Constraining the source of mantle plumes

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Abstract

In order to link the geochemical signature of hot spot basalts to Earth's deep interior, it is first necessary to understand how plumes sample different regions of the mantle. Here, we investigate the relative amounts of deep and shallow mantle material that are entrained by an ascending plume and constrain its **source region**. The plumes are generated in a viscous syrup using an isolated heater for a range of Rayleigh numbers. The velocity fields are measured using stereoscopic Particle-Image Velocimetry, and the concept of the 'vortex ring bubble' is used to provide an objective definition of the plume geometry. Using this plume geometry, the plume composition can be analysed in terms of the proportion of material that has been entrained from different depths. We show that the plume composition can be well described using a simple empirical relationship, which depends only on a single parameter, the sampling coefficient, s_c . High- s_c

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plumes are composed of material which originated from very deep in the fluid domain, while low- s_c plumes contain material entrained from a range of depths. The analysis is also used to show that the geometry of the plume can be described using a similarity solution, in agreement with previous studies. Finally, numerical simulations are used to vary both the Rayleigh number and viscosity contrast independently. The simulations allow us to predict the value of the sampling coefficient for mantle **plumes; we** find that as a plume reaches the lithosphere, 90% of its composition has been derived from the lowermost 260 – 750 km in the mantle, **and negligible amounts are derived from the shallow half of the lower mantle. This result implies that isotope geochemistry cannot provide direct information about this un-sampled region, and that the various known geochemical reservoirs must lie in the deepest few hundred kilometres of the mantle.**

Keywords:

1. Introduction

Heat flow from radiogenic elements and the cooling of the core causes the mantle to undergo continuous convection. One consequence of this is the appearance of plumes; columns of hot, buoyant material which rise through the mantle, causing intraplate volcanism at several hotspots. The formation of Large Igneous Provinces (LIPs) is generally attributed to the initial arrival of plumes at the lithosphere, while ocean island chains are thought to arise from the interaction between the remaining plume and the over-riding plate (Campbell, 2005; Morgan, 1971; Richards et al., 1989).

51 The basalts formed at these ocean islands (OIBs) tend to be highly het-
52 erogeneous, relative to those formed at spreading centres, and there is consid-
53 erable variation between the chemical signatures of OIBs found at different
54 hot spots (Hawkesworth and Scherstén, 2007; Kellogg, 1992). This indicates
55 that plumes are derived from a strongly heterogenous source - or have more
56 than one distinct source (Hofmann, 1997) - but it remains unclear where
57 these sources lie or how OIBs should be interpreted in terms of the overall
58 chemical composition of the mantle. For example, there is disagreement over
59 whether the heterogeneity in OIBs is indicative of the overall heterogeneity
60 in the mantle, or whether it simply represents the chemical state near
61 the thermal boundary layer where the plume formed (Hofmann and Farnetani, 2013).
62 **Similarly, the high $^3\text{He}/^4\text{He}$ signatures of OIBs at some**
63 **hot spots indicate the existence of reservoirs of ‘primitive’ material**
64 **that have remained isolated from the convecting mantle for billions**
65 **of years (Tackley, 2000; van Keken et al., 2002); however, it is not**
66 **known where in the mantle these reservoirs lie.**

67 The composition of a plume is determined by its entrainment process.
68 Early flow visualisation experiments of thermal plumes by Griffiths and
69 Campbell (1990) showed that plumes contain a large head followed by a
70 narrower tail, and that plumes can entrain ambient material by mechani-
71 cally ‘stirring’ it into the vortices in the plume head. However, the numerical
72 simulations of Farnetani and Richards (1995) and experiments by Cagney
73 et al. (2015) found that the contribution of mechanical stirring to the en-
74 trainment process was relatively insignificant when the plume was generated
75 from an isolated heat source. Farnetani and Richards (1995) argued that

76 the high rates observed by Griffiths and Campbell (1990) were a result of
77 their experimental system, which involved the injection of a buoyant fluid
78 into a vessel of fluid with a higher viscosity, rather than the use of a heater.
79 Hauri et al. (1994) presented a similarity solution for the steady-state struc-
80 ture of the plume stem, and suggested that entrainment occurs by the radial
81 conduction of heat away from the stem.

82 Knowledge of how plumes entrain ambient material is crucial to under-
83 standing what effect plumes will have on the creation or removal of chemical
84 heterogeneity in the mantle. Entrainment (or the lack thereof) has been
85 used to explain various geological and geochemical features associated with
86 hot spots. For example, Hart et al. (1992) and Walker et al. (1995) argued on
87 the basis of the geochemical signature of ocean island basalts that plumes do
88 not entrain significant amounts of material from the mid- or shallow mantle,
89 but are derived from the region close to the core-mantle boundary. In con-
90 trast, Bianco et al. (2008) argued that the geochemical signatures of many
91 ocean island basalts could be explained by a plume entraining heterogeneous
92 material throughout the mantle. In spite of the importance of plumes for
93 our understanding of the deep mantle, their composition remains largely un-
94 known. In part this is because there is no accepted or objective means to
95 quantify the geometry or volume of a plume, and therefore it is difficult to
96 define what material is contained within the plume and how this may change
97 with time.

98 Turner (1957) showed that plumes are a special case of vortex rings,
99 as the vortices in the plume head form a torus in three-dimensions. The
100 vortex ring structure is particularly clear in the flow visualisation images of

101 Griffiths (1986) and Griffiths and Campbell (1990). This approach was used
102 by subsequent researchers studying plumes (e.g. Chay and Schlien (1986);
103 Pottebaum and Gharib (2004); Schlien (1976); Shusser and Gharib (2000)).
104 The volume of turbulent vortex rings is often identified using the concept
105 of the ‘vortex ring bubble’ (VRB) (Shariff and Leonard, 1992). The VRB
106 is defined by the streamlines which enclose the vortex ring, calculated in
107 a reference frame moving with the vortices, as will be described in more
108 detail in Section 3.1. This concept was used by Dabiri and Gharib (2004) to
109 examine the rates of entrainment of vortex rings under various conditions.

110 This paper extends the concept of the vortex ring bubble to thermal
111 plumes to quantify their geometry and **source region**. The remainder of the
112 paper is structured as follows; the experimental system and analysis tools are
113 described in the following section; Section 3.1 demonstrates the calculation
114 of the VRB and compares it to various Lagrangian descriptions of the flow;
115 the plume composition is examined in Section 3.2; the analysis is applied to
116 numerical simulations at mantle conditions in Section 4; **the implications**
117 **for mantle dynamics and isotope geochemistry are discussed in**
118 **Section 5**; and finally some concluding remarks are made.

119 2. Experimental details

120 The experiments were performed in a sealed cubic test-section, which had
121 internal dimensions of 265 mm and 10 mm thick walls, as shown in Figure
122 1. The test-section was surrounded by a thermal bath, which was held at
123 25.2°C. The thermal plume was generated using a circular heater in the
124 centre of the test-section, which had a diameter of 20 mm and was installed

Figure 1: Elevation (top) and plan view (bottom) of the test-section and PIV system.

125 flush with the bottom surface. A closed-loop feedback controller was used to
 126 ensure that the heater temperature was held constant to within $\pm 0.25^\circ\text{C}$ of
 127 the target temperature.

128 The working fluid was corn syrup (LSI Speciality Products' Liquidose
 129 436), which has a Newtonian but strongly temperature-dependent viscosity.
 130 The density and viscosity were measured in the laboratory (see Cagney et al.
 131 (2015) for more details). The syrup had a density, ρ of 1439 kg/m^3 at 25.2°C
 132 and a coefficient of thermal expansion, α , of $3.1 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. The viscosity
 133 was found to vary as

$$\mu = 1080e^{-0.156T+6.25 \times 10^{-4}T^2} [\text{Pa.s}], \quad (1)$$

134 where T is temperature in degrees Celsius. The thermal diffusivity and spe-
 135 cific heat capacity were supplied by the manufacturer as $\kappa = 1.04 \times 10^{-7} \text{ m}^2/\text{s}$
 136 and $C_p = 2280 \text{ J/kg.K}$, respectively. These values correspond to a Prandtl
 137 number of $Pr = \mu/\rho\kappa = 2.15 \times 10^5$. **The numerical simulations of**
 138 **Whitehead et al. (2013) show that when the Prandtl number is**
 139 **greater than $\sim 10^3$, the dynamics of thermal plumes are essentially**
 140 **identical to those calculated at an infinite Prandtl number, con-**
 141 **firmed that our experiments are relevant to the mantle.**

142 The temperature difference between the heater and the ambient fluid,
 143 ΔT , is non-dimensionalised as the Rayleigh number

$$Ra = \frac{\rho\alpha\Delta TgD^3}{\mu\kappa}, \quad (2)$$

144 where g is the acceleration due to gravity, and D is the depth of the fluid
 145 domain. Four experiments were performed in which ΔT was set to 24.9°C ,
 146 34.8°C , 44.9°C and 54.9°C , which corresponded to a Rayleigh number range
 147 of $(0.62 - 1.36) \times 10^6$ and a viscosity contrast range of $\gamma = \mu/\mu_{min} = 15.1 -$
 148 141.4 (Equation 1).

149 The velocity fields associated with the thermal plume were measured us-
 150 ing stereoscopic-Particle Image Velocimetry (PIV). The syrup was uniformly
 151 seeded using a high-gloss polymer powder which had a diameter in the range
 152 $1 - 40 \text{ }\mu\text{m}$, and the measurement plane was illuminated using a halogen
 153 white light source and a fibre optic line generator. Images were acquired si-
 154 multaneously using two CCD cameras, separated by 18° , as shown in Figure
 155 1. The images from each camera were processed using a two-pass cross-
 156 correlation scheme, and the fields from the two cameras were combined to

157 find the three-component velocity field. Throughout each experiment, 41
 158 velocity fields were acquired.

159 The final velocity fields spanned the range $x = -85.5$ mm to 78.8 mm and
 160 $y = 1.39$ mm to 215.9 mm (which is referred to as the PIV domain), where
 161 x and y are measured in the horizontal and vertical directions, respectively,
 162 with the origin fixed as the centre of the heater surface. The fields had a
 163 resolution of 2.79 mm in both directions.

164 The velocity fields are also analysed using a Lagrangian approach, which
 165 has been shown to be an effective means of visualising the plume structure
 166 (Farnetani et al., 2002; Lin and van Keken, 2006; Cagney et al., 2015). The
 167 Lagrangian Coherent Structures (LCSs) can be estimated using the Finite-
 168 Time Lyapunov Exponent (FTLE) (Haller, 2015), which is a measure of the
 169 maximum rate of separation of neighbouring fluid particles over the period
 170 t_0 to t . The FTLE therefore also serves as an indicator of regions of strong
 171 fluid stretching.

172 In order to calculate the FTLE, it is first necessary to calculate the flow
 173 map

$$\psi(\mathbf{x}, t, t_0) = \mathbf{x}(t, t_0, \mathbf{x}_0), \quad (3)$$

174 which describes the dynamic position of tracers in the flow that had an
 175 initial spacing $\mathbf{x}_0$ at t_0 . This is found by numerically advecting passive
 176 tracers throughout the measured velocity fields using a fourth order Runge-
 177 Kutta scheme with linear interpolation in time and space. The tracers
 178 were uniformly distributed throughout the PIV domain with a spacing of
 179 0.93 mm $\times$ 0.93 mm, i.e. three times the spatial resolution of the vector

180 fields.

181 The FTLE field is then given by

$$\sigma_f(\mathbf{x}, t, t_0) = \frac{1}{|t - t_0|} \log \left(\left\| \frac{d\psi(\mathbf{x}, t, t_0)}{d\mathbf{x}_0} \right\| \right), \quad (4)$$

182 where $\|\cdot\|$ represents the maximum real eigenvalue of the Cauchy-Green ten-
183 sor.

184 The FTLE fields were computed in backward-time by reversing the flow
185 and advecting the tracers backwards from t to $t_0 = 0$ (i.e. until the start
186 of the start of the experiment). The regions of high σ_f represent attracting
187 lines within the flow.

188 The tracer advection process was also used to identify the original posi-
189 tion of each tracer when the experiment began. The tracers were advected
190 backwards in time from each time step until $t = 0$, and their original height
191 $y_o(\mathbf{x}, t)$ was identified. The $y_o(\mathbf{x}, t)$ fields are henceforth referred to simply
192 as the ‘tracer fields’. As will be seen, this approach is particularly useful
193 when examining the entrainment process and the composition of the plume
194 throughout its development.

195 3. Experimental Results

196 3.1. The Vortex Ring Bubble

197 It is often useful in studies of convecting vortex rings to examine the
198 flow in a reference frame that is moving with the ring (Dabiri and Gharib,
199 2004; Shariff and Leonard, 1992). In the current study, this corresponds
200 to a reference frame rising with the plume, and requires an estimate of the
201 plume’s rise velocity, which in turn requires an estimate of the plume height

Figure 2: Contours of the vertical velocity and streamlines at $t = 1797$ s and $Ra = 1.36 \times 10^6$, computed in an Eulerian reference frame (a) and one moving with the plume (b). The thick black line in (b) represents the vortex ring bubble.

as a function of time, $y_p(t)$. We derive the plume height from the tracer
fields; y_p is defined as the highest point at which the local tracers originated
from below the PIV domain ($y \leq 1.39$ mm). This was found to be consistent
with the definitions based on the maxima of σ_f (Cagney et al., 2015) and
the gradient of the radial velocity (Davaille et al., 2011), but contained less
scatter. The rise velocity signal, $u_r(t)$, was then found by differentiating $y_p(t)$
using the least-squares method.

Figure 2(a) shows the vertical component of a sample velocity field with
streamlines for the $Ra = 1.36 \times 10^6$ experiment. The streamlines were com-
puted in an Eulerian or ‘lab-based’ reference frame (i.e. one which is fixed

in space and does not move with the plume), and do not provide a clear indication of the position or geometry of the plume. Two centres of rotation are apparent at $y \approx 130$ mm, as the material is rising along the central axis and induces a return flow near the edges of the PIV domain.

The velocity field can be converted into a reference frame moving with the plume by subtracting $u_r(t)$ from the vertical velocity component, as shown in Figure 2(b). In this case the plume structure is apparent in the streamlines, and has a clear inverted teardrop shape. The heights of the centres of rotation have not changed, but they are now contained within the plume structure. A similar analysis was presented by Davaille et al. (2011).

This figure bears strong similarities to the two-dimensional measurements of a convecting vortex ring presented in Dabiri and Gharib (2004). Now that the velocity fields are in a frame moving with the plume, the vortex ring bubble can be used to define the plume geometry, as shown by the thick black lines enclosing the centres of rotation in Figure 2(b). In order to find these lines, streamlines were computed, starting from the top of the PIV domain with an initial spacing equal to that of the PIV measurement (i.e. $\Delta x = 2.79$ mm). The neighbouring streamlines which moved to either side of the plume axis were identified and the lines were joined at the top of the plume using a spline curve, which is indicated by the thick dashed line in Figure 2(b), to form the VRB.

The VRB is shown at three instances in Figures 3(a-c), overlaid on the tracer fields. The colour denotes the height of each tracer at the time when the heater was turned on, y_o . In each case, the VRB is taken to mark the boundary between the ambient fluid and material in the plume head which

Figure 3: Tracer fields showing the vertical origin of material at $y_p = 60.8$ mm (a), 122.1 mm (b), and 185.2 mm (c), for $Ra = 1.36 \times 10^6$. The vortex ring bubble is shown by the back and white dashed lines, and aligns with the boundary between ambient fluid and that derived from very deep in the fluid domain.

Figure 4: Finite-Time Lyapunov Exponent fields computed at $y_p = 60.8$ mm (a), 122.1 mm (b), and 185.2 mm (c), for $Ra = 1.36 \times 10^6$. The vortex ring bubble is shown by the black and white dashed lines, and aligns with ridge at the top of the plume head.

has risen from deep in the fluid domain. The majority of the fluid contained in the VRB has originated from very deep in the test-section (dark blue regions). This material forms a circular shape at the top of the plume head which closely matches the shape of the VRB in this region.

The VRB is compared to the FTLE fields in Figure 4. The variation in the FTLE fields throughout the growth and rise of a thermal plume is described in detail elsewhere (Cagney et al., 2015), and the current discussion focuses on its relation to the VRB. Ridges in the FTLE tend to separate qualitatively distinct regions of the flow; therefore, any definition of the structures within a flow is required to be consistent with the estimated FTLE fields.

247 The top of the plume head is marked by a circular arc of high σ_f which
 248 represents a Lagrangian Coherent Structure, and aligns closely with the top
 249 boundary of the plume. This provides further confidence that the VRB is
 250 faithfully identifying the plume geometry. It also indicates that the region
 251 at the upper edge of the VRB is characterised by strong fluid stretching and
 252 is an attracting line in the flow. A region of high σ_f is also present along the
 253 plume axis ($x = 0$), particularly close to the heater. However, this is internal
 254 to the plume and does not relate to the VRB.

255 In general, the definition of the plume geometry is consistent with the
 256 tracers and FTLE fields which were calculated independently, providing fur-
 257 ther evidence that the VRB is an effective means to define the plume geom-
 258 etry.

259 *3.2. Plume Composition and Entrainment*

260 The composition of the plume can also be examined throughout its de-
 261 velopment using the tracer fields shown in Figure 3. In order to quantify the
 262 contribution of tracers originating from different depths in the fluid domain,
 263 the tracers contained within the VRB were identified, and histograms were
 264 computed of their origin height. The histograms contain 30 bins, spaced
 265 equally over the range $y = 0$ to $y_p(t)$. For ease of comparison, the data are
 266 presented as a line graph, rather than a bar chart, and the counts for each
 267 bin, c_i , are presented in normalised form as a probability distribution:

$$\phi'_i = \frac{c_i}{\sum_i c_i}, \quad (5)$$

Figure 5: Variation in the composition of the plume for four different plume heights for $Ra = 1.36 \times 10^6$ (a); ϕ' is the normalised number of tracers which originated from a given height, y_o . The exponential distribution of tracers is clear in (b), in which the height of the tracer origin is normalised by the plume height. **The three-dimensional distribution, which accounts for the radial position of tracers (Equation 12), is shown in (c).**

268 where ' denotes a sectional property (because ϕ'_i and c_i are computed from
 269 tracers found in the measurement plane rather than from the entire three-
 270 dimensional fields). The distributions all appear to follow an exponential
 271 distribution. This is confirmed in Figure 5(b), in which the tracer heights
 272 are normalised by the plume height. In this case, all the distributions align
 273 closely, and can be described as:

$$\phi' = A' e^{-s'_c y_o/y_p}, \quad (6)$$

274 where A' and s'_c are non-dimensional constants. These values were found
 275 by performing a least-squares fit to the data in Figure 5(b) in the range
 276 $0 \leq y_o/y_p < 0.35$, which gave values of $A' = 0.22$ and $s'_c = 6.22$. The
 277 r^2 value was 0.991, and the standard deviation of the linear regression was
 278 $\sigma(s'_c) = 0.093$. The constant s'_c is henceforth referred to as the 'sampling
 279 coefficient'.

280 Equations 5 and 6 relate to two-dimensional plumes, such as those gen-
 281 erated in two-dimensional simulations. However, plumes in experiments and
 282 the mantle are three-dimensional, and this will affect the tracer distribution
 283 and the calculations of the sampling coefficient. This effect can be accounted
 284 for by weighting each point in the tracer field in proportion to the volume it
 285 occupies in a three-dimensional, axisymmetric space:

$$w_k = \pi(\Delta x_o)^2 |x_k|, \quad (7)$$

286 where w_k is the weighting factor, Δx_o is the resolution of the tracer fields
 287 and $|x_k|$ is the distance of each tracer from the plume axis. The probability
 288 distribution (Equation 5) can be modified to account for this:

$$\phi_i = \frac{\sum_{k=1}^{c_i} w_k}{\sum_i \left(\sum_{k=1}^{c_i} w_k \right)}, \quad (8)$$

$$= \frac{\sum_{k=1}^{c_i} |x_k|}{\sum_i \sum_{k=1}^{c_i} |x_k|}. \quad (9)$$

289 The three-dimensional probability distribution, ϕ (now without a prime,
 290 as it is no longer a sectional property), also follows an exponential relation-
 291 ship, as can be seen in Figure 5(c). The gradient is steeper in the three-
 292 dimensional case, i.e. $s_c > s'_c$.

293 For the four plume heights examined, no tracers are identified which had
 294 originated from above approximately half the plume height, $y_o/y_p \gtrsim 0.5$. For
 295 example, when $y_p = 166$ mm, no tracers were found for $y_o \gtrsim 82$ mm. This
 296 is likely to be a consequence of the fact that the top of the plume head is a
 297 material line in the flow (indicated by the arc-like ridge in the σ_f fields in
 298 Figure 4, Cagney et al. (2015)) and no fluid passes through it, i.e. material
 299 does not enter the plume directly from above. In theory the plume will grow
 300 by thermal diffusion, and thus may incorporate some material into the head
 301 in this manner; however, given the large Pr , this effect is likely to be very
 302 small, and negligible in the mantle, where $Pr \sim 10^{22}$.

303 The parameter A' is dependent on the number of bins used to calculated
 304 the histograms. Because no tracers are entrained for $y_p/y_o \gtrsim 0.5$, the integral
 305 of the probability distribution can be written as

$$\int_0^{0.5} \phi \, d\left(\frac{y_o}{y_p}\right) = -\frac{A}{s_c} (e^{-s_c/2} - 1) = 1, \quad (10)$$

306 which can be re-arranged to express A as a function of s_c :

$$A = \frac{s_c}{(1 - e^{-s_c/2})}. \quad (11)$$

307 Therefore, the composition of the material in the VRB can be expressed
308 using only the sampling coefficient,

$$\phi = \frac{s_c}{(1 - e^{-s_c/2})} e^{-s_c y_o / y_p}. \quad (12)$$

309 The sampling coefficient controls the degree to which the plume samples
310 material from different heights. High- s_c plumes are composed of material
311 originating almost exclusively from very deep in the fluid, while low- s_c plumes
312 entrain a greater proportion of material from different heights. Therefore, the
313 inverse of the sampling coefficient is indicative of the degree of entrainment
314 within a plume. Table 1 summarises the sectional and three-dimensional
315 sampling coefficients found for each of the four experiments. It is apparent
316 that the sampling coefficient is dependent on the Rayleigh number and the
317 viscosity contrast. However, as the two properties are both changing, it is
318 not possible to discern the effect of either variable independently.

319 The sampling coefficient appears to be independent of time and plume
320 height (Figure 5(b-c)) and Equation 12 does not contain a time-dependency
321 (other than that implicitly contained in $y_p(t)$). This suggests that the en-
322 trainment process and the form of the plume can be described by a similarity
323 solution. This feature of the plume growth is demonstrated in Figure 6, which

Ra	γ	s'_c	s_c
0.62×10^6	15.1	5.71	6
0.86×10^6	35.7	5.8	6.06
1.12×10^6	75.9	6.03	6.76
1.36×10^6	141.4	6.22	7.06

Table 1: Summary of the sectional and three-dimensional sampling coefficients found for each of the four experiments.

Figure 6: Geometry of the vortex ring bubble calculated at plume heights for $Ra = 1.36 \times 10^6$ (a), and the same geometries normalised by the plume height (b).

324 shows the VRB calculated at four times and plume heights. When the di-
325 mensions are normalised by $y_p(t)$, the four bubbles collapse (Figure 6(b)),
326 indicating that the non-dimensional geometry of the plume and its general
327 shape do not vary with time.

328 Previous researchers have proposed similarity solutions to describe the
329 steady-state stems of laminar and turbulent plumes (Batchelor, 1954; Hauri
330 et al., 1994) and isolated thermals (Griffiths, 1991; Whittaker and Lister,
331 2008; Peng and Lister, 2014). The current work empirically shows that this
332 condition is also applicable to the entire plume, and indicates that this may
333 be a fruitful approach for future analytical work.

334 4. Application to the mantle

335 4.1. Numerical Details

336 The data presented in Table 1 shows that the sampling coefficient in-
337 creases with the applied temperature difference. However, both the Rayleigh
338 number and the viscosity contrast are also varying, and it is difficult to dis-
339 tinguish between the influence of Ra and γ . In order to isolate the effect
340 of both parameters on the plume composition, numerical simulations were
341 performed in which Ra and γ were varied independently.

342 The simulations were designed to replicate conditions in the test-section.
343 The fluid domain was cubic with dimensions of 265 mm in each direction, and
344 except where otherwise stated, the fluid properties were the same as those
345 of the syrup (Section 2). The fluid was assumed to have an infinite Prandtl
346 number and the Boussinesq approximation was employed. The relevant non-
347 dimensional equations for conservation of mass, momentum and energy are:

$$\vec{\nabla} \cdot \vec{v} = 0 \quad (13)$$

$$\vec{\nabla} \cdot \varsigma_{ij} - \vec{\nabla} p = Ra \Theta \hat{e}_y \quad (14)$$

$$\frac{\partial T}{\partial t} = \nabla^2 T - \vec{v} \cdot \vec{\nabla} T \quad (15)$$

where v is the velocity vector, ς_{ij} is the deviatoric stress tensor, p the pressure, Θ the non-dimensional temperature, and $\hat{e}_y$ the vertical unit vector. The rheology of the fluid was given by Equation 1.

The model is solely heated by a circular patch at the centre of the lower surface, with the same dimensions and thermal properties as in the laboratory setup. All other boundaries had a rigid no-slip condition, and (except for the constant temperature of the circular heater) were insulating by applying a zero heat flux condition.

Two suites of simulations were performed; one in which the viscosity contrast was fixed at $\gamma = 141.4$ and the Rayleigh number varied (the Ra -suite) and one in which the Rayleigh number was held constant at 1.36×10^6 and the viscosity contrast was varied (the γ -suite). For the former suite, the Rayleigh number was varied in the range $Ra = 10^5 - 10^8$ by varying the fluid density, while in the latter suite the viscosity contrast was controlled by varying ΔT for the range $\gamma = 2 - 490$, and κ was altered to ensure that Ra remained constant. An additional iso-viscous case was examined, in which the viscosity throughout the fluid was set to the ambient value.

Calculations were performed in three-dimensional Cartesian geometry with a standard resolution of $128 \times 128 \times 128$ ($n_x \times n_y \times n_z$) grid points

and 100 Lagrangian tracers per cell has been tested and is applied. For the Ra -suite, a higher resolution (i.e., $256 \times 256 \times 256$) was required to ensure a suitable representation of the increasingly thin boundary layer with higher Ra -numbers. The physical model described above was solved by the finite-difference/volume multigrid code StagYY (Tackley, 2008), and the high resolution experiments are run on the UK's National High Performance Computing Facility ARCHER.

In order to analyse the numerical results and compare to the experimental data, the velocity field from the centre plane of the plume was extracted (i.e. the same plane that was measured using PIV in the experiments).

4.2. Sampling Coefficient in the Mantle

The results of the numerical simulations were first compared to the experimental data in order to confirm that they were adequately capturing the flow physics. Figures 7(a) and 7(b) show the tracer fields calculated from the experimental and numerical results, respectively, for $y_p = 130$ mm and $Ra = 1.36 \times 10^6$. The similarity between the two fields is clear. The scrolls in the plume head are particularly noticeable in the numerical data, which may be a result of the infinite- Pr assumption; in the experimental case, thermal diffusion ensures that the temperature gradients and viscosity contrast are small outside the vicinity of the heater. However, in the numerical case the role of thermal diffusion is weak, and larger temperature gradients and viscosity contrasts can be sustained and the tracer fields can have a much finer structure (in this case, well-defined scrolls in the plume head).

Profiles of the vertical velocity for both cases are shown in Figures 7(c) and 7(d) for $y = y_p/3$ and $y = 2y_p/3$, respectively. The

Figure 7: Tracer fields computed from the experimental (a) and numerical (b) data, for $Ra = 1.36 \times 10^6$, $\gamma = 141.4$ and $y_p = 130$ mm. **Profiles of the vertical velocity for the same times are shown in (c) at $y = y_p/3$, and (d) at $y = 2y_p/3$; experimental data (black crosses), numerical data (grey lines).**

numerical and experimental profiles align very closely. Along with the strong similarity between the tracer fields, Figure 7 indicates that the simulations are successfully replicating the flow occurring in the experiments and can be used to predict the plume dynamics.

The dependence of s_c on Ra and γ is shown in Figure 8. The error bars are given by the standard deviation found from the linear regression performed between each set of $\log\phi_i$ and y_i/y_p . The sampling coefficient has a log-linear relationship with Rayleigh number (Figure 8(a)). The trend line suggests that s_c becomes negative for $Ra \lesssim 10^{4.5}$, which is physically unrealistic (corresponding to a plume which contains very little material from deep in the domain but is primarily composed of fluid from $y \approx y_p/2$). Therefore, the log-linear relationship is likely to breakdown at some point for $Ra < 10^5$, although such values are not relevant to mantle convection. The sampling coefficient increases with Ra , implying that the more vigorous the convection, the greater the proportion of plume material is derived from near the thermal boundary layer at which the plumes formed (most likely the core-mantle boundary, but potentially also the transition zone).

A log-linear relationship also exists between the sampling coefficient and the viscosity contrast (Figure 8(b)). The trend lines indicate that Ra and γ have opposite effects on the entrainment process; increasing the viscosity contrast *decreases* the proportion of plume material derived from deep in the fluid domain.

Using a two-dimensional best-fit, the sampling coefficient can be expressed as

Figure 8: Variation in the predicted sampling coefficient of plumes as a function of Rayleigh number at $\gamma = 141.4$ (a), and as a function of viscosity contrast at $Ra = 1.36 \times 10^6$ (b). The grey lines represent the best-fit linear trend between s_c and the log values.

$$s_c = 6.97 \log_{10} Ra - 2.83 \log_{10} \gamma - 24.98. \quad (16)$$

416 The magnitude of the $\log_{10} Ra$ term is greater than that of the $\log_{10} \gamma$ term,
 417 indicating that the Rayleigh number plays a greater role in determining the
 418 composition of mantle plumes.

419 The values of the Rayleigh number and viscosity contrast in the mantle
 420 are poorly constrained, due to a combination of uncertainties in the various
 421 physical properties of mantle material and their dependence on depth and
 422 chemical heterogeneity. Estimates of the Rayleigh number are typically in
 423 the range $10^6 - 10^8$, while the viscosity contrast is likely to be somewhere
 424 between 100 and 1000. In this case, Equation 16 predicts that a plume in
 425 the mantle will have a sampling coefficient in the range 8.3 - 25.1.

426 To put this into physical terms regarding the plume composition, it is
 427 useful to consider the integral of ϕ ,

$$\Phi = \int_0^{y_o/y_p} \phi \, d\left(\frac{y_o}{y_p}\right) = \frac{1 - e^{-s_c y_o/y_p}}{1 - e^{-s_c/2}}. \quad (17)$$

428 In effect, $\Phi(y_o/y_p)$ is equal to the proportion of the plume material which
 429 originated from $y \leq y_o$. Figure 9 shows the variation in the proportion of
 430 a plume which has been derived from the deepest 10 km in the mantle as
 431 the plume rises, for $s_c = 8.3$ and 25.1. In both cases $\Phi_{(y_o=10 \text{ km})}$ decreases
 432 exponentially, but the deep mantle component remains greater for the $s_c =$
 433 25.1 case. Figure 9 indicates that as a mantle plume reaches the transition
 434 zone, between 3.7 and 10.7% of its material will have originated from the
 435 deepest 10 km in the mantle.

Figure 9: Proportion of a plume derived from the lowermost 10 km in the mantle as a function of plume height for two different values of the sampling coefficient.

436 5. Discussion

437 The analysis presented in the previous section is relevant to the role of
438 mantle plumes in redistributing siderophilic elements throughout the mantle.
439 There is conflicting evidence as to whether plumes contain material from
440 the core, as suggested by the Osmium signatures found in some hot spot
441 basalts and LIPs (Hawkesworth and Scherstén, 2007; Walker et al., 1995;
442 White, 2010). The lowermost region of the mantle, close to the core-mantle
443 boundary, is likely to contain elevated traces of siderophilic elements relative
444 to the rest of the mantle; Equation 17 describes how **siderophilic elements**
445 **near the CMB will be distributed throughout the mantle over time.**

446 These results also have very significant implications for the geo-
447 chemical interpretation of hot spot basalts. As our study exam-
448 ines a rising plume, rather than the plume tail after the head has
449 reached the surface, our results are particularly relevant to the in-
450 terpretations of LIPs. The exact composition of hot spot basalts
451 will not be the same as the composition of a rising plume (i.e. as
452 described by the sampling coefficient and Equation 17) (Ballmer
453 et al., 2011); however, as conduction plays a negligible role in man-
454 tle dynamics on the time scale of a rising plume, the material out-
455 side the plume is unlikely to experience significant melting and
456 Equation 17 therefore represents a *constraint on the source of*
457 *OIBs*. Plumes may experience some entrainment as they interact
458 with major phase transitions or the lithosphere and their dynamics
459 become too complex to be described by a simple analytical equa-
460 tion, although these contributions to the geochemical signatures

461 of basalts are thought to be small (Farnetani and Richards, 1995;
462 Farnetani et al., 2002). The results presented here show that the
463 material in OIBs is primarily derived from very deep in the mantle
464 (most likely the lowermost few hundred kilometres).

465 This supports the geochemical study of Hart et al. (1992), who argued
466 that OIBs are derived from the lower mantle, and contain little or no material
467 derived from the mid- or upper-mantle. Our analysis provides a framework
468 to explain these geochemical observations, and also quantifies the *degree* to
469 which mantle plumes are derived from the lower mantle; by the time a plume
470 is within 10 km of the surface, Equation 17 indicates that 90% of the plume
471 material has been derived from the lowermost 260 – 750 km in the mantle
472 (depending on the value of s_c).

473 These findings also have important implications for isotope geo-
474 chemistry, and what it may tell us about the chemical composition
475 of the mantle. The results presented here imply that the geochem-
476 ical signature of OIBs is derived from the deeper half of the lower
477 mantle, with some likely contribution from regions near the ma-
478 jor phase transitions and lithosphere; therefore the various known
479 geochemical reservoirs must exist in one or more of these regions.
480 These reservoirs include the Enriched Mantle (derived from sub-
481 ducted slabs) and FOZO end-members, as well as the high $^3\text{He}/^4\text{He}$
482 regions of ‘primitive’ mantle (Hofmann, 1997; Hart et al., 1992; van
483 Keken et al., 2002).

484 Perhaps more significantly, it implies that the shallower half of
485 the lower mantle will have a very small or negligible contribution to

mantle plumes and will effectively be invisible in OIBs, and isotope geochemistry cannot be used to make any direct inference about this region. Therefore, various models of mantle heterogeneity that include large-scale reservoirs of primitive mantle material in this region (see reviews by Tackley (2000) and van Keken et al. (2002)) cannot be tested using isotope geochemistry alone.

Future work is planned to investigate what effect features such as stress- and pressure-dependent rheology and phase transitions may have on the plume composition and the implications for geochemistry discussed here.

Conclusions

In this study, we used our laboratory model to examine thermal plumes rising through a viscous syrup and provided a empirical measure to constrain the **source region** of mantle plumes. The velocity fields were measured using Stereoscopic Particle-Image Velocimetry.

Following the work of Turner (1957), plumes were treated as a special case of vortex rings, and the concept of the ‘vortex ring bubble’ was used to provide the first objective definition of a plume’s geometry. This definition was shown to be consistent with the Finite-Time Lyapunov Exponents and tracer fields. This novel description of the plume geometry allows the **source region of a plume to be quantified in terms of depth; material in the plume primarily originated from deep in the fluid domain, and the distribution of the material** was found to follow a clear exponential relationship that depends on a single variable, the sampling coefficient, s_c .

510 The distribution does not have an explicit dependency on time, and our
511 results show that the plume geometry is self-similar, in support of previous
512 analytical work.

513 Additionally, numerical simulations were performed in which the Rayleigh
514 number and viscosity contrast were varied independently in order to isolate
515 their effect on the **plume’s source region**. The simulation results were
516 in good agreement to the experimental data, and showed that the sampling
517 coefficient has a log-linear relationship with both the Rayleigh number and
518 viscosity contrast. The Rayleigh number has a strong effect on the sampling
519 coefficient, implying that the more vigorous the convection in the mantle,
520 the greater proportion of plume material will be derived from close to the
521 core-mantle boundary. From the numerical results, we predicted that plumes
522 in the mantle have a coefficient in the range $s_c = 8.3 - 25.1$, **which allowed**
523 **us to constrain the source of hot spot basalts**. This crucially indicates
524 that the vast majority of plume material derives from the deep mantle, in
525 agreement with previous geochemical studies (Hart et al., 1992; Walker et al.,
526 1995). **Furthermore, it showed that isotope geochemistry studies of**
527 **hot spots basalts will provide little information on regions above**
528 **approximately 1000 km from the CMB, and that the various known**
529 **geochemical reservoirs must lie within a few hundred kilometres of**
530 **the CMB.**

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