Acoustic field modulation in regenerators

J. Y. Hu ^a, W. Wang ^{a, b}, E. C. Luo ^a, Y. Y. Chen ^a

^a Key Laboratory of Cryogenics, Technical Institute of Physics and Chemistry, Chinese

Academy of Sciences, Beijing 100190, China

^b University of the Chinese Academy of Sciences, Beijing, 100039, China

Abstract

The regenerator is a key component that transfers energy between heat and work. The conversion efficiency is significantly influenced by the acoustic field in the regenerator. Much effort has been spent to quantitatively determine this influence, but few comprehensive experimental verifications have been performed because of difficulties in modulating and measuring the acoustic field. In this paper, a method requiring two compressors is introduced and theoretically investigated that achieves acoustic field modulation in the regenerator. One compressor outputs the acoustic power for the regenerator; the other acts as a phase shifter. A RC load dissipates the acoustic power out of both the regenerator and the latter compressor. The acoustic field can be modulated by adjusting the current in the two compressors and opening the RC load. The acoustic field is measured with pressure sensors instead of flow-field imaging equipment, thereby greatly simplifying the experiment.

Key words: regenerator; acoustic field; modulation; pulse tube cryocooler

Nomenclature			
1, C1	Compressor 1	2, C2	Compressor 2
3	double-acting piston facing to the orifice, the chamber between the double-acting piston and the orifice	A	piston area (m ²)
Arg	phase of a complex number	$back$	back space of compressor
com	Regenerative component's end facing to the compression chamber	D	diameter (m)
$elec$	electric	exp	Regenerative component's end facing to the expansion chamber
i	$\sqrt{-1}$	I	current (A)
Im	imaginary part of a complex number	K	spring constant (N/m)
$mech$	mechanical	M	piston mass (kg)
opt	optimal	ori	orifice
p	fluctuating pressure (Pa)	P_a	average pressure (Pa)
res	reservoir	Re	real part of a complex number
T	transmission matrix	U	volume flow rate (m ³ /s)
v	velocity (m/s)	V	volume (m ³)
W	work (W)	Z	acoustic impedance, impedance
ΔV	voltage (V)	γ	specific heat ratio
τ	transduction coefficient (N/A)	ω	angular frequency (rad/s)
$ $	magnitude of a complex number		

1 INTRODUCTION

Qualitatively, the efficiency of a regenerator in a cryocooler or a thermoacoustic prime mover is generally maximized when the pressure and flow are in phase although the acoustic field does not determine the performance alone. Such a phase relationship minimizes the magnitude of mass flow for a given acoustic power through the regenerator [1]. Since Gifford and Longworth invented the pulse tube cryocooler, its phase shifter has been changed many times, from basic to orifice to double-inlet type, and finally an inertance-tube type [2–6]. Each change was accompanied by an improvement in efficiency. The evolution in configuration actually provides a more ideal acoustic field in the regenerators. A similar evolution is also seen with the thermoacoustic prime mover. With the realization that standing-wave thermoacoustic engines were based on an intrinsically irreversible thermodynamic cycle, attention was quickly turned to traveling-wave thermoacoustic engines with reversible thermodynamic cycles that yielded higher efficiency [7, 8].

Quantitative studies using numerical methods have shown that the acoustic field influences the power capacity and efficiency [9, 10]. However, few experiments have been performed. Indeed, the task experimentally for the experimenter is in creating a large number of different acoustic fields to test performance. For example, if an inertance tube is employed as the phase shifter, its dimensions have to be changed many times [11]. At the same time, the flow field imaging techniques such as hot-wire anemometry, particle image velocimetry, or laser Doppler velocimetry are also employed to measure the volume flow, which is an important parameter of an actual acoustic field to know; otherwise, the theoretical and experimental results cannot be compared [12]. The equipment required makes the experiment

very complicated and troublesome. A simple method that produces different acoustic fields is much needed.

In [13], Wu used a linear compressor and a linear electric generator to test the performance of a thermoacoustic prime mover. The compressor produces the pressure wave. The electric generator, which was connected to the secondary water-cooler, absorbed the acoustic power. Changing the electric resistance, compliance, and piston mass of the generator changes the impedance of the generator as well as the acoustic field in the regenerator. The volume flow rate in front of the piston can be obtained by measuring the movement of the piston, which is much simpler to perform than the above-mentioned flow field imaging techniques. In principle, this method can also be applied in a regenerative cryocooler. However, changing the parameters of the generator is still troublesome. The number of acoustic fields realized with this method is still limited. In [14], Zhou used two loudspeakers and two resonance tubes to modulate the acoustic field in a regenerator. By changing the voltages on the two loudspeakers, the movement of coils can be adjusted. Thus, it is possible for the experimenter to realize different acoustic fields easily. In this method, the resonance tubes dissipated the power and altered the phase. If the volume flow at the two ends of the regenerator is required, flow field imaging equipment is still necessary because no models can precisely predict the volume flow from the movement of the coils, especially when the pressure is high. Furthermore, changing the length of the resonance tube is very inconvenient. There are also some other modulation methods [15–18]. However, the acoustic fields obtained with these methods are very limited and the experiment processes are inconvenient.

Wu and Zhou's methods inspired the question: Can we use two linear compressors connected to the two ends of the regenerator to modulate the acoustic field? As is known, an acoustic wave flows in at one end of the regenerator and out the other. If the compressor at the other end also outputs an acoustic wave, the two waves compete because neither can be absorbed nor can the power be dissipated. This is also the reason that Wu used a generator and Zhou used a long resonator. Hence, this method seems infeasible. Although a compressor cannot absorb the acoustic power, some other dissipator such as a RC load can be employed. Therefore, it is very plausible that two compressors and a RC load can modulate an acoustic field. This paper details the concept.

2 THEORETICAL ANALYSIS

Figure 1 presents two configurations for field modulation. Each mainly comprises four parts: two linear compressors (only the pistons are shown), a regenerative component, and a RC load. The regenerative component can either be a prime mover or cryocooler. The RC load consists of an orifice and a reservoir. In Config. 1, the RC load is installed between the regenerative component and Compressor 2 (C2). In Config. 2, the compressor employs a double-acting piston. The RC-load is attached to the right of the piston. For simplicity, the volume of the compression and expansion chamber can be considered as part of the regenerative component, so it is not taken into consideration in the following analysis and calculations.

2.1. Analysis on Config. 1

A transmission matrix correlates the volume flow rates (U_{com} and U_{exp}) and pressures (p_{com} and p_{exp}) at the two ends of the regenerative component [19],

$$\begin{bmatrix} p_{exp} \\ U_{exp} \end{bmatrix} = \begin{bmatrix} T(1,1) & T(1,2) \\ T(2,1) & T(2,2) \end{bmatrix} \begin{bmatrix} p_{com} \\ U_{com} \end{bmatrix}. \quad (1)$$

If the impedance at the right end is Z_{exp} , the impedance Z_{com} at the left end can be expressed as

$$Z_{com} = \frac{p_{com}}{U_{com}} = \frac{T(1,2) - T(2,2)Z_{exp}}{T(2,1)Z_{exp} - T(1,1)}, \quad (2)$$

with the volume flow rates correlated as

$$\frac{U_{exp}}{U_{com}} = T(2,1)Z_{com} + T(2,2) = T(2,1) \frac{T(1,2) - T(2,2)Z_{exp}}{T(2,1)Z_{exp} - T(1,1)} + T(2,2). \quad (3)$$

For Compressor 1 (C1), we have [10]

$$\Delta V_1 = Z_{elec,1}I_1 + \tau_1 v_1, \quad (4)$$

$$p_{com}A_1 = \tau_1 I_1 - Z_{mech,1}v_1, \quad (5)$$

$$v_1 A_1 = U_{com}, \quad (6)$$

and similarly for Compressor 2 (C2),

$$\Delta V_2 = Z_{elec,2}I_2 + \tau_2 v_2, \quad (7)$$

$$p_{exp}A_2 = \tau_2 I_2 - Z_{mech,2}v_2. \quad (8)$$

In the expansion chamber, we have

$$v_2 A_2 = p_{exp}/Z_{ori} - U_{exp}, \quad (9)$$

where ΔV , Z , I , τ , v , and A denote voltage, impedance, current, transduction coefficient, velocity, and piston area, respectively. Subscripts 1, 2, *elec*, *mech*, *acou*, and *ori* denote C1, C2, electrical, mechanical, acoustic, and orifice, respectively. From Eqs. (5) and (6), we obtain

$$U_{com} = \frac{I_1 \tau_1 A_1}{Z_{com} A_1^2 + Z_{mech,1}} \quad (10)$$

and from Eqs. (8) and (9), we obtain

$$U_{exp} = \frac{\tau_2 I_2 A_2}{Z_{exp} A_2^2 + Z_{mech,2} Z_{exp} / Z_{ori} - Z_{mech,2}} \quad (11)$$

Substituting Eqs. (10) and (11) into Eq. (3) yields

$$\frac{\tau_2 I_2 A_2}{Z_{exp} A_2^2 + Z_{mech,2} Z_{exp} / Z_{ori} - Z_{mech,2}} = \frac{I_1 \tau_1 A_1}{A_1^2 \frac{T(1,2) - T(2,2) Z_{exp}}{T(2,1) Z_{exp} - T(1,1)} + Z_{mech,1}} \left[T(2,1) \frac{T(1,2) - T(2,2) Z_{exp}}{T(2,1) Z_{exp} - T(1,1)} + T(2,2) \right]. \quad (12)$$

Therefore, given Z_{exp} and p_{exp} , the current in C2 I_2 is obtainable from Eq. (11). Then, the current in C1 I_1 is obtainable from Eq. (12), which implies that any acoustic field can be satisfied by the two currents.

For a RC load with a reservoir of volume V_{res} , the volume flow rate into the reservoir U_{ori} can be obtained easily and accurately from measurement of the pressure in it p_{res} [20],

$$U_{ori} = \frac{i\omega p_{res} V_{res}}{\gamma P_a} \quad (13)$$

Thus, the acoustic impedance of the RC load can be calculated from pressure readings in the expansion chamber and reservoir,

$$Z_{ori} = \frac{p_{exp}}{U_{ori}} = \frac{\gamma p_{exp} P_a}{i\omega p_{res} V_{res}}, \quad (14)$$

where P_a , ω and γ denote the average pressure, angular frequency and specific heat ratio, respectively. Similarly, the volume flow rate of a moving piston can also be inferred from pressure measurement in the back space of a compressor experiencing spatially uniform pressure oscillations. Thus, the acoustic impedance in front of the piston of C2 Z_2 can be calculated from

$$Z_2 = \frac{p_{exp}}{U_2} = \frac{\gamma p_{exp} P_a}{i\omega p_{back,2} V_{back,2}}. \quad (15)$$

Subscript back denotes the back space of the compressor. The two impedances then yield the impedance Z_{exp}

$$Z_{exp} = \frac{1}{1/Z_{ori} - 1/Z_2} = \frac{\gamma p_{exp} P_a}{i\omega(p_{res}V_{res} - p_{back,2}V_{back,2})}, \quad (16)$$

Hence, for experimental analysis, the practical acoustic field can be obtained from p_{res} , p_{exp} and $p_{back,2}$ without any flow field imaging equipment. In experiment for a regenerative component, all that is needed to be done is adjusting the currents in the two compressors and monitoring the pressures.

2.2. Analysis on Config. 2

For C1 and C2, the equations are the same as Eqs. (4)–(7), but Eq. (8) should be rewritten as

$$p_3 A_3 - p_{exp} A_2 = \tau_2 I_2 - Z_{mech,2} v_2. \quad (17)$$

For simplicity, let $A_2 = A_3$, then Eq. (17) yields

$$U_{exp} = \frac{I_2 \tau_2 A_2}{Z_3 A_2^2 - Z_{exp} A_2^2 + Z_{mech,2}}. \quad (18)$$

Substituting Eqs. (10) and (18) into Eq. (3) yields,

$$\frac{I_2 \tau_2 A_2}{Z_3 A_2^2 - Z_{exp} A_2^2 + Z_{mech,2}} = \frac{I_1 \tau_1 A_1}{A_1^2 \frac{T(1,2) - T(2,2)Z_{exp}}{T(2,1)Z_{exp} - T(1,1)} + Z_{mech,1}} \left[T(2,1) \frac{T(1,2) - T(2,2)Z_{exp}}{T(2,1)Z_{exp} - T(1,1)} + T(2,2) \right]. \quad (19)$$

Given Z_{exp} and p_{exp} , the current in C2 I_2 is obtainable from Eq. (18). Then, the current in C1 I_1 is obtainable from Eq. (19), which equation implies that any acoustic field can be satisfied by the two currents.

For experimental analysis, the impedance of the RC load is found from

$$Z_{ori} = \frac{p_3}{U_{ori}} = \frac{\gamma p_3 P_a}{i\omega p_{res} V_{res}}. \quad (20)$$

The acoustic impedance at the double-acting piston facing to the orifice is

$$Z_3 = 1 / \left(\frac{i\omega V_3}{\gamma P_a} + \frac{1}{Z_{ori}} \right) = 1 / \left(\frac{i\omega V_3}{\gamma P_a} + \frac{i\omega p_{res} V_{res}}{\gamma p_3 P_a} \right) = \frac{\gamma P_a}{i\omega (V_3 + p_{res} V_{res} / p_3)}, \quad (21)$$

which then gives the acoustic impedance Z_{exp}

$$Z_{exp} = \frac{p_{exp}}{U_3} = \frac{p_{exp} Z_3}{p_3} = \frac{\gamma p_{exp} P_a}{i\omega(p_3 V_3 + p_{res} V_{res})} \quad (22)$$

Hence, for experimental analysis, the practical acoustic field can be obtained from p_{res} , p_{exp} and p_3 without any flow field imaging equipment.

3 PRACTICAL CASE DESIGN

The regenerative component is a pulse tube cryocooler consisting of a heat rejecter, a regenerator, a cold tip, a pulse tube, and a flow straightener. Table 1 lists the main parameters of this cryocooler and the two compressors [21]. The volume of the reservoir is 5 L. The design pressure amplitude in the expansion chamber p_{exp} is 0.25 MPa. Based on the numerical model in [19], we derive the transmission matrix,

$$\begin{bmatrix} T(1,1) & T(1,2) \\ T(2,1) & T(2,2) \end{bmatrix} = \begin{bmatrix} 0.990 + 0.0952i & (-58.9 - 5.59i) \times 10^5 \\ (1.35 - 25.1i) \times 10^{-9} & 0.252 + 0.132i \end{bmatrix}.$$

If $Z_{exp} = 4.4 \times 10^7 \angle 60^\circ$ Pa.m/s (with this impedance, the relative Carnot efficiency from acoustic to cooling power is about 32%) is desired, the acoustic impedance Z_1 (which equals Z_{com}) is $2.44 \times 10^7 \angle -25.7^\circ$ Pa.m/s.

3.1 Design for Config. 1

If the real part of Z_{ori} is set to 4.41×10^7 (This value is not special, but it must ensure that the acoustic power consumed by the orifice is more than that flowing out of the regenerative part. The imaginary part can be calculated from the reservoir volume), then from Eq. (16), Z_2 is $4.15 \times 10^7 \angle -62.3^\circ$ Pa.m/s. The optimum diameter for C2 is [20]

$$D_{opt,2} = 2 \sqrt{\left\{ \frac{[\text{Re}(Z_{mech,2})]^2}{[\text{Re}(Z_2)]^2} \left[\frac{\tau_2^2}{\text{Re}(Z_{elec,2})\text{Re}(Z_{mech,2})} + 1 \right] \right\}^{1/4}} / \pi = 0.0881 \text{ m}$$

and the mass of the piston at resonance is

$$M_2 = \frac{-\text{Im}(Z_2)A_2^2 + K_2/\omega}{\omega} = 4.857 \text{ kg}.$$

Similarly, for C1, the optimum diameter $D_{opt,1}$ is 0.0853 m and the mass of the piston at resonance M_1 is 1.608 kg. With these values for diameter and mass, we obtain the currents in C1 and C2 of $21.7 \angle 27.9^\circ$ and $9.4 \angle 62.3^\circ$ A, respectively.

If Z_{exp} varies and the parameters of the compressors do not change, the compressors will not resonate. How will the currents vary then? Figures 2 and 3 present the influence of the phase of Z_{exp} on the currents in the two compressors. The moduli of the currents do not vary much with the phase of Z_{exp} varying from 80° to 20° , which aids in the design of the compressors. If the currents increase too much, the diameter of the coil wires need to be especially enlarged. The efficiency of C1 (Fig. 4) is almost constant; however, the efficiency of C2 drops a little quickly as the phase of Z_{exp} decreases. From the output acoustic power of the two compressors (Fig. 5), $W_{output,1}$ of C1 is much bigger than $W_{output,2}$ of C2 when the phase of Z_{exp} is small. This is one of the reasons for the larger current modulus in C1 than in C2. With the phase of Z_{exp} varying from 30° to 80° , the phase difference between U_{com} and U_{exp} varies from 67.6° to 113.6° (Fig. 6). At the same time, the phase of I_1 first leads that of I_2 by about 38° and then lags by about 69° (Fig. 3).

If a higher pressure in the expansion chamber is wanted, C1 needs to output more acoustic power. For example, $W_{output,1}$ is 10 kW if p_{exp} is 0.4 MPa and the phase of Z_{exp} is 30° . To satisfy the experiment, the power capacity of C1 should be sufficiently large. In contrast, it is always possible for C2 to function at a relative small output power because the resistance of the orifice can be adjusted in the experiment. Figure 7 shows the influence of the phase of Z_{exp} on the modulus of the current in C2 with different output power of C2. For each curve in this figure, the orifice resistance is adjusted to maintain a constant output power

in C2. Comparing the curves, the modulus of the current is smaller with less output power. This is helpful in preventing the current in C2 from exceeding specifications.

Figure 8 shows the influence of pressure in the expansion chamber and output acoustic power on the current in C2. In this calculation, the phase of Z_{exp} is set at 30° , which implies that C2 does not acoustically match Z_{exp} . When the pressure in the expansion chamber is relatively high, the modulus of the current in C2 varies slightly even though the output power is very small. Hence, the current capacity of C2 need to be adequately sufficient if the pressure is to vary over a wide range.

3.2 Design for Config. 2

Here, there is $Z_2 = Z_{exp}$. If the real part of Z_3 be 4.37×10^7 (For simplicity, V_3 , the volume between the right piston surface of C2 and the orifice is set as zero, so Z_3 equals to Z_{ori} . The value for Z_3 is not special, but it must ensure that the acoustic power consumed by the orifice is more than that flowing out of the regenerative part), the optimum diameter for C2 is $D_{opt,2} = 0.0855$ m, and the mass of the piston at resonance is $M_2 = 4.839$ kg [10]. The optimum diameter $D_{opt,1}$ and the piston mass on resonance M_1 are the same as those in Section 3.1. With the diameter and mass, we obtain currents in C1 and C2 of $21.7 \angle 27.9^\circ$ and $9.34 \angle -60^\circ$ A.

Figures 9 and 10 show the influence of the phase of Z_{exp} on the currents in the two compressors. Figure 9 is very similar to Fig. 2, whereas Fig. 10 is quite different to Fig. 3. Figure 11 shows the dependence of current on the phase of Z_{exp} and the pressure in the expansion chamber when there is no output acoustic power from C2. When the phase of Z_{exp}

is 60° , the piston of C2 is in a resonant state. The electromagnetic force produced by the current is used to overcome the mechanical resistance. When the phase of Z_{exp} deviates from 60° , an additional electromagnetic force is needed to change the phase of the piston, so the current is larger.

4 DISCUSSION

Assuming that C2 in Config. 1 is just a mass–spring resonator coupled to the expansion chamber, then there is no coil or magnet. Equation (11) can be written as

$$Z_{ori} = \frac{Z_{mech,2}Z_{exp}}{Z_{mech,2} - Z_{exp}A_2^2}. \quad (23)$$

If $Z_{exp} = 4.4 \times 10^7 \angle 60^\circ$ Pa.m/s and $Z_{mech,2} = 30 + 3100i$ N.m/s, then $Z_{ori} = 8.95 \times 10^7 - 1.72i \times 10^6$ Pa.m/s. The corresponding reservoir volume is 9.28 L. Apparently, Z_{exp} can be achieved by adjusting the orifice and the reservoir volume, which further simplifies the modulation of the acoustic field. Figures 12 and 13 show the corresponding real part of Z_{ori} and reservoir volume when the phase of Z_{exp} varies from 30° to 80° . When $Arg(Z_{exp})$ varies from 30° to 40° , the reservoir volume changes slightly, and therefore requires very precise adjustments in reservoir volume. When $Arg(Z_{exp})$ is near 60° , the volume increases rapidly as in this instance the denominator of Eq. (23) is close to zero. When $Arg(Z_{exp})$ is larger than 60° , the reservoir volume is negative. Hence, in practice, the phase of Z_{exp} can only vary within a very limited range if $Z_{mech,2}$ is not changed.

For Config. 2, Eq. (18) can be written

$$Z_3 = Z_{exp} - Z_{mech,2}/A_2^2. \quad (24)$$

In practice, the phase of Z_{exp} can similarly vary only over a very limited range if $Z_{mech,2}$ is not changed.

5 CONCLUSIONS

With two compressors connected to the two ends of a regenerator and a RC load added to dissipate the acoustic work, an alternative method was demonstrated to modulate the acoustic field in regenerators. With this method, the acoustic field varies over a wide range by changing only the current in the two compressors. The impedance Z_{exp} can easily be obtained using three pressure sensors instead of flow field imaging equipment, thereby greatly simplifying the experiment.

The compressor connected with the expansion chamber can be further simplified using a mass-spring resonator involving no electromagnetic force. By adjusting the orifice and the reservoir volume, the acoustic field varies over a very limited range. For a wider range, other parameters such as mechanical impedance must also be adjusted. But the combination of the mass-spring resonator and the RC load could act as a new phase shifter in pulse tube cryocoolers.

ACKNOWLEDGEMENTS

This work was supported by the National Sciences Foundation of China (Grant No. 51276187 and 51576204).

REFERENCE

- [1] Radebaugh R, Lewis M, Luo EC. Inertance tube optimization for pulse tube refrigerators. *Adv Cryog Eng.* 2006; 51 (A): 59-70.
- [2] Gifford WE, Longsworth RC. Pulse tube refrigerator. *J Eng Ind, Trans ASME.* 1964; 86: 264-270.
- [3] Mikulin EI. Low-temperature expansion pulse tube. *Adv Cryog Eng.* 1984; 29: 629-637.
- [4] Radebaugh R, Zimmerman J, Smith DR, et al. A comparison of three types of pulse tube refrigerators: new methods for reaching 60K. *Adv Cryog Eng.* 1986; 31: 779-789.
- [5] Zhu SL, Wu PY, Chen ZQ. Double inlet pulse tube refrigerators: an important improvement. *Cryogenics.* 1990; 30 (6): 514-520.
- [6] Godshalk KM, Jin C, Kwong YK, et al. Characterization of 350 Hz thermoacoustic driven orifice pulse tube refrigerator with measurements of the phase of the mass flow and pressure. *Adv Cryog Eng.* 1996: 1411-1418.
- [7] Wheatley J, Hofler T, Swift GW, et al. An intrinsically irreversible thermoacoustic heat engine. *J Acoust Soc Am.* 1983; 74 (1): 153-170.
- [8] Backhaus S, Swift GW. A thermoacoustic Stirling heat engine. *Nature.* 1999; 399 (6734): 335-338.
- [9] Berchowitz DM; Lubricant free, reduced mass, free-piston, Stirling machine having reciprocating piston drivingly linked to rotary electromagnetic transducer moving in rotational oscillation. US patent, Patent No. US8713934B2. 2010.
- [10] Hu JY, Luo EC, Zhang LM, et al. A double-acting thermoacoustic cryocooler for high temperature superconducting electric power grids. *Appl Energ.* 2013; 112: 1166-1170.
- [11] Antao DS, Farouk B. Numerical and experimental characterization of the inertance effect

- on pulse tube refrigerator performance. *International Journal of Heat and Mass Transfer*. 2014; 76: 33-44.
- [12] Shiraishi M, Murakami M. Visualization of oscillating flow in a double-inlet pulse tube refrigerator with a diaphragm inserted in a bypass-tube. *Cryogenics*. 2012; 52 (7-9): 410-415.
- [13] Wu ZH, Chen YY, Dai W, et al. Investigation on the thermoacoustic conversion characteristic of regenerator. *Appl Energ*. 2015; 152: 156-161.
- [14] Zhou LH, Xie XJ, Li Q. Study of acoustic field modulation in the regenerator by double loudspeakers method. *J Acoust Soc Am*. 2011; 130 (5): 2709-2719.
- [15] Paek I, Mongeau L, Braun JE, et al. Phase of acoustic impedance and performance of standing wave thermoacoustic coolers. *J Mech Sci Technol*. 2009; 23 (5): 1476-1484.
- [16] Poignand G, Lihoreau B, Lotton P, et al. Optimal acoustic fields in compact thermoacoustic refrigerators. *Appl Acoust*. 2007; 68 (6): 642-659.
- [17] Biwa T, Tashiro Y, Mizutani U, et al. Experimental demonstration of thermoacoustic energy conversion in a resonator. *Phys Rev E*. 2004; 69 (6): 066304.
- [18] Kato Y, Baba K. Empirical estimation of regenerator efficiency for a low temperature differential Stirling engine. *Renew Energ*. 2014; 62: 285-292.
- [19] Hu JY, Dai W, Luo EC, et al. Development of high efficiency Stirling-type pulse tube cryocoolers. *Cryogenics*. 2010; 50 (9): 603-607.
- [20] Swift GW. *Thermoacoustics: A unifying perspective for some engines and refrigerators*. Sewickley PA: ASA Publication, 2002.
- [21] Zhang LM, Hu JY, Wu ZH, et al. A 1 kW-class multi-stage heat-driven thermoacoustic

cryocooler system operating at liquefied natural gas temperature range. Appl Phys Lett.
2015; 107 (3): 033905.

FIGURE CAPTIONS

Figure 1 Two configurations for acoustic field modulation.

Figure 2 Influence of the phase of Z_{exp} on the modulus of the currents in the two compressors.

Figure 3 Influence of the phase of Z_{exp} on the phase of the currents in the two compressors.

Figure 4 Influence of the phase of Z_{exp} on the efficiency of the two compressors.

Figure 5 Influence of the phase of Z_{exp} on the output acoustic power of the two compressors.

Figure 6 Influence of the phase of Z_{exp} on the phase difference between U_{com} and U_{exp} .

Figure 7 Influence of the phase of Z_{exp} on the modulus of the current in C2 with constant output power of C2.

Figure 8 Influence of pressure in the expansion chamber and the output acoustic power on the current in C2.

Figure 9 Influence of phase of Z_{exp} on the modulus of currents in the two compressors.

Fig.10 Influence of phase of Z_{exp} on the modulus of the current in C2 at constant output power of C2.

Figure 11 Influence of $Arg(Z_{exp})$ and p_{exp} on the modulus of the currents in C2.

Figure 12 Influence of $Arg(Z_{exp})$ and $Im(Z_{mech,2})$ on the real part of Z_{ori} .

Figure 13 Influence of $Arg(Z_{exp})$ and $Im(Z_{mech,2})$ on the reservoir volume.

TABLE 1 Main parameters of the pulse tube cryocooler and two compressors

Parameters	Value
Working frequency	50 Hz
Working gas	Helium
Charging pressure	3 MPa
Cooling temperature	80 K
Heat rejecter	Ø75 mm×L64 mm
Diameter and length of regenerator	Ø75 mm×L70 mm
Cold tip	Ø75 mm×L30 mm
Diameter and length of pulse tube	Ø37 mm×L150 mm
Flow straightener	Ø37 mm×L5 mm
transduction coefficient	80 N/A
Mechanical resistance	50 N.m/s
Electrical resistance	0.5 Ω
Spring constant	5×10^4 N/m

Figure 1

Config. 1

Config. 2

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13