flux, *Int. J. Heat and Mass Transfer* **30**, 405-408 (1987).
- [10] A. Mucoglu, T. S. Chen, Mixed convection about a sphere with uniform surface heat flux, *ASME J. Heat Transfer* **100**, 542 (1978).
- [11] K. Wong, S. Lee, C. Chen, Finite element solution of laminar combined convection from a sphere, *ASME J. Heat Transfer* **108**, 860-865 (1986).
- [12] P. Gopmandal, S. Bhattacharyya, Thermal buoyancy-aided flow around a Heated/Cooled spherical particle, *Heat Transfer Research* **42**, 689-710 (2011).
- [13] S. W. Churchill, A comprehensive correlating equation for laminar, assisting, forced and free convection, *AIChE Journal*, **23**, 10–16 (1977).
- [14] Tsubouchi, T., S. Sato and H. Masuda: *ibid.*, **30** (219), 1386 (1964).
- [15] C.S. Peskin, Numerical analysis of blood flow in the heart, *J. Comp. Phys.* **25**, 220–252 (1977).
- [16] J.Mohd-Yusof, Combined Immersed Boundaries/b-Splines Methods for Simulations of Flows in Complex Geometries *Annual Research Briefs, Center for Turbulence Research, Stanford University, Stanford, CA*, **161**, 35–60 (1997).
- [17] Z.-G. Feng, E. E. Michaelides, Robust treatment of no-slip boundary condition and velocity updating for the lattice-Boltzmann simulation of particulate flows. *Computers and Fluids* **38**, 370–381 (2009b)

-
- [18] Z.-G. Feng, E. E. Michaelides, Inclusion of heat transfer computations for Particle laden flows, *Phys. of Fluids* **20**, 675-684 (2008).
- [19] Z.-G. Feng, E. E. Michaelides, Heat transfer in particulate flows with direct Numerical simulation (DNS), *Int. J. Heat and Mass Transfer* **52**, 777–786 (2009a).
- [20] Z.-G. Feng, Direct numerical simulation of forced convective heat transfer from a heated rotating sphere in laminar flows. *Journal of Heat Transfer* doi:10.1115/1.4026307. (2014).
- [21] Z.-G. Feng, E. E. Michaelides, Proteus: a direct forcing method in the simulations of particulate flow, *J. Comp. Phys.* **202**, 20–51 (2005).
- [22] W. W. Ren, C. Shu., J. Wu, W. M. Yang, Boundary condition-enforced immersed boundary method for thermal flow problems with Dirichlet temperature condition and its applications. *Comp. Fluids*, **57**, 40-51 (2012).
- [23] H. Gan, J. Chang, J. J. Feng, H. H. Hu, Direct numerical simulation of the sedimentation of solid particles with thermal convection. *J. Fluid Mech*, **481**, 385-411 (2003).
- [24] S. Musong, A three dimensional immersed boundary-based method for the free and combined convective heat transfer from spherical bodies. (Order No. 1538375, The University of Texas at San Antonio), ProQuest Dissertations and Theses **182** (2013).
- [25] P. Chakraborty, S. Balachandar, R. J. Adrian, On the relationships between local vortex identification schemes. *J. Fluid Mech.*, **535**, 189-214 (2005).
- [26] R. S. Alassar, H. M. Badr, H. A. Mavromatis, Heat convection from a sphere placed in an oscillating free stream, *Int. J. Heat and Mass Transfer*, **42**, 1289-1304 (1999).
- [27] S. Whitaker, Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Spheres, and for Flow in Packed Beds and Tube Bundles, *AIChE Journal*, **18**, 361–371 (1972).

[28] Z.-G. Feng, E. E. Michaelides, Proteus: a direct forcing method in the simulations of particulate flow, *J. Comp. Phys.* **202**, 20–51 (2005).