Non-uniform distribution of a corrosion layer at a steel/concrete interface described by a Gaussian model

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Abstract

Corrosion layers at the steel/concrete interfaces of two reinforced concrete specimens subjected to chloride ion ingress were observed and measured by SEM. A Gaussian function was used to model the distribution of the non-uniform corrosion. The physical meaning of the parameters in the Gaussian model is confirmed mathematically and the relationships among these parameters are discussed. The locations of the corrosion peaks along the rebar perimeter are also discussed. When the number of cracks at the steel/concrete interface becomes two or more, the corrosion layer has the same quantity of corrosion peaks. A multi-peak Gaussian model is proposed to describe this multiple corrosion peak situation.

Keywords: A. Steel reinforced concrete; B. SEM; B. Modelling studies; C. Corrosion

1. Introduction

The corrosion of steel has been identified as the major cause of deterioration in reinforced concrete structures [1, 2]. The corrosion of steel produces pressure on the surrounding concrete because the volume of corrosion product is 2 - 6 times the volume of the original steel [3 - 5]. Aggressive agents reach the reinforcement directly from these corrosion-induced cracks and accelerate the corrosion process, leading to serious damage [6 - 8]. To accurately predict the time taken to reach the limiting state of the cracking, concrete cover is not only important for the

durability of reinforced concrete structures but also provides the scientific basis for structural maintenance. Therefore, the development of a concrete cracking model for reinforcement corrosion has received substantial attention from both scientists and practicing engineers.

Analytical models [1, 6, 9 - 17] usually assume a uniform expansion of corrosion products around the rebar circumference. As a previous study showed [18], this assumption is made primarily for two reasons. The first is that it significantly simplifies the modeling process, especially when formulating analytical and finite-difference based solution schemes. The second is that currently there is an absence of reliable information with which to characterize the actual non-uniform formation and expansion of corrosion products from the rebar surface. Because the scenarios of uniform or non-uniform corrosion of reinforcement obviously have different effects on predicting the time to corrosion-induced cracking [18 - 22] and the non-uniform corrosion around the rebar perimeter is the real situation, the variability of the thickness of the corrosion layer deposited on the circumference of the steel bar must also be determined when formulating a mathematical model.

Several models to describe the non-uniform corrosion of the steel bar have been made over the past several years: a semi-elliptical rust model by Yuan [19], linear models by B.S. Jang [20], a dual oval rust model by Liu [21] and Gaussian models by the authors [18, 22]. The authors' previous works demonstrated that a Gaussian model is the most reliable for comparing both Yuan's and the authors' tested results [22]. The authors also define the physical meaning of the parameters in the Gaussian model. However, previous works have lacked a discussion of the relation among these parameters and their functions in practical situations. Furthermore, the testing specimens in the previous study are limited.

Following up on the authors' previous work [18, 22], this paper [mathematically confirmed the physical meaning of the parameters of the Gaussian model](#) and also investigates the relationships among these parameters by measuring the thickness of the non-uniform corrosion layer around the

perimeter of the steel bar in two different types of specimens. The location of the corrosion peak along the rebar perimeter is also discussed based on which multi-peak Gaussian model is proposed to describe the multiple corrosion peak situations.

2. Experimental program

2.1 Specimens

Two specimens, which were subject to different chloride ingress environment, are used for observation and analysis in this study, i.e. TC30 and AC40. Both of these two specimens have cracked. The size of these two concrete specimens is 150 mm×150 mm×300 mm, as shown in Fig. 1. Each specimen contained three deformed carbon steel bars with a nominal number of 16 mm. The information of these two specimens is reported in Table 1.

To ensure the unidirectional diffusion of chloride ions when specimens are in the deterioration process, the side and bottom surfaces of each panel were surface treated with protective paint (which is indicated by the blue shade in Fig. 1) to ensure that chloride predominantly penetrated the top cover, with minimal penetration of the other faces of the panel.

2.2 Curing and exposure history

The flow chart of the curing and exposure history of two specimens are illustrated in Fig. 2 and Table 2, also stated in detail as follows.

2.2.1 Specimen TC30

Specimen TC30 was first soaked in a 5 wt. % sodium chloride solution at various temperatures. Each temperature-cycle included 24 days, including 4 days in 36°C, 8 days in 46°C, 4 days in 34°C and 8 days in 36°C, as listed in Table 2. This specimen underwent 20 cycles in total. The specimen was then subjected to the wetting and drying cycles. Each wetting-and-drying- cycle lasted for 7 days and consisted of soaking the blocks with a 3.53 wt. % sodium chloride solution for 3 days and subsequently allowing it to dry in a lab environment for 4 days.

2.2.2 Specimen AC40

Specimen AC40 was placed in a salt mist chamber for 111 days. Each day, the specimen was subjected to 5 wt. % sodium chloride mist for 21 h, with 1.0 - 2.0 mL / (80 cm²h) settlement of the mist, a temperature of 35±2 °C and 100% RH. The specimen was then air-dried for 3 hours at room temperature and 85% RH. The second exposure stage was the same for specimen TC30, i.e., 7-day wetting and drying cycles.

2.3 Sample preparation

The selected panel was cast into a low-viscosity epoxy resin to minimize any artificial damage that may have occurred during the sample preparation process for microscopy. The epoxy was allowed to harden for several days and then was carefully cut by a Φ355-mm concrete cutting machine to extract the corner and middle rebar while keeping the surrounding concrete intact, as shown in Fig. 3. Cutting was performed at least 15 mm away from the rebar to minimize any disturbance of the rebar-concrete interface. The cut panels were labeled L, M and R, representing the sections containing the left, middle and right corner rebar, respectively. Then, the panels were cut by a SYJ200 abrasive cutter to produce a series of 10-mm-thick cross-section slices. The slices were numbered from 1 to n.

2.4 Preparations of SEM samples

Because the samples already had some small cracks, they were cast into a low-viscosity epoxy resin to minimize any further artificial damage that may have occurred during the cutting. A SYJ200 abrasive cutter was used to cut slices into 25 mm×25 mm×8 mm samples, as shown in Fig. 4. The samples were then polished by a UNIPOL-1502 polishing machine.

2.5 Measurement of the corrosion layer thickness

The boundaries of the rebar, corrosion layer and concrete of each sample were observed by a scanning electron microscope (QUANTA FEG650) in back-scattering mode. The measuring points

were set at an interval of 700 μm on the boundary to measure the thickness of the corrosion layer using the measuring function of the microscope, as shown in Fig. 5. The relative coordinates were also recorded. Approximately 70 points around the perimeter of the steel bar were recorded in each sample.

In previous studies [23, 24], the steel corrosion accumulating at the steel/concrete interface was called the ‘corrosion layer (CL)’ and the concrete zone around the steel bar filled by corrosion products was called ‘corrosion-filled paste (CP)’, as shown in Fig. 5. [The mill scale is formed before corrosion initiation, it does not contribute to the volume expansion acting on the surrounding concrete cover, therefore,](#) when investigating the variation of the corrosion layer around the steel bar, only CL is discussed in this study. The thickness of the corrosion layer at each measuring point was tested and is recognized as T_{cl} .

3. Results

3.1 Average thickness of the corrosion layer and corrosion induced cracks

The average steel corrosion and cracking condition of the samples are shown in Table 3. Some slices of L and M of specimen TC30 can not be used because of the high degree of damage induced by the cutting process. The L part of specimen AC40 was not tested because its corrosion layer was difficult to be recognized.

3.2 Parameters in the Gaussian model to describe the non-uniform corrosion layer

As mentioned earlier in Section 1, the authors [22] have found that the Gaussian model can well describe the variation of the thickness of the corrosion layer T_{cl} around the steel bar

$$T_{cl} = \frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta - \pi}{\sqrt{2}a_2}\right)^2} + a_3 \quad (1)$$

where T_{cl} is the thickness of the corrosion layer at coordinate θ (in mm), the parameters a_1 , a_2 and a_3 are the fitting parameters, which describe various characteristics of the corrosion layer. a_1 is the

non-uniform coefficient of the corrosion layer, a_2 is the spread coefficient of corrosion layer and a_3 is the uniform coefficient of the corrosion layer.

It should be noted that to focus on the distribution of the corrosion layer, the maximum thickness of the corrosion layer is retained at a fixed position, i.e., $\theta = \pi$, which is the symmetry axis of the Gaussian model. The relative polar coordinate position of the measuring points can be converted based on the position of the x and y coordinates. Two typical cases, i.e., the partial or whole cross-section corrosion of the non-uniform corrosion layer, which could be described by the Gaussian model, are illustrated in Fig. 6.

The fitted values of the parameters of the Gaussian model from Eq. (1) of each sample are listed in Table 4. The determination coefficient R^2 is also shown in Table 4.

4. Discussion of the parameters in the Gaussian model

4.1 a_3 - Uniform coefficient of the corrosion layer

When a steel bar is in the condition of uniform corrosion, the thickness of the corrosion layer T_{cl} should be a constant value. In Eq. (1), $\frac{a_1}{a_2\sqrt{2\pi}}e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2}$ is the part related to coordinate θ , under this uniform corrosion circumstance, $\frac{a_1}{a_2\sqrt{2\pi}}e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} = 0$, then $T_{cl} = a_3$. a_3 , defined as the uniform coefficient, is independent of the value of θ and can describe the thickness of the corrosion layer, which covers the entire circumference of the steel bar.

When the steel bar is in partial corrosion (Fig. 6a), the corrosion layer a_3 is taken as 0 (i.e., the black bold part of Table 4). If the corrosion products develop across the entire circumference of the steel bar (Fig. 6b), a_3 (in mm) represents the minimum thickness of the corrosion layer T_{cl-min} , its physical meaning is illustrated in both Fig. 6b and Fig. 7.

4.2 a_1 - Non-uniform coefficient of the corrosion layer

The parameter a_3 reflects the uniform corrosion level, as discussed in 4.1. $a_3 = 0$ when the steel bar is in partial corrosion under the partial corrosion circumstance

$$T_{cl} = \frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta - \pi}{\sqrt{2}a_2}\right)^2} \quad (2)$$

To calculate the area enclosed by the function, θ is integrated from 0 to 2π

$$\begin{aligned} \int_0^{2\pi} T_{cl} d\theta &= \int_0^{2\pi} \frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta - \pi}{\sqrt{2}a_2}\right)^2} d\theta \\ &= \int_0^{2\pi} \frac{a_1}{\sqrt{\pi}} e^{-\left(\frac{\theta - \pi}{\sqrt{2}a_2}\right)^2} d\left(\frac{\theta - \pi}{\sqrt{2}a_2}\right) \\ &= a_1 \times \operatorname{erf}\left(\frac{\pi}{\sqrt{2}a_2}\right) \end{aligned} \quad (3)$$

where the error function is an increasing function from -1 to 1. Its value gets closer to 1 when the value of a_2 becomes smaller. The values of a_2 for most samples are smaller than 1.5, as shown in Table 4. Taking a_2 as 1.5, $\operatorname{erf}\left(\frac{\pi}{1.5 \times \sqrt{2}}\right) = 0.964$; therefore, for most samples, the values of $\operatorname{erf}\left(\frac{\pi}{\sqrt{2}a_2}\right)$ are larger than 0.964 so that $\int_0^{2\pi} T_{cl} d\theta$ is very close to a_1 , indicating that a_1 (in mm^2) is approximately the area of the peak of the Gaussian function as shown in Fig. 7. Hence, a_1 is defined as the non-uniform coefficient, reflecting the non-uniform corrosion level.

4.3 a_2 - Spread coefficient of the corrosion layer

Since the non-uniform corrosion part of corrosion is only related with a_1 and a_2 , a_3 will then not included in this discussion, taken as 0 in the following analysis. According to Eq. (1), the thickness of the corrosion layer T_{cl} reaches its maximum value $\frac{a_1}{a_2 \sqrt{2\pi}}$ at $\theta = \pi$. When the thickness of the corrosion layer is half of its maximum value, it can be expressed as

$$T_{cl} = \frac{1}{2} \times \frac{a_1}{a_2 \sqrt{2\pi}} = \frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} \quad (4)$$

At this moment, from Eq. (4), $e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} = \frac{1}{2}$, then

$$a_2 = \frac{|\theta - \pi|}{\sqrt{\ln 4}} \quad (5)$$

To discuss the physical meaning of a_2 , the full width at half maximum W_{hm} is introduced in this study as illustrated in Fig.8, recognized as the width of a curve measured between those points on the y-axis which are half the maximum amplitude [25]. The larger value of W_{hm} normally represents a wider distribution of the spectrum curve. For the Gaussian mode, the larger W_{hm} corresponds to a flatter curve.

As illustrated in Fig.8, when the thickness of the corrosion layer is half of its maximum value

$$W_{hm} = 2|\theta - \pi| \quad (6)$$

Substituting Eq.(6) into Eq.(5), the relationship between a_2 and W_{hm} can be established as

$$a_2 = \frac{W_{hm}}{2\sqrt{\ln 4}} \quad (7)$$

It is clear that a_2 is lineally proportional to W_{hm} . The larger a_2 , corresponding to the greater W_{hm} , physically means that corrosion products spread more extensively, leading to a flatter shape of the corrosion layer. Therefore, a_2 (in mm) is defined as the spread coefficient of the corrosion layer.

5. Relationship among a_1 , a_2 and a_3

5.1 Relationship between a_1 and a_3

The Gaussian Eq. (1) is integrated to obtain the total area of corrosion layer

$$\begin{aligned}
A_{cl} &= \int_0^{2\pi} T_{cl} \cdot R d\theta = \int_0^{2\pi} \left(\frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} + a_3 \right) R d\theta \\
&= \int_0^{2\pi} \left(\frac{a_1}{a_2 \sqrt{2\pi}} e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} \right) R d\theta + \int_0^{2\pi} a_3 R d\theta \\
&= \int_0^{2\pi} \left(\frac{a_1}{\sqrt{\pi}} e^{-\left(\frac{\theta-\pi}{\sqrt{2}a_2}\right)^2} \right) R d\left(\frac{\theta-\pi}{\sqrt{2}a_2} \right) + \int_0^{2\pi} a_3 R d\theta \\
&= a_1 R \times \text{erf} \left(\frac{\pi}{\sqrt{2}a_2} \right) + 2\pi a_3 R \\
&\approx (a_1 + 2\pi a_3) R
\end{aligned} \tag{8}$$

where A_{cl} is the area of the corrosion layer (in mm^2), R is the radius of steel bar (in mm).

According to the geometric relationship, the thickness of the corrosion layer is

$$\overline{T}_{cl} = \frac{A_{cl}}{2\pi R} = \frac{(a_1 + 2\pi a_3) R}{2\pi R} = \frac{1}{2\pi} (a_1 + 2\pi a_3) \tag{9}$$

It can be seen from Eq. (9) that the thickness of the corrosion layer shows a linear relationship with $(a_1 + 2\pi a_3)$. To test the reliability of this relationship, $(a_1 + 2\pi a_3)$ has been calculated and listed in Table 5 (where the data are in bold when $a_3 = 0$). A comparison of the fitting line from the tested data and the theoretical line from Eq. (9) has been illustrated in Fig. 9.

It can be seen from Fig. 9 that the theoretical line matches the fitting line well. The slight deviation between the two lines may due to the approximation of the area as $(a_1 + 2\pi a_3) R$ in Eq. (9).

5.2 Relationship between a_1 and a_2

To analyse the relationship between a_1 and a_2 , again let $a_3 = 0$ in the following analysis.

According to Eq. (1), the thickness of corrosion layer T_{cl} reaches its maximum value $\frac{a_1}{a_2 \sqrt{2\pi}}$ at $\theta = \pi$.

$$T_{cl-\max} = \frac{a_1}{a_2 \sqrt{2\pi}} \tag{10}$$

Therefore, the peak value of the non-uniform corrosion layer $T_{\text{cl-max}}$ is proportionally related to $\frac{a_1}{a_2}$.

To verify the reliability of this relationship, $\overline{T_{\text{cl-max}}}$, which is the average of the largest ten corrosion layer thicknesses after subtracting the thickness of the uniform corrosion layer and $\frac{a_1}{a_2}$ are calculated and listed in Table 6. The fitting line of the experimental data of $\frac{a_1}{a_2}$ and $\overline{T_{\text{cl-max}}}$ is illustrated in Fig. 10, in which the comparison of this fitting line with the theoretical line is also shown.

As shown in Fig. 10, the experimental data and fitting data of $\frac{a_1}{a_2}$ and $\overline{T_{\text{cl-max}}}$ are well matched.

Comparing Specimen TC30, Specimen AC40 has a relative larger deviation, which may be due to the measuring error, as the corrosion degree of AC40 is less than that of TC30.

6. Discussion of the application of the non-uniform rust distribution model in practical projects

Based on the relationship among a_1 , a_2 and a_3 , as discussed in section 5, a non-uniform corrosion layer model, which can be used in practical reinforced concrete projects, is proposed by measuring three key parameters: A_{cl} , $T_{\text{cl-max}}$ and $T_{\text{cl-min}}$ (which equals to a_3 , illustrated in Section 4.1). As shown in Fig. 11, the area of the corrosion layer A_{cl} , the maximum thickness of the corrosion layer $T_{\text{cl-max}}$ and the minimum thickness of the corrosion layer $T_{\text{cl-min}}$ can be measured from the cross-sections of reinforced concrete members.

According to Eq. (9), the relationship between a_1 and a_3 can be presented as

$$a_1 = \frac{A_{\text{cl}}}{R} - 2\pi a_3 \quad (11)$$

According to Eq. (10), the relationship between a_1 and a_2 are

$$a_2 = \frac{a_1}{T_{\text{cl-max}} - a_3} \sqrt{2\pi} \quad (12)$$

When a steel bar is in partial corrosion, as in Figure 11b, $a_3 = 0$, Eqs (11) and (12) then become

$$a_1 = \frac{A_{cl}}{R} \quad (11a)$$

$$a_2 = \frac{a_1}{T_{cl-max}} \sqrt{2\pi} \quad (12a)$$

After measuring three key parameters, i.e., A_{cl} , T_{cl-max} and T_{cl-min} , from the cross-section of the target reinforced concrete member, a_1 , a_2 can be calculated according to Eqs (11) and (12), so that Eq. (1) can then be established to describe the non-uniform distribution of the corrosion layer around the perimeter of steel bar. Based on this specific non-uniform corrosion layer model, the corrosion-induced concrete cover cracking prediction for the target reinforced concrete members could be more reliable.

7. Discussion of the corrosion peak locations

As mentioned in Section 2, two specimens used in this experiment were treated by anti-corrosive paint to ensure the unidirectional penetration of chloride ions from the top surface of the specimens.

Before the cracks penetrated the concrete cover, the corrosion peak was located at the area of the steel bar facing the concrete top surface, which has the shortest path for chloride ions reaching the steel bar, and the corrosion-induced cracking normally initiates from this corrosion peak area, as schematically described in Fig. 12a. This phenomenon can be observed in the samples where the corrosion induced cracks do not penetrate the concrete cover, such as samples AC40 M6 - M18 and AC40 R6 - R13, as illustrated by the images in 12a.

For the samples where the cracks have already propagated outward and reached the surface of the concrete cover, these cracks in the concrete cover provide direct paths for chloride ions to reach the steel bars and accelerate the steel corrosion, leading to the thicker corrosion layer at the end area of these cracks, as illustrated in Fig. 12b.

A corner sample of specimen TC30, which has a severe degree of steel corrosion, contains three

corrosion-induced cracks. The measured distribution of the corrosion layer in this sample is shown in Fig. 13. This sample was then observed by SEM, as illustrated in Fig. 14. It can be clearly observed that the three corrosion peaks correspond to three cracks, proving that the presence of corrosive cracks is the main cause of corrosion peak.

The multi-peak Gaussian fitting function in the commercial software Origin is utilized to model the three-peak corrosion layer in this sample. The linear superposition is found the most satisfied fitting result by trial and error. Based on the measured T_{cl} data around the perimeter of steel bar, by fixing the location of T_{cl-max} at $\theta = \pi$, the other two peaks and their locations will be regressed automatically by Origin. The regressed three-peak corrosion layer model is presented as

$$T_{cl} = \frac{1.636}{\sqrt{2\pi}} e^{-\left(\frac{\theta-\pi}{0.5\sqrt{2}}\right)^2} + \frac{1.211}{\sqrt{2\pi}} e^{-\left(\frac{\theta-0.905}{0.455\sqrt{2}}\right)^2} + \frac{1.430}{\sqrt{2\pi}} e^{-\left(\frac{\theta-5.396}{0.244\sqrt{2}}\right)^2} + 0.140 \left(R^2 = 0.874\right) \quad (13)$$

According to the above discussion, the factors that influence the location of the corrosion peaks may lie in two points.

(1) The shortest path for the penetration of harmful substances

Before concrete cover cracking, harmful substances, such as chloride ions, will reach the interface via the shortest path; the corrosion peak located at the area of steel bar facing the concrete surface subjected to the harmful substances.

(2) Corrosion-induced cracks

After concrete cover cracking, the cracks provide paths for a quicker ingress of harmful substances to the steel bars, accelerating the steel corrosion. Under this circumstance, the corrosion peaks are normally near the inner end of these cracks.

Based on the experience of three-corrosion-peak situation, the authors propose the multi-peak non-uniform distribution of the corrosion layer can be expressed as a multi-peak Gaussian model

$$\begin{aligned}
T_{cl} &= \frac{a_{11}}{a_{12}\sqrt{2\pi}} e^{-\left(\frac{\theta-\theta_1}{\sqrt{2}a_{12}}\right)^2} + \dots + \frac{a_{i1}}{a_{i2}\sqrt{2\pi}} e^{-\left(\frac{\theta-\theta_i}{\sqrt{2}a_{i2}}\right)^2} + \dots + \frac{a_{n1}}{a_{n2}\sqrt{2\pi}} e^{-\left(\frac{\theta-\theta_n}{\sqrt{2}a_{n2}}\right)^2} + a_3 \\
&= \sum_{i=1}^n \frac{a_{i1}}{a_{i2}\sqrt{2\pi}} e^{-\left(\frac{\theta-\theta_i}{\sqrt{2}a_{i2}}\right)^2} + a_3
\end{aligned} \tag{14}$$

where n is the number of cracks in the concrete cover. a_{11} , a_{i1} and a_{n1} represent the non-uniform coefficients corresponding to the 1st, i^{th} and n^{th} cracks, respectively; a_{12} , a_{i2} and a_{n2} are the spread coefficients corresponding to the 1st, i^{th} and n^{th} cracks, respectively; θ_{12} , θ_{i2} and θ_{n2} are the locations of the cracks, expressed as polar coordinate angles and a_3 is the uniform coefficient of the corrosion layer, which is equal to the minimum thickness of the corrosion layer.

There may have four corrosion peaks around the perimeter of steel bar, as shown in Fig. 15. This situation is found in a concrete slab [26], the concrete cover of both sides of the steel bar in that slab specimen are not thick. But in reality, the depth of the concrete on the opposite side of concrete cover steel bar is much larger than the thickness of concrete cover; it may not easy for the cracks to develop inwards. Therefore, four or more corrosion peaks situation is rare, three corrosion peaks are normally the maximum peaks in practical which means that the model has six non-uniform coefficients at most. To obtain these fitting parameters in the practical application needs further investigation.

8. Conclusions

Two reinforced concrete specimens that have deteriorated in different artificial environments were investigated in this study. The thickness of the corrosion layers around the perimeters of the steel bars were observed and measured by SEM. The non-uniform corrosion front was presented by a Gaussian function in a polar coordinate system. The following conclusions were drawn from this study:

(1) The physical meaning of the parameters in the Gaussian model, i.e., the non-uniform coefficient a_1 , the spread coefficient a_2 and the uniform coefficient a_3 , has been confirmed

mathematically.

(2) The discussion of the relationship among the parameters in the Gaussian model reveals that

① $(a_1 + 2\pi a_3) R$ represents the area of the corrosion layer A_{cl} ; ② the uniform coefficient a_3 equals to the minimum thickness of the corrosion layer T_{cl-min} ; ③ $\frac{a_1}{a_2}$ shows a linear relationship with the maximum thickness of the corrosion layer T_{cl-max} .

(3) In practical projects, by measuring three key parameters, i.e. A_{cl} , T_{cl-max} and T_{cl-min} , the non-uniform distribution of the corrosion layer in reinforced concrete members can be described.

(4) Before the cracks penetrate the concrete cover, the location of the corrosion peaks is determined by the shortest path for the harmful substances to reach. After concrete surface cracking, the corrosion peaks are near the inner ends of these corrosion-induced cracks.

(5) A multi-peak Gaussian model is proposed to describe the multiple corrosion peak situations.

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Table Captions:

Table 1. Information of the two concrete specimens.

Table 2. Temperature-cycle of Specimen TC30 in a sodium chloride solution.

Table 3. Average thickness of the corrosion layer and the cracking conditions of the tested samples.

Table 4. Fitted values of the parameters in the Gaussian model of each sample.

Table 5. $(a_1 + 2\pi a_3)$ of the samples.

Table 6. $\frac{a_1}{a_2}$ and $\overline{T_{cl-max}}$.

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Fig. 1. Schematic of the reinforced concrete specimen (dimensions are in mm).

Fig. 2 The flow chart of the curing and exposure history.

Fig. 3. Schematic diagram showing the sectioned concrete panels and locations of the slices.

Fig. 4. The cutting diagram of SEM samples (R15 of TC30).

Fig. 5. Schematic diagram of an SEM photo (M10 of TC30).

Fig. 6. Two typical cases of a non-uniform corrosion layer. (a) partial corrosion and (b) corrosion of the whole cross-section.

Fig. 7. Two parts of the corrosion layer.

Fig. 8. Illustration of full width at half maximum W_{hm} .

Fig. 9. Relationship between $(a_1 + 2\pi a_3)$ and T_{cl} . (a)TC30 and (b)AC40.

Fig. 10. Relationship between $\frac{a_1}{a_2}$ and $\overline{T_{cl-max}}$. (a)TC30 and (b)AC40.

Fig. 11. Key parameters in the concrete non-uniform corrosion layer. (a) partial corrosion and (b) corrosion of the whole cross-section.

Fig. 12. Schematic diagram for the non-uniform distribution of the corrosion layer accumulated at the rebar–concrete interface. (a) before cracks penetrate concrete cover; (b) after cracks penetrate concrete cover.

Fig. 13. Thickness of the corrosion layer.

Fig. 14. SEM photograph.

Fig. 15. Schematic diagram of corrosion-induced cracks and corrosion peaks.

Fig. 1 Schematic of the reinforced concrete specimen (dimensions are in mm).

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Fig.5 Schematic diagram of an SEM photo (M10 of AC40).

(a) Partial corrosion

(b) Corrosion of the whole cross-section

Fig.6 Two typical cases of a non-uniform corrosion layer.

Fig.7 Two parts of the corrosion layer.

Fig.8 Illustration of full width at half maximum W_{hm} .

(a) TC30

(b) AC40

Fig.9 Relationship between $(a_1 + 2\pi a_3)$ and $\overline{T}_{cl}$

(a) TC30

(b) AC40

Fig.10 Relationship between $\frac{a_1}{a_2}$ and $\overline{T_{cl-max}}$.

(a) Partial corrosion

(b) Corrosion of the whole cross-section

Fig.11 Key parameters in the concrete non-uniform corrosion layer.

Fig.12 Schematic diagram for the non-uniform distribution of the corrosion layer accumulated at the rebar–concrete interface.

Fig.13 Thickness of the corrosion layer.

Fig.14 SEM photograph of three-peak corrosion layer.

(a) Single peak

(c) Three peaks

(b) Two peaks

(d) Four peaks

Fig.15 Schematic diagram of corrosion-induced cracks and corrosion peaks.

Table 1

Information of the two concrete specimens.

Specimen number	Quantity	Compressive strength at 28 day age (MPa)	Ratio of water/binder	Mixture composition (kg/m ³)			
				Cement	Sand	Aggregate	Water
TC30	1	38.2	0.56	351	650	1157	195
AC40	1	49.9	0.44	439	571	1161	195

Table 2

Temperature-cycle of Specimen TC30 in a sodium chloride solution.

Temperature (°C)	36	46	34	24
Actual simulated time (day)	4	8	4	8

Table 3

Average thickness of the corrosion layer and the cracking conditions of the tested samples.

Specimen	Sample	Average thickness of	
		corrosion layer $\overline{T_{cl}}$ (mm)	Condition of cracking
TC30	R1	0.322	CP
	R3	0.223	CP
	R4	0.170	CP
	R5	0.151	CP
	R6	0.190	CP
	R7	0.159	CP
	R8	0.102	CP
	R9	0.210	CP
	R10	0.185	CP
	R11	0.143	CP
	R12	0.205	CP
	R13	0.223	CP
	R14	0.205	CP
	R15	0.210	CP
	R16	0.149	CP
	L5	0.186	CP
	L6	0.158	CP
	L10	0.107	CP
	M3	0.131	CP
	M6	0.194	CP
	M7	0.163	CP
	M10	0.146	CP
	M13	0.214	CP
AC40	M2	0.017	CP
	M3	0.014	CP
	M5	0.010	CP
	M6	0.007	NC
	M7	0.007	NC
	M8	0.006	NC

M9	0.004	NC
M10	0.010	NC
M11	0.011	NC
M12	0.005	NC
M13	0.005	NC
M14	0.007	NC
M15	0.006	NC
M16	0.005	NC
M17	0.005	NC
M18	0.004	NC
R1	0.006	CP
R2	0.008	CP
R3	0.005	CP
R4	0.004	CP
R6	0.005	NC
R7	0.006	NC
R8	0.005	NC
R9	0.007	NC
R10	0.004	NC
R11	0.0045	NC
R12	0.005	NC
R13	0.003	NC

CP means Corrosion-induced cracks penetrate the concrete cover; NC means No cracks observed by naked eye

Table 4

Fitted values of the parameters in the Gaussian model of each sample.

Sample	a_1	a_2	a_3	R^2
TC30 R1	1.479	0.742	0.042	0.860
TC30 R3	1.289	0.766	0.104	0.749
TC30 R4	0.876	0.641	0.015	0.736
TC30 R7	0.288	0.245	0.175	0.722
TC30 L5	0.923	0.763	0.058	0.763
TC30 M6	0.978	0.803	0.072	0.711
TC30 M13	1.022	0.964	0.128	0.803
TC30 R5	0.833	0.712	0	0.609
TC30 R6	1.272	0.845	0	0.827
TC30 R8	0.534	0.533	0	0.651
TC30 R9	0.779	0.627	0	0.785
TC30 R10	0.931	0.800	0	0.840
TC30 R11	0.703	0.900	0	0.687
TC30 R12	0.757	0.480	0	0.836
TC30 R13	1.026	1.018	0	0.788
TC30R14	0.605	0.677	0	0.646
TC30 R15	0.973	1.011	0	0.793
TC30 R16	0.787	0.573	0	0.716
TC30 L6	1.138	0.617	0	0.698
TC30 L10	0.736	0.843	0	0.726
TC30 M3	0.717	0.702	0	0.667
TC30 M10	0.931	0.615	0	0.767
AC40 M2	0.043	0.716	0.012	0.617
AC40 M3	0.022	0.395	0.008	0.606
AC40 M5	0.029	0.805	0.006	0.568
AC40 M6	0.007	0.485	0.006	0.641
AC40 M7	0.011	0.222	0.005	0.600
AC40 R1	0.011	0.140	0.005	0.721
AC40 R2	0.015	0.183	0.004	0.716
AC40 R3	0.004	0.121	0.005	0.830
AC40 R4	0.004	0.122	0.003	0.581

AC40 M8	0.032	0.610	0	0.721
AC40 M9	0.021	0.561	0	0.682
AC40 M10	0.054	0.890	0	0.600
AC40 M11	0.070	1.022	0	0.714
AC40 M12	0.019	0.794	0	0.738
AC40 M13	0.036	1.333	0	0.666
AC40 M14	0.036	0.905	0	0.602
AC40 M15	0.004	0.152	0	0.850
AC40 M16	0.017	1.109	0	0.835
AC40 M17	0.015	0.750	0	0.724
AC40 M18	0.016	0.928	0	0.828
AC40 R6	0.042	1.105	0	0.639
AC40 R7	0.006	0.302	0	0.774
AC40 R8	0.030	1.304	0	0.817
AC40 R9	0.036	1.895	0	0.707
AC40 R10	0.029	2.231	0	0.659
AC40 R11	0.013	0.929	0	0.643
AC40 R12	0.025	3.125	0	0.667
AC40 R13	0.018	2.571	0	0.826

Table 5 $(a_1 + 2\pi a_3)$ of the samples.

Sample	$a_1 + 2\pi a_3$
TC30 R1	1.746
TC30 R3	1.944
TC30 R4	0.968
TC30 R7	1.388
TC30 L5	1.287
TC30 M6	1.430
TC30 M13	1.826
TC30 R5	0.833
TC30 R6	1.272
TC30 R8	0.534
TC30 R9	0.779
TC30 R10	0.931
TC30 R11	0.703
TC30 R12	0.757
TC30 R13	1.026
TC30 R14	0.605
TC30 R15	0.973
TC30 R16	0.787
TC30 L6	1.138
TC30 L10	0.736
TC30 M3	0.717
TC30 M10	0.931
AC40 M2	0.118
AC40 M3	0.073
AC40 M5	0.067
AC40 M6	0.045
AC40 M7	0.043
AC40 R1	0.042
AC40 R2	0.040
AC40 R3	0.036
AC40 R4	0.022
AC40 M8	0.032

AC40 M9	0.021
AC40 M10	0.054
AC40 M11	0.070
AC40 M12	0.019
AC40 M13	0.036
AC40 M14	0.036
AC40 M15	0.004
AC40 M16	0.017
AC40 M17	0.015
AC40 M18	0.016
AC40 R6	0.042
AC40 R7	0.006
AC40 R8	0.030
AC40 R9	0.036
AC40 R10	0.029
AC40 R11	0.013
AC40 R12	0.025
AC40 R13	0.018

Table 6
 $\frac{a_1}{a_2}$ and $\overline{T_{cl-max}}$.

Sample	$\frac{a_1}{a_2}$	$\overline{T_{cl-max}}$ (mm)
TC30 R1	1.993	0.896
TC30 R3	1.683	0.561
TC30 R4	1.366	0.629
TC30 R5	1.171	0.535
TC30 R6	1.505	0.555
TC30 R7	1.174	1.004
TC30 R8	1.001	0.329
TC30 R9	1.243	0.553
TC30 R10	1.165	0.510
TC30 R11	0.781	0.368
TC30 R12	1.576	0.659
TC30 R13	1.008	0.451
TC30 R14	0.894	0.415
TC30 R15	0.963	0.448
TC30 R16	1.372	0.677
TC30 L5	1.210	0.495
TC30 L6	1.844	0.709
TC30 L10	0.873	0.308
TC30 M3	1.021	0.388
TC30 M6	1.218	0.415
TC30 M10	1.514	0.535
TC30 M13	1.060	0.401
AC40 M2	0.060	0.028
AC40 M3	0.056	0.030
AC40 M5	0.036	0.020
AC40 M6	0.014	0.015
AC40 M7	0.050	0.021
AC40 M8	0.053	0.029
AC40 M9	0.037	0.018

AC40 M10	0.061	0.025
AC40 M11	0.069	0.031
AC40 M12	0.024	0.015
AC40 M13	0.027	0.017
AC40 M14	0.040	0.020
AC40 M15	0.026	0.013
AC40 M16	0.015	0.013
AC40 M17	0.020	0.010
AC40 M18	0.017	0.008
AC40 R1	0.079	0.038
AC40 R2	0.082	0.035
AC40 R3	0.033	0.025
AC40 R4	0.032	0.021
AC40 R6	0.038	0.015
AC40 R7	0.020	0.014
AC40 R8	0.023	0.010
AC40 R9	0.019	0.008
AC40 R10	0.013	0.005
AC40 R11	0.014	0.006
AC40 R12	0.008	0.003
AC40 R13	0.007	0.003