Coupled flow and anisotropy in the UltraLow Velocity Zones

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Abstract

Seismic observations reveal a patchwork of thin and dense structures, named UltraLow Velocity Zones (ULVZs) atop the Earth's core mantle boundary. The high width to height ratio of the ULVZs, their spatial correlation with the edges of Large Low Shear Velocity Provinces (LLSVPs), and their preservation as distinct structures in the convecting mantle remains an enigmatic problem. In this article, we carry out a series of numerical simulations using Fast Multipole Boundary Elements Method (FMBEM) to address these questions and study the internal deformation within the ULVZs. Our results demonstrate that coupled flow between dense, low viscosity ULVZ patches and the LLSVP accumulates the ULVZ into stable piles along LLSVP corners, while coalescence and gravitational drainage leads to thin and wide ULVZs away from the corners. Deformation of the matrix is localized within the weaker ULVZ and the LLSVP edges, while the strain in the interior of the LLSVP remain uniform and low, explaining the observed localized anisotropy near LLSVP edges.

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Preprint submitted to Earth and Planetary Science Letters

May 30, 2016

Keywords: Core-Mantle Boundary; Two-Phase Flow; ULVZ; LLSVP;
Mantle Heterogeneity

1. Introduction

The base of the Earth's mantle is marked by a patchwork of seismically anomalous structures named UltraLow Velocity Zones (ULVZs), most commonly observed along the edges of two larger antipodal structures, termed Large Low Shear Velocity Provinces (LLSVPs). The ULVZs are typically 8 to 10% denser than the surrounding mantle and are marked by up to 30% and 10% reductions in shear and compressional wave speeds, respectively (Rost et al., 2005; Rost and Revenaugh, 2003; Williams and Garnero, 1996). Narrow, elongated ULVZs can span up to several hundreds of kilometers in length while reaching only a few tens of kilometers of height above the CMB (Cottaar and Romanowicz, 2012; Rost et al., 2005; Rost and Revenaugh, 2003; Thorne et al., 2013; Williams and Garnero, 1996). The larger LLSVPs, characterized by up to 3% reduction in shear wave speed, are dynamic and likely chemically distinct structures (McNamara et al., 2010). Despite strong internal circulation and deformation (Bower et al., 2011; McNamara et al., 2010), seismic anisotropy is strongly localized along the margins of the LLSVP (Cottaar and Romanowicz, 2013; Lynner and Long, 2014).

A number of issues arise in reconciling the physical characteristics and location of the ULVZs with the available constraints. The distinct physical properties and patchy occurrence of the ULVZs suggest that they are compositional, rather than thermal anomalies. The excess density and large shear wave speed reduction within the ULVZs are likely caused by 5-10 vol%

23 neutrally buoyant melt (Fiquet et al., 2010; Hier-Majumder, 2014; Wimert
 24 and Hier-Majumder, 2012) hosted in an iron-rich matrix (Mao et al., 2006;
 25 Muir and Brodholt, 2015a,b; Nomura et al., 2011; Wicks et al., 2010). Ana-
 26 logue material experiments and geodynamic calculations suggest that the
 27 ULVZs are likely two orders of magnitude weaker than their surrounding
 28 (Hier-Majumder and Revenaugh, 2010; Jellinek and Manga, 2004), render-
 29 ing them susceptible to mixing with the ambient mantle (Manga, 1996). The
 30 ULVZs can serve as isolated and enriched reservoirs at the base of the man-
 31 tle as required by Sm-Nd isotopic measurements (Boyet and Carlson, 2005;
 32 Coltice et al., 2011). Based on the seismic observations and these constraints,
 33 the outstanding issues are: (a) the high aspect ratio of the ULVZ patches
 34 (width is 45-80 times higher than the height (Cottaar and Romanowicz, 2012;
 35 Thorne et al., 2013)) and their preferential associations with the corner of the
 36 LLSVPs, (b) the preservation of the low viscosity ULVZs as distinct struc-
 37 tures over geologic timescales, and (c) the fact that the flow in the LLSVP
 38 does not produce internal seismic anisotropy. In this article, we demonstrate
 39 that strongly coupled flow in the ULVZ-LLSVP system stabilizes dense UL-
 40 VZs at the LLSVP corners while stretching them to small thicknesses in the
 41 other regions. We also demonstrate that matrix deformation arising from
 42 this coupled flow is strongly partitioned into the weaker ULVZ interior and
 43 along the LLSVP edges.

44 To date, majority of the discovered ULVZ locations are associated with
 45 edges of the Pacific and the African LLSVPs. There are, however, obser-
 46 vation of ULVZ patches within a ‘hole’ inside the Pacific LLSVP (Thorne
 47 et al., 2013) and isolated ULVZ patches away from the LLSVP (e.g. McN-

48 mara et al., 2010, and references therein). The occurrence of these ULVZs
49 can be explained by lateral migration and coalescence of segments of the
50 LLSVP, spillage of ULVZ material outside the LLSVP, or the presence of
51 ULVZ material inside and outside the LLSVPs. In this work, we do not
52 address these processes and focus, instead, on the more commonly observed
53 instance of the association between the LLSVP edges and the ULVZs.

54 Tracking the evolution of the ULVZ shape while monitoring the internal
55 deformation has been difficult in the existing scope of work. While analog
56 material experiments are able to provide a wealth of information on the
57 evolution of the ULVZ topography, a quantitative description of internal
58 deformation is unattainable in these experiments (Jellinek and Manga, 2004).
59 In contrast, high resolution convection models can quantify and map the
60 strain within the ULVZ, but due to the lack of an explicit interface between
61 the ULVZ and the ambient mantle, tracking the shape of the ULVZ and
62 calculating the viscous drag on the ULVZ surface are not straightforward in
63 these models (Bower et al., 2011; McNamara et al., 2010).

64 In this article, we present a new geodynamic model that tracks the shape
65 and interaction between multiple ULVZ patches while monitoring their in-
66 ternal velocity and strain fields. The system is driven by the flow within
67 the LLSVP. The circulation within the LLSVP and the ULVZs are modeled
68 using a Fast Multipole Boundary Elements Model (FMBEM). A detailed
69 description of the governing equations, boundary conditions, and the nu-
70 merical techniques are presented in the online supplementary material. In
71 this model, the velocity fields are calculated using a set of coupled boundary
72 integral equations, accelerated by fast multipole expansion of the integral

73 kernels (Liu and Nishimura, 2006). As the shape of the ULVZ, the velocity,
 74 and pressure fields are variables in this system, quantitative evaluation of the
 75 shape and internal deformation can be performed directly.

76 2. Methods

Figure 1: A schematic diagram outlining the scope of the problem. The ULVZ patches within the LLSVP are shaded in light gray. Density and viscosity within each of the entities are shown in the figure. The figure in the inset shows the no-slip and stress jump boundary conditions at the ULVZ-LLSVP interface. Velocity at the LLSVP-Mantle interface are shown by the arrows marked by $\mathbf{u}_{mantle}$.

77 In our model, we treat the ULVZs as dense, low viscosity patches, embed-
 78 ded within a larger LLSVP. In our calculation in two-dimensional Cartesian
 79 coordinates, the coupled convective flow within the LLSVP and the ULVZs
 80 are driven by density contrast between the LLSVP and the ULVZ patches
 81 and the prescribed velocity at the boundary between the LLSVP and the
 82 surrounding mantle.

83 The numerical model in this work builds on a previous boundary el-
 84 ements model developed to study the microstructure in partially molten

85 rocks (Drombosky and Hier-Majumder, 2015; Hier-Majumder, 2008; Hier-
 86 Majumder and Abbott, 2010; Hier-Majumder and Drombosky, 2015). In
 87 this method, the governing partial differential equations and the boundary
 88 conditions are combined into a vector Boundary Integral Equation (BIE). In
 89 the next subsection, we present the governing equations.

90 *2.1. Governing equations*

91 We treat the ULVZs and LLSVP as individual bodies of incompressible
 92 fluid, with distinct densities and viscosities. Conservation of mass within
 93 each entity requires,

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

94 where $\mathbf{u}$ is the velocity. Conservation of momentum within each ULVZ and
 95 the LLSVP requires, in the presence of gravitational acceleration,

$$\nabla \cdot \mathbf{T} + \rho \mathbf{g} = 0, \quad (2)$$

96 where $\mathbf{T}$ is the stress tensor of the fluid with density ρ and the vector $\mathbf{g}$
 97 represents gravity. The schematic diagram in Figure 1 outlines the bound-
 98 ary conditions associated with the coupled flow within each of these bodies.
 99 The governing conservation equations must be supplemented with boundary
 100 conditions. We impose a no-slip boundary condition at the interface between
 101 the ULVZ and the LLSVP, expressed as,

$$\Delta \mathbf{u} = 0, \quad (3)$$

102 where Δq indicates the difference between the physical quantity q evaluated
 103 within the ULVZ and evaluated within the LLSVP. In addition, we impose a
 104 stress jump boundary condition across each interface, such that the difference

105 in traction is balanced by buoyancy contrast between the ULVZ and the
 106 LLSVP. This condition is given by,

$$\Delta \mathbf{T} \cdot \hat{\mathbf{n}} + \Delta \rho (\mathbf{x} \cdot \mathbf{g}) \hat{\mathbf{n}} = 0, \quad (4)$$

107 where $\hat{\mathbf{n}}$ is the unit normal to the interface, as depicted in Figure 1.

108 We solve for velocities at the boundary and within each entity by con-
 109 verting the governing partial differential equations into a Boundary Integral
 110 Equation (BIE). We provide the details of this conversion, the nondimen-
 111 sionalization scheme, and implementation of the fast multipole expansion in
 112 the supplementary material. For the sake of brevity, we next proceed to the
 113 integral equation describing the coupled flow over multiple domains.

114 In a collection of P ULVZ patches embedded within an LLSVP (marked
 115 as domain $P + 1$), the dimensionless BIE for the velocity $\mathbf{u}$ at point $\mathbf{x}_0$ on
 116 the surface of the q -th ULVZ patch is given by:

$$\begin{aligned} \mathbf{u}(\mathbf{x}_0) = & \frac{2}{1+\lambda_q} \left[- \sum_{p=1}^{P+1} \frac{\mathcal{R}}{4\pi} \int_{\Gamma_p} \mathbf{J}(\mathbf{x}, \mathbf{x}_0) \cdot \Delta \mathbf{f}(\mathbf{x}) d\Gamma_p \right. \\ & \left. + \sum_{p=1}^{P+1} \frac{1-\lambda_p}{4\pi} \int_{\Gamma_p} \mathbf{u}(\mathbf{x}) \cdot \mathbf{K}(\mathbf{x}, \mathbf{x}_0) \cdot \hat{\mathbf{n}}(\mathbf{x}) d\Gamma_p \right], \end{aligned} \quad (5)$$

117 where the point $\mathbf{x}_0$ is called the pole point while the point $\mathbf{x}$ is called the field
 118 point (Pozrikidis, 2001), λ_q is the viscosity ratio between the q -th ULVZ patch
 119 and the LLSVP, $\mathcal{R}$ is the nondimensional compositional Rayleigh number,
 120 $\Delta \mathbf{f}(\mathbf{x})$ is the buoyancy force arising from density contrast between the ULVZ
 121 and the LLSVP, and the kernels $\mathbf{J}(\mathbf{x}, \mathbf{x}_0)$ and $\mathbf{K}(\mathbf{x}, \mathbf{x}_0)$ in the two integrals
 122 on the right hand side are known as the Stokeslet and the Stresslet tensors.
 123 Each of these integrals are taken over the boundary of the P -th domain.

124 The system of linear algebraic equations arising from discretization of
 125 (5) is dense and generally asymmetric. Following the methods outlined in

126 Drombosky and Hier-Majumder (2015) and Hier-Majumder and Drombosky
 127 (2015), we use the Fast Multipole (FM) expansion to approximate the kernels
 128 in the integrands. After discretization and implementing FM expansion, we
 129 can recast the BIE as a system of algebraic equations given by

$$\left[\left(\frac{1+\lambda}{2} \right) \mathbf{I} - \left(\frac{1-\lambda}{4\pi} \right) \mathbf{\mathcal{K}} \right] \mathbf{u} = -\frac{\mathcal{R}}{4\pi} \mathcal{J} \Delta \mathbf{f}, \quad (6)$$

130 where $\mathcal{J}$ and $\mathbf{\mathcal{K}}$ are the matrices generated from the kernels $\mathbf{J}(\mathbf{x}, \mathbf{x}_0)$ and
 131 $\mathbf{K}(\mathbf{x}, \mathbf{x}_0)$, respectively. The accelerated multiplication, in conjunction with
 132 iterative methods such as the GMRES (Saad, 2003), solves the linear system
 133 of equations in (6) in $\mathcal{O}(N)$ time.

134 The nondimensional BIE consists of two dimensionless quantities, $\mathcal{R}$, the
 135 compositional Rayleigh number and λ , the viscosity ratio. We define $\mathcal{R}$ as,

$$\mathcal{R} = \frac{\Delta \rho g x_c^2}{u_c \mu}, \quad (7)$$

136 where μ is the viscosity of the LLSVP, x_c is the characteristic length, and
 137 u_c is the characteristic velocity, respectively. The compositional Rayleigh
 138 number is the inverse of the intrusion number sometimes used in nondi-
 139 mensional models of spreading viscous gravity currents (Hier-Majumder and
 140 Revenaugh, 2010; Olson, 1990). It signifies the ratio between buoyancy and
 141 viscous drag. If the magnitude of $\mathcal{R}$ is high, buoyancy dominates over vis-
 142 cous resistance to flow and the ULVZ patches drain rapidly. Notice also that
 143 the sign of $\mathcal{R}$ is controlled by the difference between densities of the ULVZ
 144 and the LLSVP. We present further discussion about the relation between
 145 these two competing forces in Section 2.2. We discuss the value of $\mathcal{R}$ in our
 146 simulations in Section 2.3.

147 Once the boundary velocities are calculated by solving equation (6), and
 148 the velocities within the ULVZ patches and the LLSVP are calculated by
 149 the methods outlined in the supplementary section, we update their shapes.
 150 We describe the shape of the P -th ULVZ patch by the function $F^P(\mathbf{x}, t) = 0$
 151 at time t . To ensure the volume of each grain remains constant during
 152 deformation of the grain, we use the kinematic equation

$$\frac{\partial F^P(\mathbf{x}, t)}{\partial t} + \mathbf{u}^P(\mathbf{x}, t) \cdot \nabla F^P(\mathbf{x}, t) = 0. \quad (8)$$

153 Using the updated shape functions, we calculate the normal vectors at each
 154 node of the new ULVZ-LLSVP boundary to be used in the BIE (5) for calcu-
 155 lating the boundary velocities at the next time step. This process is iterated
 156 in time to model the evolution of the ULVZ shape.

157 2.2. Linear analysis

158 The rate of gravitational drainage of the dense ULVZs is modulated by the
 159 viscosity ratio between the ULVZ and the mantle. To understand the nature
 160 of the relationship between the two key nondimensional parameters, $\mathcal{R}$ and
 161 λ , we carried out a linear stability analysis for the ULVZ shape subjected to
 162 a straining and drainage flow.

163 The linear solution consists of a base state and a perturbed state. The
 164 base state is characterized by zero motion within a neutrally buoyant, hemi-
 165 spherical ULVZ. The perturbation is caused by both density contrast and
 166 flow induced by circulation within the LLSVP. We want to characterize, to
 167 the first order, a relation between the two nondimensional parameters, λ and
 168 $\mathcal{R}$, to identify the boundary between two regimes, gravitational drainage and
 169 entrainment. The set up for the linear problem is outlined in Figure 2. Note

170 that this analysis is applicable to the stability of the ULVZ far away from
171 the LLSVP corners.

172 With this goal, we can write the entire velocity fields within the ULVZ
173 and the LLSVP as $\mathbf{u}^j = \epsilon(\mathbf{u}_e^j + \mathbf{u}_d^j)$, where $\epsilon \ll 1$ is a constant, the superscript
174 j refer to either the ULVZ (U) or the LLSVP (L), and the subscripts e and d
175 refer to two linear flows arising from entrainment and drainage, respectively.

176 To build analytical solutions for each perturbing flow, we use the method
177 of solid spherical harmonics. We use the well-known Hadamard-Rybczynski
178 solution for gravitational settling of a viscous drop (see, for example, Leal,
179 1992, Ch. 5). For the entraining flow, we build a set of solutions forced by a
180 straining flow in the LLSVP, $\mathbf{u}_e^L \propto \dot{\mathbf{E}} \cdot \mathbf{r}$, where $\dot{\mathbf{E}}$ is a constant strain rate and
181 $\mathbf{r}$ is the distance from the ULVZ center. We present a detailed description
182 of the method for solution building in Appendix A.

183 For a small perturbation f , the shape of ULVZ is given by $F = r - h(1 +$
184 $\epsilon f)$, where f is the perturbed shape function. The $\mathcal{O}(\epsilon)$ kinematic condition
185 (8), then becomes,

$$\frac{\partial f(\mathbf{x}, t)}{\partial t} = \frac{1}{h} \mathbf{u}^U(\mathbf{x}) \cdot \hat{\mathbf{n}}, \quad (9)$$

186 where $\hat{\mathbf{n}}$ is the unit normal to the hemispherical ULVZ. We present a
187 schematic diagram for the linear analysis in Figure 2(a) and solutions to
188 equation (9) in Section 3.1.

189 2.3. Simulations

190 The geometry of the simulation is divided into two sets of interfaces: the
191 LLSVP-mantle interface and LLSVP-ULVZ interfaces. For each simulation,
192 we fix the LLSVP-mantle boundary as a trapezoid with rounded corners.

193 Roundness around the corner prevents singularities in the simulation arising
 194 from high curvature of the boundary elements. The width, height, interior
 195 angle, and radius of curvature for the corners are specified to create the
 196 LLSVP geometry similar to structures observed in whole-mantle models. The
 197 LLSVP is 1,000 km across and 100 km high. The lower acute corner of the
 198 trapezoid has an angle of $\tan^{-1}(1/3)$. Both the corners are rounded with a 10
 199 km radius of curvature. The density of the LLSVP is $\rho_L = 3500 \text{ kg/m}^3$, while
 200 the density of the ULVZ, ρ_U , is 3850 kg/m^3 . The viscosity of the LLSVP is
 201 $\mu_L = 5 \times 10^{20} \text{ Pa s}$, and the viscosity of the ULVZ is two orders of magnitude
 202 less at $\mu_U = 5 \times 10^{18} \text{ Pa s}$. Acceleration due to gravity is $g = 9.8 \text{ m/s}^2$. We
 203 impose a tangential velocity of 1 cm/yr at the LLSVP-mantle interface.

204 We simulate three stages of ULVZ evolution within the LLSVP. Each
 205 simulation begins with ULVZs configured in a unique initial geometry. The
 206 first stage simulates the evolution of ULVZ material distributed in the center
 207 of the LLSVP. The second simulation begins with the ULVZ material dis-
 208 tributed along the CMB. The third and final simulation demonstrates the
 209 steady state behavior of the ULVZ material in the corner of the LLSVP.

210 We set the characteristic length $x_c = 25 \text{ km}$ and the characteristic ve-
 211 locity $u_c = 1 \text{ cm/yr}$, resulting in a time scale of 2.5 Ma. Taken together
 212 with the physical parameters for the density, gravitational acceleration, and
 213 viscosity of the LLSVP, the dimensionless compositional Rayleigh number at
 214 the LLSVP-ULVZ boundary is $\mathcal{R} = -13.5$, where the negative sign indicates
 215 that the ULVZ is more dense than the surrounding LLSVP.

216 These numerical simulations using the FM accelerated BEM allows us
 217 to specifically address the three questions raised in the introduction. In the

following section, we present our results in light of these questions.

3. Results

3.1. Results from the linear analysis

Figure 2: Linear analysis of drainage within the ULVZ. (a) A schematic diagram outlining the problem set-up for the linear analysis. (b) Plots of R_c/E_1 from equation (11) as a function of λ . In the plot we assume $h = 1$.

As discussed in section 2.2, we build solutions for the entraining flow in terms of a known strain rate $\dot{\mathbf{E}}$. In 2D, this tensor can be expressed as $\dot{\mathbf{E}} = [(E_1, 0), (0, -E_1)]$. Combining the solutions for the entraining and drainage flow and evaluating equation (9) atop the hemispherical ULVZ patch, the $\mathcal{O}(\epsilon)$ equation becomes

$$\frac{df}{dt} = \left(\frac{h^2}{2} - 1\right) E_1 + \frac{2}{3h} \left(\frac{1 + \lambda}{2 + 3\lambda}\right) \mathcal{R}. \quad (10)$$

The first term on the right hand side arises from entraining flow, while the second term arises from drainage. This equation provides a direct way to

228 compare the relative contribution of these two types of flows in determining
 229 the shape of the ULVZ.

230 The critical state at which the drainage and entraining flows balance each
 231 other is marked by $df/dt = 0$. Notice that in the absence of a straining flow
 232 in the ULVZ ($E_1 = 0$), the shape change will be determined only by the
 233 sign of $\mathcal{R}$. For a dense ULVZ, $\mathcal{R} < 0$, rendering $df/dt < 0$, such that the
 234 top of the ULVZ drains towards the core-mantle boundary. The presence of
 235 a straining flow, however, counters this drainage. The critical value $\mathcal{R}_c$ for
 236 which $df/dt = 0$, thus becomes,

$$\mathcal{R}_c = \frac{3E_1}{4} \left(\frac{2 + 3\lambda}{1 + \lambda} \right). \quad (11)$$

237 The two regimes of flow are defined by the critical compositional Rayleigh
 238 number. The plot in Figure 2(b) depicts $\mathcal{R}_c/E_1$ as a function of λ from
 239 equation (11). For values of $\mathcal{R}/E_1$ above the curve, the shape of the ULVZ
 240 is dominated by density contrast-driven drainage flow, while values of $\mathcal{R}/E_1$
 241 below the curve lead to the entrainment of the ULVZ by the flow within the
 242 LLSVP. In the numerical simulations described in Section 3.2, the flow takes
 243 place in the drainage regime.

244 In the absence of density contrast, the flow takes place entirely in the
 245 entrainment regime. We carried out a set of simulations for a neutrally
 246 buoyant ULVZ patch located in the corner of the LLSVP. Similar to previ-
 247 ously observed behavior of neutrally buoyant, low viscosity blobs embedded
 248 in a circulating mantle (Manga, 1996), we found that the neutrally buoyant
 249 ULVZs are easily entrained by the mantle circulation and will likely ho-
 250 mogenize with the surrounding mantle over geologically significant periods.
 251 Results from these simulations are reported in the supplementary material.

3.2. Shape of the ULVZs

While it has been known for some time that the ULVZs are characterized by relatively low thickness (Rost et al., 2006; Rost and Revenaugh, 2003; Williams and Garnero, 1996), recent discoveries of very large (~ 800 kms or higher in lateral extent) ULVZs indicate that very high width:height ratio (45:1 to 80:1) can characterize many of the ULVZs (Cottaar and Romanowicz, 2012; Thorne et al., 2013). To understand the development of such flattened shapes as well as the lack of observed ULVZs high above the CMB, we carried out a series of simulations starting with two ULVZ patches suspended above the CMB.

The evolution of the initially suspended ULVZ patches is marked by two stages, leading to the coalescence of the patches. In Figure 3, two initially circular ULVZ patches of 25 km radius and separated by 100 km, coalesce in two phases. During the initial descent phase spanning over the first 0.75 Ma, the dense patches rapidly sink to the bottom of the LLSVP. The vertical drainage within the patches is marked by the mostly vertical streamtubes within each patch. During the next 11 Ma of gravity current phase, the patches spread laterally with a lobate front characteristic of spreading gravity currents (Koch and Koch, 1995; Leahy and Bercovici, 2007). In contrast to the top two panels, the streamtubes within the patches are now almost entirely horizontal. This gravity current phase is also marked by coalescence of the initially distinct patches. As the snapshots in time indicate, the leading patch coats the lobate front of the trailing patch. While such coalescence of dense spreading multiple layers were not observed in previous single gravity current models (Koch and Koch, 1995; Leahy and Bercovici, 2007), the

277 coalescence of buoyant viscous blobs rising through a dense liquid follow a
278 similar pattern of coating (Manga and Stone, 1995).

279 After 12 Ma, flow within the LLSVP continues to drag the ULVZ patches
280 towards the corner. At this stage of the simulation, however, highly curved
281 boundary elements lead to the development of numerical instabilities in the
282 simulations. The horizontal stream tubes intersecting the surface of the
283 ULVZ patches indicate that the ULVZs are yet to reach the steady state, as
284 they creep towards the LLSVP corner. To study the steady-state location
285 and shape of the ULVZs, we start the next set of simulations from an initially
286 flat ULVZ patch with rounded corners, described in the following section.

287 The lack of observed ULVZs far above the CMB and the high width to
288 height ratio is explained by the evolution outlined in Figure 3. While the
289 density contrast rapidly drains any ULVZ patches that might have been cre-
290 ated above the CMB, subsequent coalescence of multiple patches explain the
291 recently observed ULVZs of very large lateral extent (Cottaar and Romanow-
292 icz, 2012; Thorne et al., 2013). This observation of ULVZ coalescence from
293 our results are qualitatively similar to those obtained by thermochemical
294 convection calculations of McNamara et al. (2010). As described above, the
295 presence of a thin, flat ULVZ patch away from the corner of the LLSVP is
296 likely to be a transient feature of the coupled flow.

297 *3.3. Stability of the ULVZs*

298 Upwelling LLSVP circulation sweeps an initially flat ULVZ patch into
299 a steady-state pile near the corner over a geologically short period of time.
300 The second stage of simulation, depicted in Figure 4, starts with a flat ULVZ
301 that would result from the merger of two ULVZ patches to form an approx-

302 imately 111 km long and 9 km high basal layer. At 8.75 Ma, the leading
 303 edge of the basal layer is lifted up to a height of 19 km along the edge of
 304 the LLSVP. The change in the topography during the last 3.75 Ma of this
 305 simulation is substantially muted compared to the first 1.5 Ma. At 8.75 Ma,
 306 the establishment of a closed loop circulation at the left corner of the ULVZ
 307 and stream tubes near parallel to the right edge ensures that the shape of the
 308 ULVZ has reached near steady-state. The attainment of such a steady-state
 309 implies the ULVZ can survive mixing by mantle convection and remain as
 310 isolated, chemically distinct entity over geologically significant periods.

311 To test the stagnation of the ULVZ patch at the corner of the LLSVP, we
 312 numerically evaluated the rate of change in topography of the ULVZ patch
 313 in the simulation in Figure 4 as a function of time. As the plots in Figure 5
 314 indicate, the initially flat ULVZ patch is uplifted at a velocity of 1.5 km/Ma
 315 for the first 3.5 Ma. Subsequently the rate of uplift declines rapidly, dropping
 316 to zero at 8 Ma, indicating the attainment of steady-state topography.

317 In the third series of reverse experiment, we started with an initial ULVZ
 318 patch located at the corner of the LLSVP. The series of simulations in Figure
 319 6 depict the evolution of the shape of the patch. In the beginning of the sim-
 320 ulation, the velocity field indicates that the height of the ULVZ is unstable.
 321 At the corner, concentric stream tubes depict the closed circulation that was
 322 observed in the steady-state in Figure 4 at 8.75 Ma. The circulation on the
 323 vertical right edge, however, is quite distinct, as streamtubes enter and leave
 324 the ULVZ at the top and the bottom of this edge, respectively. Notice that
 325 the stream tubes are tightly bunched over a small fraction of the area of the
 326 ULVZ-LLSVP interface, indicating that the flow is focused within narrow

327 regions. This counterclockwise flow, driven primarily by buoyancy forces,
 328 drains ULVZ material by a gravity current flowing upstream with respect to
 329 the entraining flow in the LLSVP. Over the next 2.5 Ma, the gravity current
 330 develops a nose-like structure near the bottom, where the outward normal
 331 velocity is the highest, indicated by the high angle subtended by the stream
 332 tubes to the interface. The opposing flow within the LLSVP lifts the bottom
 333 of the right edge of the ULVZ by a small amount, as the gravity current
 334 continues spreading upstream. By 5 Ma, two closed circulation cells are es-
 335 tablished within the ULVZ. Both of these cells are highlighted in the figure.
 336 The stream tubes on the ULVZ-LLSVP interface is now mostly tangential
 337 to the interface. As a result, the shape of the ULVZ remains practically
 338 unchanged during subsequent iterations of the simulation. The steady-state
 339 ULVZ has a height of 23.5 km and width of 67 km within the corner of the
 340 LLSVP. The rate of change in height in Figure 7 demonstrates that a brief
 341 initial period of gravitational drainage of the ULVZ is followed by a brief
 342 period of uplift, eventually reaching near steady-state around 5Ma.

343 *3.4. Flow and anisotropy in the ULVZs*

344 During the coupled flow, the weaker ULVZ absorbs the larger amount
 345 of deformation. The plot in Figure 8(a) maps the magnitude of the largest
 346 eigenvalue of the strain-rate tensors within the ULVZ and the LLSVP at the
 347 steady-state in Figure 4. The warmer colors, corresponding to high strain-
 348 rate are confined within and around the ULVZ. The strain rate in the interior
 349 of the LLSVP is relatively uniform and has a substantially lower value. Low
 350 stress levels associated with such low strain rate will confine deformation in
 351 diffusion creep regime, preventing the formation of strong anisotropy induced

352 by dislocation creep, explaining the observed lack of anisotropy in the LLSVP
 353 interior. Strong deformation within and immediately around the ULVZ will
 354 lead to the development of a strong textural fabric. If partially molten,
 355 such deformation will lead to wetting of grain boundaries by thin films of
 356 melt (Daines and Kohlstedt, 1997; Hier-Majumder, 2011; Hier-Majumder
 357 and Drombosky, 2015). The map in Figure 8(b) Shows the orientation of
 358 strain-rate ellipses within the ULVZ matrix. Partial melt will segregate in
 359 films, depicted by the black bars, oriented along the principal axis of shorten-
 360 ing (Daines and Kohlstedt, 1997; Hier-Majumder, 2011; Hier-Majumder and
 361 Drombosky, 2015) at angles between 30° and 40° to the CMB. In addition,
 362 high stress levels within the ULVZ can also cause dislocation motion and
 363 lattice-preferred orientation in postperovskite crystals.

364 **4. Assumptions and implications for the lower mantle**

365 We make two assumptions in our simulations. First, the LLSVP remains
 366 stationary over the time period of the simulations. LLSVPs are dynamic
 367 structures and may change shape, coalesce, or break-up over geological time
 368 scales. Our assumption implies that the timescale for LLSVP migration is
 369 larger than the rapid stabilization of the ULVZs. In addition, the boundary
 370 condition of tangential velocity at the LLSVP-mantle interface restricts the
 371 location of the ULVZ within the physical boundaries of the LLSVPs. Seismic
 372 observations of sharp and steep boundaries between the LLSVP and the man-
 373 tle and geodynamic simulations suggest that the flow within the LLSVP is
 374 likely largely self-contained, prompting our selection of the boundary condi-
 375 tion at the LLSVP-mantle interface. Isolated observations of ULVZ patches

376 away from the LLSVP (McNamara et al., 2010), however, are difficult to
 377 explain based on our simulations. To address these issues, future FMBEM
 378 models incorporating motion of the LLSVPs need to be constructed. Sec-
 379 ond, we use a Newtonian linear viscosity for the ULVZ and the LLSVP, any
 380 anisotropy arising from the realignment of melt is not taken into account for
 381 the simulations. Bearing these assumptions in mind, we discuss the implica-
 382 tions of our results for the lower mantle dynamics.

383 The coupled flow and deformation in the LLSVP-ULVZ system bears
 384 a number of implications for the Earth’s lowermost mantle. If the ULVZs
 385 formed from late stage accumulates of a basal magma ocean (Labrosse et al.,
 386 2007; Nomura et al., 2011), they will act as isolated reservoirs of incompati-
 387 ble heat producing elements in the lower mantle (Abe, 1997; Boyet and Carl-
 388 son, 2005). Such fertile ULVZ reservoirs will also stay close to their solidus
 389 temperature at the CMB (Fiquet et al., 2010), containing small amounts of
 390 melt, which causes the observed drastic shear wave velocity reduction. As
 391 deformation within the ULVZ segregates the melt into thin films subparallel
 392 to the CMB, the ULVZs will likely become strongly anisotropic in elastic
 393 strength, viscosity, and electrical conductivity. One important outcome of
 394 formation of anisotropic melt films will be the development of anisotropic
 395 melt permeability. Two recent models of redistribution of melt within the
 396 ULVZ demonstrate that shearing from the LLSVPs exert significant influ-
 397 ence on compaction and localization of melt (Hernlund and Jellinek, 2010;
 398 Hier-Majumder, 2014). Future work on the effect of anisotropic melt dis-
 399 tribution within the ULVZ will be crucial in addressing the important issue
 400 of melt drainage within the ULVZ. Finally, here we explore the commonly

401 observed association between LLSVP corners and ULVZ patches. But thin,
402 undetectable ULVZ layers can exist outside the LLSVPs. ULVZs observed
403 outside the LLSVP are likely associated with regions of localized mantle
404 upwelling (Jellinek and Manga, 2004).

405 In conclusion, our results demonstrate that dense, low-viscosity ULVZs
406 stagnate at the corner of LLSVPs remaining chemically unmixed over geolog-
407 ically long periods of time. Away from the LLSVP corners, initially distinct
408 ULVZ patches can coalesce within a few Ma into flat structures near imme-
409 diately above the CMB. During the coupled flow of the surrounding mantle
410 and the ULVZ, most of the deformation is partitioned within and around
411 the ULVZ, explaining the lack of observed anisotropy in the interior of the
412 LLSVP.

413 **Acknowledgments**

414 This work was supported by NSF grant EAR 1215800 and XSEDE com-
415 putational grant EAR130030. Mark Jellinek, John Brodholt, and anonymous
416 reviewer provided insightful reviews.

417 Appendix A. Linear stability analysis

418 In this section, we present the method for building an analytical solution
419 to the velocity and pressure fields within the ULVZ and the evolution of
420 ULVZ shape subjected to a flow within the surrounding mantle.

421 Treating the ULVZ and the mantle as separate entities, we notice that
422 mass and momentum is conserved within each of them. The conservation
423 of mass is described by equation (1). Since we treat pressure as a primary
424 variable in this analysis, we can recast the equation for conservation of mo-
425 mentum as,

$$-\nabla p^j + \mu^j \nabla^2 \mathbf{u}^j = 0, \quad (\text{A.1})$$

426 where superscript $j = U$ for the ULVZ and L for the LLSVP. We can write
427 the perturbed velocity field within each entity as $\mathbf{u}^j = \epsilon(\mathbf{u}_e^j + \mathbf{u}_d^j)$, where
428 $\epsilon \ll 1$ and the subscripts e and d refer to two linear flows characteristic of
429 entrainment and drainage. We build analytical solutions for both of these
430 velocity fields separately. Since the overall nature of the flow is linear, su-
431 perposition of two different flow fields is justified. This formulation enables
432 us to determine the relative importance between drainage and entrainment.

433 For the draining flow, we use the Hadamard-Rybczinski solution, for a
434 translating flow past a viscous sphere (Leal, 1992, Ch. 5). For the entrain-
435 ing flow, we build analytical solutions for velocity and pressure fields using
436 solid vector harmonics. The system of solution building using the method
437 of solid harmonics is described in a number of previous articles (Drombosky
438 and Hier-Majumder, 2015; Hier-Majumder, 2011; Hier-Majumder and Drom-
439 bosky, 2015).

440 To study the influence of deformation of the ULVZ, it is useful to build

441 the solutions in terms of a dimensionless, constant strain rate tensor $\dot{\mathbf{E}}$. We
 442 set the origin at the center of the hemispherical ULVZ, such that the position
 443 vector $\mathbf{r}$ describes the distance of a point from the center of the ULVZ. The
 444 nondimensional velocity and pressures within the ULVZ and the LLSVP are
 445 then given by

$$\mathbf{u}_s^U = -a_1 (1 - r^2) \dot{\mathbf{E}} \cdot \mathbf{r} + a_2 \mathbf{r} \dot{\mathbf{E}} : \mathbf{r} \mathbf{r} \quad (\text{A.2})$$

$$p_s^U = a_3 \frac{\lambda}{2} \dot{\mathbf{E}} : \mathbf{r} \mathbf{r} \quad (\text{A.3})$$

$$\mathbf{u}_s^L = c_1 \dot{\mathbf{E}} \cdot \mathbf{r} \quad (\text{A.4})$$

$$p_s^L = c_2 \dot{\mathbf{E}} : \mathbf{r} \mathbf{r}. \quad (\text{A.5})$$

446 In the Cartesian coordinates, we can describe the strain tensor $\dot{\mathbf{E}}$ in terms
 447 the principal strain rates as,

$$\dot{\mathbf{E}} = \begin{bmatrix} E_1 & 0 \\ 0 & -E_1 \end{bmatrix}. \quad (\text{A.6})$$

448 The solution for $\mathbf{u}_s^L$ ensures the conservation of mass within the LLSVP. We
 449 also rewrite the stress jump condition as,

$$\lambda \hat{\mathbf{n}} \cdot \mathbf{T}_s^U \cdot \hat{\mathbf{n}} - \hat{\mathbf{n}} \cdot \mathbf{T}_s^L \cdot \hat{\mathbf{n}} = 0, \quad (\text{A.7})$$

450 at $r = h$.

451 We determine four of the five unknown constants from mass and momen-
 452 tum conservation within the ULVZ (equations (1) and (A.1)), momentum
 453 conservation within the LLSVP (equation (A.1)) and continuity of normal
 454 traction across the ULVZ-mantle interface (equation (A.7)). Then, setting

455 $c_1 = \lambda$, we obtain the following constants,

$$a_1 = -1 \quad (\text{A.8})$$

$$a_2 = \frac{1}{2} \quad (\text{A.9})$$

$$a_3 = -6 \quad (\text{A.10})$$

$$c_1 = \lambda \quad (\text{A.11})$$

$$c_2 = 0. \quad (\text{A.12})$$

456 Solution for the axisymmetric Hadamard-Rybczinski flow is given in spherical
 457 polar coordinates (Kim and Karilla, 2005, Ch. 4). For brevity, we only
 458 provide the solutions for flow internal to the ULVZ, which will be used in the
 459 next step. This solution is given by,

$$\begin{aligned} \mathbf{u}_d^L = & \left[-\frac{1}{2(1+\lambda)} \left(\frac{r}{h} \right)^2 + \frac{3+2\lambda}{2(1+\lambda)} \right] U \cos \theta \, \hat{\mathbf{r}} \\ & - \left[\frac{1}{1+\lambda} \left(\frac{r}{h} \right)^2 + \frac{3+2\lambda}{2(1+\lambda)} \right] U \sin \theta \, \hat{\boldsymbol{\theta}}, \end{aligned} \quad (\text{A.13})$$

460 where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors and U is a constant terminal velocity, respec-
 461 tively. Notice that atop the ULVZ ($r = h, \theta = 0$) this velocity reduces to
 462 $U \hat{\mathbf{r}}$. The dimensional magnitude of the terminal velocity, U' , can be modified
 463 from (Leal, 1992, Eq. 5.45) using our definitions

$$U' = \frac{2}{3} \left(\frac{1+\lambda}{2+3\lambda} \right) \frac{x_c^2 g \Delta \rho}{\mu}. \quad (\text{A.14})$$

464 Using the characteristic velocity for nondimensionalization, we can rewrite
 465 U in terms of the dimensionless compositional Rayleigh number as,

$$U = \frac{2}{3} \left(\frac{1+\lambda}{2+3\lambda} \right) \mathcal{R}. \quad (\text{A.15})$$

466 The solution for straining flow leads to the following expression for the normal
 467 component of the velocity atop the ULVZ ($r = h, \theta = 0$)

$$\mathbf{u}_s^U \cdot \hat{\mathbf{r}} = \left(\frac{h^3}{2} - h \right) E_1. \quad (\text{A.16})$$

468 To determine the change of the ULVZ shape atop, we rewrite equation (8)
 469 in $\mathcal{O}(\epsilon)$ as,

$$\frac{\partial f(\mathbf{x}, t)}{\partial t} = \frac{1}{h} \mathbf{u}^U(\mathbf{x}) \cdot \hat{\mathbf{n}}. \quad (\text{A.17})$$

470 Decomposing the total velocity $\mathbf{u}^U$ in terms of the entraining velocity
 471 $\mathbf{u}_s^U$ and the draining velocity $\mathbf{u}_d^U$, and using the expressions from equations
 472 (A.13) and (A.16) for $r = h, \theta = 0$, we can rewrite equation (A.17) as,

$$\frac{df}{dt} = \left(\frac{h^3}{2} - h \right) E_1 + \frac{2}{3} \left(\frac{1 + \lambda}{2 + 3\lambda} \right) \mathcal{R}. \quad (\text{A.18})$$

473 In the main article, we discuss the nature of the above equation in deter-
 474 mining the regime boundary between entrainment and drainage.

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Figure 3: Evolution of circular ULVZ reservoirs placed in the LLSVP. The five snapshots show the evolution of the system between 0 and 12 Ma. The streamlines are colored by the magnitude of the velocity with light blue corresponding to 1 cm/yr and dark red to 3 cm/yr. The 10% denser and two orders of magnitude weaker ULVZ patches are shaded in gray. Only half of the symmetric LLSVP flow is shown in the figure.

Figure 4: Evolution of rectangular ULVZ reservoirs with rounded corners placed in corners of the LLSVP. The five snapshots show the evolution of the system between 0 and 8.75 Ma. The streamlines are colored by the magnitude of the velocity with light blue corresponding to 1 cm/yr and dark red to 2 cm/yr. The ULVZ patches are separated from the LLSVP by a black outline and shaded in gray. At 8 Ma, the rate of change of ULVZ topography becomes zero, indicating steady-state.

Figure 5: Rate of change of the ULVZ height in km/Ma is plotted from the numerical simulation in Figure 4 in solid line. The dashed line represents no change in height, i.e. steady state. The derivative is passed through a triangle smoothing filter to reduce noise in the measurement.

Figure 6: Evolution of ULVZ reservoirs wedged into the corners of the LLSVP. The five snapshots show the evolution of the system between 0 and 5 Ma. The streamlines are colored by the magnitude of the velocity with light blue corresponding to 1 cm/yr and dark red to 2 cm/yr. The ULVZ patches are separated from the LLSVP by the black outline and shaded in gray. The last snapshot highlights the streamlines, in green and orange, completely within the ULVZ indicating a steady state geometry has been achieved.

Figure 7: Change in height of the ULVZ reservoir as it falls down the wall of the LLSVP in Figure 6 is plotted in the solid line. The dashed line represents no change in height, i.e., steady state. The derivative is passed through a triangle smoothing filter to reduce noise in the measurement.

Figure 8: (a) Map of the magnitude of the largest principal strain rate within the coupled ULVZ-LLSVP system. The dotted line outlines the steady-state ULVZ-LLSVP interface at the end of the series of simulations in Figure 2 of the supplementary material. (b) Orientation of principal strain ellipses, colored by the magnitude of the largest principal strain rate. The black bars are oriented parallel to the principal shortening axis, and serve as a proxy for the orientation of melt films. The rose diagram in the inset depicts the frequency of angles subtended by the melt films with the CMB.