Bayesian estimation of magma supply, storage, and eruption rates using a multiphysical volcano model: Kīlauea Volcano, 2000–2012

Kyle R. Anderson^{a,b,*}, Michael P. Poland^c

^a*U.S. Geological Survey Hawaiian Volcano Observatory, Hawaii National Park, HI. 96718 USA*

^b*Now at: U.S. Geological Survey California Volcano Observatory, Menlo Park, CA. 94025 USA*

^c*U.S. Geological Survey Cascades Volcano Observatory, Vancouver, WA. 98683 USA*

Abstract

Estimating rates of magma supply to the world’s volcanoes remains one of the most fundamental aims of volcanology. Yet, supply rates can be difficult to estimate even at well-monitored volcanoes, in part because observations are noisy and are usually considered independently rather than as part of a holistic system. In this work we demonstrate a technique for probabilistically estimating time-variable rates of magma supply to a volcano through probabilistic constraint on storage and eruption rates. This approach utilizes Bayesian joint inversion of (1) diverse datasets using predictions from a multiphysical volcano model, and (2) independent prior information derived from previous geophysical, geochemical, and geological studies. The solution to the inverse problem takes the form of a probability density function which takes into account uncertainties in observations and prior information, and which we sample using a Markov Chain Monte Carlo algorithm. Applying the technique to Kīlauea Volcano, we develop a model which relates magma flow rates with deformation of the volcano’s surface, sulfur dioxide emission rates, lava flow field volumes, and composition of the volcano’s basaltic magma. This model accounts for effects and processes mostly neglected in previous supply rate estimates at Kīlauea, including magma compressibility, loss of sulfur to the hydrothermal system, and potential magma storage in the volcano’s deep rift zones. We jointly invert data and prior information to estimate rates of supply, storage, and eruption during three recent quasi-steady-state periods at the volcano. Results shed new light on the time-variability of magma supply to Kīlauea, which we find to have increased by 35–100% between 2001 and 2006 (from 0.11–0.17 to 0.18–0.28 km³/yr), before subsequently decreasing to 0.08–0.12 km³/yr by 2012. Changes in supply rate directly impact hazard at the volcano, and were largely responsible for an increase in eruption rate of 60–150% between 2001 and 2006, and subsequent decline by as much as 60% by 2012. We also demonstrate the occurrence of temporal changes in the proportion of Kīlauea’s magma supply that is stored versus erupted, with the supply “surge” in 2006 associated with increased accumulation of magma at the summit. Finally, we are able to place some constraints on sulfur concentrations in Kīlauea magma and the scrubbing of sulfur

by the volcano’s hydrothermal system. Multiphysical, Bayesian constraint on magma flow rates may be used to monitor evolving volcanic hazard not just at Kīlauea but at other volcanoes around the world.

Keywords: Bayes; joint inverse; magma supply rate; eruption rate; Kīlauea; volatile content

1. Introduction

The rate at which magma is supplied to a volcano plays a direct and fundamental role in controlling eruptive style, duration, and volume (Dvorak & Dzurisin, 1993). Yet, magma supply rate (MSR) – and changes in MSR which drive evolving eruption hazard – must be inferred indirectly and as a result may be known only approximately.

Multiphysical, physics-based forward models can be used to relate the physics of magma ascent and eruption with a wide range of observations (Anderson & Segall, 2011). Most simply, conservation of mass dictates that MSR be balanced by rates of magma storage and eruption; the former may therefore be estimated by constraint on the latter two (Swanson, 1972; Dvorak & Dzurisin, 1993). At reasonably well-monitored volcanoes, magma storage rates may be obtained from observations of ground deformation, and eruption rates obtained from the volumes of erupted products or through a proxy, such as gas or thermal emission rates. These calculations additionally require independent knowledge of various properties of the volcanic system, such as the magma’s volatile content (needed to relate gas emissions with eruption rate). A probabilistic framework makes it possible to include uncertainties associated with both data and independent information in estimates of MSR, and to quantitatively separate the effects of uncertainties from actual changes in supply rate, enhancing our ability to understand and forecast changing volcanic hazard.

Bayes’ Theorem allows us to combine a model of the volcanic system – capable of predicting observed data – with independent *a priori* (prior) information in order to formulate posterior probability density functions (PDFs) that characterize our knowledge of model parameters, such as magma supply rate, and their associated uncertainty. In the Bayesian approach, uncertainties in data and prior information are described probabilistically. In contrast with the common approach of fixing “nuisance” model parameters to assumed values, prior distributions for all parameters take the form of PDFs. In this work we construct prior PDFs directly from the results of previous quantitative studies; as such, these priors are not subjective, except to the extent that they must usually be constructed from discrete and sometimes conflicting published

*Corresponding author

Email address: kranderson@usgs.gov (Kyle R. Anderson)

values. Posterior PDFs describe our ability to resolve model parameters given the ability of the model to fit observed data, and the independent constraint offered by the prior PDFs. Multiphysical volcano models are well-suited for Bayesian analysis, as they are capable of relating magma physics with a wide range of observations using a relatively simple model parameterization (Anderson & Segall, 2011, 2013).

Kīlauea Volcano, believed to be fed by an upwelling plume originating within the mantle (Wolfe et al., 2009), is the youngest sub-aerial volcano in the Hawaiian-Emperor chain. Kīlauea (Figure 1) has been in semi-continuous eruption from East Rift Zone (ERZ) vents since 1983 and is one of the world’s most comprehensively-monitored volcanoes, offering an outstanding opportunity to estimate time-variable rates of magma supply, storage, and eruption at an extraordinarily active volcano. Observations suggest that the volcano’s MSR has waxed and waned during 2000–2012. Following long-term subsidence of the broader summit region and Southwest Rift Zone (SWRZ) during the 1990s at up to ~ 5 cm/yr (Figure 2), rapid inflation and high rates of CO_2 and SO_2 emissions were believed to be associated with a surge in magma supply beginning in ~ 2003 (Poland et al., 2012); summit deflation associated with eruptive activity followed the supply surge, and although inflation resumed in ~ 2010 it was associated with low rates of ERZ effusion (Poland, 2014) possibly caused by low rates of magma supply. Existing constraints on MSR are largely empirical, however, since the data sets on which these inferences are based are treated independently rather than as a part of a coherent model. Coupling a multiphysical model of the system with diverse observations in a Bayesian inverse procedure – and incorporating recent advances in our conceptual understanding of Kīlauea along with new techniques for measuring the volcano’s storage and eruption rates – should yield improved quantitative understanding of the volcano’s time-variable magma flow rates.

2. Model and Observations of Kīlauea Volcano

The basic conceptual model of Kīlauea Volcano was introduced by Eaton & Murata (1960). Magma supplied to Kīlauea from a source in the mantle rises into summit storage reservoirs, in which it accumulates and from which it may erupt at the surface or move laterally along rift zones, resulting in intrusions and eruptions tens of km from the summit. Although punctuated by dike intrusions and other transient behavior, during eruptions the system often enters a state of quasi-steady magma flux (Swanson, 1972; Tilling et al., 1987). Since the 1983 onset of Kīlauea’s currently ongoing ERZ eruption, magma has, for the most part, passed from depth through the summit reservoir and migrated subhorizontally along the ERZ for ~ 20 km before erupting as lava flows near the Pu’u ‘Ō’ō eruptive vent. A second eruptive vent opened in 2008 at Kīlauea’s summit; this vent exhibits a clear magmatic connection with the summit reservoir and ERZ vent

(Rowe et al., 2015; Thornber et al., 2015; Patrick et al., 2015) and currently hosts an active lava lake which emits a persistent plume of gas and minor amounts of tephra (Orr et al., 2012).

Decades of ground deformation, seismic, gravity, and petrologic data at Kīlauea have revealed that magma storage at the volcano occurs principally in the summit reservoir system, the shallow rift zones, and possibly within the deeper rifts. Summit storage occurs in a complex of reservoirs at different depths (Baker & Amelung, 2012; Poland et al., 2014). In the shallow rift zones, magma storage is indicated by ground deformation (Okamura et al., 1988; Baker & Amelung, 2015), the petrology of erupted products (Thornber et al., 2003, 2015), and direct sampling via drilling (Teplow et al., 2009), but some of these areas appear to be small bodies relict from past intrusive episodes, and here we focus on time periods which avoid geodetic transients in the ERZ suggestive of shallow storage (section S8.1.1). Finally, geodetic evidence of quasi-continuous seaward-motion of Kīlauea’s south flank suggests that Kīlauea’s deep rift system is opening by tens of cm/yr, which could accommodate a significant fraction of the magma supplied to the volcano (Owen et al., 2000; Cayol et al., 2000). South flank motion is complex, however (e.g., Denlinger & Morgan (2014)), and evidence that deep rift opening is associated with magma storage is contradictory (Poland et al., 2014); it thus remains unknown just how much, if any, magma is stored in this region of the volcano.

With this general framework in mind, we develop a model of Kīlauea Volcano (Figure 1) in which the mass rate of magma supplied to the volcano is balanced by the mass rates of magma stored in the subsurface plus that which is erupted at the surface (e.g., Gonnermann et al. (2012)). Mass flow rates are related to volumetric flow rates using magma densities throughout the system, and volume flow rates are related to observations of ground deformation, accounting for compressibility of the magma. Magmatic sulfur concentrations are used to relate eruption rate with observed SO₂ emissions. We model the system during three quasi-steady-state time periods which allow us to examine possible months- to years-long changes in magma fluxes at the volcano (Table 1 and Figure 2): (1) t_1 , spanning 2000–2001, (2) t_2 , from Dec. 2005 to Dec. 2006, and (3) t_3 , from Oct. 2011 to May 2012. Start and end dates are chosen based on the availability of data and prior information necessary to constrain storage and eruption rates (eruption rates in particular are not well known during many time periods at Kīlauea).

Denoting mass flow rates of non-volatile bulk magma (melt and crystals) by Q ,

$$Q_i = Q_s + Q_r + Q_o, \quad (1)$$

(c.f. Gonnermann et al. (2012)) where subscripts represent $[i]$ nflow (supply), $[s]$ ummit storage, $[r]$ ift storage, and $[o]$ utflow (eruption). (Symbols are defined in Table 2.) In this sign convention, positive Q represent flow of magma from Kīlauea’s deep supply into storage reservoirs and towards the ERZ for eruption; $Q_i \geq 0$,

$Q_r \geq 0$ (because deformation indicates persistent rift opening), and $Q_o \geq 0$ always, but Q_s may be positive or negative according to whether or not magma accumulates in or is withdrawn from summit reservoirs. This mass balance holds regardless of whether or not all of Kilauea’s magma supply passes through the summit before entering the rifts (an ongoing source of debate; see Poland et al. (2014)).

To relate mass flow rates Q with volume flow rates q we use

$$q_j = Q_j / \rho_j \quad ; \quad j = i, s, r, o \quad (2)$$

where the magma’s bulk density ρ depends on the density of the basaltic melt and crystals and varies as magma moves through the volcano’s plumbing system. We derive prior distributions for ρ_i , ρ_s , ρ_r , and ρ_o from previous geophysical, geochemical, and geological studies (prior distributions used in this study are mathematically defined in the Supplementary Material and shown in Figure 3). These distributions account for storage of magma at different density throughout the system; for instance, magma in the summit reservoir system is probably mostly melt, while in the deep rift it may take the form of a dense ultrabasic cumulate mush (Ryan, 1988).

2.1. Magma Storage

Addition or subtraction of magma from storage reservoirs will generally alter reservoir pressure and, because magma is compressible, change the density of the stored magma. As a result, the rate of volumetric magma storage q in a reservoir does not in general equal the rate of volume change V' (prime terms denote rates of change) of the reservoir itself in the crust, to which geodetic data are sensitive (e.g., Johnson et al. (2000)). For Kilauea’s summit, we relate these using

$$V'_s = q_s / (1 + R_s), \quad (3)$$

(section S1.2), where R_s is the ratio of magma compressibility to reservoir compressibility (section S4.2). In the case of large R_s (e.g., for a very bubbly magma in a spherical reservoir), a significant disparity will be introduced between the reservoir volume change estimated from geodetic data and the actual volume of magma added or subtracted (Johnson et al., 2000; Rivalta & Segall, 2008). R_s may be obtained by computing **magma and reservoir compressibilities** directly, but such calculations are strongly dependent on host rock strength, reservoir shape, and especially exsolved bubble fraction – none of which are well known. We therefore leave R_s as a free parameter which we constrain using a prior distribution derived from theoretical modeling and observations of past eruptions (section S4.2).

For simplicity, we directly constrain V'_s during t_1 (denoted by $V'_{s|t_1}$) using a prior distribution derived from the results of a previous GPS and leveling study during 1996–2002 (section S4.4; Cervelli & Miklius

(2003)). For other periods we compute ground deformation at the summit from V'_s and compare directly to data. Because our goal is only to place first-order constraint on rates of magma storage and not to carefully constrain the location and geometry of storage reservoirs, we treat the summit storage system as a single reservoir and compute ground deformation using simple elastic models, in which rates of ground deformation $\mathbf{u}'_s$ in three dimensions are directly proportional to V'_s and functions f of reservoir location ξ , observation coordinates $\mathbf{x}$, and additional parameters $\mathbf{g}$ governing reservoir shape and host rock properties:

$$\mathbf{u}'_s(\xi, \mathbf{x}, \mathbf{g}) = V'_s f(\xi, \mathbf{x}, \mathbf{g}). \quad (4)$$

Because of temporal changes in the style of magma storage in Kilauea's summit, we use different f for different time periods. For t_2 , following Myer et al. (2008), Baker & Amelung (2012), and Poland et al. (2012), we use a model of an opening rectangular sill (Okada, 1985) beneath the summit and upper SWRZ, while for t_3 we use a model of a spherical magma reservoir (Mogi, 1958). In the first case $\mathbf{g}$ is a function of the sill's length, width, and strike, while in the second it is only a function of Poisson's ratio ν . Model predictions in t_2 are compared to a single ascending-mode ENVISAT radar interferogram spanning 12 Dec. 2005 to 31 Dec. 2006, while for t_3 we use one ascending-mode and one descending-mode COSMO-SkyMed interferogram spanning 21 Oct. 2011 to 16 May 2012 and 20 Oct. 2011 to 15 May 2012, respectively. InSAR data processing is described in section S3.1.

For storage in Kilauea's deep rift, we argue that R must be negligible (section S4.3), so

$$V'_r k_r = q_r \quad ; \quad 0 \leq k_r \leq 1, \quad (5)$$

where k_r is a scaling factor that accounts for the unclear relation between deep rift opening and magma storage. Varying k_r between 0 and 1 allows the model to simulate the range of possible deep rift storage (DRS) conditions, from all deep rift opening being associated with magma accumulation ($k_r = 1$) to no magma storage in the deep rift zones ($k_r = 0$). Although V_r may be modeled directly using deformation data, this approach is complex and computationally expensive when used in an inverse procedure. Because DRS has occurred at a near-constant rate during the time periods over which our model is applied (Poland et al., 2014), and observations constraining k_r are ambiguous (section S4.5), we apply a prior distribution on the product $V'_r k_r$ that is uniform between 0 (no DRS) and an upper limit on V'_r obtained from previous inversions of south-flank deformation data ($\sim 0.045 \text{ km}^3/\text{yr}$; section S4.5).

2.2. Magma Output

Only recently have direct, high-precision estimates of Kilauea’s time-averaged lava eruption rates become available through the TanDEM-X radar mission (Poland, 2014). To constrain eruption rate in t_3 we use a TanDEM-X estimate of bulk eruption rate for 15 Sept 2011 through 25 May 2012 (ignoring a brief period of ocean entry), converted to volume flow rate using equation (2). To be conservative, for the TanDEM-X data we use a relative measurement uncertainty of 2%, although actual measurement errors are likely much smaller.

TanDEM-X data are unavailable for t_1 and t_2 . For these periods we use the rate of SO_2 emissions from the eruption site as a proxy for eruption rate (Sutton et al., 2003), accounting for minor summit emissions (prior to the 2008 onset of Kilauea’s summit eruption, more than 90% of SO_2 emissions were recorded from the ERZ, where magma first neared the surface). We use SO_2 emissions data from Elias & Sutton (2007); see section S3.2. We do not use SO_2 emissions data after 2008, since the opening of the summit vent at that time has complicated interpretation of traditional spectrographic measurements (Elias & Sutton, 2012). Sulfur dioxide emissions at the summit and ERZ are related to magma and sulfur flow rates using

$$E_{es}^{SO_2} = (f^{SO_2}/f^S) Q_{es}^S \quad (6)$$

$$E_{ee}^{SO_2} = (f^{SO_2}/f^S) Q_o \chi_{ee}^S, \quad (7)$$

respectively, where f^{SO_2} and f^S are molecular weights of sulfur dioxide and sulfur, respectively, $f^{SO_2}/f^S \approx 2$, and

$$Q_{es}^S = (Q_o + Q_r) (\chi_i^S - \chi_o^S - \chi_{ee}^S - \chi_{sb}^S) \quad (8)$$

from conservation of sulfur throughout the system (section S1.1). Here, Q_{es}^S represents the mass rate of sulfur emitted to the atmosphere at the summit, and χ^S terms are **sulfur concentrations**. This equation is derived under the assumption that DRS magma passes through the summit (section S1.1), but allowing DRS magma to bypass the summit little affects the sulfur mass balance because summit SO_2 emissions are minor during modeled time periods.

From equation (7), constraint on the mass fraction of sulfur emitted from erupted ERZ lavas, χ_{ee}^S , is required in order to relate SO_2 emissions with eruption rate. χ_{ee}^S has been previously estimated utilizing observed carbon/sulfur ratios in ERZ gas emissions together with a modeled concentration of CO_2 in the ERZ (Sutton et al., 2003) (see section S4.9), but the latter may not be well known. Instead, we utilize equations (6) and (8) together with observed summit SO_2 emissions and constraint on concentrations of sulfur χ^S at

165 various points in the system. Sulfur concentration χ_i^S in the primary melt is constrained using the results
 166 of previous studies of melt inclusions and submarine-erupted basalts (which are quenched under pressure
 167 prior to significant degassing) (section S4.7). Similarly, we use geochemical observations to constrain the
 168 small amount of sulfur that remains in solidified basalts χ_o^S (section S4.8). We are unable to constrain χ_{sb}^S ,
 169 the loss of sulfur to ground or surface water (“scrubbing”) before emission into the atmosphere (although
 170 it is unlikely that χ_{sb}^S is significant at Pu’u ‘Ō’ō, the importance of scrubbing at Kīlauea’s summit is not
 171 well known; section S2). We therefore use previous estimates of Q_o during t_1 (see section S4.6) obtained
 172 from very-low-frequency (VLF) electromagnetic estimates of flow through lava tubes, geologic mapping,
 173 and summit ground tilt (Sutton et al., 2003; Heliker & Mattox, 2003) to estimate χ_{ee}^S and χ_{sb}^S during t_1 .
 174 Assuming that these concentrations did not change significantly between t_1 and t_2 , eruption rate in t_2 may
 175 be estimated utilizing only SO₂ emissions data (equation (7)).

176 3. Inverse

177 We hypothesize that observed data $\mathbf{d}$ can be described using $\mathbf{d} = \mathbf{G}(\mathbf{m}) + \epsilon$, where $\mathbf{G}$ is a nonlinear
 178 function (the forward model) of the model vector $\mathbf{m}$, and errors ϵ follow a Gaussian distribution with mean
 179 $\mathbf{0}$ and covariance Σ . We manually weight Σ for gas data such that the variation in model predictions
 180 reasonably matches variation (scatter) in the data.

181 We simultaneously invert the data from t_1 , t_2 , and t_3 , allowing some parameters – such as magma
 182 supply, storage and eruption rates – to vary between time periods but assuming that others remain constant
 183 (including $V_r' k_r$ and χ^S , as suggested by the near-constant rate of south flank deformation and the lack of
 184 temporal trends in petrologic data and following Greenland et al. (1985) and Poland et al. (2012)). Under
 185 these assumptions, the model vector is given by

$$\mathbf{m} = [\mathbf{Q}_o, \mathbf{V}_s', V_r' k_r, R_s, \chi^S, \chi_{sb}^S, \chi_{ee}^S, \chi_o^S, \rho_j, \xi, \mathbf{g}]^T, \quad (9)$$

186 where $j = s, r, o$ (we do not invert for ρ_i directly but use its distribution in postprocessing to compute q_i from
 187 Q_i), $\mathbf{Q}_o = [Q_{o|t_1}, Q_{o|t_2}, Q_{o|t_3}]$, $\mathbf{V}_s' = [V_{s|t_1}', V_{s|t_2}', V_{s|t_3}']$, $\xi = [\xi_{|t_2}, \xi_{|t_3}]$, and $\mathbf{g} = [\mathbf{g}_{|t_2}, \nu]$. The deformation
 188 source position vector ξ contains east, north, and depth parameters, and $\mathbf{g}_{|t_2}$ contains sill length, width,
 189 and strike. Note that we postprocess to obtain MSR and certain other distributions of interest.

190 We follow the general inverse approach of Anderson & Segall (2013) and use Bayes’ Theorem to compute
 191 the posterior PDF using

$$P(\mathbf{m}|\mathbf{d}) \propto P(\mathbf{d}|\mathbf{m}) P(\mathbf{m}). \quad (10)$$

The likelihood function $P(\mathbf{d}|\mathbf{m})$ is a measure of the fit of model predictions to observed data, accounting for data uncertainties, while $P(\mathbf{m})$ is the prior probability function. Thus, model parameters are constrained by the fit of model predictions to the data and by prior probability distributions. To sample (evaluate) the posterior PDF, we perform a standard Markov Chain Monte Carlo simulation with the Metropolis algorithm (e.g., Mosegaard & Tarantola (1995)), which is allowed to run until 48 independent Markov chains converge to similar distributions, which we combine to produce the final PDF (section S6).

We emphasize that our goal is not to find a single best-fitting model but rather to explore the full range of model parameters consistent with observations. To ensure that uncertainties are fully considered, we let all 25 model parameters vary in the inverse procedure. We do not expect the inverse to offer improved constraint on many of these parameters, such as Poisson’s ratio – however, by allowing these parameters to vary freely over their *a priori* range, we ensure that uncertainties are propagated fully into estimates of magma supply, storage, and eruption rates. We consider this a key element of the Bayesian inverse approach utilized here.

4. Results

Prior and posterior PDFs for estimated model parameters are shown in Figure 3. It is important to remember that posterior PDFs include uncertainties in data and prior information, but cannot account for the extent to which the model $\mathbf{G}$ misrepresents reality (so-called model error). Although we have attempted to reduce model error by constraining certain model parameters (such as R_s) using rough prior constraints rather than computing them directly, uncertainties below should be considered minimums. Possible sources of remaining model error in this study include unmodeled magma storage away from the summit, the influence of secular motion on deformation data, viscoelastic behavior of summit magma reservoirs, the simple idealized deformation sources we use, the storage of magma in void space in the summit, and the assumption of a direct relationship between magma flow rate and gas emission rates. Some of these uncertainties are discussed more fully in section S8.

The model is able to fit observations for all time periods to within uncertainties (Figure 4). Model predictions fall within the scatter in gas emissions data and explain the majority of InSAR-observed deformation.

In the following sections, the notation a – b is generally used to represent posterior PDFs, where a and b indicate lower and upper 68% confidence bounds, respectively. Median values and 95% confidence bounds are provided in Table 3. We emphasize, however, the importance of considering the full shape of the posterior PDFs presented in the figures.

4.1. Eruption Rates and Sulfur Concentrations

Estimated eruption rates are given in Table 3 and shown in Figure 5. Eruption rate during t_1 is constrained by its prior PDF, and in t_2 by SO_2 emissions data. (In section S5 we investigate placing prior constraint on χ_{ee}^S rather than on eruption rate in t_1 as described in section 2.2; we find that results are similar, but parameter estimates are less well resolved.) Eruption rate during t_3 is very tightly resolved by TanDEM-X data, and is consistent with other evidence for low rates during 2010–2013 (Poland, 2014; Orr et al., 2015). Relative changes in eruption rates between time periods are discussed in section 5.2.

Results place some constraint on sulfur concentrations in Kīlauea’s magma. Posterior PDFs for χ^S and χ_o^S are not improved over prior PDFs, but we are able to constrain χ_{es}^S , χ_{ee}^S and χ_{sb}^S (during t_1 and t_2). We estimate $\chi_{ee}^S=0.06\text{--}0.10$ wt%, in agreement with Sutton et al. (2003), who used $\chi_{ee}^S=0.075$ wt% with SO_2 emissions to estimate eruption rates consistent with our prior constraint. Although we are able to fit observations without any scrubbing of sulfur by Kīlauea’s hydrothermal system, neither can we rule out high rates of scrubbing; of 12.1–19.2 kg/s and 19.7–31.7 kg/s of sulfur that entered Kīlauea’s magma system in t_1 and t_2 , respectively, as much as 7.6 kg/s and 10.7 kg/s (43% and 36%, at 68% confidence) may have been scrubbed by the hydrothermal system prior to detection, respectively.

Such high rates seem inconsistent with qualitative arguments against significant scrubbing at Kīlauea, but are difficult to rule out *a priori*. We perform inversions in which we allow for no loss of sulfur to Kīlauea’s hydrothermal system; unsurprisingly, results indicate higher erupted sulfur concentrations χ_{ee}^S in the ERZ (closer to 0.1 wt%, which is higher than estimated in Sutton et al. (2003)), in turn yielding slightly lower eruption rates (at the lower end of the prior range for $Q_{o|t_1}$). The influence of scrubbing on estimated MSR is discussed briefly in section 4.3.

4.2. Magma Storage Rates

Volume change rate V' of Kīlauea’s active summit magma reservoir during t_1 is constrained here by its prior PDF (Figure 3; section S4.4). During t_2 , InSAR data (Figure 4) suggest a long, thin, southwest-striking sill at a depth of 4.5–4.9 km. We estimate a volume change rate of 0.45–0.48 m^3/s , in general agreement with a variety of previous studies across different time periods in 2006 yielding anywhere from 0.25 to 2.8 m^3/s (with variation due in part to variations in the rate and location of inflation that occurred during 2006) (Myer et al., 2008; Poland et al., 2012; Baker & Amelung, 2012). During t_3 , InSAR data suggest a spherical reservoir beneath the northeast margin of Halema‘ūma‘u Crater at 1.9–2.5 km depth with a volume change rate of 0.04–0.05 m^3/s , roughly an order of magnitude lower than in 2006 (Figure 5).

Although these models adequately fit the InSAR data (Figure 4), we emphasize again the simplicity of our deformation modeling, which is designed to resolve first-order rates of reservoir volume change rather than provide detailed constraint on storage locations or geometries.

Reservoir volume change rates V' yield estimates of magma storage rates to within uncertainties on the magma compressibility ratio R_s . We estimate low rates of magma withdrawal during t_1 at -0.16 – 0.02 m^3/s , storage during t_2 at 0.73 – 1.87 m^3/s , and storage during t_3 at 0.07 – 0.19 m^3/s (Figure 5). Thus, up to several times more magma may have been stored in the summit than geodetic constraint on V'_s might indicate (and highlighting the importance of future work to constrain compressibility).

Results here do not improve constraint on DRS above that already offered by the prior PDF. However, it is clear that DRS ($k_r > 0$) is not necessary to fit the data. In the next section we investigate the influence of uncertainty in DRS on estimated MSR.

4.3. Magma Supply Rates

We estimate magma supply rates in t_1 , t_2 , and t_3 of 0.11 – 0.17 km^3/yr , 0.18 – 0.28 km^3/yr , and 0.08 – 0.12 km^3/yr , respectively (Figure 5; mass flow rates in tons/day and volume flow rates in m^3/s are given in Table 3). These values can be interpreted as the rate of magma supply from the mantle source region through the lithosphere to Kīlauea Volcano, exclusive of supply to other volcanoes such as Mauna Loa (Miklius & Cervelli, 2003; Gonnermann et al., 2012) or storage in deep, geodetically-invisible storage reservoirs (Gonnermann et al., 2012).

Over millions of years, average supply rates from the Hawaiian hot spot to all of Hawai‘i’s active volcanoes has probably averaged 0.05 – 0.2 km^3/yr , possibly increasing over the last 30 Ma (Robinson & Eakins, 2006; Poland et al., 2014). Contemporary estimates of MSR to Kīlauea range from <0.05 km^3/yr to nearly 0.20 km^3/yr during various **time intervals between the 1950s and the onset of the ongoing Pu‘u ‘Ō‘ō eruption** (e.g., Swanson (1972); Dzurisin et al. (1980); Dvorak & Dzurisin (1993); Cayol et al. (2000); Wright & Klein (2008)), and from 0.08 to 0.18 km^3/yr during various parts of 1983–2002 (e.g., Wolfe et al. (1987); Denlinger (1997); Cayol et al. (2000); Heliker & Mattox (2003); Sutton et al. (2003)). Most recently, Poland et al. (2012) estimated **at least 0.2 km^3/yr** and Wright & Klein (2014) ~ 0.4 km^3/yr for the 2006 supply surge. Interested readers are referred to **reviews in Poland et al. (2014) and Klein and Wright (in review).**

Our posterior PDFs fall within the wide range of these contemporary estimates, but should improve upon them by including additional constraints, propagation of uncertainties, and important factors mostly neglected in previous work. No previous studies have accounted for magma compressibility, and of the handful that have attempted to include DRS, all have fixed it to the volume determined from models of

deep rift opening under the assumption that all rift opening is produced by magma storage (Cayol et al., 2000; Wright & Klein, 2014). Here, we argue that DRS cannot be constrained *a priori* any better than 0– ~ 0.045 km³/yr (Figure 3) on the basis of current understanding of Kīlauea (section S4.5), so we allow it to vary freely over this range such that our posterior PDFs encompass both end-member possibilities as well as all intermediate values. Failure to account for uncertainty in DRS could greatly bias estimated MSR; for instance, under the assumption of no DRS ($k_r = 0$), we estimate a rate of 0.088–0.13 km³/yr in t_1 , while with 0.045 km³/yr of DRS, estimated MSR increases to 0.13–0.18 km³/yr (Figure S1). Thus, DRS represents one of the most important sources of uncertainty in any mass-balance based estimate of MSR (section S8.2), and explains in part why our estimates are generally lower than those in Cayol et al. (2000) (0.14–0.18 km³/yr for 1983–1991) and Wright & Klein (2014) (~ 0.2 km³/yr for 1983–2003 and 0.4 km³/yr for 2006), who assumed that all flank motion was driven by DRS. Magma compressibility and scrubbing of sulfur by Kīlauea’s hydrothermal system also contribute importantly to uncertainty. Although we suspect that scrubbing is probably minor (as is usually the case where magma is at or very close to the surface), we cannot discount the possibility of significant scrubbing and are reluctant to apply a prior constraint based on our qualitative suspicions. Unsurprisingly, under the assumption of no scrubbing, we estimate eruption rates and MSRs that are lower and better resolved (Figure S1). We hope that these results will help motivate future work to resolve these critical components of Kīlauea’s magma plumbing system.

5. Discussion

5.1. Is Kīlauea an Open System?

Previous studies have suggested that Kīlauea’s supply and eruption rates tend to track one another, at least when averaged over a number of eruptions (Dvorak & Dzurisin, 1993), and also that shallow storage rates during the ongoing Pu’u ‘Ō’ō eruption have usually been less than either supply or eruption rates (Johnson, 1992; Kauahikaua & Miklius, 2003). These results have suggested that Kīlauea has been a relatively open system since at least 1983, but DRS and magma compressibility complicate the picture.

We find that the distribution of Kīlauea’s magma supply (by mass) was similar during t_1 and t_3 , with less than 10% of MSR stored or withdrawn from the summit reservoir system, up to $\sim 50\%$ entering the deep rifts, and the remainder ($> \sim 50\%$) erupted from the ERZ (Figure 6). During both time periods DRS exceeded the rate of summit storage at $> 90\%$ confidence (given our prior constraint on DRS), which leads us to conclude that although Kīlauea was a relatively open system, storage of magma in the deep rifts may have represented a significant fraction of supply. The situation differed during the supply surge in t_2 , when high

rates of summit storage **likely matched or exceeded the rate of DRS**. Distributions under the assumption of DRS at either 0 or 0.045 km³/yr are shown in Figure S2.

We define an eruption efficiency as the ratio of eruption rate to supply rate (Q_o/Q_i ; this differs somewhat from the definition in Wright & Klein (2008)) and find efficiencies above ~50% for all time periods (Figure 6b). **Efficiency decreased at ~90% confidence in association with the supply surge in t_2 , but this decrease was probably less than 30% (Figure 6c). Change in efficiency is not well resolved between t_2 and t_3 , but we find that efficiency in t_3 was reduced from t_1 at 99.9% confidence. Computing ratios of mass summit storage rate to eruption rate (Q_s/Q_o), we find that summit storage rates in t_3 were ~3–9% of eruption rate, down from ~13–38% in t_2 . The modest decrease in efficiency and high rates of summit storage and rapid uplift in t_2 may indicate that the ERZ conduit was able to absorb some, but not all, of the increased supply during the surge. Additional variations in efficiency may have been caused by unrelated changes in Kilauea’s shallow magma plumbing system, which could perhaps explain why efficiency did not return to t_1 levels following the end of the surge.**

5.2. Changes in Kilauea’s Magma Flow Rates and Implications for Eruptive Activity

Marginal posterior PDFs suggest temporal changes in magma flow rates, but computing relative changes between time periods can eliminate some constant error terms and bring changes into tighter focus. We compute an unambiguous >99% chance of some increase in MSR between t_1 and t_2 ; this increase in supply was probably between 35% and 99% (Figure 5), roughly consistent (especially at the high end of the range) with the observed increase in summit CO₂ emissions (Poland et al., 2012). Eruption rate likewise increased, by between 63% and 151%. By t_3 , Kilauea’s magma supply surge had ended; relative to t_2 , MSR had decreased by -67%–-46% and eruption rate by -57–-14%, and both parameters were decreased from t_1 at >95% confidence. These results support the idea of a post-surge lull in supply following t_2 (Poland, 2014).

The surge and subsequent lull were clearly manifested at the surface of Kilauea by variations in magmatic and volcanic activity. The magma supply surge was accompanied by an increase in eruption rate, resulting in a more vigorous lava flow field. Changes in the mineralogy and chemistry of erupted products were also documented, suggesting an increasing component of “fresh” mantle-derived magma within the volcano (Poland et al., 2012; Thornber et al., 2015). The eruption was unable to keep pace with the burgeoning supply of magma, however, and the rate of storage beneath the summit increased, stressing nearby faults and causing a few moderate-magnitude earthquakes (Wauthier et al., 2013). In June 2007 the magma system along the ERZ ruptured in the “Father’s Day” ERZ intrusion and eruption, with the formation of new eruptive vents along the ERZ (Poland et al., 2008).

Magmatic and eruptive activity during the magma supply lull in t_3 , in contrast, was considerably subdued, even compared to pre-surge levels. Lava flows failed to reach the south coast of the island during most of 2012 (Poland, 2014), whereas ocean entries in the same area were common prior to and during the surge. Based on evidence for continued low rates of lava effusion, the MSR lull appears to have persisted into at least 2015 (although not modeled here). It is tempting to speculate that the lull in supply might therefore have influenced the ability of lava flows to advance far enough from their source vents to cause considerable damage to communities on the east side of the Island of Hawai'i during the Pāhoa lava flow crisis of 2014–2015 (Poland et al., 2016). Were supply higher, the impact on populated areas might have been more severe.

6. Conclusions

The technique developed in this study may be suitable for constraining magma supply rates at a variety of volcanoes for which storage and eruption rates can be probabilistically estimated. Direct measurements of ground deformation and erupted volumes from space- or air-borne platforms are suitable in this regard, and might make the technique applicable even to volcanoes without extensive ground-based monitoring networks. Where direct measurements of eruption rate are not available, proxies such as SO_2 emissions may be used, but require knowledge of system properties such as the magma's volatile content and the extent of hydrothermal scrubbing of sulfur, and may therefore result in greater uncertainties. In such cases, the technique may be expected to work best at reasonably well-understood volcanoes with good monitoring networks.

Our model of Kīlauea is simple – based most fundamentally on mass conservation – but relates steady-state magma flow rates with observed ground deformation, SO_2 emissions, flow field volumes, and magma composition. With this model and the Bayesian approach used in this work, we are able to quantitatively account for important sources of uncertainty that have mostly been neglected in previous studies. Despite uncertainties, changes in the rate of magma supply to Kīlauea (by perhaps as much as a factor of 2 or more) over sub-decadal time scales are unambiguous in the context of our model. The “surge” in magma supply to Kīlauea during 2006 was followed by a reduction of supply to rates that were lower than pre-surge (2000–2001) levels at $>95\%$ confidence. Whether these changes in supply rate are quasi-random, the post-2006 lull represents a temporary equilibration following the surge, or the lower rates represent the early stages of the end of the ongoing Pu'u Ō'ō eruption, remains unknown.

Although MSR to Kīlauea may be modulated by summit reservoir pressure (Dvorak & Dzurisin, 1993;

Gonnermann et al., 2012), the changes observed in this study were likely driven predominantly by deeper (bottom-up) processes, as suggested by petrologic data, CO₂ emissions, and other observations (Poland et al., 2012). How changes in deep supply are manifested as changes in storage and eruption rates has implications for lava flow advance rates, the formation or abandonment of eruptive vents, and the intrusion of magma into country rock, but is complicated by changes in the volcano’s shallow plumbing system. We find that despite substantial variations in Kīlauea’s supply rate, at least half – and perhaps nearly all, depending on the rate of deep rift storage – of the magma that entered the system erupted onto the surface during all three studied time periods, and summit storage rates were generally only a fraction of eruption rates. Thus, even a minor reduction in eruption efficiency can result in greatly increased rates of summit inflation, as observed during the 2006 supply surge. Changes in efficiency may occur as a response to changes in MSR or be due to unrelated variations in the shallow magma plumbing system, but we speculate that modestly reduced efficiency during 2006 could be consistent with limitations in the carrying capacity of the ERZ conduit during the surge. Regardless of the cause, tracking probabilistic changes in magma supply, storage, and eruption rates may provide an important tool for monitoring evolving volcanic hazard.

Multiphysical volcano models, used in Bayesian inverse procedures with a wide range of data and prior information, show great promise for probabilistically illuminating otherwise difficult-to-constrain properties of volcanic systems. This work represents only a first step in the use of this approach at Kīlauea. Estimates of magma flow rates at the volcano must be updated and refined as new observations become available, as understanding of the volcanic system improves, and as the volcano model is further developed. We emphasize the importance of future work on Kīlauea’s mobile south flank, as well as alternative means of quantifying lava discharge rate, since significant uncertainty in rates of deep rift storage and eruption translate into significant uncertainty in magma supply rates. If storage and eruption rates can be estimated over shorter time periods it should be possible to estimate MSR with correspondingly improved temporal resolution, with the caveat that propagation delays of magma throughout the system place a lower bound on the time durations over which simple models such as the one developed in this study may be used. Our volcano model is straightforward, fast, and robust, requiring relatively few complex (and often poorly-understood) physical processes to be directly modeled. In the future it will be useful to include additional physical processes, such as the effect of volatiles on magma compressibility, which will relate model parameters in ways not currently considered.

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Table 1: **Time periods** used in this study and primary source of constraint on summit deformation and eruption rate.

Name	Time period	Δt (days)	Summit Deformation	Eruption rate
t_1	01 Jan 2000 – 31 Dec 2001	730	Prior from Cervelli & Miklius (2003)	Pior constraint
t_2	11 Dec 2005 – 31 Dec 2006	385	InSAR (ENVISAT)	SO ₂ emissions
t_3	20 Oct 2011 – 16 May 2012	208	InSAR (COSMO-SkyMed)	TanDEM-X

Table 2: **Model parameters and symbols**

Symbol	Description	Units
Model parameter and data		
Q, q	Magma flow rate (mass, volume)	kg/s, m ³ /s
Q^s	Sulfur flow rate (mass)	kg/s
ρ	Magma density	kg/m ³
V'	Reservoir volume change rate	m ³ /s
R	Compressibility ratio	-
$\mathbf{u}'$	Ground displacement rate	cm/yr
ξ	Reservoir location	m
$\mathbf{x}$	Deformation observation coordinates	m
$\mathbf{g}$	Reservoir model parameters (vector)	-
k_r	Rift/magma scaling factor	-
E^{SO_2}	SO ₂ emission rate	kg/s
χ^S	Sulfur concentration	wt%
Location symbols (subscripts)		
i	Inflow (supply)	-
s	Summit storage	-
r	Rift storage	-
o	Outflow (eruption)	-
sb	Scrubbing	-
es	Erupted at summit	-
ee	Erupted in ERZ	-

Table 3: **Magma flow rates.** Median of accepted samples, and 68% and 95% confidence bounds for magma supply, summit storage, and eruption rates. Volumetric eruption rates are dense-rock equivalent (DRE); tons are metric. For consistency, all volumetric results are presented in m³/s.

Time	Median (tons/s)	68% (tons/s)	95% (tons/s)	Median (m ³ /s)	68% (m ³ /s)	95% (m ³ /s)
MSR						
t_1	12.22	[9.71,14.83]	[7.65,17.30]	4.36	[3.46, 5.31]	[2.72, 6.24]
t_2	19.72	[15.68,24.67]	[12.29,30.90]	7.05	[5.59, 8.84]	[4.37, 11.10]
t_3	8.39	[6.66,10.31]	[5.81,11.67]	3.00	[2.38, 3.69]	[2.06, 4.21]
Summit storage rate						
t_1	-0.19	[-0.42,-0.04]	[-0.68,0.11]	-0.07	[-0.16, -0.02]	[-0.26, 0.04]
t_2	3.36	[1.88,4.86]	[1.30,5.78]	1.29	[0.73, 1.87]	[0.50, 2.21]
t_3	0.32	[0.18,0.49]	[0.12,0.66]	0.12	[0.07, 0.19]	[0.04, 0.26]
Eruption rate						
t_1	9.88	[7.94,11.87]	[6.50,13.70]	3.53	[2.83, 4.26]	[2.31, 4.94]
t_2	13.66	[10.29,18.17]	[7.68,24.09]	4.88	[3.67, 6.51]	[2.73, 8.65]
t_3	5.58	[5.21,5.93]	[4.94,6.21]	1.99	[1.85, 2.14]	[1.73, 2.28]

Figure 1: Map of Kilauea's summit (a) and schematic cross section (b) showing conceptual model geometry. Magma transport and storage regions and lava flows are schematic only. Dashed line in (b) indicates possible alternate path of magma into the deep rift (both are consistent with the mathematical model design). The lava lake shown in (b) did not exist prior to 2008.

Figure 2: Summit deformation and relationship to time periods examined in this study (shaded gray bars). GPS station locations are shown in Figure 1.

Figure 3: Posterior PDFs (defined in Supplementary Material) for all 25 estimated model parameters, with prior PDFs shown by dotted red lines (uniform prior PDFs, which are nonzero within the figures and zero outside them, are not shown). Vertical axes indicate relative probability, which is not scaled between parameters. Limits of horizontal axes correspond to *a priori* lower and upper bounds. Model and data cannot hope to constrain many model parameters such as magma density ρ (note that posterior PDFs are unchanged from prior PDFs), but by estimating these parameters their uncertainty is incorporated into estimates of other important model parameters such as MSR. “Sill” and “chamber” parameters are associated with deformation sources during t_2 and t_3 , respectively (ξ and $\mathbf{g}$ in Equation (4)), and east and north positions are given relative to 19.4047° N, 155.2805° W, roughly the SE margin of the active lava lake during 2012. Sill coordinates reference a point at the center of the north (long) edge, and strike is degrees clockwise from north.

Figure 4: Fit of model predictions to data. The range of MCMC predictions are shown by horizontal blue bars in (a)–(c); because the forward model is steady-state, all rates that it predicts are constant within each time period. (a) Summit SO₂ emissions during t_1 and t_2 . (b) ERZ SO₂ emissions during t_1 and t_2 . (c) TanDEM-X time-averaged bulk eruption rate during t_3 . (d) Observed InSAR displacements **in the direction of the satellite** (top row) and **maximum-probability** predictions of InSAR displacements (bottom row) for t_2 (first column) and t_3 (second and third columns). Orthogonal arrows show satellite flight and look directions. InSAR data are decimated using the quadtree algorithm (section S3.1).

Figure 5: (a) Eruption rates, (b) summit storage rates, and (c) magma supply rates. First and second rows show posterior PDFs for mass and volumetric flux rates, respectively (eruption rate is DRE). Colored and gray horizontal bars above PDFs show 68% and 95% confidence bounds, respectively. Third row shows PDFs of relative changes in flow rates between the three studied time periods (only t_2 to t_3 is shown for storage rates since storage in t_1 is negative). Fourth row shows complimentary cumulative distribution functions, which indicate the probability that a change in flow rate exceeded a particular value.

Figure 6: (a) Posterior PDFs (relative probability on vertical axis) showing percentage of MSR by mass entering summit storage, entering the deep rift, and erupted onto the surface for the three examined time periods. For distributions under different assumptions about DRS, see Figure S2. (b) Eruption efficiency. (c) Relative change in eruption efficiency.