TITLE

Stress Intensity Factors of Various Size Single Edge-Cracked Tension Specimens: A Review and New Solutions

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ABSTRACT

Non-dimensional geometric correction factor, β , data for stress intensity factor, K , calculations are generated through finite element analyses and collected from the literature for axially loaded single-edge cracked plates with rotationally free and constrained loaded edges typically used in fatigue crack growth experiments. The solutions are valid for crack length-to-width ratio, a/W , in the range of $0.01 \leq a/W \leq 0.975$ and plate height-to-width ratio, H/W , in the range of $0.8 \leq H/W \leq 10.0$. Comparisons are made between the literature reported data and the present effort's results with generally satisfactory correlation. However, differences as large as 12.7% exist for short cracks in small H/W valued plates.

1. KEYWORDS

Fatigue Crack Growth; Fracture Mechanics; Stress Intensity Factor; Finite Element Analysis

2. NOMENCLATURE

a	- crack length
B	- plate thickness
CMOD	- crack mouth opening displacement
E	- Young's (elastic) modulus
H	- plate height
K	- mode-I stress intensity factor
ΔK	- mode-I stress intensity factor range
MSE(T)	- modified single edge-cracked specimen under tensile loading
P	- remote axial tensile load
SE(T)	- single edge-cracked specimen under tensile loading
u	- x-direction (transverse / horizontal) displacement
v	- y-direction (axial / vertical) displacement
W	- plate width
β	- non-dimensional geometric correction factor
ν	- Poisson ratio
σ	- average far-field axial stress

3. INTRODUCTION

The use of fatigue crack growth simulations is fundamental in the modern assessment of a structural component's serviceable life. Generation of this vital crack growth rate data falls to the many researchers and labs that have been able to provide tremendous amounts of data for use by analysts. However, the connection between crack growth rate measurements and a fracture mechanics parameter, such as the stress intensity range, ΔK , is undertaken by several different methods [1]. The use of single edge-cracked specimens subjected to tensile loading is commonly desired for use in experimental testing due to its ease of use and minimal amount of material needed in specimen fabrication. Furthermore, there is an additional benefit of having a single crack tip where only the front to back symmetry is of concern. As such, the relationship between pertinent geometric features (e.g. height-to-width (aspect) ratio, H/W), applied loading, and the stress intensity factor, K , at the crack tip is essential.

The aim of the present paper is to collate data from various researchers and present some comparisons and discussion about the applicability of the available data with regards to single edge-cracked monolithic specimens subjected to tensile loading. Numerical results, generated by the author, obtained through an *hp*-version finite element analysis (FEA) program [2] are also presented with comparisons to available solutions.

The primary loading, or boundary condition (BC), investigated here is the Clamped-End scenario. This BC precludes the rotation and lateral contraction of the specimen along the upper and lower edges, which idealizes a uniaxial loading apparatus with friction grips, as shown in Fig. 1. This specimen and loading scenario is referred to as the Modified Single Edge-Crack specimen, MSE(T), as opposed to the SE(T) designation which is commonly used to refer to a similarly designed specimen with uniform applied remote tensile stress and tested in a Pinned-End load fixture. SE(T) solutions for the non-dimensional geometric correction factor, $\beta(a/W)$, are readily available in the literature [3,4]. The eccentrically-loaded single edge-crack tension, ESE(T), is another commonly used experimental configuration [1]. The MSE(T) prevents rotation of the specimen, which precludes significant compression at the un-cracked edge and is thus ideally suited for thin specimens.

The total applied tensile force, along the y -direction, P , is used in conjunction with the plate cross-sectional dimensions of width, W , and thickness, B , to determine the average "far-field" axial stress, σ , shown in Eq. (1). Note that the stress distribution can be highly non-uniform for small aspect ratio plates and/or large crack lengths.

This far-field stress, along with β and the crack length, a , is commonly used to calculate the stress intensity factor, K , through Eq. (2). Here, K refers to the mode-I opening mode, or K_I , stress intensity factor.

Fig. 1. Schematic and geometric definitions for the MSE(T) specimen.

$$\sigma = P/BW \quad (1)$$

$$K = \sigma\beta\sqrt{\pi a} \quad (2)$$

4. LITERATURE REVIEW

Multiple investigators have approached the problem of the MSE(T) specimen, though only a few are included here for comparison, as they are more frequently referenced in the literature. Several have used an FEA approach; either with p -version [6] or low order hp -version [7,8] codes. Both an analytical weight function [9] and a crack compliance method [5] have also been proposed. Crack mouth opening displacement (CMOD) solutions for the MSE(T) specimen have been previously reported for $H/W = 4.0$ in [8] and later for a wider range of plate aspect ratios in [7]. The tabulated β factors from [6] are included in Table A-2 and those from [7] are included in Table A-4. The reader is directed to the appropriate references for the formulation of the other methods.

SE(T) specimen, pinned-end BC, solutions are presented in [3,6] and again in [4] with the added influence of fatigue crack starter notches on small cracks.

More recently, results of both the SE(T) and MSE(T) specimens were presented in [10]. Tabulated β factors from [10] are included in Table A-6. However, their use of a far-field stress value calculated from the specimen's axial displacement and elastic modulus, Eq. (3), for the MSE(T) dataset may not be practicable in the laboratory with regard to the separation of specimen and loading system displacement components. Note that Eq. (3) has been reformatted by the authors to align with the geometric definitions used herein.

$$\sigma = \frac{2\nu}{H} E \quad (3)$$

5. PRESENT WORK

5.1 METHOD / VALIDATION

An *hp*-version FEA program, StressCheck® [2], was used to generate the K solutions for a range of non-dimensional crack lengths, a/W , of 0.01 to 0.975, and a range of plate aspect ratios, H/W , from 0.8 to 10.0 for the MSE(T) specimen and an $H/W = 20.0$ for the SE(T) specimen. The finite element mesh consisted of highly shape controlled tetrahedral elements of polynomial order 8, though convergence was achieved to within $\pm 0.1\%$ in β with polynomial order of 6 or 7, for all models. The numerical simulations were performed using a plane stress and linear-elastic constitutive model with a Poisson ratio, ν , of 0.33. Geometrically non-linear considerations were ignored; and are believed to have minimal impact on high-cycle fatigue crack growth rates. The simulations' degree-of-freedom, DOF , counts ranged from 78k to 274k. The K values are automatically extracted from the program using the contour integral method (CIM) [11,12], also known as the J-integral. The K values and applied axial load levels are reported from the FEA software. Combined with the *a priori* known plate and crack geometry, the geometric correction factor can be calculated through rearrangement of Eqs. (1) and (2), and is presented in Eq. (4).

$$\beta = \frac{KBW}{P\sqrt{\pi a}} \quad (4)$$

The variation, or fluctuations, in the calculated β values as a function of the radius used in the CIM data extraction can be seen in Fig. 2 overlaid upon a typical finite element mesh. Over a small radius range, the CIM path intersects similar elements. Since the stress, strain, and displacement results are fitted with polynomial shape functions it is expected that the solution may fluctuate around the real value; especially in the areas of larger gradients. The stress singularity influence, at the crack tip, is completely contained within the first several layers of elements around the tip. Note that any CIM radius which is larger than the

largest crack tip element length produces numerically minor fluctuations in the calculated β values. In all cases, the reported β values have been calculated using K extractions from a sufficiently large CIM radius, relative to the crack tip element sizes, to minimize this phenomenon.

Fig. 2. Effect of radius on the *CIM* path integral on β results.

The accuracy of the present FEA method is benchmarked against the well-known analytical form presented in [3], and reproduced with current variables in Eq. (5), for the SE(T), pin-loaded, specimen configuration. The benchmark equation is reported to be accurate to within 0.5% for any a/W . The relative difference in the present FEA method results as compared to the benchmark equation is shown in Fig. 3 as a function of a/W . The SE(T) FEA simulations DOF counts ranged from 104k to 177k; this dataset is also presented in the last column of Table A-1. Note that the requirement for plate aspect ratio, as reported in [3,4], is $H/W \geq 2.0$ for the SE(T) loading BC with uniform remote tensile stress. The small variation in the relative differences indicates that the present FEA method is acceptable. It is unknown at this time whether the primary fluctuations in percent difference are attributable to the FEA method inaccuracies or the coefficient precision and/or trigonometric functional form of Eq. (5).

$$\beta\left(\frac{a}{W}\right) = \sqrt{\frac{2W}{\pi a} \tan\left(\frac{\pi a}{2W}\right)} * \frac{0.752 + 2.02\left(\frac{a}{W}\right) + 0.37\left(1 - \sin\left(\frac{\pi a}{2W}\right)\right)^3}{\cos\left(\frac{\pi a}{2W}\right)} \quad (5)$$

Fig. 3. Percent difference between [3] and present work for SE(T).

5.2 NEW RESULTS

The variation of β values as a function of a/W and H/W can be seen in Figs. 4 and 5 for both the MSE(T) and SE(T), from Eq. (5), configurations. This dataset is also presented in Table A-1. It is apparent that there are marked differences between the MSE(T) and SE(T) specimen loading configurations, as the clamped edges of the MSE(T) specimen tend to reduce the crack mouth opening, or prying, action. This is especially true above a/W values of about 0.20. Fig. 5 shows the effect of H/W on small cracks, and the general convergence to a single point value of 1.1215 for H/W values ≥ 3.0 . As Figs. 5 and 6 show, this is not true for sufficiently small H/W plate geometries where significant influence of the BCs is felt at the crack tip. This phenomenon is easily attributable to the well-known St. Venant's principle of characteristic loading influence decay lengths [13].

Fig. 4. Effect of a/W and H/W on the geometric correction factor, β , for MSE(T).

Fig. 5. Extreme values of the geometric correction factor, β , for MSE(T).

Fig. 6. Effect of plate aspect ratio, H/W , on short crack β values.

6. DISCUSSION

All of the existing literature solutions have varying degrees of correlation with test data and one another. Comparisons are made based upon relative percent differences between the reported geometric correction factors from [6], [7], and [10] and the present results in Tables A-3, A-5, and A-7, respectively. Not all cracking scenarios that are referenced from the available literature were recreated in the present effort. While generally good correlation is seen in all dataset comparisons, the best correlation is seen from [7] where the differences lie within $\pm 1.23\%$. There are several larger differences in the comparisons with [6 and [10]], where differences are as high as 12.69% for the smallest crack and plate aspect ratio. It is noted that the SE(T), pinned-end, comparison of [6] with the present work varies no more than $\pm 0.66\%$ and no more than $\pm 0.82\%$ when compared to [10]. However, the significant variation in β values for short cracks in small aspect ratio plates (see Fig. 6) does not appear in the data presented in [6], [7], or [10]. It should be noted that the MSE(T) comparisons made with [10] used the far-field stress definition presented in Eq. (3); therefore, the resulting geometric correction factors of the present work differ from those presented in Table A-1 in this modified form. However, the pinned BC case of [10] uses the present far-field stress definition, Eq. (1).

The existence of an asymptotic limit of the geometric correction factor as the crack length tends towards zero, $a/W \rightarrow 0.0$, is to be expected. As the crack shrinks, the effective position of the load line returns to the mid-plane, at $x = W/2$, of the specimen, and thus, no secondarily induced in-plane bending moment is

generated. The commonly quoted geometric correction factor value of 1.122 [3] is found to be 1.1215 by both [7] and the present effort, 1.12 in [10], and 1.118 in [14]. Though, all of these values are identical when rounded to two decimal places of precision. It should be noted that the asymptotic value is applicable for $H/W \geq 2.0$ per [7] and $H/W \geq 0.2$ per [10], whereas the present work indicates that this value is only obtained with $H/W \geq 3.0$, approximately, for the MSE(T) specimen.

As presented in [4], the presence of a starter notch will decrease the stress intensity factor of short cracks due to the ability of the load to more smoothly transfer around the crack tip. The length of influence of the starter notch is on the order of the length of the notch itself. This phenomenon was not explored herein.

It should be noted that the solutions presented herein are independent of the elastic modulus of the material since a linear-elastic constitutive material model is utilized. Furthermore, variation in the value of Poisson's ratio over typical structural metal ranges, 0.2-0.4, produces negligible variations in the results.

The solutions presented herein have been generated with the basic assumption that the loaded edges of the specimen are unable to rotate or slip within a loading fixture causing a change in the effective plate aspect ratio. The relative stiffness of the test specimen should be weighed against that of the test apparatus to ensure the relative rigidity of the BC imposed in the FEA solutions is appropriate. Furthermore, the proper plate height must be determined in the experimental setup, which may not be at the initial specimen / loading apparatus interface.

The assumptions of linear-elastic fracture mechanics (LEFM) require that the plastic zone size of the loaded material's crack tip region be appreciably small compared to any geometric feature of the specimen (e.g. crack length, remaining ligament length ($W-a$)). As such, a practical use of the asymptotic value of the geometric correction factor at $a/W = 0.0$ may be non-existent. It is the duty of the researcher to ensure applicability of the reported data to real-world utilization.

7. CONCLUSION

Non-dimensional geometric correction factor, β , data for stress intensity factor, K , calculations have been generated and collected from the literature for axially loaded single-edge cracked plates with rotationally free and constrained loaded edges. The solutions are valid for a wide range of a/W and H/W ratios which are typical of common laboratory specimens. Comparisons were made between the literature reported data and the present effort's results with generally satisfactory correlation. However, significant differences were found for short cracks and plate aspect ratios of $H/W < 3.0$ for the rotationally constrained specimen.

8. APPENDIX

Table A-1. New MSE(T) and SE(T) geometric correction factor, β , values.

	H/W								
a/W	0.8	1	1.5	2	4	6	8	10	Pinned
0.000	0.9554	0.9976	1.0735	1.1084	1.1215	1.1215	1.1215	1.1215	1.1215
0.010	0.9597	1.0007	1.0751	1.1097	1.1248	1.1249	1.1255	1.1250	1.1222
0.020	0.9641	1.0037	1.0768	1.1111	1.1271	1.1265	1.1274	1.1269	1.1245
0.030	0.9668	1.0053	1.0772	1.1118	1.1291	1.1295	1.1304	1.1306	1.1283
0.040	0.9673	1.0058	1.0780	1.1126	1.1321	1.1335	1.1344	1.1342	1.1334
0.050	0.9660	1.0057	1.0775	1.1135	1.1357	1.1375	1.1388	1.1398	1.1397
0.075	0.9570	1.0009	1.0781	1.1169	1.1471	1.1527	1.1553	1.1569	1.1609
0.100	0.9423	0.9921	1.0771	1.1216	1.1623	1.1721	1.1770	1.1806	1.1890
0.125	0.9247	0.9820	1.0763	1.1262	1.1809	1.1959	1.2024	1.2078	1.2238
0.150	0.9060	0.9693	1.0747	1.1323	1.2024	1.2244	1.2350	1.2409	1.2649
0.175	0.8882	0.9571	1.0727	1.1380	1.2263	1.2555	1.2700	1.2789	1.3126
0.200	0.8721	0.9455	1.0709	1.1443	1.2527	1.2893	1.3091	1.3208	1.3670
0.225	0.8586	0.9349	1.0695	1.1509	1.2806	1.3276	1.3521	1.3672	1.4283
0.250	0.8482	0.9270	1.0690	1.1584	1.3106	1.3680	1.3991	1.4185	1.4971
0.275	0.8403	0.9200	1.0697	1.1664	1.3425	1.4120	1.4500	1.4740	1.5738
0.300	0.8349	0.9160	1.0722	1.1759	1.3760	1.4589	1.5037	1.5342	1.6591
0.325	0.8326	0.9146	1.0743	1.1860	1.4114	1.5089	1.5626	1.5991	1.7542
0.350	0.8332	0.9152	1.0797	1.1978	1.4485	1.5616	1.6269	1.6681	1.8602
0.375	0.8364	0.9189	1.0856	1.2096	1.4873	1.6181	1.6943	1.7415	1.9783
0.400	0.8420	0.9250	1.0943	1.2249	1.5278	1.6772	1.7661	1.8246	2.1102
0.425	0.8501	0.9348	1.1047	1.2409	1.5701	1.7394	1.8423	1.9112	2.2578
0.450	0.8623	0.9449	1.1191	1.2583	1.6142	1.8049	1.9233	2.0039	2.4237
0.475	0.8765	0.9600	1.1357	1.2756	1.6599	1.8729	2.0087	2.1020	2.6107
0.500	0.8934	0.9788	1.1536	1.3006	1.7076	1.9443	2.0989	2.2069	2.8226
0.525	0.9136	0.9988	1.1770	1.3260	1.7571	2.0186	2.1938	2.3184	3.0638
0.550	0.9374	1.0236	1.2014	1.3545	1.8087	2.0962	2.2933	2.4366	3.3402
0.575	0.9625	1.0518	1.2310	1.3860	1.8624	2.1765	2.3966	2.5612	3.6590
0.600	0.9944	1.0850	1.2647	1.4228	1.9181	2.2587	2.5055	2.6919	4.0296
0.625	1.0347	1.1231	1.3035	1.4612	1.9761	2.3451	2.6177	2.8289	4.4642
0.650	1.0767	1.1667	1.3471	1.5072	2.0375	2.4325	2.7316	2.9728	4.9793
0.675	1.1260	1.2187	1.3978	1.5569	2.1022	2.5239	2.8507	3.1209	5.5970
0.700	1.1847	1.2772	1.4553	1.6135	2.1709	2.6172	2.9726	3.2734	6.3485
0.725	1.2504	1.3436	1.5195	1.6794	2.2459	2.7113	3.0977	3.4269	7.2776
0.750	1.3304	1.4250	1.5995	1.7544	2.3247	2.8100	3.2238	3.5828	8.4463
0.775	1.4232	1.5164	1.6898	1.8418	2.4126	2.9147	3.3397	3.7379	9.9496
0.800	1.5343	1.6280	1.7968	1.9447	2.5105	3.0232	3.4769	3.8916	11.943
0.825	1.6720	1.7619	1.9301	2.0704	2.6268	3.1370	3.6094	4.0401	14.679
0.850	1.8419	1.9293	2.0892	2.2254	2.7616	3.2699	3.7439	4.1895	18.609
0.875	2.0668	2.1459	2.2954	2.4265	2.9345	3.4302	3.8971	4.3467	24.613
0.900	2.3648	2.4369	2.5786	2.6958	3.1772	3.6359	4.0879	4.5258	34.598
0.925	2.7934	2.8642	2.9870	3.0924	3.5238	3.9416	4.3571	4.7636	53.585
0.950	3.5130	3.5686	3.6676	3.7612	4.1280	4.4832	4.8349	5.1895	99.033
0.975	5.1035	5.1410	5.2105	5.2791	5.5416	5.8064	6.0576	6.3259	281.77

Table A-4. MSE(T) geometric correction factor, β , values [7].

H/W												
a/W	2	3	4	4.5	5	5.5	6	6.5	7	8	9	10
0.00	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150	1.12150
0.10	1.12523	1.14394	1.15342	1.15661	1.15919	1.16127	1.16306	1.16454	1.16586	1.16793	1.16961	1.17091
0.15	1.13343	1.17223	1.19249	1.19943	1.20501	1.20961	1.21352	1.21680	1.21967	1.22430	1.22798	1.23090
0.20	1.14345	1.20703	1.24154	1.25354	1.26330	1.27140	1.27825	1.28410	1.28920	1.29751	1.30408	1.30938
0.25	1.15555	1.24704	1.29893	1.31733	1.33245	1.34510	1.35589	1.36515	1.37322	1.38651	1.39709	1.40564
0.30	1.17062	1.29170	1.36377	1.38993	1.41166	1.43005	1.44582	1.45945	1.47140	1.49125	1.50716	1.52012
0.35	1.18985	1.34097	1.43559	1.47084	1.50053	1.52588	1.54784	1.56698	1.58387	1.61216	1.63506	1.65388
0.40	1.21468	1.39513	1.51423	1.55985	1.59881	1.63251	1.66198	1.68789	1.71096	1.75000	1.78193	1.80845
0.45	1.24665	1.45486	1.59973	1.65682	1.70642	1.74985	1.78826	1.82241	1.85305	1.90554	1.94904	1.98558
0.50	1.28757	1.52112	1.69233	1.76192	1.82327	1.87781	1.92666	1.97057	2.01037	2.07946	2.13758	2.18702
0.55	1.33956	1.59535	1.79257	1.87521	1.94932	2.01618	2.07688	2.13213	2.18274	2.27192	2.34819	2.41405
0.60	1.40538	1.67957	1.90135	1.99713	2.08452	2.16461	2.23834	2.30634	2.36938	2.48224	2.58061	2.66695
0.65	1.48886	1.77679	2.02035	2.12864	2.22918	2.32276	2.41022	2.49194	2.56867	2.70840	2.83274	2.94390
0.70	1.59575	1.89183	2.15268	2.27193	2.38446	2.49088	2.59175	2.68735	2.77825	2.94683	3.10017	3.23999
0.75	1.73562	2.03299	2.30460	2.43192	2.55397	2.67109	2.78371	2.89187	2.99605	3.19278	3.37580	3.54623
0.80	1.92614	2.21619	2.48940	2.62033	2.74757	2.87134	2.99190	3.10918	3.22350	3.44320	3.65215	3.85083
0.85	2.20485	2.47644	2.73863	2.86660	2.99241	3.11620	3.23816	3.35805	3.47624	3.70687	3.93062	4.14762
0.90	2.67101	2.90858	3.14207	3.25761	3.37220	3.48593	3.59899	3.71107	3.82253	4.04266	4.25978	4.47372

Table A-6. MSE(T) and SE(T) geometric correction factor, β^T , values [10].

	H/W*					
a/W	0.8	1	2	4	10	Pinned ^T
0.010	1.1203	1.1209	1.1204	1.1209	1.1209	1.1163
0.020	1.1167	1.1191	1.1206	1.1223	1.1230	1.1162
0.030	1.1096	1.1149	1.1204	1.1242	1.1261	1.1190
0.040	1.0995	1.1085	1.1200	1.1266	1.1302	1.1244
0.050	1.0868	1.1002	1.1191	1.1293	1.1353	1.1318
0.075	1.0467	1.0727	1.1156	1.1377	1.1516	1.1577
0.100	0.9985	1.0379	1.1098	1.1479	1.1725	1.1910
0.125	0.9464	0.9986	1.1017	1.1592	1.1975	1.2295
0.150	0.8937	0.9570	1.0914	1.1713	1.2261	1.2723
0.175	0.8426	0.9147	1.0790	1.1837	1.2580	1.3194
0.200	0.7946	0.8729	1.0647	1.1958	1.2928	1.3716
0.225	0.7507	0.8328	1.0488	1.2073	1.3302	1.4299
0.250	0.7113	0.7949	1.0313	1.2178	1.3701	1.4955
0.275	0.6765	0.7597	1.0127	1.2269	1.4119	1.5699
0.300	0.6460	0.7274	0.9930	1.2342	1.4555	1.6543
0.325	0.6195	0.6980	0.9726	1.2396	1.5002	1.7498
0.350	0.5965	0.6716	0.9518	1.2428	1.5457	1.8576
0.375	0.5763	0.6478	0.9306	1.2435	1.5913	1.9787
0.400	0.5586	0.6265	0.9093	1.2416	1.6363	2.1141
0.425	0.5426	0.6075	0.8881	1.2371	1.6800	2.2651
0.450	0.5281	0.5905	0.8671	1.2297	1.7216	2.4334
0.475	0.5147	0.5751	0.8464	1.2196	1.7601	2.6214
0.500	0.5022	0.5612	0.8262	1.2067	1.7945	2.8322
0.525	0.4905	0.5485	0.8064	1.1911	1.8240	3.0704
0.550	0.4796	0.5368	0.7872	1.1729	1.8475	3.3423
0.575	0.4694	0.5260	0.7685	1.1523	1.8640	3.6563
0.600	0.4602	0.5159	0.7505	1.1295	1.8728	4.0234
0.625	0.4519	0.5065	0.7330	1.1047	1.8731	4.4578
0.650	0.4445	0.4976	0.7161	1.0783	1.8641	4.9775
0.675	0.4379	0.4893	0.6998	1.0506	1.8455	5.6049
0.700	0.4317	0.4816	0.6841	1.0221	1.8171	6.3675
0.725	0.4251	0.4743	0.6691	0.9930	1.7790	7.2987
0.750	0.4170	0.4674	0.6550	0.9640	1.7316	8.4386

^TThe definition of β differs from that used in the present work.

*The authors have converted the data presented in [10] to the definitions used herein.

^TPinned data for H/W $\geq$ 2.0.

Table A-7. Relative difference between MSE(T) and SE(T), β^* , values [10] and present work (%).

a/W	H/W					Pinned
	0.8	1	2	4	10	
0.010	12.69	8.95	-0.26	-0.83	-0.39	-0.52
0.020	11.94	8.58	-0.23	-0.79	-0.37	-0.74
0.030	11.16	8.17	-0.22	-0.77	-0.35	-0.82
0.040	10.39	7.73	-0.21	-0.75	-0.33	-0.79
0.050	9.62	7.28	-0.21	-0.74	-0.31	-0.69
0.075	7.77	6.12	-0.24	-0.73	-0.28	-0.27
0.100	6.04	4.99	-0.30	-0.72	-0.26	0.17
0.125	4.40	3.89	-0.37	-0.72	-0.25	0.46
0.150	2.82	2.84	-0.45	-0.71	-0.25	0.58
0.175	1.30	1.81	-0.54	-0.70	-0.26	0.52
0.200	-0.13	0.82	-0.62	-0.68	-0.26	0.34
0.225	-1.45	-0.14	-0.70	-0.66	-0.27	0.11
0.250	-2.59	-1.05	-0.78	-0.64	-0.28	-0.10
0.275	-3.54	-1.89	-0.85	-0.63	-0.28	-0.24
0.300	-4.28	-2.64	-0.92	-0.61	-0.27	-0.29
0.325	-4.79	-3.28	-0.99	-0.59	-0.25	-0.25
0.350	-5.11	-3.80	-1.05	-0.57	-0.24	-0.14
0.375	-5.25	-4.19	-1.10	-0.56	-0.23	0.02
0.400	-5.27	-4.46	-1.15	-0.54	-0.21	0.19
0.425	-5.21	-4.60	-1.19	-0.52	-0.20	0.32
0.450	-5.11	-4.65	-1.21	-0.50	-0.19	0.40
0.475	-4.99	-4.60	-1.22	-0.48	-0.19	0.41
0.500	-4.88	-4.48	-1.22	-0.45	-0.19	0.34
0.525	-4.77	-4.31	-1.20	-0.43	-0.19	0.21
0.550	-4.66	-4.08	-1.17	-0.41	-0.19	0.06
0.575	-4.51	-3.83	-1.13	-0.39	-0.19	-0.07
0.600	-4.29	-3.55	-1.08	-0.38	-0.18	-0.15
0.625	-3.98	-3.25	-1.01	-0.36	-0.16	-0.14
0.650	-3.55	-2.92	-0.95	-0.36	-0.14	-0.04
0.675	-3.03	-2.55	-0.88	-0.35	-0.12	0.14
0.700	-2.50	-2.16	-0.81	-0.34	-0.11	0.30
0.725	-2.10	-1.73	-0.74	-0.33	-0.11	0.29
0.750	-2.11	-1.28	-0.66	-0.31	-0.13	-0.09

*Comparisons are made based upon the definition of β presented in [10].

ACKNOWLEDGEMENTS

This material is based on research sponsored by the US Air Force Academy under agreement number FA7000-11-2-0011. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon.

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