A local criterion for ductile fracture under low-triaxiality axisymmetric stress states

Christopher Smith^a, Amit Kanvinde^{b*}, Gregory Deierlein^c

¹ Catastrophe Engineering & Analytics, Berkshire Hathaway Specialty Insurance, Boston, MA, 02110, USA

² Department of Civil and Environmental Engineering, University of California at Davis; Davis, CA 95616, USA

³ Department of Civil and Environmental Engineering, Stanford University; Stanford, CA 94305, USA

*Email: kanvinde@ucdavis.edu; Tel:530-752-2605

ABSTRACT

A local criterion is presented for ductile fracture of metals under a wide range of stress triaxiality. In contrast to previous criteria, whose accuracy is generally limited to the narrower stress ranges over which they were originally validated, the proposed criterion is rigorously supported through (1) hypothesis testing against 352 experiments conducted during previous studies of high- and low-triaxiality fracture, and (2) micromechanical models of the underlying fracture process. The proposed criterion implies a low-triaxiality fracture cutoff at a stress triaxiality of approximately zero, which is supported by experimental test data.

Keywords:

Ductile fracture; Low triaxiality; Steel; Probabilistic fracture mechanics

1. Introduction

Various models and criteria for predicting ductile fracture initiation are available to evaluate the fracture resistance of metal structures. In contrast to conventional far-field fracture parameters, such as the J-integral or the stress intensity factor K, that are evaluated at some distance away from the crack tip, local models [1] predict fracture in terms of the continuum stress and strain at the point of fracture initiation. Local models are especially useful in situations where large-scale yielding prevents the application of traditional far-field measures, as traditional fracture criteria are limited by assumptions of small scale yielding, monotonic loading, or the presumption of a sharp initial crack [2]. Local fracture criteria are often supported by an understanding of the underlying fracture mechanism and its relationship to the material microstructure. Hence, they are often termed micromechanical models.

There is strong evidence that the formation of ductile cracks in polycrystalline metals (e.g., steel) depends strongly on the accumulated plastic deformation, represented by the equivalent plastic strain ($\overline{\epsilon}_P$), and on the degree of hydrostatic confinement. The hydrostatic confinement is typically represented by the dimensionless stress triaxiality, defined as the ratio between the mean (hydrostatic) stress and the von Mises (deviatoric, or equivalent) stress ($T = \frac{\sigma_m}{\sigma_{VM}}$).

A significant distinction has been noted [3,4] between the regimes of high and low stress triaxiality, in terms of both the micromechanical fracture mechanism and the empirical measurements of fracture toughness. High values of triaxiality (i.e., $T > 1.5$) are found in highly constrained regions such as at crack tips, or at the center of necking or notched tension specimens. Low triaxiality conditions are often found at surfaces and protruding corners, where the principal shear stress is high relative to the tensile hydrostatic pressure. Dating back to the 1940's, Bridgman [5] conducted landmark experiments to demonstrate that low triaxiality strongly inhibits fracture initiation. Subsequent studies investigated the effect of triaxiality in more detail, through analytical models of the growth of inter-granular voids [6,7] and tension tests of pre-notched round bars [8,9]. These studies have found strong evidence that under high triaxiality conditions ($T > 1.5$), the local strain at fracture is related to the triaxiality by a decreasing exponential function (e.g., $\overline{\epsilon}_{P_{crit}} = \alpha * e^{-B*T}$), where the exponent coefficient B is in

the range of 1.0 to 2.0 [8,10,11].

More recently, researchers in various engineering disciplines have applied this exponential functional form to conditions outside of those for which it was originally developed. These conditions include cyclic loading typical of earthquakes [2], non-axisymmetric deviatoric stress states [12], and stress states with low ($T < 1.0$) triaxiality [13]. These efforts have achieved mixed levels of success, especially in the prediction of fracture under low triaxiality conditions.

When loaded under high hydrostatic compression, steel samples have been observed to withstand extraordinarily high plastic strains (logarithmic strains over 3.0) without fracturing. This effect was demonstrated by Bridgman [5] in high-pressure test chambers, and Kudo and Aoi [14] in compressive “upsetting” tests (tests of compressive cylinders where fracture occurs after specimen bulging). Building on this evidence, recent research has sought to define a low-triaxiality fracture cutoff below which fracture is very unlikely or practically impossible[4]. Quantifying this type of a cutoff is especially important when low triaxiality stress states are of interest.

This paper proposes an improved local criterion for ductile fracture in steel under both high-triaxiality and low-triaxiality conditions. The criterion is supported by objective maximum-likelihood based hypothesis testing and data from 352 experiments reported in two independent previously published studies [5,11]. The next section outlines the sources of data that validate the new criterion, including low-triaxiality experiments [5], high-triaxiality experiments [11], analytical void models [6], and computational void models [15]. Subsequent sections describe a probabilistic framework for the proposed fracture criterion and apply the method of maximum likelihood estimation to compare the proposed and existing models.

2. Current local fracture criteria and their application to low-triaxiality stress states

Examination of dimpled fracture surfaces has indicated that ductile fracture in metals is precipitated by the growth of microvoids (on the order of 10 μm) until the voids coalesce to form a macroscopic crack [16]. Several successful local criteria are based in simulations of the growth of these microvoids. The closely related Stress-Modified Critical Strain (SMCS) and Void Growth Model (VGM) [17] belong to a

family of “exponential” ductile fracture criteria. The specific form of both criteria are based on analytical derivations by Rice and Tracey [6], who showed that the instantaneous radial growth rate ($\frac{dR}{R}$) of a spherical void in an infinite, elastic-plastic continuum is controlled by the stress triaxiality and the rate of applied equivalent plastic strain ($d\bar{\epsilon}_p = d\epsilon_p : d\epsilon_p$). For highly triaxial ($T > 1.5$) and axisymmetric (two identical principal stresses) stress states, Rice and Tracey derived the following expression for the void growth rate (Equation 1). McClintock [7] derived similar results for cylindrical voids.

$$\frac{dR}{R} = 0.283 * e^{1.5*T} * d\bar{\epsilon}_p \quad (1)$$

If it is assumed that the stress triaxiality in Equation 1 remains relatively constant throughout the growth history of the voids, and that fracture occurs when the voids grow to a critical size, then Equation 1 may be simplified to reveal that the plastic strain at fracture is inversely related to the void growth rate. Following this, Hancock and Mackenzie [10] proposed the SMCS fracture criterion (Equation 2). The SMCS is referred to as a “critical strain” criterion because fracture is predicted when the strain exceeds a critical value. This type of criterion has been widely used and adapted [17,18] including a more general form by Johnson and Cook [8] that has been adopted in commercial finite element software [19].

$$\bar{\epsilon}_p \geq \bar{\epsilon}_{p,crit} = \alpha * e^{-1.5*T} \quad (2)$$

Despite the simplicity of the SMCS, large plastic strains are accompanied by evolution of the geometry (e.g., necking) and consequently, the evolution of the local stress state. This invalidates the constant-triaxiality assumption of the SMCS criterion [2], motivating a fracture criterion that considers the entire history of the local parameters. The Void Growth Model (VGM) is designed to account for the loading history by integrating the void growth rate over the local strain history to predict the total void growth. In this model, the void size is assumed to quantify the fracture “damage” (D). The fracture damage calculation for the VGM is shown in Equation 3, and fracture is predicted when the damage exceeds the void-growth capacity, which is considered to be a fundamental material property.

$$D = \int_{\bar{\epsilon}_p} e^{1.5*T} d\bar{\epsilon}_p \geq D_{crit} \quad (3)$$

The VGM and SMCS are both based on the same exponential relationship between the void growth rate and plastic strain rate, and were originally developed and applied to predict fracture at high triaxiality (e.g. in the vicinity of cracks and notches). Rice and Tracey [6] briefly extend their derivation of microvoid growth to low triaxiality but focus on a simplified equation valid for $T > 2.0$. Johnson and Cook [8], Rousselier [20], and Panontin [17] all validated an SMCS-type criterion using tests of round circumferentially notched tension specimens, under characteristic triaxiality levels between 1.0 and 2.0. However, the available literature does not support the extrapolation of exponential criteria to low levels of stress triaxiality (i.e., $T < 1$).

The exponential fracture criteria in Equations 2 and 3 predict rather attainable fracture strains for stress states with low and even negative triaxiality. For example, Equation 3 predicts that under pure compression ($T = -1/3$) fracture would occur at only four times the plastic strain that would initiate fracture in a standard tension coupon ($T = 0.8$). However, many experimental and analytical results suggest that ductile fracture becomes increasingly unlikely at these very low triaxiality levels.

Bridgman [5], among others [3,21], reports the results of several hundred round tension coupons in a chamber filled with high-pressure hydraulic fluid. While these coupons were pulled in “tension”, the externally applied pressure was sufficient to maintain an internal mean stress that was compressive, and thus a negative stress triaxiality. In many of these tests of conventional mild steels, the specimens with negative triaxialities withstood local logarithmic strains in excess of 3.0 (engineering strains over 2000%) without fracturing. A second experimental example is that of compressive cylinder tests, known as “upset” tests, commonly used for characterizing material properties for metal forming [22]. Upset specimens can be deformed in compression to significant plastic strain levels ($\epsilon_p > 3.0$). When fracture does occur in the upset compression tests, it only occurs after the specimens “barrel” and the resulting hoop stress increases the stress triaxiality [22,23]. Further examples of this effect are routinely noted in metal-working processes such as wire drawing, where the material can undergo very large plastic strains without fracture, since the metal-working

equipment is designed such that the stress triaxiality remains negative [24].

The inhibition of fracture at very low triaxiality has also been suggested by analytical and computational models of microvoid growth. For example, the derivations by Rice and Tracey [6] found the void growth rate to be antisymmetric about the zero triaxiality axis, i.e. plastic strain under a negative triaxiality level will cause voids to shrink at the same rate as the voids would grow if the sign of the triaxiality was inverted ($\frac{dR}{R*d\varepsilon_p}(T) = -\frac{dR}{R*d\varepsilon_p}(-T)$). This result suggests that for negative triaxiality levels the void growth and coalescence fracture mechanism is not active. Similarly, computational void cell modeling by Cooke [15] found that groups of neighboring voids, when deformed under negative triaxiality, do not grow or coalesce as observed under positive triaxiality.

Thus, an objective of this paper is to provide a fracture criterion that captures the observed ductile fracture behavior at low-triaxiality more accurately than the existing models, while maintaining the accurate fracture predictions provided by the aforementioned exponential models at high positive triaxiality. Other alternatives to the exponential models have been proposed, either based on an adaptation of the exponential equation or an entirely different functional form. For example, efforts to extend the SMCS model to much lower triaxiality levels ($T < 0.5$) have involved significant modifications. Johnson and Cook [8] found that fitting the exponential function to fracture observations in straight (un-notched) tension coupons, which fracture at moderate triaxiality levels ($T = 0.8$), requires significant changes to the 1.5 value of the exponent coefficient. Ohata and Toyoda [13] observed fracture at the free surface of sharply notched tension specimens, where the fracture strains were much lower than predicted with the SMCS model. Based on their observations, Ohata and Toyoda proposed adding a maximum “capping” strain to the model.

In addition to these adaptations of the SMCS model, several researchers have proposed fracture models based on alternative functional forms. As per the original derivations of McClintock [7] and Rice and Tracey [6], the void growth rate was shown to be proportional to a hyperbolic sine function of the stress triaxiality for positive and negative triaxiality levels. This full (hyperbolic sine) form of the equation is written in exponential form in Equation 4. The more commonly used interpretation of this

equation, drops the term with the negative exponent (see Equations 2, 3), which is a reasonable approximation for high triaxiality, but loses the triaxiality range versatility of the original hyperbolic sine functional form.

$$\frac{dR}{R} = 0.283 * d\bar{\varepsilon}_p * \sinh(1.5 * T) = 0.283 * d\bar{\varepsilon}_p * (e^{1.5*T} - e^{-1.5*T}) \quad (4)$$

The popular Gurson model [25] also relies on the hyperbolic sine to express the relationship between void growth (i.e., porosity) and stress triaxiality. In the Gurson model, porosity acts as a surrogate for fracture damage in addition to influencing the constitutive response. In an effort to better model stress regimes with lower triaxiality, Huang [26] reevaluated the analytical solutions of Rice and Tracey for the void growth by accounting for more complex displacement fields (expanding the approximation using more terms) and proposed replacing the void growth rate of Equation 1 with the piecewise expression shown in Equation 5.

$$\frac{dR}{R} = \begin{cases} 0.427 * d\varepsilon_p * e^{1.5*T} & \text{for } T \geq 1 \\ 0.427 * T^{0.25} d\varepsilon_p * e^{1.5*T} & \text{for } \frac{1}{3} < T < 1 \end{cases} \quad (5)$$

Kuwamura and Yamamoto [27] studied ductile fracture of round notched bars and tension coupons from four structural steels, and provide an alternative quadratic form, shown by Equation 6. This criterion was based partly on prismatic tension coupons, which fracture at relatively low levels of triaxiality, in contrast with the SMCS criterion, which has typically been calibrated using only notched specimens.

$$\bar{\varepsilon}_{p,frac} = C * \frac{1}{T^2} \quad (6)$$

Recently, Bao and Wierzbicki [4] also observed the disagreement between the current high-triaxiality models and test data from high pressure and upset tests, proposing another piecewise function for the fracture strain, given by Equation 7. Note that this criterion incorporates a self-described fracture “cutoff” at $T=-1/3$, below which the critical strain is assumed infinite, where fracture is not possible.

$$\bar{\varepsilon}_{P,crit} = C * \begin{cases} \frac{1}{T} & \text{for } T > \frac{1}{3} \\ \frac{1}{T + \frac{1}{3}} & \text{for } -\frac{1}{3} < T < 0 \end{cases} \quad (7)$$

The models for ductile fracture initiation described above can be separated into three groups, (1) the SMCS and VGM, which are based on a common underlying

(exponential) functional form and have found wide use in many fields, (2) models that have been adapted from the SMCS or VGM for low triaxiality, and (3) independent model forms. Each of these models were developed from a specific set of experiments or analyses, and has been validated within that narrow context. However, a unified criterion tested against these disparate data sets is not available in literature. Overcoming this limitation is the primary motivation for the proposed criterion described in the next section.

As described in the next section, the proposed model is restricted to fracture under axisymmetric tension stress states, which specifically describes the condition where two local principal stresses are identical and are less than the third principal stress. This is a special case of the three-dimensional stress state that is found (among other places) at the center of round and square tension specimens, and so has received a disproportionate level of study. Recent studies have found that the degree of axisymmetry in the local stress state, as represented by the Lode angle, has an influence on the fracture process, although much less than the influence of the stress triaxiality [28,29]. This influence of varying axisymmetry is not mentioned in the development of several of the models presented above [4,13], even though these models are based on experimental data sets which mix axisymmetric and non-axisymmetric test data. For the remainder of this paper, the proposed fracture criteria and supporting data represent fracture under exclusively axisymmetric tension conditions. A more general fracture model is reported by the authors in Smith et al. [29].

2223. Proposed criterion for ductile fracture under axisymmetric stress states

Motivated by the need for a unified fracture criterion for a large range of stress triaxiality, Equation 8 proposes the following local fracture criterion:

$$D_{crit} \leq D = \int_0^{\bar{\epsilon}_{p,crit}} (B^+ * e^{(A^+)*T} - B^- * e^{-(A^-)*T}) d\bar{\epsilon}_p \quad (8)$$

where, the fracture damage parameter D grows with plastic strain according to a stress-dependent damage rate. Similar to other void growth models, fracture initiation is predicted when the accumulated damage exceeds some critical value, denoted as the fracture damage capacity D_{crit} . The terms inside the integral are the rate form of the fracture criterion, expressed separately in Equation 9:

$$\frac{dD}{d\bar{\epsilon}_p} = (B^+ * e^{(A^+)*T} - B^- * e^{-(A^-)*T}) \quad (9)$$

This criterion differs from the previous VGM by introducing a second exponential term that reduces the damage rate as the stress triaxiality decreases. As shown in Figure 1, the implied damage rate decreases to zero as triaxiality tends to zero and becomes negative for triaxiality less than zero. Considering that fracture is predicted by the criterion when the damage exceeds some positive value, a fracture cutoff is implicit at the point where the damage rate (Equation 9) becomes negative. The constants in the equations are B , representing the magnitude of the damage rate and A , representing the influence of the stress triaxiality on the damage rate. Each set of constants is separated into a positive and negative instance (e.g. B^+ and B^-) to allow for differences in the damage process under tension and compression. Other researchers have investigated this issue of tension and compression damage to find slight differences in the appropriate constants [15,30,31], but these variations appear to be small relative to the uncertainty involved in their assessment. The separation of these constants does allow the presence and location of the fracture cutoff to be defined parametrically by modifying the ratio B^+/B^- and observing the point where the damage rate is zero. As B^+/B^- approaches infinity, the form of the fracture criterion reduces to the VGM.

The origins of the proposed functional form lie in the hyperbolic sine models previously discussed by Rice, Tracy, McClintock and Gurson [6,7,25]. The additional treatment of the formulation shown in Equations 8 and 9 serves to bring these models in line with the current understanding of the fracture process. Specifically, flexibility is provided through the constants in the criterion because (1) the theoretical growth predictions are for a single ideal spherical void, whereas actual void growth involves multiple voids with irregular shapes, and (2) flexible constants allow the criterion to account for events in the fracture process (e.g. void nucleation [32] and coalescence [15]) that are somewhat independent of void growth.

4. Evidence to support new local fracture criterion

Sources of experimental data from two different specimen types support the proposed fracture criterion. The first type of specimen is a standard round tension coupon tested in a high-pressure chamber, which is ideal for inducing fracture under

low-triaxiality ($T < 1/3$) conditions. The second type of specimen is a round bar with a circumferential groove cut into it, termed the Cylindrical Notched Tension (CNT) specimen. CNT specimens are commonly used to study fracture under higher triaxiality levels ($1 < T < 2$).

Local fracture criteria are further supported through their relation to the micro-scale fracture process. The controlling fracture mechanism in ductile steels is that of void growth and coalescence, which may be studied through analytical and computational models. The proposed fracture criteria will be compared to the most detailed results from each of these analysis methods.

4.1 Bridgman high-pressure tests

The most detailed source of experimental data for fracture at very low triaxiality ($T < 1/3$) is in the hydrostatically pressurized tension specimens tested by Bridgman [5], who developed an apparatus for conducting round tension tests in a chamber filled with pressurized hydraulic fluid. The applied hydraulic pressure could be maintained at several times the yield stress of the steel, such that when the coupons yielded in tension, the stress triaxiality at their center was negative. Other researchers have conducted similar tension tests with externally applied hydrostatic pressure (e.g. [3,33]). Balzer [34] provides a survey of high-pressure test data and methodology.

Test data selected from the Bridgman study include 290 round bars tested at hydrostatic compressive pressure levels between atmospheric and 3000 MPa. The set includes eight materials that were designated materials 2 through 9 in the Bridgman report. Bridgman considered multiple heat treatments for each material, and the resulting 33 combinations of material and heat treatment are treated as independent materials in this paper.

Figure 2 illustrates the extent of the Bridgman specimens by comparing the plastic strain and the stress triaxiality at the end of each test. Both quantities are reported at the assumed point of fracture initiation, the center of the specimen. As shown in the figure, about half of the specimens did not fracture despite reaching large plastic strain levels. Several qualitative conclusions can be made from this data before further examination. First, there is a significant decrease in the likelihood of observing fracture for specimens

in which the stress triaxiality remains low, an observation which supports the existence of a ductile fracture cutoff. Second, even below the supposed cutoff triaxiality, fracture observations are made in cases where the plastic strain is extremely high. These results can be explained by considering that true strain levels greater than 3.5 (350%) are associated with a 97% reduction in the cross sectional area of a tension coupon. Such highly deformed specimens are very sensitive to perturbations or eccentricity in the applied loading that might cause inaccurate fracture observations. As such, Bridgman's designation of fracture at these points does not necessarily contradict the fracture cutoff, around $T=0$, observed in more moderately deformed specimens.

4.2 Detailed analysis of the Bridgman data set

The fracture criteria described in Equation 8 is based on an integration of the damage rate at the location where a ductile crack may initiate. Evaluating this integral requires the history of the stress triaxiality and plastic strain at that location. In modern tests, the stress-strain history is determined through complementary finite element simulations, but this level of detail is not available for historical test data, where complete load-deformation plots and material parameters are not reported. To consider the effects of the continuum parameter history without the use of finite element analysis, an approximate method is applied, based on derivations by Bridgman. The reported data from the Bridgman tests are limited to the following: the axial stress at initial yielding (σ_Y), necking (σ_{max}), and fracture (σ_f), the test chamber pressure p , the reduction in specimen area (A_i/A_f), and the ratio between the neck longitudinal radius and tangential diameter (D_{NR}/R_N) measured after the specimen fractured.

From this information, three points in the specimen history can be calculated, as illustrated in Figure 3. These points are (A) the initial undeformed condition, (B) the initiation of necking, and (C) the failure of the specimen through fracture or area reduction. The locations of these three points are calculated according to Equations 10-15.

Point A:

$$T = \frac{1}{3} + \frac{p}{\sigma_y} \quad (10)$$

$$\bar{\varepsilon}_p = 0 \quad (11)$$

318 Point B:

$$T = \frac{1}{3} + \frac{p}{\sigma_{max}} \quad (12)$$

$$\bar{\varepsilon}_p = 0.2 \quad (13)$$

319 Point C:

$$T = \frac{1}{3} + \frac{p}{\sigma_f} + 1.4 \ln \left(1 + \frac{D_{NR}}{4R_N} \right) \quad (14)$$

$$\bar{\varepsilon}_p = \ln \frac{A_i}{A_f} \quad (15)$$

320 Equations 10 and 11 calculate the point of initial yielding in the coupon. For tests
 321 without a reported initial yield stress, the stress at the point of maximum load (σ_{max})
 322 was substituted for the yield stress. The externally applied pressure is negative
 323 (compressive) and remained constant throughout each test.

324 The initiation of necking in the tension specimens generally coincides with the point
 325 of maximum load, and represents an important change in the deformation mechanism
 326 (from uniaxial tension and distributed yielding to a localized and sharpening notch). The
 327 conditions at the initiation of necking are determined from the point of maximum load
 328 by Equations 12 and 13, where Bridgman [5] found the plastic strain to be consistently
 329 about 0.2 at the initiation of necking.

330 Bridgman developed an analytical relationship between the geometry of the necked
 331 section (i.e. notch radius and diameter) and the internal stress state. Using modern
 332 finite element tools, Bao and Wierzbicki [4] proposed a refined version (Equation 14)
 333 of the Bridgman equation, accounting for the effects of strain hardening and non-
 334 uniform stress fields across the neck. Assuming a relatively smooth strain distribution
 335 across the notch root (validated experimentally by Bridgman), the plastic strain at the
 336 center of the specimen is related to the reduction in cross-sectional area by Equation
 337 15.

338 By interpolating between the above points, the approximate evolution of the
 339 triaxiality-strain relationship, shown in Figure 3, was constructed for each of the

Bridgman specimens. In the subsequent sections, this history will be used to evaluate the proposed damage-based fracture criteria (e.g. Equation 8).

4.3 Cylindrically Notched Tension data set

A database of Cylindrically Notched Tension (CNT) tests and standard round tension bar tests, conducted by Kanvinde and Deierlein [30] and Myers et al. [11], and compiled in the report by Myers et al. [11], is used to evaluate the fracture models at high stress triaxiality. A total of 62 CNT specimens and 5 standard round tension coupons are used for this analysis.

The stress triaxiality and plastic strain in the smooth tension specimens were calculated using Equations 10 through 15. Myers calculated the corresponding values for the CNT specimens using finite element simulations. In contrast to the smooth tension coupons, where the stress state changes due to the evolution of the neck geometry, the stress triaxiality in CNT specimens is driven by the preexisting notch and remains relatively constant throughout each test. The triaxiality and strain conditions at the end of each test are shown in Figure 4.

4.4 Supporting micromechanical evidence for fracture criteria

4.4.1 Analytical void mechanics

Micro-scale models of the ductile fracture process (e.g. void growth and coalescence) provide strong evidence to support ductile fracture criteria. Following the original work of Rice and Tracey [6] discussed in Section 2, void growth models have been extended to more complex materials and stress states by Huang [26] and others. Specifically, Huang reevaluated the solutions for the void growth rate by expanding the Rice-Tracey approximation using more terms to account for more complex displacement fields around the voids. Huang proposed replacing the Rice and Tracey void growth rate (Equation 1) with the piecewise equation shown in Equation 16.

$$\frac{dR_v}{R_v} = \begin{cases} 0.427d\varepsilon_p e^{1.5T} & \text{for } T \geq 1 \\ 0.427T^{0.25}d\varepsilon_p e^{1.5T} & \text{for } \frac{1}{3} < T < 1 \end{cases} \quad (16)$$

Compared to the findings of Rice and Tracey, the Huang solution finds a general increase in the void growth rate, along with a more complex solution for triaxiality

levels as low as one-third (whereas Rice and Tracey limited their solution to triaxiality levels above 2.0). It should be noted that Equation 16 is a prediction of the average (volumetric) void growth rate. As the lower hydrostatic tension provided by lower stress triaxiality levels causes prolate void growth, the volumetric void growth may become less correlated with fracture initiation at lower triaxiality.

Figure 5 illustrates the agreement between the proposed damage growth rate in Equation 9 and the void growth rates derived by Rice and Tracey [6] and Huang [26]. Both of these results for the void growth rate agree well with the proposed damage model.

4.4.2 Computational void models

There are several assumptions inherent in the analytical models for void growth proposed by Rice and Tracey [6] and Huang [26] that inhibit their application at low stress triaxiality levels. First, these models only incorporate the void growth stage of the fracture process. Other studies have shown (1) that a relatively small amount of void growth is required to cause strain localization and void coalescence [29,35,36], and (2) that the critical void growth level is not necessarily independent of the stress triaxiality [15]. As a result, models based purely on void growth predictions may fail to capture significant parts of the fracture process. Second, the assumption of spherical void growth is violated at low triaxiality levels, as voids are observed to grow into long prolate shapes. This aspherical effect may decrease the ability of the average void radius (predicted by the Rice-Tracey equation) to predict fracture.

Finite element simulations of void growth and coalescence seek to avoid these issues by directly modeling the micro-scale deformations of the void and surrounding matrix [37–39]. Cooke [15] conducted two such analyses with an emphasis on the low-triaxiality fracture process which are useful in this application. The first set of simulations measured the volumetric growth rate of a single void that was deformed under various levels of stress triaxiality. As indicated in Figure 6, the void growth rates are closely represented by a model following the form of Equation 9. The simulated void growth rate decreases as the stress triaxiality decreases, becoming negative for values of the stress triaxiality which are below zero. These simulations validate the void growth predictions of Rice and Tracey for low-triaxiality conditions, but they do not

consider the effects of void interaction on the coalescence and fracture process, an effect that may be more significant at low triaxiality levels.

In a second set of simulations, Cooke modeled a multi-void matrix that was loaded until the point of void coalescence. The results from these simulations are shown in Figure 7. The strain required to cause the voids to coalesce increases as the applied stress triaxiality decreases (Figure 7a). Void coalescence is indicated at the micro-scale by the onset of unstable void growth and material softening (Figure 7b). For triaxiality levels below 0.3, no void coalescence was observed before numerical nonconvergence forced the termination of the analysis.

4.5 Probabilistic evaluation of the new fracture criteria

Ductile fracture in metals is a random process, owing to uncertainty in the local stress and strain fields, material properties, and the fracture process itself. Typical fracture models provide only a deterministic prediction of fracture, where the inherent randomness is considered implicitly using regression to minimize the error between model predictions and observed data. These deterministic models provide good predictions for a single uniform material with high quality test data under monotonic loading [40,41]. However, the challenges in fitting a deterministic fracture model to complex cyclic test data have recently been demonstrated by (1) Myers et al. [42] who applied fracture models to specimens with inhomogeneous material properties around welded connections and (2) Smith et al. [29] in specimens with sharp gradients of local parameters caused by low triaxiality and complex specimen geometry.

To account explicitly for the uncertainty in fracture predictions, and to facilitate more robust calibration, it is necessary to formulate and calibrate the fracture criterion in a probabilistic sense [42]. A probabilistic formulation, first proposed by Myers et al. [42], is based on the assumption that the combined uncertainty can be treated as a random distribution of the fracture capacity (D_{crit}). While the true uncertainty in fracture predictions can be attributed to several sources (e.g., experimental error, material properties, or the fracture process itself), treating these sources separately would require an unrealistic amount of test data and analyses. In the “lumped uncertainty” model, validated by Myers et al. [42], the evolution of the fracture damage

(D) can still be calculated deterministically (i.e. Equation 8), producing a single deterministic value to be compared with the stochastic damage capacity.

Recall that in the deterministic formulation of the fracture criteria (Equation 8), fracture initiation is predicted at the instant that the fracture damage (D) exceeds the fracture capacity (D_{crit}). Alternatively, in the probabilistic formulation of the criteria, there is a non-zero probability of fracture initiation at all levels of the fracture damage (D). The associated probability distribution of the fracture capacity is a random variable defined as $\widetilde{D_{crit}}$, and is assumed to be normally distributed [42], with mean μ_D , standard deviation σ_D , and coefficient of variation $COV_D = \frac{\mu_D}{\sigma_D}$.

Maximum likelihood estimation (MLE) is a commonly used tool for estimating the parameters of a probabilistic model[43]. MLE is a method of parameter estimation, and operates by searching for the model parameters that are most likely given a set of observations. In the context of this study, the observations may include any physical or computational test whose result (e.g., a measured or simulated fracture event) can be predicted by the fracture model. The most likely parameter set is that which maximizes the probability of the observed results. For statistically independent tests, the likelihood function is written as:

$$L(\text{parameter set} | \text{observation set}) = \prod P(\text{obs}_n | \text{parameter set}) \quad (17)$$

The equation determines the likelihood that a candidate parameter set is accurate given a set of observed data (which may be the results of physical or computational experiments). This likelihood is equal to the product of the probability of making each observation, were the candidate parameter set true. Per the above description of the probabilistic fracture criteria, the probability of making such an observation is equal to the value of the probability density function ($\widetilde{D_{crit}}$) at the value of D where fracture was observed. Note that D_{crit} is not directly observable, since it is derived from FE simulations. However, for the purposes of maximum likelihood estimation, it may be treated as such since it bears a unique correspondence with the observable quantity (i.e., failure displacement) and is consequently a unique indicator of the failure displacement.

For example, consider the results of Bridgman's high pressure tests on one material, reported as steel #2, heat treatment #7 by Bridgman [5]. The local history of the stress

triaxiality and plastic strain in two typical specimens are determined by Equations 10-15, and this history is input into the fracture criteria (Equation 8) to evaluate the fracture damage parameter (D) for each test. Figure 8 illustrates the evaluation of the likelihood function for two tests of the material. The upper half of the figure shows the evolution, in triaxiality-strain space, of each of the tested specimens as determined by Equations 10-15. The lower half of the figure shows the same tests in triaxiality-damage space, where the damage is determined according to the proposed fracture criteria.

Given an assumed distribution of the material fracture toughness (indicated by the normal probability distributions in the figure), the probability of observing a given test is proportional to the Probability Density Function (PDF) of the fracture toughness distribution evaluated at the end of the test: $P(obs_n) = f_{\bar{D}}(D)$, where D is the accumulated fracture damage calculated at the end of the test.

Figure 8 shows one test that fractured and one test that did not fracture, even though the second was pulled to a higher level of strain. Aside from representing actual measured data, this second example is also an important observation that can be tested against the candidate fracture criteria. The probability of not observing fracture by the end of one of these tests is equal to the complement of the CDF at the end of the test, i.e. $P(obs_n) = 1 - F_{\bar{D}}(D)$. Then, according to Equation 17, the likelihood of the assumed parameter set given the two tests shown in the figure is equal to the product of the probability of seeing the observed outcome in each test. As illustrated in this example, inclusion of non-fracture (survival) data through maximum likelihood estimation are important contributors to the outcome – which is why this practice is common in clinical trials [44]. Using the MLE method, the search for the fracture model parameters that maximize this likelihood is typically done using numerical optimization.

All of the test observations described in the preceding section were evaluated by a similar procedure. The physical observations from the CNT fracture tests mirror those of the Bridgman tests, where the plastic strain, triaxiality, and resulting damage history are calculated for each specimen.

In the computational models of void coalescence, the triaxiality and strain history (and thus the damage history) are imposed by the boundary conditions of the finite element model. The “observation” of fracture initiation is recorded at the point of void

coalescence, as measured by the point that the stiffness of the finite element model becomes negative. The modeling parameters that result in the six different curves in Figure 7a represent six separate materials, and the results are analyzed in the same way as separate materials in the high-pressure or CNT experimental data sets.

Fracture “observations” are made from the analytical models of void growth by selecting a critical void size which is assumed to represent fracture initiation. Virtual experiments are conducted to evaluate the critical plastic strain at many values of the stress triaxiality. This critical plastic strain and stress triaxiality are combined to calculate the observed damage index (D) at fracture, per Equation 8.

The distribution of the fracture toughness parameter ($\widetilde{D_{crit}}$) is a material property that must be separately calibrated for each material. However, the other parameters in the fracture model (i.e. A^+, A^-, B^+, B^-) are considered to be fairly consistent for all mild steel materials. Other researchers have similarly found these parameters to be relatively material invariant[17,18]. Therefore, calibrating the individual fracture toughness parameters and the model shape parameters requires a two-step process.

For a candidate set of characteristic parameters, (e.g. the familiar VGM: $D = \int e^{1.5T} d\varepsilon_p$) we calibrate the fracture model to each material using MLE. The likelihood returned from this calibration indicates the likelihood that the candidate set of model generic parameters is accurate for each material. Taking the product of the likelihood value for each material yields the combined likelihood that the candidate model parameters are valid given the results of all of the materials and data sources. A mathematical optimization routine is used to find the model parameters that maximize this combined likelihood.

4.6 Calibration and discussion of model form

Within the Maximum Likelihood framework described in the previous section, the likelihood value may be used to both calibrate the internal parameters of a proposed fracture criteria and evaluate the relative accuracy of multiple independent criteria. In this way, a wide variety of fracture criteria can be compared objectively. The following figures and table will illustrate the calibration of the newly proposed fracture criteria in Equation 8, and demonstrate the improvement in fracture prediction relative to the

models available in literature. Following the approach described in the previous section, these comparisons will be made in terms of the likelihood of a given model being correct given the available supporting evidence.

It is convenient to discuss the likelihood in terms of its logarithm, as they are both maximized under the same conditions and the so-called log-likelihood produces more computationally tractable values. The log-likelihood can be interpreted by considering that for a given data set, a candidate prediction model that increases the log-likelihood by 1.0 is 10 times more likely than the alternative. However, one is cautioned against placing too much weight in the absolute magnitude of the log-likelihood values, as they are dependent on many modeling assumptions and the diversity of the calibration data set. In developing a robust criterion, a reasonable fit to a large set of empirical fracture observations is more important than a perfect fit to a theoretical model or a limited data set.

4.6.1 Calibration of the proposed fracture criterion

Figures 9 and 10 demonstrate the calibration of the proposed fracture criteria (Equation 8) within the MLE framework. In both figures, the log-likelihood of observing each data source (e.g. high-pressure test data, computational void models) is indicated as a function of a parameter of the fracture criteria. The most likely version of the fracture criteria is that which maximizes the likelihood of making all of these collected observations, judged qualitatively as the likelihoods have slightly different maxima. Figure 9 illustrates the likelihood of observing each data source as a function of the fracture cutoff location (mapped to the ratio B^+/B^-), while holding the value $A=1.5$, with two implications. First, the most likely value for the fracture cutoff, when separately considering each source of observations, falls in the range $(-0.1 < T < 0.15)$. Second, the CNT tests and computational void cell models are relatively tolerant of fracture cutoffs that are highly negative (tending in the extreme to the traditional exponential model). This is the expected result, as the CNT and computational data sets mainly contain data for triaxiality greater than one-half. The choice of fracture cutoff has little influence on the shape of the fracture criteria for these higher triaxiality levels.

Figure 10 shows the variation in the log-likelihood for values of the exponential parameter A while holding the value $(B^+/B^- = 1.0)$. With the exception of the high

pressure tests, the plots suggest that the mostly likely value for the A parameter falls between 1.2 and 1.7, a range consistent with other reported values under different test conditions. The likelihood of observing Bridgman's high-pressure test data, however, was almost unaffected by the choice of the A parameter. This is a consequence of the fact that Bridgman's high-pressure test data contained almost exclusively triaxiality levels below $T=0.8$, such that the shape of the fracture criteria in this region is driven mostly by the presence and location of the fracture cutoff, rather than the A parameter.

A separate parametric study was conducted to explore using separate values for the constants A^+ and A^- . Decoupling the values had minimal impact on the overall likelihood, owing to the relative lack of ability in this data set to separate the rates of positive and negative damage accumulation. Cyclic test data and simulations would have more statistical power in this regard.

Based on the results of the two parametric calibrations described above, and considering the uncertainty and relative value of the various data sources, reasonable values for the model shape parameters were selected ($A^+ = A^- = 1.3$; $B^+/B^- = 1.0$)

$$D = \int (e^{1.3*T} - e^{-1.3*T}) d\bar{\epsilon}_p \quad (18)$$

as shown in Equation 18.

These values were chosen to fall within the range of values which maximized the likelihood of observing each data set, while giving preference to matching the observations from physical experiments rather than theory or simulations. The calibrated value of the A parameters is unsurprising, and fits within the range of historical values for similar parameters. The calibrated value of $B^+/B^- = 1.0$ indicates the existence of a fracture cutoff at a stress triaxiality level of zero. This cutoff value is higher than that proposed by Bao and Wierzbicki [4], owing to the fact that their study did not consider the history effect on fracture damage, but the cutoff location is identical to the fracture cutoff proposed by Reddy et al. [24] for wire drawing operations.

4.6.2 Comparison of the proposed fracture criterion with alternatives

Table 1 shows a comparison of the proposed fracture criterion with several other models found in the literature. The values reported are the log-likelihood, relative to the

proposed model, of each model for each source of supporting data. Therefore, negative values in the table indicate that the specified model is less likely than the proposed criterion, Equation 18; and positive values indicate that the specified model is more likely than the proposed criterion.

$D = \int e^{1.3T} - e^{-1.3T} d\varepsilon_p$
$D = \int e^{1.5T} d\varepsilon_p$
$\varepsilon_{p,crit} = \alpha e^{-1.5T}$
$\varepsilon_{p,crit} = CT^{-2}$
$\varepsilon_{p,crit} =$
$\begin{cases} T < 0 & C \left(T + \frac{1}{3}\right)^{-1} \\ 0 < T < \frac{1}{3} & \dots \\ T > \frac{1}{3} & CT^{-1} \end{cases}$

Analytical void growth

When compared to the analytical void growth relationship by Huang [26], the SMCS and VGM are somewhat more likely than the newly proposed criterion. This is an expected result, as derived void growth relationships form the basis for these models. The proposed criterion is significantly more likely than the Kuwamura and Yamamoto [27] and Bao and Wierzbicki [7] models, as they have an independent functional form from the analytical void growth predictions.

FEM void coalescence

The available observations of computational void coalescence [15] tend to parallel the analytical void growth predictions, except that they also simulate the micro-mechanical effects of void shape change and coalescence. As indicated by the negative likelihood data, the proposed model is more closely aligned to the observed FEM void coalescence than any of the other four models.

CNT tests

Recall that the CNT data set includes both CNT specimens (triaxiality at fracture

between 1.0 and 2.0) and standard tension coupons (triaxiality at fracture between 0.5 and 0.8). The VGM and SMCS were developed for triaxiality levels above 1.0, and they are found to be less likely than the proposed model or the other two empirical models. In contrast, Kuwamura and Yamamoto developed their model empirically from test data of tension coupons and CNT specimens, and thus, it is not surprising that their model has slightly better correlation with the proposed model. The proposed model also provides a higher likelihood than the Bao and Wierzbicki model.

High-pressure tension tests

An important motivation of this paper is to address the conflict between existing fracture criteria and the very low-triaxiality test data published by Bridgman, among others. As evidenced by the likelihood parameters in the last column of Table 1, the proposed fracture criterion is a significant improvement over other models to simulate fracture under low-triaxiality. Compared to the proposed model, the VGM is the second most likely model of those investigated, and is the only other damage-based criteria. As discussed earlier, critical strain based criteria are expected to be less reliable for lower triaxiality levels, due to significant variation in the fracture damage rate which those models cannot capture. Of the remaining three critical strain based criteria, Bao and Wierzbicki [4] as well as Kuwamura and Yamamoto [27] are more likely than the SMCS, which does not incorporate a fracture cutoff.

As indicated by the table, the newly proposed criterion is shown to have the highest likelihood in predicting both the experimental test data and the theoretical micromechanical observations.

5. Conclusions

This paper has examined the use of local criteria to predict fracture under axisymmetric conditions, including the consideration of the historical justification for such models and their deficiencies. Models for ductile fracture under axisymmetric stress conditions have traditionally been based on the idea that material damage is proportional to an exponential function of the stress triaxiality. These exponential models are particularly important because in addition to their original use for axisymmetric high-triaxiality conditions, they have been extended to alternative

conditions including cyclic loading [2] and non-axisymmetric stress conditions [18].

A significant limitation of exponential fracture models is their failure to capture the low-triaxiality fracture cutoff, a triaxiality level below which fracture cannot occur. Evidence from multiple data sources that have been reviewed in this paper suggests the cutoff to occur at a triaxiality level of zero. This implies that ductile fracture is impossible without the presence of tensile hydrostatic pressure (positive triaxiality).

The objectives of this study are to find a fracture damage model that (1) closely approximates the results of established models at high stress triaxiality, (2) captures the effect of the low triaxiality fracture cutoff, 3) provides flexibility to allow accurate fits to higher quality data sets, and 4) requires relatively few calibration parameters. These objectives are achieved through the adaptation of the void growth model originally proposed by Rice and Tracey based on the hyperbolic sine of the stress triaxiality. The model was further adapted to accommodate features such as dissimilar void growth and shrinkage rates. The proposed model (Equation 8 and 9) contains four parameters, three of which are shown to be fairly constant across various steel material tests. Recommended values of these parameters are determined based on a combined analysis of the available data. This results in a criterion (Equation 18) with only one parameter, D_{crit} , that needs to be calibrated for specific materials.

The rediscovery of the hyperbolic sine based fracture prediction model that was proposed by Rice and Tracey in 1969 is an interesting development. The Rice-Tracey exponential model has previously been adapted to simulate fracture under cyclic loading [2] and low-triaxiality stress states [28], including the effect of the triaxiality fracture cutoff [4]. The hyperbolic sine formulation by Rice and Tracey encompasses these objectives in a simple and theoretically justified form. Only minor modification of their model is necessary to fit it accurately to a diverse set of experimental data.

Acknowledgments

This research was supported by the National Science Foundation under Grant Number CMMI-0825339 (NSF Project #0825339). Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views or policies of the National Science Foundation.

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758

Table 1: Normalized log-likelihood of several proposed fracture models

Source	Model Fracture criteria	Log-likelihood by supporting data set			
		Analytical void growth [26]	FEM -void coalescence [15]	CNT tests [11]	High- pressure tension tests [5]
Proposed (Eq. 18)	$D = \int e^{1.3T} - e^{-1.3T} d\varepsilon_p$	0.0	0.0	0.0	0.0
VGM [2]	$D = \int e^{1.5T} d\varepsilon_p$	1.2	-2.7	-6.8	-12.9
SMCS [4]	$\varepsilon_{p,crit} = \alpha e^{-1.5T}$	1.2	-2.7	-23.6	-135.2
Kuwamura and Yamamoto [27]	$\varepsilon_{p,crit} = CT^{-2}$	-405.6	-9.1	1.0	-83.4
Bao and Wierzbicki [4]	$\varepsilon_{p,crit} = \begin{cases} T < 0 & C \left(T + \frac{1}{3}\right)^{-1} \\ 0 < T < \frac{1}{3} & \dots \\ T > \frac{1}{3} & CT^{-1} \end{cases}$	-576.8	-16.3	-4.0	-62.2

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763 Figure Captions:

764 Figure 1: Proposed relationship between fracture damage rate and stress triaxiality

765 Figure 1: Ultimate conditions in smooth notched bars tested by Bridgman [1]

766 Footnote:

767 ¹ Fracture observations at strains above $\bar{\varepsilon}_p = 3.5$ are deemed unreliable due to near-complete area
768 reduction of specimens by the time of failure

769 Figure 3: Schematic history of continuum parameters in Bridgman specimens

770 Figure 4: Conditions at fracture in tension coupons and CNT specimens

771 Figure 5: Damage rate of the proposed criteria compared with void growth predictions of
772 Rice and Tracey [6] and Huang [26]773 Figure 6: Damage rate of the proposed criteria fit to void growth measurements from
774 single void cell simulations by Cooke et al. [15]775 Figure 7: Use of multi-void models to inform fracture criteria: (a) localization strains in
776 six materials under varying stress triaxiality, (b) bulk stress-strain response of
777 voided material illustrating the determination of the localization strain. Adapted
778 from Cooke [15]

779 Figure 8: Plastic strain and calculated damage history for a hypothetical steel

780 Figure 9: Likelihood of models with varying fracture cutoffs

781 Figure 10: Likelihood of models with varying A parameters

a)

b)