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5 1 **Calibration of empirical models considering model fidelity and model robustness –**
6 2 **Focusing on predictions of liquefaction-induced settlements**

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11 7 **Abstract:** Most data-driven empirical models adopted in the geotechnical design have various
12 8 degrees of uncertainty. Consequently, it is important to properly calibrate this uncertainty prior
13 9 to its application in the geotechnical analysis to ensure the integrity of the final design. The
14 10 conventional probabilistic model calibration approaches, such as maximum likelihood estimate
15 11 and Bayesian method, focus only on model fidelity (measured through, for example, likelihood).
16 12 These approaches often yield models with a large model uncertainty, especially when the
17 13 observed data are scattered. A large model uncertainty leads to a large variation in the model
18 14 prediction, which poses a significant challenge in a geotechnical design. In this paper, we
19 15 propose a new model calibration approach that can consider both model fidelity and model
20 16 robustness. The new model calibration approach yields a more sensible model by balancing the
21 17 objectives of model fidelity and model robustness. Because model fidelity and model robustness
22 18 are two conflicting objectives, the new approach involves a multi-objective optimization that
23 19 leads to a Pareto front, which defines a tradeoff relationship between model fidelity and model
24 20 robustness. By enforcing a certain level of robustness, the variation in the model prediction can
25 21 be reduced, which overcomes the major weakness of the traditional probabilistic approaches that
26 22 focus solely on model fidelity. The new approach is demonstrated through applications to the
27 23 problems of liquefaction-induced settlement prediction.
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31 25 **Keywords:** empirical models; liquefaction; model fidelity; model robustness; multi-objective
32 26 optimization; probabilistic model calibration; settlement.
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1. Introduction

Despite the advances in numerical methods and testing techniques during the past decades, data-driven empirical models are still widely used in geotechnical design. For example, case histories often serve as the platform for studying the problems of earthquake-induced soil liquefaction and liquefaction effects (e.g., settlement and lateral spread), and the empirical models that are derived from a database of case histories are often used for prediction of a future case in a design situation (e.g., Seed and Idriss 1971; Seed et al. 1985; Robertson and Wride 1998; Andrus and Stokoe II 2000; Youd et al. 2001; Juang et al. 2002; Zhang et al. 2002; Cetin et al. 2004; Moss et al. 2006; Boulanger and Idriss 2012; Juang et al. 2013a&b; Goh and Zhang 2014; Khoshnevisan et al. 2015b). Uncertainty does exist in these empirical models, however, regardless of whether it is acknowledged and/or calibrated (Zhang et al. 2009). If the model uncertainty is not calibrated and not included in the analysis, the model prediction would be biased and not reliable.

In theory, the uncertainty of an empirical model can be calibrated through a systematic comparison between model predictions and observed data. To account for the uncertain nature of the solution model, this comparison is often conducted in a probabilistic manner (Ang and Tang 1984; Cheung and Tang 2005). The conventional probabilistic approaches that are often adopted to calibrate the model uncertainty include the maximum likelihood estimate (MLE) (e.g., Wang et al. 2013a; Gong et al. 2014a) and Bayesian method (e.g., Zhang et al. 2009; Wang et al. 2013a). In the context of these probabilistic approaches, the accuracy (or fidelity) of a solution model is measured with the likelihood of observed data, and the objective of the uncertainty calibration (or model calibration) is to maximize the likelihood of that data.

The conventional probabilistic approaches are typically implemented as a single objective optimization problem, in which model fidelity (i.e., likelihood of observed data) is maximized. The model that is calibrated based *solely* on model fidelity could be highly uncertain; and thus, the model prediction (i.e., prediction by the calibrated model) for a future case can vary in a wide range. For example, Figure 1 illustrates the predicted liquefaction-induced settlement using the empirical model that has been calibrated based on fidelity versus the observed settlement (Table 1; Juang et al. 2013b). Because of the uncertainty of the solution model, the model prediction is not a fixed value, but rather a random variable. As such, both the mean and the 95% confidence interval of the model prediction are plotted in Figure 1. In a perfect model with perfect observed data, the data points of observation versus prediction would rest on the 1:1 line (meaning that the prediction agrees perfectly with the observation). According to the statistical theory, the confidence interval of the model prediction should bracket the observation if the model is “accurate”. In other words, the 1:1 line in the graph of prediction versus observation (Figure 1) should fall within the confidence interval of the model prediction. Thus, the model developed by Juang et al. (2013b) based solely on fidelity (in terms of the likelihood of observed data) is “accurate”, but not perfect. The 95% confidence interval of the model prediction in this case, however, is too large to be meaningful in a design situation. For instance, the solution model developed by Juang et al. (2013b) is applied to a case designated as “TK-J2” (Table 1) at site J of Adapazari city (Sancio 2003), and the derived probability distribution of the settlement prediction is plotted in Figure 2. Although the 95% confidence interval contains the observation, the wide distribution in the predicted settlement for this case poses a great challenge in a design situation. For example, it would be difficult to ascertain whether the predicted settlement exceeds a specified limiting settlement (Ishihara and Yoshimine 1992).

The wide distribution of the predicted settlement, shown in Figure 1 and Figure 2, is mainly attributed to the large model uncertainty. At this point, it is helpful to recall that the solution model by Juang et al. (2013b) was developed through a maximum likelihood analysis of a database of case histories (Table 1), in which only model fidelity is considered and optimized. Although the solution model calibrated with MLE is deemed “accurate” (i.e., the observation falls within the 95% confidence interval of the model prediction), it is not “precise” (i.e., the range of the 95% confidence interval is not narrow), which makes the model prediction difficult to use in a design situation.

Intuitively, a solution model that yields a smaller variation in the model prediction would be ideal if the fidelity of this model is acceptable. In fact, from the viewpoint of usability, the variation of the model prediction should be as small as possible. Here, a smaller variation in the model prediction implies a higher degree of model robustness. Thus, the robustness of the solution model (reflected by the level of variation of the model prediction) should be considered and optimized along with the model fidelity. This is a step beyond the focus of *fidelity* in the model calibration, as is adopted in the conventional probabilistic model calibration methods. Therefore, a new multi-objective optimization approach that considers both model fidelity and model robustness in the geotechnical model calibration is proposed in this study. Note that the desire to maximize the model fidelity and the desire to maximize the model robustness are two conflicting objectives. Thus, the concepts of Pareto front (Deb et al. 2002; Khoshnevisan et al. 2015a) and knee point (Branke et al. 2004; Deb and Gupta 2011; Gong et al. 2014b&2015) are further introduced to interpret the outcome of the model calibration.

2. Conventional Probabilistic Approaches

In that the solution models adopted in the geotechnical design are usually an abstraction of the real world, the discrepancy between the model prediction and the observed performance almost always exists. If the discrepancy or model error is not calibrated using a representative database and included in the analysis for a future case, the prediction from this solution model could be biased. For illustration purposes in this study, the discrepancy between the model prediction and that observed in the field is represented with a multiplicative model bias factor, denoted as M . The adjusted model prediction can be symbolically expressed as

$$d = M \cdot g(\theta) \quad (1)$$

where $g(\theta)$ represents a general solution model adopted in the geotechnical design; θ represents the model input parameters, which can be either fixed parameters or random variables; and, d represents the observed performance, which can be assessed through laboratory or in-situ tests. In principle, the model bias factor M in Eq. (1) can be determined by a systematic comparison between model predictions and observed performances. Although this comparison can be conducted in either a deterministic or a probabilistic manner, the deterministic approaches are inadequate for addressing the high uncertain nature in a geotechnical problem. Oftentimes, the input parameters of a solution model cannot be characterized exactly; and, the observations may also be subjected to the error. Thus, in addition to the model uncertainty, the uncertainties in the input parameters and the observations can also contribute to the discrepancy between the model prediction and the observed performance. The deterministic approaches are not capable of studying the problem of the geotechnical model calibration in the face of uncertainties. Hence, the probabilistic approaches are adopted in this study.

2.1 Maximum likelihood estimate (MLE) of the statistics of the model bias factor

The likelihood of observed data is a built-in element of the conventional probabilistic approaches. Let's assume that both model bias factor M and model prediction $g(\boldsymbol{\theta})$ follow lognormal distributions, the adjusted model prediction $M \cdot g(\boldsymbol{\theta})$ would be a lognormal variable. The problem of the model calibration is then reduced to the estimation of the mean and standard deviation of the model bias factor M , denoted as μ_M and σ_M , respectively. Note that the mean and standard deviation of the model prediction $g(\boldsymbol{\theta})$, denoted as $\mu_{g(\boldsymbol{\theta})}$ and $\sigma_{g(\boldsymbol{\theta})}$, respectively, can readily be derived from the model input parameters $\boldsymbol{\theta}$. The mean and standard deviation of the adjusted model prediction $M \cdot g(\boldsymbol{\theta})$, denoted as $\mu_{[M \cdot g(\boldsymbol{\theta})]}$ and $\sigma_{[M \cdot g(\boldsymbol{\theta})]}$, respectively, can then be approximated.

$$\mu_{[M \cdot g(\boldsymbol{\theta})]} = \mu_M \cdot \mu_{g(\boldsymbol{\theta})} \quad (2)$$

$$\sigma_{[M \cdot g(\boldsymbol{\theta})]} = \sqrt{\mu_M^2 \sigma_{g(\boldsymbol{\theta})}^2 + \sigma_M^2 \sigma_{g(\boldsymbol{\theta})}^2 + \sigma_M^2 \mu_{g(\boldsymbol{\theta})}^2} \quad (3)$$

The likelihood (or chance) to observe data d , denoted as $l[d | (\mu_M, \sigma_M)]$, is obtained and expressed as:

$$l[d | (\mu_M, \sigma_M)] = \frac{1}{d^\xi \sqrt{2\pi}} \exp \left\{ -\frac{[\ln(d) - \lambda]^2}{2\xi^2} \right\} \quad (4)$$

where λ and ξ are the mean and standard deviation of $\ln[M \cdot g(\boldsymbol{\theta})]$, respectively, and estimated as:

$$\lambda = \ln \left[\frac{\mu_{[M \cdot g(\boldsymbol{\theta})]}}{(1 + \delta_{[M \cdot g(\boldsymbol{\theta})]}^2)^{0.5}} \right] \quad (5)$$

$$\xi = \sqrt{\ln(1 + \delta_{[M \cdot g(\boldsymbol{\theta})]}^2)} \quad (6)$$

where $\delta_{[M \cdot g(\boldsymbol{\theta})]}$ represents the coefficient of variation (COV) of the adjusted model prediction $M \cdot g(\boldsymbol{\theta})$, which is estimated from Eqs. (2-3).

$$\delta_{[M \cdot g(\boldsymbol{\theta})]} = \frac{\sigma_{[M \cdot g(\boldsymbol{\theta})]}}{\mu_{[M \cdot g(\boldsymbol{\theta})]}} \quad (7)$$

In most cases, multiple observed data such as $\mathbf{d} = [d_1, d_2, \dots, d_n]$ are available, where n is the number of observations. The likelihood of observed data $\mathbf{d}$, denoted as $L[\mathbf{d} | (\mu_M, \sigma_M)]$, can be written as:

$$L[\mathbf{d} | (\mu_M, \sigma_M)] = \prod_{i=1}^{i=n} l[d_i | (\mu_M, \sigma_M)] = \prod_{i=1}^{i=n} \frac{1}{d_i \xi_i \sqrt{2\pi}} \exp \left\{ -\frac{[\ln(d_i) - \lambda_i]^2}{2\xi_i^2} \right\} \quad (8)$$

where d_i represents the i^{th} observed data; and, λ_i and ξ_i are the mean and standard deviation of $\ln[M \cdot g(\boldsymbol{\theta}_i)]$, respectively. In reference to the likelihood function in Eq. (8), the log-likelihood function of observed data $\mathbf{d}$ is:

$$\ln \{L[\mathbf{d} | (\mu_M, \sigma_M)]\} = \sum_{i=1}^{i=n} \ln \{l[d_i | (\mu_M, \sigma_M)]\} = \sum_{i=1}^{i=n} \left\{ -\ln [d_i \xi_i \sqrt{2\pi}] - \frac{1}{2} \left[\frac{\ln(d_i) - \lambda_i}{\xi_i} \right]^2 \right\} \quad (9)$$

With MLE, the statistics of the model bias factor M , in terms of μ_M and σ_M , are readily obtained by maximizing the log-likelihood function of observed data $\mathbf{d}$ in Eq. (9). An improved model prediction, which yields the observed data $\mathbf{d}$ in a probabilistic manner, could then be achieved. It is noted that the likelihood function of the observed data $l[d | (\mu_M, \sigma_M)]$, shown in Eq. (4), is only available when the observed data d is a fixed value. In situations where the observed data d is a range bounded by an upper bound d_{up} and a lower bound d_{low} , the likelihood function of the observed performance $l[d | (\mu_M, \sigma_M)]$, in Eq. (4), should be modified as follows:

$$l[d | (\mu_M, \sigma_M)] = \Phi \left(\frac{\ln(d_{\text{up}}) - \lambda}{\xi} \right) - \Phi \left(\frac{\ln(d_{\text{low}}) - \lambda}{\xi} \right) \quad (10)$$

where $\Phi(\cdot)$ represents the cumulative distribution function (CDF) of the standard normal variable. The likelihood function of observed data $\mathbf{d}$ and the log-likelihood function of observed data $\mathbf{d}$, shown in Eq. (8) and (9), respectively, are then modified accordingly.

2.2 Bayesian method for calibrating the statistics of the model bias factor

The limited availability of observed data $\mathbf{d}$ (i.e., the number of observations is limited) prevents an accurate determination of the statistics of the model bias factor M , in terms of μ_M and σ_M ; thus, the uncertainty of the model bias factor statistics must be calibrated. The Bayesian method (Zhang et al. 2009; Wang et al. 2013a; Juang et al. 2013c) is summarized herein for calibrating the uncertainty of the model bias factor statistics. Within the framework of that method, the posterior probability density function (PDF) of the model bias factor statistics, denoted as $f''(\mu_M, \sigma_M)$, can be evaluated as:

$$f''(\mu_M, \sigma_M) = k f'(\mu_M, \sigma_M) L[\mathbf{d} | (\mu_M, \sigma_M)] \quad (11)$$

where k is a normalization constant to make the derived posterior PDF valid; $f'(\mu_M, \sigma_M)$ is the prior PDF of the model bias factor statistics, which might be estimated empirically; and, $L[\mathbf{d} | (\mu_M, \sigma_M)]$ is the likelihood of observed data $\mathbf{d}$. Several approximation methods such as Markov Chain Monte Carlo (MCMC) simulation (Juang et al. 2013c) and system identification method (Zhang et al. 2010) have been used to cope with the difficulty in deriving the closed-form solution in Eq. (11). Based on the derived posterior PDF of the model bias factor statistics, the statistics of the model bias factor statistics, in terms of μ_{μ_M} , σ_{μ_M} , μ_{σ_M} and σ_{σ_M} , can easily be determined. The statistics of the model bias factor M , in terms of μ_M and σ_M , are then obtained and used for adjusting the model prediction.

$$\mu_M \approx \mu_{\mu_M} \quad (12)$$

$$\sigma_M \approx \sqrt{\sigma_{\mu_M}^2 + \mu_{\sigma_M}^2 + \sigma_{\sigma_M}^2} \quad (13)$$

3. Geotechnical Model Calibration Considering Fidelity and Robustness

The conventional probabilistic approaches focus on model fidelity, which is measured with the likelihood of observed data $\mathbf{d}$, $L[\mathbf{d} | (\mu_M, \sigma_M)]$. Note that this likelihood function is indeed a probability function. If the mean of the model prediction $g(\theta)$ differs significantly from the observed data d , the variability of the model bias factor M would be relatively large, which can lead to a significant variation in the adjusted model prediction $M \cdot g(\theta)$. The wide distribution of the adjusted model prediction poses a great challenge in a geotechnical design. To overcome this issue, model robustness should be considered along with model fidelity in the model calibration.

To illustrate the concept of model fidelity and model robustness, let's look at predictions of two calibrated models, $M_1 \cdot g(\theta)$ and $M_2 \cdot g(\theta)$, shown in Figure 3. Because of the uncertainty in the model bias factors, the adjusted model predictions are probability distributions, rather than fixed values. Note that the model fidelity can be interpreted as the likelihood of the observed data falling within the certain confidence interval of the model prediction. Thus, a higher degree of fidelity is evident in $M_2 \cdot g(\theta)$ than $M_1 \cdot g(\theta)$, reflected by the wider distribution of $M_2 \cdot g(\theta)$. The model robustness, on the other hand, is a measure of the level of variation of the model prediction. A higher degree of model robustness is evident in $M_1 \cdot g(\theta)$ than $M_2 \cdot g(\theta)$, reflected by the narrower distribution of $M_1 \cdot g(\theta)$. A model with a higher level of robustness is more usable than the one with a lower level of robustness, particularly in matters of design. However, raising the level of robustness to an extreme at the expense of model fidelity may be a poor option, as the likelihood of the observed data falling within the prescribed confidence interval might

decrease drastically, which is not desirable. Thus, a sensible criterion in the model calibration involves balancing the desire of maximizing the model fidelity with that of maximizing the model robustness.

3.1 Measure of model robustness

Inspired by the concepts of signal-to-noise ratio (SNR) (Park et al. 2006) and the residual sum of squares (RSS) (Geladi and Kowalski 1986), the authors propose the following index, denoted as $R[\mathbf{d} | (\mu_M, \sigma_M)]$, to measure model robustness:

$$R[\mathbf{d} | (\mu_M, \sigma_M)] = \sum_{i=1}^{i=n} r[d_i | (\mu_M, \sigma_M)] \quad (14)$$

where $r[d_i | (\mu_M, \sigma_M)]$ is the robustness index of the solution model attributed to the i^{th} observation d_i , which is defined as follows.

$$\begin{aligned} r[d_i | (\mu_M, \sigma_M)] &= E\{[d_i - M \cdot g(\boldsymbol{\theta}_i)]^2\} \\ &= E\{d_i^2 + [M \cdot g(\boldsymbol{\theta}_i)]^2 - 2d_i \cdot [M \cdot g(\boldsymbol{\theta}_i)]\} \\ &= E(d_i^2) + E[M \cdot g(\boldsymbol{\theta}_i)]^2 + V[M \cdot g(\boldsymbol{\theta}_i)] - 2E(d_i)E[M \cdot g(\boldsymbol{\theta}_i)] \end{aligned} \quad (15)$$

Here $E[M \cdot g(\boldsymbol{\theta}_i)]$ and $V[M \cdot g(\boldsymbol{\theta}_i)]$ are the mean and variance, respectively, of the adjusted model prediction $M \cdot g(\boldsymbol{\theta}_i)$ attributed to the i^{th} observation d_i , which are readily estimated with Eqs. (2-3).

If the observed data d_i is a fixed value, $E(d_i) = d_i$ and $E(d_i^2) = d_i^2$; and, if the observed data d_i is a range bounded by an upper bound $d_{i,\text{up}}$ and a lower bound $d_{i,\text{low}}$, the “two-sigma” rule may be used for approximating $E(d_i)$ and $E(d_i^2)$ in Eq. (15).

$$E(d_i) \approx \frac{d_{i,\text{up}} + d_{i,\text{low}}}{2} \quad (16)$$

$$E(d_i^2) \approx \left(\frac{d_{i,\text{up}} + d_{i,\text{low}}}{2} \right)^2 + \left(\frac{d_{i,\text{up}} - d_{i,\text{low}}}{4} \right)^2 \quad (17)$$

Note that the robustness index $R[\mathbf{d}|(\mu_M, \sigma_M)]$ in Eqs. (14-15) can be interpreted as the expected variance between model predictions $[M \cdot g(\boldsymbol{\theta}_1), M \cdot g(\boldsymbol{\theta}_2), \dots, M \cdot g(\boldsymbol{\theta}_n)]$ and observed data $\mathbf{d} = [d_1, d_2, \dots, d_n]$. Further, the model robustness measure $R[\mathbf{d}|(\mu_M, \sigma_M)]$ in Eqs. (14-15) is also consistent with the info-gap robustness concept (Ben-Haim 2006). Here, a smaller value of the robustness index, $R[\mathbf{d}|(\mu_M, \sigma_M)]$, implies a lower variation of the model prediction and thus a higher model robustness.

3.2 Multi-objective optimization-based approach for calibrating geotechnical models

For simplicity and consistency with the conventional probabilistic approaches, the negative log-likelihood function of observed data $\mathbf{d}$, $-\ln\{L[\mathbf{d}|(\mu_M, \sigma_M)]\}$, is defined herein as the model fidelity index. A smaller value of the model fidelity index, $-\ln\{L[\mathbf{d}|(\mu_M, \sigma_M)]\}$, indicates a higher fidelity. Thus, in the measures of both the robustness index and fidelity index, “*smaller is better*”.

Note that fidelity index, $-\ln\{L[\mathbf{d}|(\mu_M, \sigma_M)]\}$, and robustness index, $R[\mathbf{d}|(\mu_M, \sigma_M)]$, are both constructed herein as functions of the statistics of the model bias factor M , in terms of μ_M and σ_M . The model calibration is reduced to the task of seeking optimal statistics of the model bias factor such that model fidelity and model robustness are maximized simultaneously (i.e., to minimize the fidelity index and the robustness index simultaneously). Hence, the model calibration is implemented as a multi-objective optimization problem, in which the optimization parameters are the model bias factor statistics and the objectives are model fidelity and model robustness. Symbolically, the multi-objective optimization-based geotechnical model calibration is set up as:

$$\begin{aligned}
&\text{Find:} && \mu_M \text{ and } \sigma_M \\
&\text{Subject to:} && \mu_M > 0 \text{ and } \sigma_M > 0.01 \\
&\text{Objectives:} && \text{Minimize fidelity index, } -\ln\{L[\mathbf{d} | (\mu_M, \sigma_M)]\} \\
&&& \text{Minimize robustness index, } R[\mathbf{d} | (\mu_M, \sigma_M)]
\end{aligned} \tag{18}$$

It should be mentioned that while, in principle, the constraint of $\sigma_M \geq 0$ should be adopted in Eq. (18), a constraint of $\sigma_M > 0.01$ is used here to avoid the numerical difficulty of processing the fidelity index.

As the objectives with respect to model fidelity and model robustness conflict with each other, the above multi-objective optimization will not yield a single best solution, but rather a set of non-dominated optimal solutions that collectively form a Pareto front showing a tradeoff between the conflicting objectives (Deb et al. 2002; Chen et al. 1999). The authors solved the multi-objective optimization, formulated in Eq. (18), using the non-dominated sorting genetic algorithm version II (NSGA-II) (e.g., Deb et al. 2002; Wang et al. 2013b), a proven fast optimization algorithm. Alternatively, this multi-objective optimization can be solved through a series of single-objective optimizations (e.g., Chen et al. 1999).

The Pareto front is seen as a tool to aid in making an informed decision. For example, either the model of the highest fidelity that is above a target level of robustness or the model of the highest robustness that is above a target level of fidelity can be selected as the most preferred model. However, the determination of an appropriate target level of robustness (or fidelity) may not be trivial. Thus, the knee point concept is applied, which yields the best compromise between model fidelity and model robustness. The readers are referred to Branke et al. (2004), Deb and Gupta (2011) and Gong et al. (2015) for detailed procedures for identifying the knee point on the Pareto front.

4. Example Application: Liquefaction-Induced Settlement Prediction

In this section, the proposed model calibration approach is applied to the problems of predicting liquefaction-induced settlement. The subject of liquefaction-induced settlement has been investigated by many researchers and many empirical solution models have been developed (e.g., Ishihara and Yoshimine 1992; Zhang et al. 2002; Wu and Seed 2004; Robertson 2009a&b; Dashti et al. 2010). These empirical models all experienced significant model bias or uncertainty. Juang et al. (2013b) applied the principle of maximum likelihood (fidelity concept) to calibrate the uncertainty of the Zhang et al. (2002) model. Based on the results of this model calibration, they derived a probabilistic model for evaluating the liquefaction-induced settlement. However, the coefficients of variation (COV) of the model bias factor M in these probabilistic models are 0.3175 and 0.7839 for predicting the liquefaction-induced free-field settlement (type A) and building settlement (type B), respectively. Consequently, the large variation in the model bias factor leads to a large variation in the model prediction, which makes it difficult to use by a practicing engineer in a design situation. Here, the model calibration conducted by Juang et al. (2013b) based solely on the fidelity requirement is repeated using the proposed approach, in which both model fidelity and model robustness are considered and optimized.

To begin with, the formulation shown in Eq. (1) is modified as follows for predicting liquefaction-induced settlement.

$$S_a = M \cdot S_p \quad (19)$$

where S_a represents the observed liquefaction-induced settlement; S_p represents the liquefaction-induced settlement prediction from the empirical deterministic model; and M represents the model bias factor, the statistics of which, in terms of μ_M and σ_M , are to be determined.

4.1 Database of case histories

For illustration purposes, the case histories listed in Table 1 were used, in which 64 case histories of liquefaction-induced settlements are available. This data set consists of 32 type A (i.e., free-field settlement) case histories and 32 type B (i.e., building settlement) case histories. For each case history, the following information is readily available: (1) the ground water table G_w , the maximum ground surface acceleration $a_{\max}$ and the moment magnitude M_w ; (2) the observed liquefaction-induced settlement S_a , which can be either a fixed value or a range; and (3) the mean and variance of the predicted liquefaction-induced settlement S_p , in terms of μ_{sp} and σ_{sp}^2 . Note that the mean and variance of the predicted settlement S_p are estimated using the CPT-based model developed by Zhang et al. (2002) and Robertson (2009a&b). The readers are referred to Juang et al. (2013b) for a detailed derivation of the mean and variance of the settlement prediction.

4.2 Type A: Liquefaction-induced free-field settlement

Within the proposed approach, the calibration of the solution model, or the determination of the statistics of the model bias factor M in Eq. (19), is constructed as a multi-objective optimization problem. With the aid of nondominated sorting genetic algorithm version II (NSGA-II) (Deb et al. 2002), the statistics of the model bias factor M in Eq. (19) can readily be determined using the optimization setting in Eq. (18). Here, the population size and the generation number are set at 100 and 20,000, respectively, in NSGA-II. This parameter setting is obtained through a series of trial and error exercises, which yields a converged Pareto front.

In this section, only case histories of type A are analyzed, in which the model calibration is focused on the liquefaction-induced free-field settlement (type A). Figure 4 depicts the derived Pareto front with the two objectives: fidelity index, $-\ln\{L[\mathbf{d}|\mu_M, \sigma_M]\}$, and robustness index, $R[\mathbf{d}|\mu_M, \sigma_M]$. As can be seen from the results in Figure 4, there is a tradeoff between

maximizing the model fidelity (or minimizing the fidelity index) and maximizing the model robustness (or minimizing the robustness index). Thus, a Pareto front is obtained and a knee point is identified (Figure 4). The knee point offers the best compromised solution (i.e., the most preferred model) for predicting liquefaction-induced free-field settlement. On the left side of the knee point, a slight reduction in the robustness index can drastically increase the fidelity index, which is not desirable; on the right side of the knee point, a slight reduction in the fidelity index can yield a huge increase in the robustness index, which is also not desirable. The statistics of the model bias factor of this best compromise solution, in terms of the knee point on the Pareto front, are: $\mu_M = 1.023$ and $\sigma_M = 0.104$.

For the convenience of discussion, the model with the highest fidelity (or lowest fidelity index) among all designs on the Pareto front shown in Figure 4 is known as Model 1, the model with the highest robustness is known as Model 3, and the knee point (with the best compromise between fidelity and robustness) is known as Model 2. The statistics of the model bias factor for these models calibrated through the multi-objective optimization (Eq. 18) are listed in Table 2, along with those obtained from the traditional probabilistic approaches such as the maximum likelihood estimate (MLE) and MCMC-based Bayesian method. Two findings are derived based on the results shown in Table 2. First, as the traditional model calibration focuses only on fidelity, the derived statistics are, as expected, identical to those of Model 1 (highest fidelity) obtained by the proposed model calibration approach. Second, among the three multi-objective optimization-based models, Model 1 has the highest σ_M value and the highest likelihood [i.e., $\ln(L)$] value, Model 3 has the smallest σ_M value and the smallest likelihood value, while the σ_M value and the likelihood value for Model 2 are in between. These values are consistent with the trend revealed by the robustness index (R) and the fidelity index [$\ln(L)$] of the three models.

The three multi-objective optimization-based models are further compared in Figure 5, where the predicted settlements versus the observed settlements for all cases (type A) are plotted for the respective three models. In these plots, if the observed settlement is a range instead of a fixed value, the mean is plotted; and for the predicted settlement, which is a lognormal random variable (Juang et al. 2013b), both the mean and the 95% confidence interval are plotted.

In Figure 5, if the 1:1 line is within the 95% confidence interval, it means the model prediction is consistent with the observed data and hence the model is of high fidelity. On the other hand, the range of the confidence interval, which shows the variability in the model prediction, measures the model robustness. In Figure 5(a), the 1:1 line always falls within the 95% confidence interval, which is desirable from the traditional view of probabilistic calibration that focuses solely on fidelity. However, in this case the 95% confidence interval could be too wide for use in a design situation, as it would be difficult to make a design decision when the predicted settlement varies in a wide range. Thus, Model 1 is accurate (with high fidelity) but not precise (low robustness). In fact, from a user's perspective, Model 1 is the least robust among all models on the Pareto front, as it yields the largest variation in the predicted settlement. On the other hand, Model 3, the performance of which is shown in Figure 5(c), is precise but less accurate, as the 95% confidence interval is much narrower but does not always bracket the 1:1 line. This model has the highest robustness (i.e., least variation in the predicted settlement) but is not as accurate as Model 1. Figure 5(b) shows the performance of Model 2, the knee point on the Pareto front, which is the best compromise with respect to both fidelity and robustness.

4.3 Type B: Liquefaction-induced building settlement

In this section, only case histories of type B are studied, in which the model calibration is focused on the liquefaction-induced building settlement (type B).

Figure 6 shows the derived Pareto front and knee point with respect to the two objectives: fidelity index $-\ln\{L[\mathbf{d}](\mu_M, \sigma_M)\}$ and robustness index $R[\mathbf{d}](\mu_M, \sigma_M)$. Similar to that shown in Figure 4, there is a tradeoff between model fidelity and model robustness. The best compromise model is the knee point of the Pareto front, which yields the following statistics for the model bias factor: $\mu_M = 1.023$ and $\sigma_M = 0.476$. As a reference, the statistics of the model bias factor determined based on the maximum likelihood analysis of the same data set are: $\mu_M = 1.658$ and $\sigma_M = 1.300$. Similarly, three models are identified on the Pareto front: Model 4 (highest fidelity), Model 5 (knee point), and Model 6 (highest robustness). The statistics of the model bias factor for these models calibrated through the multi-objective optimization (Eq. 18) are listed in Table 3, along with those obtained from the traditional probabilistic calibration approaches such as maximum likelihood estimate (MLE) and MCMC-based Bayesian method. The findings or implications of the results shown in Table 3 are the same as those shown in Table 2.

The performance of the three multi-objective optimization-based models, Model 4 (highest fidelity), Model 5 (knee point), and Model 6 (highest robustness) is illustrated in Figure 7. For each model, if the observed liquefaction-induced building settlement is a range instead of a fixed value, the mean is plotted; and for the predicted settlement, which is a lognormal random variable, both the mean and the 95% confidence interval are plotted. The predicting performance of Model 4, Model 5, and Model 6, are illustrated in Figures 7(a), 7(b), and 7(c) respectively, with findings similar to that in Figure 5. The model with the highest fidelity, Model 4, is accurate but not precise. From a user's perspective, this model is the least robust, as it yields the largest variation in the predicted settlement. The model with the highest robustness, Model 6, is precise (with the least variation in the predicted settlement) but not accurate. Model 5 is the best compromise with respect to both fidelity and robustness.

The statistics of the model bias factor derived based on free-field settlement (Type A) and building settlement (Type B) are different; a much larger variation (in terms of σ_M) is observed for Type B model ($\sigma_M = 0.476$) than for Type A model ($\sigma_M = 0.104$). The implication is that the uncertainty in the predicted liquefaction-induced building settlement is much greater than that in the free-field settlement prediction, which agrees well with the findings reported in Dashti et al. (2010) and Juang et al. (2013b). The larger variability in the derived model for liquefaction-induced building settlement may be attributed to the larger uncertainty in the field observations and the difficulty in characterizing the behaviors of soil, structure, and soil-structure interaction during a seismic event.

5. Further Discussions

The significance of the proposed model calibration approach considering both model fidelity and model robustness, as opposed to the traditional fidelity-based calibration approach, is further illustrated by applying the derived models to a couple of cases of liquefaction-induced settlement predictions.

5.1 Case 1: M4 (Marina District site)

The first case history concerns a free-field case designated as “M4” in the database (Table 1) at the Marina District site that is located on the north side of San Francisco, California, and which suffered significant ground settlements during the 1989 Loma Prieta earthquake (Zhang et al. 2002). The ground water table and seismic parameters are listed in Table 1 and the CPT sounding profiles at this site can be found in Juang et al. (2013b). Following the procedures

in Juang et al. (2013b), the mean and variance of the settlement prediction are calculated as: μ_{Sp}
 $= 10.75 \text{ cm}$ and $\sigma_{Sp}^2 = 0.35 \text{ cm}^2$.

For a future case such as M4, the model bias factor M can be used to adjust the settlement prediction as per Eq. (19). Based on the statistics of the model bias factor M (in terms of μ_M and σ_M) that has been calibrated with the proposed approach, the mean and standard deviation of the adjusted settlement prediction are calculated as per Eqs. (2-3). The probability distribution of the predicted settlement is then obtained with the assumption that the settlement is lognormally distributed. Plotted in Figure 8(a) are the probability distributions of the predicted liquefaction-induced free-field settlements obtained with Model 1 (highest fidelity), Model 2 (knee point; the best compromise between model fidelity and model robustness), and Model 3 (highest robustness); also plotted in Figure 8(a) is the observed settlement (9.6 cm).

As can be seen in Figure 8(a), the observed settlement falls in the range of the settlement predicted by each and every of the three models. Model 1 is seen to have the highest likelihood to be “accurate” in predicting the observed settlement (reflected by the widest distribution of the settlement prediction), while Model 3 is seen to have the lowest likelihood to be “accurate” in predicting the observed settlement (reflected by the narrowest distribution of the settlement prediction). However, from the perspective of model robustness, Model 3 is seen to have the highest robustness (least variation) in its prediction, while Model 1 has the lowest robustness in its prediction. The answer to the question of “Which is the better model” certainly depends on what is being emphasized (e.g., fidelity or robustness). The traditional probabilistic model calibration emphasizes fidelity, and with the principle of maximum likelihood, the calibration process yields Model 1. However, from a user’s perspective and in a design situation, Model 1 would be more difficult to apply, as its predicted settlement varies in a wide range; whereas,

Model 2 is seen as a model that provides a sensible balance between accuracy (fidelity) and precision (robustness).

5.2 Case 2: TK-J2 (site J at Adapazari)

The second case history concerns a case designated as “TK-J2” in the database (Table 1) at Site J of Adapazari city, which suffered widespread ground failure and building damage in the 1999 Kocaeli earthquake (Sancio 2003). The ground water table and seismic parameters are listed in Table 1 and the CPT sounding profiles at this site can be found in Juang et al. (2013b). Following the procedures in Juang et al. (2013b), the mean and variance of the settlement prediction are calculated as: $\mu_{sp} = 19.20$ cm and $\sigma_{sp}^2 = 0.01$ cm².

Let’s assume that TK-J2 is a future case and the above settlement prediction is adjusted by the model bias M as per Eq. (19). Based on the calibrated statistics of the model bias factor M (in terms of μ_M and σ_M), the mean and standard deviation of the adjusted settlement prediction are calculated as per Eqs. (2-3); then, the probability distribution of the predicted settlement is obtained with the assumption that the settlement is lognormally distributed. Plotted in Figure 8(b) are the probability distributions of the predicted liquefaction-induced building settlements obtained with Model 4 (highest fidelity), Model 5 (knee point; the best compromise between model fidelity and model robustness), and Model 6 (highest robustness); also plotted in Figure 8(b) is the observed settlement (26 cm). Similar observation as in Figure 8(a) can be found in Figure 8(b). Model 5 is seen to offer a compromise between Model 4 that emphasizes fidelity and Model 6 that emphasizes robustness.

6. Concluding Remarks

The traditional probabilistic model calibration approaches, such as the maximum likelihood estimate (MLE) and Bayesian method, focus only on the fidelity. Although these approaches can produce a model with the highest likelihood (or fidelity) with respect to the given set of observed data, the practical applicability of the derived model, especial in the design situation, may be limited, as the confidence interval of the model prediction is often too wide. This is evidenced in the model calibration study of the liquefaction-induced settlements in both free-field condition and under the influence of buildings presented in this paper. Thus, model robustness should be considered along with model fidelity in the model calibration.

The proposed model calibration approach that considers both model fidelity and model robustness in a multi-objective optimization provides a tradeoff relationship and selection of a sensible solution model that balances model fidelity and model robustness. Based on the results presented, the variation of the model prediction can be greatly reduced by enforcing an optimal degree of model robustness. The developed robustness index is proven effective, with which the variation of the model prediction can be explicitly considered in the model calibration.

The proposed optimization-based model calibration approach that explicitly considers both model robustness and model fidelity is quite refreshing. Though the proposed approach is only demonstrated for the problems of liquefaction-induced settlement prediction, the formulation is set up in a way that is suitable for other geotechnical problems. Further studies to confirm the general applicability of the proposed approach are warranted.

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Table 1. Summary of case histories for post-liquefaction settlement (after Juang et al. 2013b)

CPT designation	G_w (m)	$a_{\max}$ (g)	M_w	S_a (cm)	μ_{sp} (cm)	σ_{sp}^2 (cm ²)	Type*	Reference	Earthquake and Site
C2	2.3	0.24	7.0	9.6-10.7	10.82	0.64	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
C8	2.7	0.16	7.0	1.9	1.31	0.29	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
C9	2.6	0.16	7.0	0-3.4	0.02	0	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
C12	2.3	0.24	7.0	7.0-10.7	6.83	0.73	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
M1	2.3	0.16	7.0	0-3.4	4.19	0.29	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
M2	2.7	0.16	7.0	0-3.4	0.94	0.09	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
M3	2.7	0.16	7.0	1.1	0.6	0.06	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
M4	2.4	0.24	7.0	9.6	10.75	0.35	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
M6	5.5	0.12	7.0	0-1.6	0.04	0	A	Zhang et al. (2002)	1989 Loma Prieta Marina District
C28	2	0.16	7.0	5-10	10.42	0.5	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C32	2	0.16	7.0	5-10	9.6	0.4	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C33	2	0.16	7.0	5-10	6.82	0.38	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C34	2	0.16	7.0	5-10	5.48	0.32	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C35	2	0.16	7.0	5-10	5.66	0.28	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C37	2	0.16	7.0	5-10	5.78	0.3	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C42	2	0.16	7.0	5-10	10.28	0.53	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone one
C29A	2	0.16	7.0	10-15	9.26	0.48	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone two

C30	2	0.16	7.0	10-15	14.59	0.72	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone two
C31	2	0.16	7.0	10-15	21.82	0.92	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone two
C39	2	0.16	7.0	10-15	15.56	0.63	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone two
UM10	2	0.16	7.0	10-15	18.35	0.64	A	Zhang et al. (2002)	1989 Loma Prieta Treasure Island Zone two
TK-A1	0.7	0.40	7.4	10-15	8.01	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site A
TK-A2	0.7	0.40	7.4	10-15	6.29	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site A
TK-A5	0.7	0.40	7.4	30-35	8.21	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site A
TK-E1	0.34	0.40	7.4	20-25	14.91	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site E
TK-E2	0.34	0.40	7.4	5-6	1.24	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site E
TK-E3	0.34	0.40	7.4	20-25	9.13	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site E
TK-E4	0.34	0.40	7.4	5-6	8.87	0.01	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site E
TK-G2	0.41	0.40	7.4	10	10.33	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site G
TK-H1	1.72	0.40	7.4	32-33	8.29	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site H
TK-H2	1.72	0.40	7.4	32-33	11	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site H
TK-H3	1.72	0.40	7.4	32-33	9.75	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site H
TK-I2	0.71	0.40	7.4	17	9.55	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site I

TK-I4	0.71	0.40	7.4	12	6.71	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site I
TK-J1	0.41	0.40	7.4	26	19	0.01	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site J
TK-J2	0.41	0.40	7.4	26	19.2	0.01	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site J
TK-J3	0.41	0.40	7.4	21	12.63	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site J
TK-J4	0.41	0.40	7.4	21	14.07	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site J
TK-L1	0.68	0.40	7.4	25	9.22	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site L
TK-L2	0.68	0.40	7.4	25	9.38	0	B	Sancio (2003); PEER (2000)	1999 Kocaeli Adapazari Site L
TW-WF-C7	3.2	0.79	7.6	20	12.51	0.01	A	Chang (2001); NCREE (2000)	1999 Chi-Chi Wufeng
TW-WF-C9	1.38	0.79	7.6	14-28	15.03	0	A	Chang (2001); NCREE (2000)	1999 Chi-Chi Wufeng
TW-WF-C10	1.4	0.79	7.6	15	14.13	0	A	Chang (2001); NCREE (2000)	1999 Chi-Chi Wufeng
TW-WF-C15	2	0.79	7.6	15-16	11.16	0.01	A	Chang (2001); NCREE (2000)	1999 Chi-Chi Wufeng
TW-NT-C7	3.6	0.43	7.6	15-25	21.35	0.07	B	NCREE (2000)	1999 Chi-Chi Nantou
TW-NT-C15	3.4	0.43	7.6	12-35	19.38	0.04	B	NCREE (2000)	1999 Chi-Chi Nantou
TW-YL-C6	1.78	0.19	7.6	10	26.19	0.28	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C12	1.58	0.19	7.6	8-13	13.72	0.23	A	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C17	0.6	0.19	7.6	5-40	24.74	0.23	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C18	1.78	0.19	7.6	10	14.53	0.23	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin

TW-YL-C19	0.8	0.19	7.6	5-15	26.59	0.27	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C20	1.4	0.19	7.6	20	21.76	0.15	A	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C22	0.71	0.19	7.6	5-30	28.43	0.36	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C23	1.3	0.19	7.6	10-20	14.77	0.22	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C24	2	0.19	7.6	5-35	25.5	0.54	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C25	2.3	0.19	7.6	10-20	19	0.4	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C26	2.3	0.19	7.6	5-13	22.22	0.35	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C32	0.7	0.19	7.6	10-20	22.56	0.16	A	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C41	1.4	0.19	7.6	5-20	19.87	0.31	B	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL-C45	2.3	0.19	7.6	15-20	20.34	0.47	A	Su et al. (2000); NCREE (2000)	1999 Chi-Chi Yuanlin
TW-YL- WA2	1	0.19	7.6	36	34.23	0.41	B	Ku et al. (2004)	1999 Chi-Chi Yuanlin
TW-LW-C1	2	0.12	7.6	36.8	21.37	1.2	A	Ku et al. (2004)	1999 Chi-Chi Lunwei
TW-LK-EQ3	1.5	0.12	7.6	43	24.59	1.33	A	Ku et al. (2004)	1999 Chi-Chi Lukang
TW-DN-D1	1.5	0.18	7.6	10	13.94	0.35	A	Chang (2000)	1999 Chi-Chi Dounan

*Type A: liquefaction induced free-field settlement; Type B: liquefaction-induced building settlement.

Table 2. Statistics of the model bias factor of the selected three models on the Pareto front and those obtained with the traditional probabilistic approaches based on Type A settlement data

Selected model/calibration method	<i>Model factor statistics</i>		<i>Calibration objective</i>	
	μ_M	σ_M	<i>Fidelity index, $-\ln(L)$</i>	<i>Robustness index, $R (\times 10^3)$</i>
Model 1 (highest model fidelity) on Pareto front	1.041	0.326	48.13	1.624
Model 2 (knee point) on Pareto front	1.023	0.104	86.34	1.086
Model 3 (highest model robustness) on Pareto front	1.024	0.037	278.86	1.032
Model obtained with MLE	1.045	0.332	–	–
Model obtained with MCMC-based Bayesian method	1.045	0.334	–	–

Table 3. Statistics of the model bias factor of the selected three models on the Pareto front and those obtained with the traditional probabilistic approaches based on Type B settlement data

Selected model/calibration method	<i>Model factor statistics</i>		<i>Calibration objective</i>	
	μ_M	σ_M	<i>Fidelity index, $-\ln(L)$</i>	<i>Robustness index, $R (\times 10^3)$</i>
Model 4 (highest model fidelity) on Pareto front	1.650	1.290	89.76	2.410
Model 5 (knee point) on Pareto front	1.023	0.476	107.45	0.666
Model 6 (highest model robustness) on Pareto front	1.014	0.173	301.26	0.479
Model obtained with MLE	1.658	1.300	—	—
Model obtained with MCMC-based Bayesian method	1.642	1.316	—	—

Figure 1. Model perditions versus observed data for cases in Table 1

Figure 2. Probability distribution of the model prediction for TK-J2

Figure 3. Conceptual illustration of the robustness concept of solution models

Figure 4. Pareto front between model fidelity and model robustness (Type A) (note: in both measures of robustness index and fidelity index, “smaller is better”)

Figure 5. Comparison between the settlement prediction from the resulting solution model and the observed settlement (Type A): (a) Model 1 (highest fidelity); (b) Model 2 (knee point); (c) Model 3 (highest robustness)

Figure 6. Pareto front between model fidelity and model robustness (Type B) (note: in both measures of robustness index and fidelity index, “smaller is better”)

Figure 7. Comparison between the settlement prediction from the resulting solution model and the observed settlement (Type B): (a) Model 4 (highest fidelity); (b) Model 5 (knee point); (c) Model 6 (highest robustness)

(a) Case 1 – M4 at the Marina District site

(b) Case 2 – TK-J2 at site J of Adapazari city

Figure 8. Predictions of liquefaction-induced settlements at two sites