1 Title:
2 **Modeling frictional melt injection to constrain coseismic physical conditions**

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8 Abstract:

9 Pseudotachylite, a fault rock formed through coseismic frictional melting, provides an
10 important record of coseismic mechanics. In particular, injection veins formed at a high angle to
11 the fault surface have been used to estimate rupture directivity, velocity, pulse length, stress
12 drop, as well as slip weakening distance and wall rock stiffness. These studies have generally
13 treated injection vein formation as a purely elastic process and have assumed that processes of
14 melt generation, transport, and solidification have little influence on the final vein geometry.
15 Using a pressurized crack model, an analytical approximation of injection vein formation based
16 on dike intrusion, we find that the timescales of quenching and flow propagation may be similar
17 for a subset of injection veins compiled from the Asbestos Mountain Fault, USA, Gole Larghe
18 Fault Zone, Italy, and the Fort Foster Brittle Zone, USA under minimum melt temperature
19 conditions. 34% of the veins are found to be flow limited, with a final geometry that may reflect
20 cooling of the vein before it reaches an elastic equilibrium with the wall rock. Formation of these
21 veins is a dynamic process whose behavior is not fully captured by the analytical approach. To
22 assess the applicability of simplifying assumptions of the pressurized crack we employ a time-
23 dependent finite-element model of injection vein formation that couples elastic deformation of
24 the wall rock with the fluid dynamics and heat transfer of the frictional melt. This finite element

model reveals that two basic assumptions of the pressurized crack model, self-similar growth and a uniform pressure gradient, are false. The pressurized crack model thus underestimates flow propagation time by 2-3 orders of magnitude. Flow limiting may therefore occur under a wider range of conditions than previously thought. Flow-limited veins may be recognizable in the field where veins have tapered profiles or smaller aspect ratios than expected. The occurrence and shape of injection veins can be coupled with modeling to provide an independent estimate of minimum melt temperature. Finally, the large aspect ratio observed for all three populations of injection veins may be best explained by a large reduction in stiffness associated with coseismic damage, as injection vein growth is likely to far exceed the lifetime of dynamic stresses at any location along a fault.

Keywords:

Fault; Earthquake; Pseudotachylyte; Injection vein; Frictional melt; Coseismic environment;

1 Introduction

Earthquakes occur when a fault patch suddenly slips (ruptures), releasing high frequency ($> \sim 0.1$ Hz) seismic waves. Though a defining characteristic of earthquakes, radiated seismic waves contain only a small portion of the total earthquake energy, and provide an incomplete view of the earthquake source region (Beeler, 2006). Much of the energy released is absorbed by physicochemical processes (Shipton et al., 2006) driven by slip-induced changes in stress and temperature within the fault zone (Reches and Dewers, 2005; Sibson, 1973). By integrating theoretical and experimental modeling and field studies we can obtain a more complete view of

the active physicochemical processes, parameters, and physics at the earthquake source (Niemeijer et al., 2012).

Studies of pseudotachylyte injection veins (Fig. 1), opening-mode fractures originating from a fault surface filled with solidified frictional melt (Sibson, 1975), have been particularly successful at recovering earthquake source parameters from exhumed fault zones. In order to form frictional melt, slip rates at seismogenic depth must exceed $\sim 10^{-2}$ m/s (Sibson, 1973, 1975), within the range of rates that is unique to earthquakes (Cowan, 1999; Rowe and Griffith, 2015). The rapid quench times of these thin melt veins, ~ 100 seconds or less (Rowe et al., 2012; Sibson, 1975), is on the order of the source duration for moderate to large earthquakes (Kanamori and Brodsky, 2004), suggesting that they solidify during seismic slip or soon after slip ceases. Pseudotachylyte injection veins thus form under the physical conditions present during and immediately after earthquake slip, and their crack-like shape provides a well-developed theoretical framework for interpreting these conditions.

An evolving conceptual model for the formation of pseudotachylyte injection veins (Di Toro et al., 2005a; Griffith et al., 2012; Griffith and Prakash, 2015; Sibson, 1975) suggests a general process whereby 1) a propagating rupture front creates a dynamic stress field that generates tensile cracks. 2) Slip along the fault produces frictional melt. 3) Melt flows from the fault surface into the recently formed cracks due to pressure gradients generated by an anisotropic stress tensor (Fig. 2). 4) The melt solidifies preserving the coseismic structural geometries.

This conceptual model has been coupled with theoretical and experimental work to estimate a variety of source parameters and conditions from measurement of pseudotachylyte injection veins along exhumed faults. Rupture directivity and velocity (Di Toro et al., 2005a), as well as velocity weakening behavior (Ngo et al., 2012), have been inferred from the inclination

of injection veins relative to the fault surface. Rupture size, stress drop, and fracture energy have been found using injection vein length (Griffith and Prakash, 2015). The near-fault stress field (Rowe et al., 2012) and coseismic wall rock softening (Griffith et al., 2012) have been investigated using length to aperture ratios.

These studies generally assume that injection veins preserve coseismic fracture orientations and lengths and that melt flows freely to the end of cracks before quenching. Rowe et al. (2012) evaluated this assumption using an approximate analytical solution derived by Rubin (1995b) for modeling the intrusion of igneous dikes and found that quench times far exceed flow propagation times for a set of likely physical conditions. Here we revisit this analytical model and show that flow limiting (i.e. quenching before reaching equilibrium) may occur under some conditions. We then develop a more complete finite element model that couples elastic wall rock deformation, viscous flow of the melt, and cooling. Using this model we find that key assumptions of the analytical solution are likely unrealistic and that flow limiting may therefore occur under a wider range of conditions than previously thought.

2 Field Surveys of pseudotachylyte injection veins

Two previous studies have cataloged length and base aperture of pseudotachylyte injection veins. Rowe et al. (2012) report 215 tonalite-hosted injection veins from the Asbestos Mountain Fault, USA. Griffith et al. (2012) report two sets of injection veins from the Gole Larghe Fault Zone, Italy: 22 cataclasite-hosted and 28 tonalite-hosted. We combine these with a new set of 49 gneiss-hosted injection veins from the Fort Foster Brittle Zone, USA to form a composite data set of 314 injection veins used in this analysis (Fig. 3). Aperture and length data from all three sites overlap, with aspect ratios (ratio of aperture to length) ranging from 0.014 to 1.94, generally larger than aspect ratios typical of magmatic dikes (10^{-2} to 10^{-4} , (Rubin, 1995a)).

The composite data set also reveals a weak trend toward decreasing aspect ratio with length. Here we briefly describe each locality, while noting that factors relevant to injection vein formation (Table 1) are similar among all three sites.

2.1 Fort Foster Brittle Zone

The Fort Foster brittle zone has been the focus of studies by Swanson (2006; and references therein) who used the pseudotachylyte bearing fault surfaces to infer earthquake rupture geometries. The zone is defined by abundant strike-slip fault surfaces developed within a narrow (~50 m) zone of mylonitic dextral shear. Pseudotachylyte-bearing faults are most common within fine-grained amphibolitic ultramylonites (Swanson, 1988). The continuum of ductile to brittle deformation (Swanson, 1988) and the dominance of adhesive wear mechanisms (Swanson, 1992) suggest that the fault was active at a depth approaching the brittle-ductile transition (~12 km). Deformation within the fault zone has been correlated to the later phases of the regional scale Norumbega fault (Bothner and Hussey, 1999; Swanson, 2006), which was active from Late Devonian (~380 Ma) to Permian (~280 Ma) (West, 1999).

Pseudotachylyte within the Fort Foster brittle zone is dark grey and glassy with sharp wall-rock contacts. Pseudotachylyte-bearing structures form an intersecting network including fault veins parallel to the mylonitic layering as well as abundant obliquely oriented fault veins, localized dilatant reservoirs, and injection veins. Slip along individual layer-parallel fault veins is difficult to measure, but cumulative offsets of linking structures for several faults sum to a minimum of several meters (Swanson, 2006).

We measured a set of 49 injection veins from the northwest boundary zone, a well-exposed 40-m long section of ultramylonite that has an especially high density of pseudotachylyte in dominantly layer-parallel fault veins with relatively few oblique and

connecting structures (Swanson, 2006). We selected injection veins along isolated layer-parallel fault veins to minimize effects of nearby structures. Field measurements of injection vein length, base aperture, and fault vein thickness were made in the plane of the outcrop. The nearly horizontal exposure is approximately fault perpendicular and slip parallel. We constructed a photorealistic model of the outcrop using the structure from motion methodology (Johnson et al., 2014) to correct field measurements to true lengths and thicknesses. Vein traces were projected into a plane perpendicular to the average intersection line between fault and injection veins. Injection vein aperture was measured at the base of the vein parallel to the local fault surface. Injection vein length was measured as the straight-line distance from the midpoint of the aperture on the fault vein to the tip, as at other locations (Griffith et al., 2012; Rowe et al., 2012).

2.2 Asbestos Mountain fault

The Santa Rosa Mylonite Zone of the Eastern Peninsular Ranges is a classic region for the study of large strain mylonitic deformation (Goodwin and Renne, 1991; Wenk, 1998). The upper part of the sequence contains a system of spatially associated faults, including the east-dipping Asbestos Mountain Fault (Todd et al., 1988). Here, abundant pseudotachylyte has been used to investigate the seismic record and deformation processes (Jiang et al., 2015; Kirkpatrick and Rowe, 2013; Rowe et al., 2012; Wenk et al., 2000). The youngest phase of brittle deformation can be attributed to late Miocene extension (Axen and Fletcher, 1998).

Fault parallel pseudotachylyte veins occur concentrated in 3 -5 m wide zones in the hanging wall (tonalitic to granodioritic Asbestos Mountain Suite) and are subparallel to foliation but locally crosscut both foliation and pegmatite veins (Rowe et al., 2012). Fault veins are up to

10 mm thick, thinner where off fault injection veins are common (Rowe et al., 2012; Wenk et al., 2000).

2.3 Gole Larghe Fault Zone

The pseudotachylyte veins of the Gole Larghe Fault Zone have provided data and served as a natural analog for both experimental and theoretical studies of earthquake source parameters and physics (Di Toro et al., 2005a; Di Toro and Pennacchioni, 2004; Ngo et al., 2012; Pittarello et al., 2008). This right-lateral strike-slip zone is composed of hundreds of fault strands, spanning a ~500 m wide swath running east-west through the Adamello tonalitic batholith and accommodating ~1100m of slip in total (Di Toro and Pennacchioni, 2005). Pseudotachylyte occurs as millimeter to centimeter thick fault veins often located at the boundaries of cataclastic horizons (Di Toro and Pennacchioni, 2005). Pseudotachylyte veins are easily identified by sharp boundaries and a uniform black color with a lighter inner layer in thicker (>6mm) veins (Di Toro and Pennacchioni, 2005).

3 Pressurized crack model

Injection vein geometry has been modeled as a finite, blade-like crack in an infinite medium (Rowe et al., 2012) or an edge crack in a semi-infinite medium (Griffith et al., 2012). In both cases, the final geometry is determined by elastic equilibrium between the internal pressure of the vein and remote stresses. To achieve this state of elastic equilibrium, the time to flow melt to the end of an opening crack (flow propagation time) must be shorter than the time to quench the melt. If the timescales for flow propagation and quenching are similar, coupling of these two processes may result in a final vein geometry that reflects an intermediate, flow-controlled state rather than elastic equilibrium. Here we estimate these relative timescales using a modified version of a 2D pressurized crack model of dike injection (Rubin, 1995b) to evaluate if

flow limiting might occur within the range of possible thermal conditions for pseudotachylite-bearing faults. In this analysis we follow the approach of Rowe et al. (2012), noting major differences in individual subsections as well as in the summary (section 3.4). Key parameters and results of this analysis are summarized in Table 2.

3.1 Vein geometry and driving stress

The aspect ratio of a linear elastic mode-I fracture is approximately equal to the elastic strain developed in the wall rock (Freund, 1998; Lawn, 1993), and can be related to the balance between fluid pressure acting to open the crack and wall rock stress acting to close it. Under uniform stress in an isotropic, linear elastic medium, the maximum opening (a) of an edge crack with length l is given by (Tada et al., 2000):

$$a = 1.454 * \frac{\Delta P}{M} 2l \quad (1)$$

where the driving pressure (ΔP) is the difference of the melt pressure and the remote stress perpendicular to the injection vein. For a high-angle injection vein, the pressure of the melt along the fault (P_{melt}) is approximately equal to the fault normal stress (σ_y) (Nielsen et al., 2008) and the remote stress perpendicular to the vein is the fault parallel stress (σ_x). The stiffness (M) is, in the case of plane strain, given by $G/(1 - \nu)$ where ν is the Poisson's ratio, and G is the shear modulus. We use an edge crack formulation rather than the whole space solution used by Rowe et al. (2012) as it is more appropriate for an injection vein propagating from a melt-filled fault (Griffith et al., 2012).

Using Equation 1 on individual measurements of aperture and length we estimate the ratio $\Delta P/M$ for each injection vein (Table 2), as the driving pressure and stiffness are not independently constrained. Values of $\Delta P/M$ range from 4.7×10^{-3} to 6.7×10^{-1} with a median of 5.1×10^{-2} (Table 2). This requires either that ΔP be significantly greater than reasonable local

differential stress, that M be significantly lower than laboratory measurements of intact rock, or a combination of the two.

Assuming intact stiffness ($M = 25$ GPa), values for ΔP range from 1.2×10^2 to 1.7×10^4 MPa with a median of 1.3×10^3 MPa. Assuming 80% softening ($M = 5$ GPa, within range of damage-induced softening (Griffith et al., 2009)), values for ΔP range from 2.4×10^1 to 3.3×10^3 MPa with a median of 2.5×10^2 MPa. We prefer the second case as it does not require high tensile stress and results in lower crack tip stress intensity factors (Table 2).

3.2 Flow propagation

The flow propagation velocity of melt in an injection vein can be derived based on the analogous estimation for magma flow in dikes (Rubin, 1995b). This analysis assumes that the velocity of the melt front at the tip of the injection vein is approximately equal to the average inflow velocity at the base of the injection vein and that the geometry of the vein remains self-similar during growth.

Flow at base of the injection vein is approximated as plane Poiseuille flow, which assumes pressure driven laminar flow between two parallel plates. The average inflow velocity is found by integrating over the width of the opening (Batchelor, 1967):

$$u_{avg} = \frac{a^2}{12\mu} \frac{dP}{dl} \quad (2)$$

For a crack of finite out-of-plane length, this approximation represents an upper limit for the flow velocity and therefore a lower limit on the flow propagation time. The channel width is taken as the aperture (a) at the base of the injection vein and the viscosity (μ) and pressure gradient along the length of the injection vein (dP/dl) are assumed to be constant. The pressure gradient thus simplifies to $(P_{fault} - P_{tip})/l$ where P_{tip} is the pressure in the small

cavity between the melt front and the fracture tip (Fig. 2) that exists due to the viscosity and surface tension of the melt (Barenblatt, 1962). For dikes, this cavity is most likely filled by exsolved magma volatiles or pore fluids at low pressure (Rubin, 1993).

Rubin (1995b) and Rowe et al. (2012) use $P_{tip} = \sigma_x$, a valid assumption for compressive stress normal to injection veins. However, in cases where σ_x is tensile, possible for injection veins in a coseismic stress state, this assumption implies a negative absolute pressure. We therefore use $P_{tip} = 0$ and the pressure gradient term reduces to P_{fault}/l . This formulation keeps our solution in terms of $\Delta P/M$, but will conservatively underestimate the propagation time if $P_{tip} > 0$. Substituting this term for the pressure gradient and Equation 1 for the aperture into Equation 2 gives a modification of Rubin's (1995b) Equation 8 for the flow propagation velocity:

$$v_{tip} = \frac{1}{1.42\mu} (\Delta P/M)^2 (P_{fault}) l \quad (3)$$

By equating $v = dl/dt$ and assuming an initial crack length l_0 , integrating Equation 3 yields the flow propagation time for an injection vein of final length l_f :

$$t_{flow}(l_f) = 1.42 \frac{\mu \ln(l_f/l_0)}{(\Delta P/M)^2 P_{fault}} \quad (4)$$

The injection vein must initiate on a pre-existing defect with length (l_0) in order to propagate because the velocity approaches zero as length does. Physically, this defect is expected to be on the order of grain or asperity size. A length of 100 microns is used for the initial defect here. The final flow time is relatively insensitive to the exact choice of initial length.

A constant melt viscosity (μ) is estimated by modeling temperature-viscosity relationships (Giordano et al., 2008) for each melt composition and selecting a representative value. A significant overstep of the melt temperature is possible during frictional melting due to the high

rate of heat production and non-equilibrium nature of the melting process (Dixon and Dixon, 1989). In this analysis, however, we chose a lower limit for the initial melt temperature of 1200 °C, based on average frictional melt temperature estimates (Sibson and Toy, 2006). During flow the melt is cooling and its viscosity will range from 10^2 to 10^7 Pa s before quenching at the solidus (~700°C). Most flow will occur early while the temperature is high and thus a representative viscosity at 1100 °C is used.

The flow propagation time for each injection vein is calculated by Equation 4 using $\Delta P/M$ from Section 3.1. The propagation time ranges from 7.0×10^{-5} to 6.4×10^0 s with a median of 2.64×10^{-2} s (Fig. 4, Table 2).

3.3 Quenching

The solidification of an injection vein is approximated as the 1D cooling of an infinite planar body. The quenching time for a body at its melt temperature is given by (Turcotte and Schubert, 2002):

$$t_{quench}(a) = \frac{(a/4)^2}{4\alpha\lambda^2} \quad (5)$$

where a is the aperture at the widest point, α is the thermal diffusivity (10^{-6} m²/s for gneiss (Eriksson et al., 2003)), and λ is a dimensionless constant determined by the thermal conditions. λ is equal to 0.71 given an ambient temperature of 250 °C (corresponding to ~10 km depth), melt initial temperature of 1200 °C, latent heat of 300 kJ/kg, and specific heat of 1.2 kJ/kg K. As the initial temperature of the melt may be significantly higher, our estimate is approaching a lower limit on quench time.

The amount of melt that needs to solidify in order to halt propagation is not well constrained. Rowe et al. (2012) estimate that the geometry will be locked in once the

solidification front is half way to the center of the injection vein, a distance of $a/4$. For an elliptic opening profile, this condition is equivalent to the complete quenching of the tip for 15% of the injection vein length. For consistency, we use the same condition.

Using Equation 5, the quench time is calculated for each injection vein, ranging from 6.3×10^{-3} to 5.0×10^1 s, with a median of 7.4×10^{-1} s (Table 2). Since quench time is only dependent on aperture for a given set of thermal conditions, equation 5 describes a straight line with a slope of two in Fig. 4.

3.4 Potential for flow limiting

Timescales of flow propagation and quenching are similar for many injection veins in the composite data set, especially in the Fort Foster and Gole Larghe populations. This difference is indicated in Fig. 4 by the vertical distance between a point (a specific injection vein's flow propagation time) and the quench time line – points falling within the shaded regions indicate an injection vein whose flow and quench times are within an order of magnitude.

28 injection veins have overlapping timescales, meaning the estimated flow propagation time is longer than the quench time – a physical impossibility that indicates these veins would not have reached equilibrium before quenching under the assumed thermal conditions. An additional 79 (totaling 34% of the composite data set) have quench times less than one order of magnitude greater than their propagation times. For this subset of injection veins, the final geometry may reflect a flow-limited state, and the pressurized crack model may not adequately describe their formation process.

Rowe et al. (2012) reached the opposite conclusion after finding that quench times exceeded flow propagation times by 3 to 6 orders of magnitude for the Asbestos Mountain injection veins. While this location has the greatest difference in timescales of those used here,

we still find 33.5% of the injection veins have quench times within an order of magnitude of their propagation times.

Five main differences in our analysis explain this discrepancy. First, we use the edge crack, rather than interior crack, formulation for aperture (Equation 1), resulting in a factor of 1.454 discrepancy of $\Delta P/M$. Second, as the viscosity and flow propagation time are directly related, our times are approximately an order of magnitude greater than those of Rowe et al. (2012) due to higher viscosities associated with assuming a lower initial melt temperature. Third, Rowe et al. (2012) determine the along-vein pressure gradient using a total pressure drop ($P_{fault} - P_{tip}$) equal to ΔP by assuming $P_{tip} = \sigma_x$, which implies a negative tip pressure for tensile remote stress states. We set $P_{tip} = 0$, and thus a total pressure drop of σ_y . With $\sigma_y = 100$ MPa this change reduces the pressure gradient and increases the flow propagation time by approximately an order of magnitude. Fourth, Rowe et al. (2012) use the propagation velocity at the final length as the average propagation velocity, rather than integrating over time. Depending on length, the final velocity is 5 to 10 times larger than the average. Fifth, quenching times are about half those of Rowe et al. (2012) from the use of a larger lambda (0.71 instead of 0.5), based on conditions deemed more representative of the assumed thermal environment.

4 Numerical model

Our results in Section 3 suggest that coupling between melt temperature, viscosity, and rock deformation may create a dynamic process whose behavior is not fully captured by the analytical approximations of the pressurized crack models. To investigate these effects, we employ a time-dependent finite-element model of injection vein formation with explicit modeling of a temperature dependent viscous melt. The model is implemented using the

COMSOL Multiphysics® 5.2 finite-element software package. In the following section we briefly detail model physics, geometry, initial and boundary conditions, and numerical implementation.

The numerical model has a large number of input parameters that reflect the coseismic environment. Rather than undertaking a full exploration of this parameter space we have chosen to model a representative injection vein from the composite data set with a final length of 20 mm and an aperture of 3mm. Where specific parameters are required we have chosen values from the Fort Foster location, although similar conditions of the Asbestos Mountain Fault and Gole Larghe Fault Zone allow for comparison. The results of the pressurized crack model outlined in section 3 indicate that formation of this representative injection vein requires a high driving pressure to wall rock stiffness ratio ($\Delta P/M$). These conditions are simulated in the numerical model with a relatively soft wall rock and a uniaxial load equivalent to lithostatic stress.

4.1 Geometry

An injection vein is modeled as a 20 mm-long pre-existing edge crack within a uniform block of wall rock (Fig. 5). Only one half of the space is modeled, as the geometry is symmetric about the long axis of the injection vein. The wall rock and fault vein are in contact along the lower boundary. The fault vein is not explicitly modeled, but rather represented using spatially uniform, time dependent boundary conditions on the lower boundaries of the wall rock and injection vein where they would be in contact with the fault vein. To avoid the topologic challenge of modeling the propagating fluid-solid boundary explicitly, the injection vein is modeled as a 200 micron thick pre-existing fluid domain along the left symmetry boundary

315 4.2 Material properties

316 The solid domain is uniform, isotropic, linear-elastic with a Young's modulus of 9.6 GPa
317 and Poisson's ratio of 0.2, or $M = 5$ GPa, reflecting 80% softening from intact stiffness, and
318 thermal conductivity of 3.24 W/m K . The fluid domain is linear viscous with a temperature-
319 dependent viscosity. We approximate the solid and fluid domains as having uniform density and
320 thermal diffusivity, and neglect variation with temperature and pressure.

321 The viscosity of the melt as a function of temperature is calculated from Fort Foster
322 pseudotachylyte composition using the method of Giordano et al. (2008). The solidus is
323 calculated from the composition using MELTS (Ghiorso and Sack, 1995). Model viscosities for all
324 field locations are within a factor of ~ 2 throughout the temperature range and are similar to
325 those used by Rowe et al. (2012). For numerical stability, the melt viscosity is capped at 10^6 Pa s ,
326 approximately the predicted viscosity at solidus, 700°C . Therefore, solidified melt is treated as a
327 highly viscous fluid. A latent heat of crystallization of 300 kJ/kg is approximated as an increase in
328 specific heat near the solidus.

329 4.3 Thermal boundary and initial conditions

330 The entire domain is initially at the ambient temperature, T_{amb} . At this temperature, the
331 melt in the pre-existing fracture is highly viscous and behaves essentially as a solid. Thermal
332 boundary conditions are summarized in italic text on Fig. 5. The heat flux is set to zero on the
333 left and right boundaries to impose symmetry. The top boundary is held at T_{amb} . The boundary
334 condition for the lower boundary simulates contact with a fault vein of finite thickness that is
335 initially at a higher temperature, $T_{melt} = T_{amb} + \Delta T$. Assuming the fault vein is planar, and the
336 thermal properties of the melt and wall rock are similar, the temperature of the fault vein

($T_{FV}(y, t)$) can be solved for analytically assuming a Dirac delta function initial condition and 1D thermal diffusion perpendicular to the fault vein:

$$T_{FV}(y, t) = T_{amb} + \frac{\Delta T_{FV_t}}{2\sqrt{\pi\alpha(t+t_1)}} \exp\left(\frac{-y^2}{4\alpha(t+t_1)}\right) \quad (6)$$

where t_1 is a time constant and FV_t is the fault vein thickness (Eq. 4-170, Turcotte and Schubert, 2002). The temperature difference and fault vein thickness are proportional to the total excess thermal energy available. As the majority of melt flowing into the injection vein will come from the center of the fault vein (due to a viscosity-gradient enhanced laminar parabolic flow field) the temperature of melt inflow is set using Equation 6 with $y = 0$. Similarly, the wall rock contact uses Equation 6 with $y = FV_t/2$ to simulate the edge of the fault vein. Letting $t_1 = FV_t^2/4\pi\alpha$ sets the initial temperature equal to the melt temperature, $T_{FV}(0,0) = T_{melt}$. A gradient smooths the transition between these boundary conditions near the injection vein.

The median fault vein thickness, measured at the base of injection veins, at Fort Foster is 1.7 mm. Similar values are reported from the other two locations. This number represents the final thickness of the fault vein melt reservoir, reduced from the coeval thickness by melt loss to off-fault features (Di Toro et al., 2005b; Sibson, 1975). Taking this reduction into account, we use a fault vein thickness of 5 mm for this temperature calculation.

4.4 Mechanical boundary and initial conditions

Mechanical boundary conditions, summarized regular text on Fig. 5, are used to simulate the domain's location as a small section of a uniform linear fault with no nearby off-fault features. The left side has symmetry mechanical boundary conditions (simulating the other half of the injection vein). The right side is subject to the remote stress (σ_x), reflecting the continuation of the fault. The fault boundary is represented by unconstrained displacement and

an applied stress from the melt pressure (P_{fault}). A remote stress $\sigma_y = 250 \text{ MPa}$, loosely simulating lithostatic stress at seismogenic depth, and $\Delta P = 250 \text{ MPa}$ were imposed. Therefore, $\sigma_x = 0 \text{ MPa}$, $P_{fault} = 250 \text{ MPa}$, and $\Delta P/M = 0.05$.

The boundaries between the melt and wall rock couple the behavior of the two domains. This interface is subject to a normal load equal to the melt pressure and a shear load from the viscous effect of fluid flow. A no-slip condition on this interface sets the fluid velocity equal to the velocity of the boundary, which deforms with the solid domain.

In order to create a defect that can be exploited by melt to initiate crack growth (Rubin, 1995a), a low viscosity boundary condition is also set along the symmetry boundary in the fluid. The viscosity at the boundary is set to equal the current fault vein viscosity. Over a distance of 10^{-4} m away from the boundary it increases linearly to the thermally correct viscosity. During growth, melt propagates down the axis of the injection vein, exploiting this defect, and then expands laterally, displacing the pre-existing fluid with the wall rock. The incoming melt fluid is at approximately the fault melt temperature, so this condition has minimal effect on the fluid flow once the melt front has passed an area. Without such a defect, the front of high viscosity melt at initialization effectively blocks the initiation of fluid flow into the fracture as if it were a continuous solid boundary.

4.5 Numerical implementation

A finite element implementation of this model was created using COMSOL Multiphysics® 5.2. All calculations use streamline and crosswind diffusion to increase stability and convergence. The thermal equations are also stabilized using isotropic diffusion with a coefficient of one.

The boundary loads, set to zero for the initial time step to improve convergence, are ramped to their steady state value over 0.07 s, approximately 2 to 10 % of the formation times of model runs. The maximum viscosity limit is also increased from 10^3 to 10^6 Pa s over the same period. Solutions were computed with the fully coupled linear solver using the PARDISO method (Schenk and Gartner, 2004) and backward differentiation.

The approximation of the solidified melt phase as a highly viscous fluid means a small amount of erroneous flow will continue to occur even as the temperature falls below the solidus. As cooling progresses, flow velocities in the cooling and shrinking liquid melt region decrease, and the solidified melt region expands, resulting in an increase of erroneous solidified melt flow relative to liquid melt flow. Changes in the overall flow pattern during this transition are used as a stop condition to end the model run. Models were therefore stopped when the second derivative of inflow velocity is zero.

4.6 Numerical Model Results

The model was run for a range of initial melt temperatures between 1150 and 1500 °C. The opening profile of the 20 mm long fracture during the formation of injection veins is shown in Fig. 6a. The final profile is taken when the model stop condition is reached, and represents the final geometry of the injection vein. Intermediate profiles are given at uniform intervals of 0.5 s. Aperture and length at intermediate times show an increase in aspect ratio during growth and at higher initial temperatures (Fig. 7). Below an initial temperature of ~1200 °C the opening is negligible and injection vein formation does not occur, indicating a minimum melt temperature for these environmental conditions. Above some threshold temperature (~1500 °C in this case) flow is able to fully exploit the 20 mm fracture and the injection vein reaches elastic equilibrium. The range of intermediate temperatures, ~1200 °C to 1500 °C is where injection veins may form,

but are flow limited (i.e. quenching before reaching equilibrium) under the assumed geometry and loading conditions.

Stress and flow velocity fields from intermediate times of the 1500 °C run (Fig. 6b) show the growth of an injection vein to elastic equilibrium. Flow velocity is greatest along the centerline near the base of the vein and decreases toward the vein margins and toward the vein tip due to viscous drag and cooling of the melt near the vein boundary. The basal centerline velocity increases throughout the simulation as the growing aperture reduces the effects of viscous drag. The vein creates a moderate stress concentration (~350 MPa at 3.0 s) during propagation that rapidly increases when the flow reaches the tip of the pre-existing fracture (4.0 s). The tip velocity, mean inflow velocity, and base pressure gradient were computed for this initial temperature for comparison with analytical results under the same conditions (Fig. 8).

5 Discussion

Results from the numerical model have several significant differences with those estimated from the pressurized crack model, for both injection veins that are flow limited and those that reach elastic equilibrium. Here these differences are discussed and used to draw insights into the injection vein formation process and to provide new constraints on the coseismic environment.

5.1 Implications for IV formation

For a 20 mm initial crack, the pressurized crack model predicts an aperture of 2.9 mm, while the largest aperture our numerical model predicts is 2.5 mm, for an initial melt temperature of 1500 °C (Fig. 6a). The aperture in the numerical model, however, is quite sensitive to the choice

of stop condition, which was chosen to minimize effects of deformation after solidification and may therefore underestimate final aperture.

5.1.1 Flow Propagation

Estimating the time required for injection vein propagation is important for determining if veins will quench before filling pre-existing fractures and relating the conditions of their formation to those of the coseismic environment. The analytical model estimates a range of flow propagation times within 10^{-3} to 10^{-1} s for ~20 mm injection veins with varying apertures, while the numerical model suggests flow propagation times of ~1 s for the same length and conditions. This difference can be seen in the discrepancy between the two models' tip velocities of 2 to 3 orders of magnitude (Fig. 8b). The large difference appears to be due to two assumptions of the analytical model that are not supported by the numerical model results: a uniform pressure gradient along the length of the vein and self-similar growth.

The pressurized crack model assumes a pressure gradient that is uniform and inversely proportional to vein length. This assumed pressure gradient is 2 to 4 orders of magnitude greater than the gradient observed in the numerical model (Fig. 8a). Furthermore, the pressure gradient in the numerical model is not uniform along the vein length due to viscosity-driven variations in the flow field. The orders of magnitude variation in viscosity, higher near the tip and lowest at the base, requires a pressure gradient with similar distribution to support inflow during growth.

The pressurized crack model also assumes self-similar growth in order to equate inflow velocity with tip propagation velocity. Numerical model results suggest that the growth of injection veins is, in fact, not self-similar. Indeed, the aspect ratio is not constant throughout growth but increases to the final value (Fig. 7). Length and aperture both exhibit self-similar

growth but at different rates, so the geometry as a whole is self-affine, not self-similar. This implies, and provides a mechanism for, the length dependence of aspect ratio which is seen in field measurements (Fig. 3).

5.1.2 Field evidence of flow limiting

The results of the numerical model suggest several criteria that may be used to identify the occurrence of flow limiting in the field. The elastic equilibrium opening profile (Fig. 6a, 1500 °C) is convex and rounded at the tip. This profile requires significant tensile stress just ahead of the tip (Fig. 6b). In contrast, the flow-limited profiles have straight or concave sides with an angular tip. The angularity reflects the lack of cohesion or tensile strength in the material ahead of the tip due to the continuing fracture. These differences are subtle but may be observable in the field for large, well-preserved injection veins.

The conditions required for an injection vein to reach elastic equilibrium in a fracture are less likely to be met for longer fractures. Therefore, in a given population of injection veins, the longest veins are most likely to be flow limited. The numerical model predicts that flow-limited injection veins have smaller aspect ratios (Fig. 7), suggesting that longer injection veins will have systematically smaller aspect ratios. A reduction in aspect ratio of up to one third is seen among the runs in Fig. 7. A large range in aspect ratio and slight length dependence is observed in all 4 populations of injection veins (Fig. 3). The range in variation that can be attributed to the flow-limited effect is not clear, and the aspect ratio may also be influenced by spatial variations in stress, stiffness, or orientation. Griffith et al. (2012) indirectly observed this trend in the length dependence of their stiffness estimates for the Gole Large Fault Zone populations. They attribute it to a stiffness gradient in the fault normal direction caused by increasing damage near the fault surface. Flow limiting may be an alternative explanation.

5.2 Implications for earthquake source conditions

The initial melt temperature relative to ambient temperature and the ratio of driving stress to rock stiffness ($\Delta P/M$) are the strongest controls on injection vein growth.

5.2.1 Initial melt temperature

The initiation and cessation of injection vein growth is highly dependent on the initial melt temperature, and can be divided into three categories using this parameter. Below a minimum viable temperature the melt is too viscous and cools too quickly for significant growth to occur. Above a threshold temperature the melt is able to exploit a fracture and grow to elastic equilibrium. Between these temperatures growth initiates, but is flow limited.

The initiation of growth is controlled by the ability of fault vein melt to exploit the defect at the base of the fracture. This is controlled by the initial opening rate, a function of the driving pressure and viscosity. Results shown in Fig. 6a indicate that initiation is strongly controlled by the initial temperature through viscosity. Cessation of growth is dependent on melt cooling. In order to reach elastic equilibrium, the growth rate must be faster than this cooling rate. Therefore the threshold temperature for elastic equilibrium is influenced by the length of the fracture and the ambient temperature. The threshold temperature increases for longer fractures or lower ambient temperatures.

The presence and shape of injection veins can thus be coupled with modeling to provide an independent estimate of minimum initial melt temperature. For example, using conditions representative of the Fort Foster environment, the minimum temperature for injection vein formation is $\sim 1200^\circ\text{C}$ and the threshold temperature is $\sim 1500^\circ\text{C}$ for a 20 mm injection vein (Fig. 6a). The presence of injection veins longer than 20 mm with elliptical shapes, consistent with elastic equilibrium, suggest an initial melt temperature greater than 1500°C .

5.2.2 Wall rock stiffness and stress estimates

The large aspect ratio of pseudotachylite injection veins at the three study sites ($10^{-2} - 10^0$) requires large $\Delta P/M$ values under the assumptions of the pressurized crack model described in section 3. Griffith et al. (2012) attribute the large aspect ratios to reduction in wall rock stiffness of 1 to 2 orders of magnitude due to coseismic damage (low M). Rowe et al. (2012) attribute the large aspect ratios to high driving pressure associated with dynamic tensile stresses generated during earthquake rupture (high ΔP). We emphasize here that the pressurized crack model cannot uniquely discriminate between these end-member solutions or combinations of the two. Our results further suggest that some veins may arrest prior to reaching elastic equilibrium. If equilibrium conditions are assumed for these veins it could lead to a 50% overestimation of stiffness, or underestimate of driving pressure.

For our numerical model we have favored a mixed solution with significant softening (80%) combined with a high driving stress equivalent to a complete reduction in fault-parallel stress (σ_x). Attaining higher driving pressures would require fault zone overpressure, which is unlikely to be sufficient (Rowe et al., 2012), or tensile stresses of similar magnitude to ΔP . These tensile stresses would exceed the intact strength of the rock and are thus likely to cause significant off-fault damage and pulverization (Reches and Dewers, 2005) rather than elastic deformation. Furthermore, the transient dynamic stresses associated with a propagating rupture are short-lived. While source durations of earthquakes are 1 to 10 s, depending on the magnitude of the event (Kanamori and Brodsky, 2004), the slip pulse duration at a location along the fault may only be ~ 0.1 s (Griffith and Prakash, 2015). Our numerical model of injection vein growth predicts formation times on the order of seconds for relatively small injection veins. Dynamic stresses are thus relevant to the initiation of the process of injection veins, but the growth and arrest of veins is likely to occur under post-seismic stress conditions.

The large aspect ratios of injection veins may also reflect significant inelastic deformation associated with vein growth. Bestmann et al. (2012) describe microfracturing and micro shear-zone development at the tip of an injection vein from the Gole Larghe fault zone. A significant portion of the fracture energy may thus be dissipated in the process zone of injection veins in a manner similar to map-scale igneous dikes (Pollard, 1987). The effective fracture toughness (K_c^{eff}) may therefore be orders of magnitude higher than typical laboratory values. The median crack-tip stress-intensity factor (K_I) predicted by the analytical model, is $\sim 430 \text{ MPa m}^{1/2}$ for the intact stiffness and $\sim 89 \text{ MPa m}^{1/2}$ for the 80% reduced stiffness (Table 2). Both values fall within the range of field-based estimates of K_c^{eff} for igneous dikes, but the lower value is closer to estimates for cm- to m-scale veins (Olson, 2003). In our numerical model we assume that the vein intrudes a pre-existing crack of limited extent. During growth of the vein the crack-tip stress-intensity is thus relatively low, but non-zero due to the presence of the thin fluid domain included for model implementation reasons. Maximum crack-tip stress-intensity factors for these models, calculated using a numerical approximation of the J-integral, range from 20-32 $\text{MPa m}^{1/2}$. In the case where the vein reaches the full extent of the pre-existing crack the stress intensity factor is only slightly higher (34 $\text{MPa m}^{1/2}$). In the numerical model, under the chosen physical conditions, it therefore seems likely that veins can fully form in a pre-existing crack without a stress intensity that would indicate further propagation. The use of a linear-elastic material thus appears reasonable, but future models may attempt to include wall-rock plasticity to more fully capture the processes associated with injection vein intrusion.

6 Conclusions

The formation of pseudotachylite injection veins is a complex physical process controlled by coupling between elastic deformation, melt flow, and melt solidification. The traditional

model of formation as a purely elastic process holds for short injection veins with relatively high initial melt temperature but breaks down at lower temperatures when fluid and thermal processes dominate. Applying a modified version of the analytical dike model of Rubin (1995b), first applied to injection veins by Rowe et al. (2012), we find the difference in cooling and propagation timescales, assuming typical melt conditions, is within 1 order of magnitude for ~30% of injection veins measured at three different localities. It is likely that this subset of veins is flow limited, while the majority reaches elastic equilibrium.

To further investigate the fluid and thermal processes, we have developed a numerical model of injection vein formation incorporating fluid dynamics and heat transfer. According to our simulations, two major underlying assumptions of the pressurized crack model do not hold for injections veins. 1: The growth of injection veins is self-affine rather than self-similar with aspect ratio increasing during growth. 2: The pressure gradient along the length is not uniform, but is concentrated at the tip and relatively low elsewhere, resulting in a mean internal pressure close to the driving pressure. For an average driving pressure to stiffness ratio ($\Delta P/M$), the pressure gradient at the base during growth is 2 to 3 orders of magnitude less than assumed by the pressurized crack model. The reduced pressure gradient results in a propagation time discrepancy of the same order, further reducing the difference in propagation and cooling timescales and increasing the likelihood of flow-limited emplacement. The model results suggest that the occurrence, length and geometry of injection veins may be used to estimate minimum melt temperatures from field observations. We estimate a lower limit of 1500 °C for the Fort Foster Brittle Zone for our assumed ambient conditions. Finally, the longer timescales estimated from the numerical model indicate that injection veins provide an integrated record of the coseismic and immediately post-seismic environment. If high coseismic stresses do not persist

for the duration of the formation process, a significant reduction in M is needed to explain the large aspect ratios observed in the field.

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9. Tables, Table and Figure Captions

Table 1.

Location	Wall rock	Depth (km)	Temp (°C)	Setting	Age (Ma)
Fort Foster Brittle Zone	ultramylonitic gneiss (Swanson, 2006)	10-15 (Swanson, 1992)	300 to 450 (Swanson, 1992)	dextral strike slip (Swanson, 1988)	280 - 380 (West, 1999)
Asbestos Mountain Fault	tonalite to granodiorite (Rowe et al., 2012)	<10 (Wenk et al., 2000)	<250 (Wenk et al., 2000)	Normal reactivation (Axen and Fletcher, 1998)	<20 (Axen and Fletcher, 1998)
Gole Larghe Fault Zone	tonalite or cataclasite (Griffith et al., 2012)	9 - 11 (Griffith et al., 2012)	250 – 300 (Di Toro and Pennacchioni, 2004)	dextral strike slip (Di Toro and Pennacchioni, 2005)	30 (Del Moro et al., 1983)

Table 1. Estimates of pertinent environmental factors for each data set location.

718 Table 2.

		M = 25 GPa		M = 5 GPa						
Location:	$\Delta P/M$	ΔP (MPa)	K (MPa m ^{1/2})	ΔP (MPa)	K (MPa m ^{1/2})	Flow (ms)	μ (Pa s)	Cool (ms)	λ	$\log(\Delta time)$
AMF-Rowe*	0.071	2064	767.7	-	-	0.03	100	1562.50	0.5	4.67
AMF	0.049	1228	456.7	246	91.3	29.23	816	774.90	0.71	1.42
FFBZ	0.063	1585	394.4	317	78.9	11.99	678	270.46	0.71	1.25
GLFZ-t	0.038	937	429.0	187	85.8	81.39	1299	1061.45	0.71	1.14
GLFZ-c	0.091	2279	648.1	456	129.6	10.84	1299	774.90	0.71	1.89
Composite	0.051	1264	434.2	253	86.8	26.42	-	740.22	0.71	1.38

719 Table 2. Key results from the pressurized crack model calculations, reported as medians for
720 regional injection vein populations. Viscosity is the effective viscosity used in Eq. 4. $\Delta time$ is
721 ratio of quench time and flow propagation time. Stress intensity calculations assume a uniform
722 pressure distribution equal to ΔP . *The AMF-Rowe calculations were done using equivalent
723 equations and parameters from Rowe et al. (2012) to illustrate the differences in methods, thus
724 a single stiffness, $M = 28.9$ GPa, is used. Gole Larghe measurements are reported separately for
725 tonalite-hosted (GLFZ-t) and cataclasite-hosted (GLFZ-c).

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Fig. 1. A large injection vein from the Fort Foster Brittle Zone showing dominantly mode-I opening. Especially in the anisotropic wall rock of Fort Foster, many injection veins show non-idealities, including small shear displacements, grain removal (erosion), and arcuate centerlines. Banded coloration is interpreted as evidence for concurrent cooling and flow. *[Single column, black and white in print]*

Fig. 2. The injection vein is idealized as a semi-elliptical fluid filled crack of length l and maximum aperture a . Pressures in the fluid are determined by the remote stresses: fault vein pressure is equal to the fault normal stress, opening is a function of the deviatoric stress and wall rock stiffness. *[Single column, black and white in print]*

Fig. 3. Length and aperture of 314 injection veins are shown. Length ranges from 1.5 to 1500 mm with a median of 31 mm, aperture from 0.5 to 40 mm with a median of 5 mm. The median aspect ratio is 0.147 (central black line), with most veins having aspect ratios between 0.05 and 0.5 (outer black lines). A power law fit (bold line) shows the scale dependence of geometry – longer veins tend to have smaller aspect ratios than shorter veins. *[Single column, black and white in print]*

Fig. 4. The difference in flow duration for an injection vein (points, symbols as in Fig. 3) and the quench time for its aperture (black line, slope of 2) is the vertical distance between the point and the line. The shaded region includes injection veins with flow propagation times within +/- one order of magnitude of their quench time. 34% of veins are within the shaded region, which we interpret as evidence for interaction between flow and cooling processes. *[Single column, black and white in print]*

Fig. 5. (A) The full geometry of the numerical model showing relevant mechanical and thermal (italicized) boundary conditions. (B) Inset showing details of the pre-existing fluid filled domain that forms the injection vein. *[Single column, black and white in print]*

Fig. 6. (A) Intermediate and final opening profiles for a range of initial melt temperatures between 1150 and 1500 °C. The final profile is taken when the model stop condition is reached.

Only at 1500 °C does the profile begin to resemble the semi-elliptic opening profile expected for elastic equilibrium. (B) Melt velocity, dominantly in the along-vein direction, and Von Mises stress at four times during formation with an initial melt temperature of 1500°C initial melt. [1.5 column, color in print]

Fig. 7. Aspect ratio as a function of crack length during growth for a range of initial melt temperatures between 1150 and 1500 °C. The analytical aspect ratio (constant during growth) is also plotted. Our numerical model shows aspect ratio increases during growth with initial melt temperature. The final aspect ratio and length also increase with initial melt temperature. [Single column, black and white in print]

Fig. 8. (A) Pressure gradient at the base is determined analytically (dot-dashed line) by assuming a uniform pressure gradient, constant remote stress state, and length of the vein increasing with time. The average pressure gradient across the opening of the crack (dashed line) from our numerical model (1500 °C initial melt temperature) is two to four orders of magnitude less. (B) The analytical tip velocity (dot-dashed line) is assumed equal to average inflow velocity at the base of the vein, found using equation 3. Each of these quantities is also calculated directly from our numerical model. The tip velocity (solid line) is found by differentiating the location of the tip, here defined as the furthest point where opening is non-zero. Fluid velocity (dashed line) is the mean velocity at the base of the crack. [Single column, black and white in print]

A **B**

$$\Delta P = \sigma_y - \sigma_x$$

$$0 < P_{\text{tip}} < \sigma_x$$

A

B

Figure 8: Pressure and velocity
[Click here to download Figure: 8_pressure_velocity](#)