Title: Rapid Compression Machine Study of a Premixed, Variable Inlet Density and Flow Rate, Confined Turbulent Jet

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Abstract

The behavior of transient, compressible and combusting pre-mixed methane-air jets was experimentally studied with high speed imaging in a rapid compression machine. The jets were generated with a turbulent jet ignition system, which is a prechamber initiated combustion system. The absence of physical analyses of the characteristics of premixed turbulent jets was the motivation for the present study. Experiments were completed for turbulent jet ignition system orifice diameters of 2.0, 2.5 and 3.0 mm each at lean-to-stoichiometric equivalence ratios of $\phi=0.67$, 0.8 and 1.0. The hot jet velocity at the orifice exit was calculated using mathematical correlations. The Mach number and Reynolds number were also computed. The high speed imaging shows the influence of orifice diameter on the flame propagation and the shape and structure of vortices resulting from the turbulent jet. Results revealed a direct relationship between orifice exit area reduction and a decrease in hot jet penetration speed. There was a reduction in hot jet penetration speed with an increase in the equivalence ratio. For the orifice diameters and equivalence ratios tested here, results showed that the jet evolved downstream of the orifice exit in partial agreement with existing correlations. Moreover, the jet was turbulent with calculated Reynolds numbers of around 20,000 or greater.

Introduction

The understanding and modeling of the unsteady turbulent jet in the Turbulent Jet Ignition (TJI) system is a challenge due to very high mixing and reaction rates. The TJI system is a prechamber initiated combustion enhancement system that can be used in place of a spark plug in a spark ignition (SI) engine. Combustion enhancement systems for spark ignited engines have been studied for decades, with reviews completed by Dale and Oppenheim [1] and Dale et al. [2]. Enhanced ignition systems include technologies such as microwave spark ignition [3], plasma-assisted ignition, [4, 5], divided chamber stratified charge [6] and prechamber ignition systems. Among all of these, prechamber ignition systems are of great interest as they are capable of enabling low temperature combustion, through both lean and/or dilute combustion.

A comprehensive study of the literature of prechamber ignition systems is presented in Toulson et al. [7]. This review covers a variety of systems ranging from the two-stroke Ricardo Dolphine engine [8], the three-valve engines of Summers [9] and Mallory [10], the LAG engine of Gussuk [11, 12] and the flame jet ignition investigation of Oppenheim et al. [13]. Recent studies on prechamber ignition systems have been carried out by Watson et al. [14, 15] and Attard et al. [16-20]. Combustion in the unfueled TJI system, similar to conventional spark ignition engines, occurs in the premixed mode which is one of the two categories of turbulent reacting flows [21]. The prechamber is a small volume chamber where an injector and spark plug, or an alternative ignition system, are located. This prechamber is connected to the main combustion chamber through either one or multiple small orifices. The combustion that initiates in this prechamber, increases the mixture pressure which is the necessary driving force for the flow of gases from the prechamber into the main chamber. The turbulent jet increases the turbulence and mixing rate in the main chamber and as a result, the dilute fuel-air mixture in the main chamber has a higher chance of igniting through the action of spatially distributed ignition sites. The unsteady turbulent jet plays a dominant role in combustion duration, flame propagation and burn rate. A parameter strongly affecting turbulent jet structure is the nozzle orifice area. A recent study on the influence of nozzle orifice was performed by Gentz et al. [22], using combustion visualization and characterization of TJI in a Rapid Compression Machine (RCM). The RCM, a device often used to study the fuel decomposition and ignition behavior at high temperatures and pressures produces conditions similar to those experienced in the engine combustion environment [23]. Gentz et al. showed that for near-stoichiometric conditions, a wider nozzle produced more spatially distributed jets that produced a faster rate of combustion. However at leaner conditions, a smaller diameter orifice yielded a faster and more vigorous jet that was required to initiate combustion. These opposing trends suggest that there may exist an optimum nozzle diameter.

Although TJI has been investigated over many years and broad experimental and computational studies have been recently performed, a shortage of experimental analysis has motivated the current study of transient turbulent jets in TJI systems. This study focuses on the observation and estimation of the hot jet exit velocity and its penetration speed at various locations downstream of the nozzle exit. The experiments are performed using high speed imaging of combustion in the TJI system.

The turbulent jet of the TJI system has a similar structure to the Pulsed Jet Combustion (PJC) system [24]. Maxson et al. [25] defined the three stages for the PJC process as the “jet”, the “plume” and the “puff”. The “jet” is defined as the initial core of the jet that consists of the incomplete combustion products at high temperature in which the mixing is disabled by shear at the orifice exit. This produces a time delay for the onset of combustion. The most active stage of PJC is the “plume”, where the large scale structures of the turbulent field in the combustion chamber serve to enhance combustion. Finally, the turbulent cloud of products surrounded by the flame front is termed the “puff.” Maxson et al. stated that a 2.0 *mm* single orifice diameter,

produced unreliable operation of the system. The triple orifice system showed better performance. They also investigated the performance of two simultaneously opposed jets and observed a higher rate of pressure rise with the opposed jet system in contrast to the single jet [25].

The configuration of the TJI system also has some similarities with the Piloted Premixed Jet Burner (PPJB) [26]. PPJB consists of a central jet of high velocity lean premixed methane/air mixture that is piloted by a low-velocity stoichiometric premixed pilot of hot reaction products from a stoichiometric Fuel/air mixture (Methane was used as the fuel in [26]). A large diameter coflow of lean premixed hydrogen-air combustion products surrounds the pilot and the central jet, isolating them from the surrounding air stream [26-28]. The major difference between TJI and PPJB is that with PPJB the high velocity central jet enters into a laterally confined open-ended environment whereas the central TJI jet enters into a bounded combustion chamber where the jet does not experience the fully developed region. Secondly, the PPJB central jet is a cold mixture of methane/air and the surrounding pilot and coflow mixtures are hot products of methane/air or hydrogen/air. By contrast, the TJI central jet mixture consists of hot products of fuel/air combustion that enter into a colder quiescent fuel/air mixture. While the nozzle exit velocity and density of the PPJB are fixed, the nozzle exit velocity of the hot jet in TJI will be estimated in the current study, using analytical correlations from the literature.

Round Turbulent Jets

Round turbulent jet flows have been extensively studied experimentally and numerically. A schematic description of a jet penetrating into a fluid at rest is shown in Figure 1 [29].

Figure 1. Schematic description of a jet penetrating a fluid at rest. The virtual origin is located at $x=0$, $5d/2$ upstream from the nozzle exit plane [29].

In Figure 1, a polar coordinate system is used to describe the round jet, the x -direction being aligned with the flow direction. In the present study, the orifice diameter is constant, d , and the jet exit velocity is defined as v_e . Gushamn-Roisin [29] stated that because the initial jet diameter is not zero but consists of a finite size orifice or opening, the stream-wise distance, x , should be

measured from a virtual source inside the nozzle at a distance $5d/2$ upstream from the nozzle exit (see Figure 1). Naber and Siebers [30] gave a slightly different definition for the virtual origin that is a function of orifice exit diameter, density ratio and jet cone angle of the outer boundary of the jet (a measure of spray dispersion). As the fluid exits the orifice, it experiences different flow regions starting with a developing region with an initial velocity profile ranging to a fully developed region with a self-similar velocity profile [31]; that is independent of the initial profile.

Based on the characteristics of the velocity field, Fellouah et al. [32] identified three separate and distinct regions in the jet flow field. The near-field, where x/d is between zero and six, is the initial flow or development region and has a top-hat velocity profile. In the far-field region, where x/d is greater than 25, the flow is fully developed. This region is termed the self-similar region. The intermediate field occurs between the near-field and the far-field. In this transition region, the developing jet encompasses characteristics of both the near and the far fields [33].

As described by Ball et al. [31], an unstable laminar shear layer is produced near the orifice exit of a non-reacting axisymmetric jet. This shear layer grows rapidly to form ring vortices that cause flow interchange from the irrotational ambient air into the jet and vice-versa. The turbulent jet examined in the present study is not simple and cannot be described easily because it consists of burning gases that enter a stagnant, homogeneous, and not-yet-reacting surrounding fluid. Here, the hot gas jet not only flows into the surrounding mixture but also serves as its ignition source.

To shed light on the influence of Reynolds number on the near and intermediate field regions of a round jet, Fellouah et al. [34] used flying and stationary hot wire measurements. They considered the velocity spectra and turbulence length scale distribution in the range of $6,000 \leq Re_D \leq 100,000$ in order to explore the idea of mixing transition proposed by Dimotakis [35]. Their study did not examine reacting jets. Hrycak et al. [36] experimentally examined the nominal and the actual length of the potential core of an axisymmetric jet and showed that in the potential core region, the nominal core length is independent of Reynolds number except in the transition region.

Another dominant parameter of the turbulent jet is the entrainment rate and spray dispersion. The local entrainment rate in the initial region of the axisymmetric air jet was studied by Hill [37] who used the porous wall technique introduced by Ricou and Spalding [38]. Here, the jet flows through a chamber that is open at one end to atmospheric pressure. Hill showed that the local entrainment rate, although independent of the Reynolds number for $Re_D > 60,000$, which is about half of the value specified by Fellouah et al. [34], depends on the axial distance from the jet exit. Hill's results show that the jet flow reaches the fully developed condition at an axial distance of about thirteen nozzle diameters from the exit; between the top hat (6) and the self-similar (25) regimes described by Fellouah et al. [34].

Penetration and dispersion of diesel sprays have similarities with reacting transient compressible gas jets (i. e., the turbulent jet of the current study). Studies on various aspects of gas and liquid sprays are abundant. A pertinent example is the study of a helium spray in an air chamber (Johansen et al. [39]). Schlieren visualization was used to help investigate the mean coefficient of variation of the penetration length, jet width, and jet angle. Johansen et al. found that if the jet is under expanded, a structure of expansion and shock waves is highly likely to occur at the nozzle exit. These expansion and shock waves are required for the jet to reach ambient pressure. Expansion waves may occur in the present TJI system as well.

Naber and Siebers [30] performed experiments in a constant volume combustion vessel using Schlieren imaging to investigate the influence of ambient gas density and fuel vaporization on the penetration and dispersion of diesel sprays. The authors derived a data correlation based on integral control surface techniques that was used to explain the effects of ambient gas density on the jet.

A physical model that predicts the velocity distribution in round jets with time varying injection profiles was developed by Abani and Reitz [40]. In this work the authors tested various injection profiles and ambient densities and obtained a steady state solution for the velocity profile within a turbulent gas jet using similarity analysis of the *2D* axisymmetric Navier-Stokes equations. This model can be used to study transient jet and spray injection processes and to predict the jet tip penetration speed.

The purpose of this study is to investigate the behavior of transient compressible gas jets in a TJI system and to perform a structural analysis of the jet using existing correlations from the literature. As an important component of this investigation, the normalized hot jet penetration length is measured and compared to literature values. The nozzle exit velocity is estimated by using several correlations. The hot jet penetration speed is measured from the processed images captured by a high speed camera during the RCM experiments.

Experimental Setup

The experimental turbulent jets being analyzed in the present study were generated in the Michigan State University (MSU) RCM using methane fuel. A schematic of the RCM is shown in Figure 2. The RCM consists of three pistons, the pneumatic piston, hydraulic piston and combustion chamber piston which are all on the same shaft. The combustion cylinder piston compresses the homogeneous fuel-air mixture and stops at top dead center (TDC) enabling combustion to occur at a constant volume condition. The compression ratio and combustion chamber wall temperature used for these experiments are 8.5 and 80° C, respectively. The combustion chamber is insulated to minimize temperature fluctuations. The entire cylinder bore is visible through a 50.75 mm diameter sapphire optical window. LabVIEW software is used for the data acquisition of the pressure traces, collected via two Kistler piezoelectric pressure

transducers, one installed at the bottom of main chamber and the other installed in the prechamber. Further details of the RCM experimental setup can be found in [41].

Figure 2- Schematic of the RCM, highlighting the combustion cylinder and turbulent jet ignition system. The overall apparatus shown has a length of $\sim 1.4\text{ m}$.

The TJI system used for this work consists of a spark plug located in a small prechamber that is connected to the main chamber via a nozzle having a single orifice. The different-diameter nozzles are easily interchangeable to examine different inlet configurations. The prechamber volume is approximately 5% of the clearance volume of the RCM. A schematic of the TJI system is shown in Figure 3. This image is an end-on (bore view) close-up of the turbulent jet igniter shown on the left hand side of Figure 2.

Figure 3- Close-up view of TJI with a nozzle installed.

Since the flow rate and hot jet Reynolds number depend on the orifice diameter, three single orifice nozzles were tested and their jet characteristics compared. The orifice diameters were 2.0, 2.5 and 3.0 mm. The experiments were run at stoichiometric ($\phi=1$) and lean conditions ($\phi=0.8$ and 0.67).

Combustion visualization of the TJI process was performed with a Photron SA4 High Speed color camera coupled with a Nikon $f/2.8$ objective lens. The resolution and frame rate were 512×512 pixels and 20,000 frames per second, respectively. The camera lens aperture was completely open for all experimental tests to capture the initial illumination of the turbulent jet.

The images were processed with ImageJ software available through the National Institutes of Health [42]. Their contrast and brightness were enhanced in order to facilitate the measurement of the hot jet penetration length, referred here as the jet tip.

Theory

In order to understand the TJI process, it is necessary to analyze the hot jet that generates the flame and changes the flow structures in the main chamber, which subsequently influences the burn duration and flame propagation. The hot jet velocity at the nozzle exit is not experimentally measurable. However, using high speed imaging and image enhancement techniques, the jet tip location is measurable as the jet travels toward the lower (see Figure 2) main chamber wall. The hot jet penetration speed on the centerline is calculated based on the jet travel distance divided by the time interval of travel.

The hot jet velocity at the nozzle exit was estimated from the penetration speed as a function of jet penetration length and by using available mathematical correlations. In the following section, three mathematical correlations [30, 40, 43] are described briefly. Their corresponding estimated nozzle exit velocities are shown in the results section.

Correlation I: Laminar Jet Analysis [43] and Inverse Ratio Correlation

A straightforward laminar jet analysis provides a non-dimensional axial velocity distribution as a function of Reynolds number at the nozzle exit. Further details of the correlation can be found in [43]. The result is

$$\frac{v(x,r)}{v_e} = 0.375 \left(\frac{\rho_e v_e r_e}{\mu} \right) \left(\frac{x}{r_e} \right)^{-1} \left[1 + \frac{\xi^2}{4} \right]^{-2}. \quad (1)$$

Similar to the jet description shown in Figure 1, a polar coordinate system is used in which the x -axis aligns with the flow direction. In Eq. (1), $v(x)$ is the jet velocity of a flow particle at a specific point in the main chamber with axial and radial distances (x, r) from the nozzle exit. The quantities ρ_e , v_e and r_e are the jet density, velocity and radius at the nozzle exit, respectively, and the dimensionless quantity ξ is defined as

$$\xi = \left(\frac{3\rho_e J_e}{16\pi} \right)^{1/2} \frac{1}{\mu} \frac{r}{x}, \quad (2)$$

where J_e is the jet exit momentum defined as follows:

$$J_e = \rho_e v_e^2 \pi r_e^2. \quad (3)$$

Since $r=0$ on the centerline, $\xi=0$ and the velocity there reduces to

$$\frac{v_{x0}}{v_e} = 0.375 \left(\frac{\rho_e v_e r_e}{\mu} \right) \left(\frac{x}{r_e} \right)^{-1}. \quad (4)$$

Here v_{x0} is the centerline jet velocity at every stream-wise x location. It is important to note that the preceding laminar jet analysis is based on the hypothesis of a steady non-reacting laminar jet that issues into an infinite reservoir of quiescent fluid (oxidizer) [43]. In TJI, however, the transient turbulent jet issues into a wall-bounded finite chamber. Consequently, the laminar jet analysis is likely not a precise correlation for the hot jet exit velocity. In fact, the exit velocity estimated by the laminar jet correlation is an order of magnitude smaller than the experimentally measured hot jet tip penetration speed. However, the laminar jet analysis suggests a functional form of a correlation in which the nozzle exit velocity is inversely proportional to the hot jet penetration length. It is convenient to apply a descriptive name for this correlation, which is referred to henceforth as the “inverse ratio” correlation,

$$v_{x0} = \frac{C}{x}. \quad (5)$$

Here, C is a constant coefficient that can, in principle, be deduced from the experimental data.

Correlation II: Similarity Analysis, [40]

The second correlation that will be used here is based on the steady state solution for the velocity profile within a turbulent gas jet. This correlation uses a similarity analysis of the two dimensional axisymmetric Navier-Stokes equations [40], to find

$$v(x, r) = \min \left[v_e, \frac{3v_e^2 d_{eq}^2}{[32\vartheta_t x (1 + \frac{3v_e^2 d_{eq}^2 r^2}{256\vartheta_t^2 x^2})^2]} \right]. \quad (6)$$

As in Eq. (1), v_e is the injection velocity at the nozzle exit. Abraham [44] defined a characteristic length called the “effective” or “equivalent” diameter of the gas jet, $d_{eq} = d_{noz} \sqrt{\frac{\rho_e}{\rho_{amb}}}$, in which d_{noz} is the nozzle diameter and ρ_e and ρ_{amb} represent the jet and ambient fluid densities, respectively. Also, ϑ_t is the turbulent viscosity, defined as $\vartheta_t = C_t \pi^{0.5} v_e d_{eq} / 2$. Here C_t is a constant coefficient equal to 0.0161 as proposed by Schlichting [45].

Abani and Reitz [40] stated that the velocity decay rate at the centerline is generalized as

$$v_{x0} = \frac{dx}{dt} = \frac{3v_e d_{eq}}{Kx} \quad (x \geq \frac{3d_{eq}}{K} = x_0) \quad (7)$$

Here, K is the entrainment constant, $K=16\pi^{0.5}C_t$. Abani and Reitz [40] calculated the jet penetration length as a function of time, by integrating Eq. (7) and assuming a constant injection velocity. The resulting equation is

$$x(t) = \left(\frac{6}{K}\right)^{1/2} (v_e d_{eq})^{1/2} t^{1/2} \quad (for \ x \geq x_0). \quad (8)$$

It should be noted that although the hot jet tip penetration speed is a function of $1/x$ in both Eq. (4) and Eq. (7), simplifying both of these equations in terms of v_e , redefines them as follows:

$$v_e = \frac{1}{r_e} \left[\frac{v(x) \cdot \mu}{0.375 \rho_e} \cdot x \right]^{1/2}, \quad (4-b)$$

$$v_e = \frac{v(x) \cdot K}{3d_{eq}} \cdot x \quad (7-b)$$

where x is measured from the virtual origin of the jet. Eq. (4-b), in the laminar flow analysis, gives $v_e \propto c_1 x^{1/2}$ whereas the turbulent jet similarity analysis gives $v_e \propto (c_2 \times x)$, or, in other words, $v_e \propto (c_3 \times t^{\frac{1}{2}})$. In what follows it will be demonstrated that the predictions of these two correlations may differ by an order of magnitude.

Correlation III: Non-Vaporizing Spray Penetration Correlation, [30]

The third correlation here is the non-vaporizing spray penetration correlation of Naber and Siebers [30]. Their analysis is a modification of the spray penetration analyses of Wakuri et al. [46] and Hays [47] in which an inverse relationship (penetration distance versus time) is introduced. To establish the correlation, the integral control surface technique is used for the injected and ambient fluids with different densities. Major assumptions of the correlation are the uniform functional form of the jet velocity profile in the domain, constant injection velocity, no slip velocity between the fuel and entrained air, and quasisteady flow with a uniform growth rate (i.e. constant dispersion angle). The dimensionless penetration time, $\tilde{t} = t/t^+$, as a function of dimensionless penetration distance, $\tilde{s} = x/x^+$, is obtained as;

$$\tilde{t} = \frac{\tilde{s}}{2} + \frac{\tilde{s}}{4} \times \sqrt{1 + 16\tilde{s}^2} + \frac{1}{16} \ln[4 \times \tilde{s} + \sqrt{1 + 16\tilde{s}^2}] \quad (9)$$

where $t^+ = \frac{d_{eq} \sqrt{\rho}}{a \times \tan(\frac{\theta}{2}) v_e}$ and $x^+ = \frac{d_{eq} \sqrt{\rho}}{a \times \tan(\frac{\theta}{2})}$ are the time and length scales used in the nondimensional correlation of Eq. (9). The density ratio is defined as $\tilde{\rho} = \frac{\rho_e}{\rho_{amb}}$, where ρ_e and ρ_{amb} are the jet and the ambient gas densities, respectively. The effective jet diameter is defined as $d_{eq} = \sqrt{c_a} \times d_{noz}$ where c_a is the area contraction coefficient. The fuel velocity at the orifice exit is a function of jet pressure, ambient pressure, jet density, and velocity coefficient, (which is the ratio of discharge coefficient, c_d , and contraction coefficient, c_a , ($c_v = \frac{c_d}{c_a}$)). However since the jet pressure is unknown in the present study, v_e is a variable that must be estimated (in Naber and Siebers [30], the spray's velocity and pressure were both known). The cone angle, θ , of the outer boundary of the jet is termed the dispersion angle since it measures spray dispersion. The constant a accounts for the difference between the real jet dispersion angle and the angle used in the model. Based on the best fit correlation provided by Naber and Siebers [30], $a=0.66$. More details regarding the derivation of the correlation are available in [30].

In order to apply the penetration correlation in the current study, the penetration length was measured from the orifice exit and the elapsed time was measured from the first appearance of the hot jet. The half dispersion angle is 10 degrees and the area contraction coefficient is 0.8.

Results

The prechamber size and orifice diameter were examined in several studies [48-50]. It was determined that if the orifice diameter was too small, ($d < 1\text{ mm}$), the jet was not capable of igniting lean mixtures in the main chamber. In addition, smaller orifices, although having greater hot jet penetration, had higher heat losses [7]. The commonly used orifice diameters for prechamber ignition systems in general, and TJI systems in particular, are in the range of 1-4 mm [7, 22]. In the present study, experiments were performed for orifice diameters of 2.0, 2.5 and 3.0 mm for equivalence ratios of $\phi=0.67$, $\phi=0.8$ and $\phi=1.0$ for a test matrix of 9 possible cases. A nozzle of 1 mm diameter was also examined, however, the exiting hot jet, even with image enhancement, was not sufficiently luminous to measure its penetration length in the region close to the orifice exit. As a result, the exit velocity estimation was not experimentally viable. Murase and Hanada [48] showed that a faster pressure rise is attainable with the smaller orifice diameter ($\sim 2.5\text{ mm}$ compared to 4 mm), which greatly enhances the combustion process. In order to produce a substantial pressure rise in the prechamber, a smaller orifice is used and thus the use of a large orifice ($d > 3\text{ mm}$) is avoided here.

Ten runs at each test point were performed to ensure repeatability. The initial pressure in all cases was 1.04 bar which resulted in a compressed pressure of $\sim 15\text{ bar}$ and a peak combustion pressure of $\sim 47\text{ bar}$ (with a higher peak pressure for the stoichiometric condition). Figure 4 shows an example of the pressure trace and spark trigger signal from an RCM test (note: spark occurs on the falling edge of the square wave signal).

Figure 4- Recorded pressure traces in both pre and main chambers together with the spark trigger signal for the 3.0 mm orifice at $\phi=0.8$. The inlay shows images of the locations of the jet. The images correspond to the time sequence of Table 1. ($t=0.1\text{ ms}$, $t=0.25\text{ ms}$ and $t=0.45\text{ ms}$ respectively).

An annotated view of the pressure, which correlates to the experimental images of the issuing hot jet appears in Figure 4. This plot shows that the hot jet dynamics occur when the pressure in both chambers is close to the TDC pressure (15-20 bar). Numerical studies of the TJI system [51, 52] have shown that a relatively cold jet of unburned prechamber mixture issues into the main chamber prior to the hot jet of combustion products. This colder non-reacting jet cannot be seen in the chemiluminescence imaging. The cold jet has a strong effect and both increases the turbulent intensity and the mixing rate in the main chamber. Figure 4 shows that the initial 15% of the pressure rise is due to the issuance of the hot combusting jet. The influence of the TJI concept on the burn rate compared to direct SI combustion was investigated by Gentz et al. [41]. Detailed images of the issuing jets are provided in the next section.

The temperature at TDC is calculated by assuming polytropic compression. Compressed temperatures at $\phi=1.0$, 0.8 and 0.67 as a function of equivalent ratio values are 755, 757.4 and 758 K, respectively. In order to estimate the nozzle exit density and viscosity, the major assumption made here is that the jet consists of the hot products of methane/air combustion, which is a gross simplification of the actual complexity of the real injection process. The jet density and viscosity were calculated using this assumption, which results in a comparable order of inaccuracy in the values of these quantities. The viscosity of the mixture is calculated as the summation of the partial viscosities of all species, $\sum x_i \mu_i$, (x_i =mass fraction of species, i) while μ_i is the viscosity of every species at the adiabatic flame temperature and is calculated using the formula

$$\ln(\mu_i) = A \ln(T) + \frac{B}{T} + \frac{C}{T^2} + D. \quad (10)$$

The coefficients A , B , C and D are provided by McBride et al. [53]. The jet density is calculated using the ideal gas equation of state at the adiabatic flame temperature.

In order to estimate the uncertainty of the desired/intermediate variables based on the contribution of the uncertainty of each parameter, a sensitivity analysis was performed. The sensitivity analysis has two parts, the uncertainty of the temperature and ambient density at TDC and the uncertainty of each correlation of the exit velocity.

In order to conduct the uncertainty analysis, each variable is considered to be composed of a mean and an error, viz.,

$$\alpha = \bar{\alpha} + \delta\alpha. \quad (11)$$

Substituting these expressions for the initial temperature, initial pressure, and the compressed pressure in the polytropic compression process equation, and knowing the related errors, ($\frac{\delta T_{BDC}}{T_{BDC}} < 1.5\%$), ($\frac{\delta P_{BDC}}{P_{BDC}} < 0.7\%$) and ($\frac{\delta P_{TDC}}{P_{TDC}} < 7\%$), the compressed temperature error is estimated as 3.5%. Equivalently, using the ideal gas equation of state, the uncertainty of the main

chamber mixture density at TDC, $(\frac{\delta \rho_a}{\rho_a})$ is found to be less than 6.5 % for all conditions studied here.

For the second part of the analysis, a considerable error ($\frac{\delta T_{adiabatic}}{T_{adiabatic}} = 10\%$) is assumed for the jet temperature since the experimental measurement of the jet temperature is unfeasible. This adiabatic flame temperature error leads to $\sim 10\%$ error in the jet density, (varying due to the air/fuel ratio). Substituting for all variables of Eq. 7 in the form of Eq. 11, the uncertainty calculation of the nozzle exit velocity based on the similarity analysis results in the exit velocity errors, $(\frac{\delta v_e}{v_e})$, of 11.4%, 7% and 12.5% for 2 mm, 2.5 mm and 3 mm nozzles, respectively. The uncertainty calculation for the non-vaporizing spray correlation produces estimated exit velocity errors $(\frac{\delta v_e}{v_e})$ of less than 4% for all nozzle diameters. Details of the uncertainty analysis are provided in Appendix A.

Jet Penetration Illustration

Experimental images of the penetrating hot jet and combustion initiation in the main chamber are shown in Tables 1 and 2. In Table 1, variations of the orifice diameter are compared at $\phi=0.8$, whereas Table 2 shows the variation in equivalence ratio for the 2.5 mm orifice. To make the early flame development clearly visible for comparison, the image brightness was enhanced.

Table 1 shows that the turbulent jet exiting the 3.0 mm orifice is the fastest. The span-wise growth of the larger orifice jet is also quicker than the growth of the jets from the smaller orifices. The higher mass flow rate that passes through the 3.0 mm orifice enables the development of a more powerful ignition source in the main chamber. By contrast to the 3.0 mm orifice, the 2.0 mm orifice jet has the slowest speed, probably caused by flow resistance, which restricts the flow and therefore diminishes the flow rate.

In Table 2 it can be seen that the stoichiometric jet flame is brighter than the lean jets due to the difference in oxygen concentration. Decreasing the equivalence ratio results in lower soot production. The images in Table 2 show that the lean jet is faster than the stoichiometric jet. By the time the stoichiometric jet reaches the center of the main chamber, the leanest jet ($\phi=0.67$) has reached the opposite chamber wall. The images show that the jet does not experience self-similarity as it impinges on the wall after leaving the orifice exit. The jet tip shape illustrates that the combustion propagation is strong in both the radial and the axial direction at the beginning and that it strongly spreads radially after the jet impinges on the wall. The images exhibit three processes that are made possible by the TJI: distributed combustion; improved flame propagation, and cavity radical dissemination. An additional fundamental feature of the turbulent hot jet that can be seen from Tables 1 and 2 is the large scale structure of the turbulent jet, which plays a crucial role in combustion enhancement. Images of Tables 1 and 2 illustrate the existence of a laminar flame structure surrounding the reacting core of the flame in the near field region,

($x/d < 6$). In addition, similar to [54], vorticity generation and destruction is present in the intermediate region ($6 < x/d < 25$) where density gradients, produced by heat release, are the main reason for shear layer reduction and potential core expansion. In the intermediate region, a shear layer is formed between the hot jet core and the surrounding mixture: large vortices are located at the edge of this layer

Table 1- Experimental images of hot jet penetration into the main chamber for varying orifice diameters ($d=2.0, 2.5, 3.0$ mm) at a constant equivalence ratio condition ($\phi=0.8$).

t (ms)	$\phi=0.8$ $d=2.0$ mm	$\phi=0.8$ $d=2.5$ mm	$\phi=0.8$ $d=3.0$ mm
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

Table 2- Experimental images of hot jet penetration into the main chamber for varying equivalence ratios ($\phi=1.0, 0.8, 0.67$) for an orifice diameter of 2.5 mm .

t (ms)	$d=2.5\text{ mm}$ $\phi=1.0$	$d=2.5\text{ mm}$ $\phi=0.8$	$d=2.5\text{ mm}$ $\phi=0.67$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1.2

Penetration Speed Measurements

Figure 5 shows a plot of the changes in the direction x (abscissa) of the hot jet tip penetration speed (ordinate) for the three different orifice diameters with the same equivalence ratio, $\phi=0.8$. The hot jet penetration length is normalized by the main chamber diameter, D . The error bars were calculated based on ten experimental runs completed for each test point. The initial variation is highest due to the variability in the flame kernel formation in the prechamber. The variability reduces as the hot jet propagates towards the far combustion chamber wall. As can also be seen in Table 1, the 3.0 mm orifice presents a lower flow restriction, enabling a higher mass flow rate into the main chamber at a higher jet tip penetration speed. In addition, for $d=3.0$ mm orifice, the rate of penetration speed change stays constant until the jet impinges on the far main chamber wall. By contrast, the 2.5 and 2.0 mm orifice jet velocities have a steeper slope in the region of $x/D < 0.3$ (where D is the main chamber bore) and then remain approximately constant between $0.3 < x/D < 1.0$. The steeper slope of the penetration speed decrease for $x/D < 0.3$ is due to the viscosity effect that induces higher shear forces in the gas close to the orifice exit. This observation is in agreement with a previous turbulent jet diffusion flame study [54]. In this study Pitsch [54] showed that there is a laminar region close to the nozzle exit which results in strong diffusion effects and that molecular diffusion is the main physical process affecting mixing in the vicinity of the orifice exit. As the hot jet reaches the center of the main chamber, its penetration speed increases slightly due to heat released by the flame. Diffusion of heat and mass far downstream of the orifice exit contributes to the constant hot jet penetration speed in the region $0.5 < x/D < 1.0$.

Figure 5- Jet tip penetration speed versus normalized jet penetration length at $\phi=0.8$.

Figure 6 shows the jet penetration speed and its penetration length as functions of time. For consistency, the data for $\phi=0.8$ for the three nozzle orifices is shown. Results show that the hot jets of the $d=3.0$ mm and $d=2.5$ mm orifices reach the main chamber wall the fastest. In addition, the hot jet penetration speed decreases rapidly with the increase of x/D in the region $x/D < 0.5$.

This is the hot jet development region, where the rate of entrainment is low and the radial extension is limited, but the axial propagation of the hot jet is notable. On the other hand, in the region $x/D > 0.5$, defined as the confining region, constant diffusivity effects lead to a constant mixing rate [54], that results in a gradual change in the hot jet tip penetration speeds. In this region, the high temperature reacting jet forces the surrounding mixture to react, which expands the flame surface area. In addition, large scale vortices of the turbulent hot jet that are associated with higher turbulent intensity induce additional entrainment of the main chamber unburnt mixture into the jet. The momentum entrainment rate is lower than the mass entrainment rate, which results in a semi-constant tip penetration speed for the jet in the region $0.5 < x/D < 1.0$.

Figure 6 demonstrates a very obvious difference between the penetration from the smallest orifice compared to the two wider orifices. As previously discussed, a smaller nozzle cross sectional area applies greater contraction effects on the jet flow, leading to a slower hot jet. The required time to reach the chamber wall for the smaller orifice jet is 1.5 *ms* longer than the two larger orifices.

Figure 6- Jet tip penetration speed and normalized jet penetration length versus time at $\phi = 0.8$, here, D is the main chamber bore.

In order to demonstrate the dependence of the hot jet penetration on time mathematically, Equation (8) provides a linear relationship between the jet penetration length and the square root of time for a steady state injection velocity. In a similar manner, Naber and Siebers [30] introduced a short and a long time limit in Eq. 9. With the short time limit as $\tilde{t}$ approaches zero near the injector (i. e., $\tilde{x}$ approaches zero), the jet penetration shows a linear dependence on time, $\lim_{\tilde{t} \rightarrow 0} |\tilde{s}| = \tilde{t}$, and with the long time limit where $\tilde{t}$ approaches infinity, (i. e., $\tilde{x}$ approaches infinity), the penetration length becomes a function of the square root of time, $\lim_{\tilde{t} \rightarrow \infty} |\tilde{s}| = \tilde{t}^{1/2}$. Naber and Siebers [30] also defined a transition region where the spray changes from one dominated by the injected fluid to one dominated by entrained air. The results of the present study also show a linear dependency of penetration length on time in the development region x/D

≤ 0.5 and a square root dependence in the region of $0.5 \leq x/D$, as the hot jet approaches the chamber wall, see Figure 7.

Figure 7- Dependence of jet penetration length on time, (a) short ($\xi = \tilde{t}$), and (b) long ($\xi = \tilde{t}^{1/2}$), limits, for $\phi = 0.8$. Note that correlation (a) remains linear throughout but correlation (b) breaks down as time increases and the hot jet approaches the opposite wall.

Figure 7 shows that the initial trends of the jets in the present study are in good agreement with the suggested literature correlations [30, 55]. The transition region from linear to square root dependence does not occur at a specific time for all cases; it does, however, occur at the center of the main chamber.

Figure 8 depicts the hot jet tip penetration speeds of all mixture conditions ($\phi = 1.0, 0.8, 0.67$) exiting each of the three orifices. In general, lean hot jets have higher speeds than stoichiometric jets. This observation is explained by differences in jet density. Although the main chamber density increases with the increase in the air/fuel ratio, the differences between main chamber densities at various air/fuel ratio conditions is not notable. However at the orifice exit (not the main chamber), a decrease from $\phi = 1.0$ to 0.8 results in about 17% increase in the jet density. The additional decrease from $\phi = 0.8$ to 0.67 results in extra 10% jet density increase. Higher density jets have higher penetration speeds. The jet density changes across the equivalence ratio conditions are due to changes in the burned gas temperature, while the differences in burned gas pressures are relatively minor.

A distinct characteristic of the larger orifice injector is the more moderate slopes of the penetration speed plots for all equivalence ratios, from the nozzle exit to the far wall. The 2.0 mm orifice is notable for its steep initial penetration speed slope which becomes nearly zero in the region $x/D > 0.3$. Results of the 2.5 mm orifice lie between the 2.0 mm and the 3.0 mm orifice velocities. Figure 8 also shows that for the 3.0 mm orifice, there is more variation in the hot jet tip penetration speed for different mixture compositions as the jet travels toward the main chamber end wall. This penetration speed variation decreases with the decrease in orifice diameter. The 2.0 mm orifice shows the least variation in penetration speed with respect to the equivalence ratio.

Figure 8- Jet tip penetration speed versus normalized jet penetration length for all nine cases. Here, D is the main cylinder bore.

Figure 9 shows the experimental penetration speed data in the near and intermediate regions. The turbulent jets studied here satisfy the inequality $x/d < 25$ (note: d is the orifice diameter). In this range of x/d , the flow is not self-similar according to the jet flow regimes identified by Fellouah et al. [32]. In the region $x/d < 25$, both the viscosity and momentum of the mixture determine its flow behavior. The structure and size of the vortices depend on the turbulent intensity of the jet and the combustion rate in the main chamber. Two phenomena can be put forth for the combustion rate in the main chamber. Firstly, the amount of turbulence that forms in the main chamber due to the piston motion. Secondly, the turbulent intensity of the initial cold jet of the unburnt mixture, followed by the hot jet of the combustion products of the prechamber mixture that issues into the main chamber. The turbulent intensity can be influenced by the volume of the prechamber, the size and number of orifices, the spark trigger event, and the amount of released energy, as well as the equivalence ratio. The measurement of the turbulent intensity and all its determining factors is not within the scope of the current study. In addition as stated before in the near field region (i. e., vicinity of the orifice exit), diffusion of heat and mass dictates the structure of the flow field, whereas diffusion effects remain constant in the intermediate field [54]. Here, entrainment determines the speed and shape of the hot jet and also the structure of turbulent vortices.

Figure 9- Jet tip penetration speed versus x/d (d is the nozzle diameter) for three nozzle diameters at $\phi=0.8$

Orifice Exit Velocity Estimation

The measurement of orifice jet exit velocity is not viable in the RCM due to the experimental difficulties corresponding to unsteady combustion in a very small high pressure vessel. Thus, a major goal of the current study was to *estimate* the orifice exit velocity using existing literature correlations. Figure 10 illustrates the differences between the inverse, the similarity [40], and the penetration [30] correlations in the prediction of hot jet velocity at the nozzle exit. Due to viscosity effects and entrainment, the hot jet penetration speed decreases as it travels downstream of the nozzle exit. Consequently, the exit velocity is expected to be higher than the jet tip penetration speeds measured in the experiments. The similarity analysis, however, under predicts the exit velocity. The similarity analysis uses the jet and ambient densities in the calculation of the equivalent diameter, $d_{noz} \sqrt{\frac{\rho_e}{\rho_{amb}}}$, and in the jet exit velocity prediction. A substandard estimate of the jet density is a possible reason for the under prediction of the velocity by the similarity model.

The jet may be a combination of combustion products and unburnt gases from the prechamber. Since the composition of the hot jet exiting the orifice was not measured, the presumption of complete combustion in the prechamber and a jet of combustion products that leaves the orifice is a reasonable assumption. It has been observed experimentally, and in numerical simulations [51, 56], that the hot burning gases exit the orifice after an initial jet of cold, unburnt mixture. This initial cold jet changes the main chamber temperature in the vicinity of the orifice exit and also increases the turbulence intensity of the ambient gas, resulting in higher entrainment and radial spread of the hot jet. This strongly influences the hot jet penetration speed.

The inverse ratio correlation predicts a higher velocity at the orifice exit since it borrows the model's constant coefficients from the experimental results. However, the predicted exit velocity for $d=3.0$ mm seems high. Because of the use of constant coefficients in this correlation, its uncertainty is higher than the two other correlations.

The penetration correlation [30] has a discrepancy in the prediction of the $d=2.0 \text{ mm}$ orifice exit velocity. As shown in Figure 10, the predicted exit velocity based on the penetration correlation does not agree with the other correlations and with the first experimental data point (i. e., the measured penetration speed based on the experimental image in which the initial hot jet is observed). Uncertainties in density ratio, $\frac{\rho_e}{\rho_{amb}}$, area contraction coefficient, c_a , and the dispersion angle are the main reasons for the inaccurate estimation by the penetration correlation.

The Mach number at the nozzle exit is calculated using the exit velocity computed by the inverse ratio analysis. The Mach numbers calculated for all the cases in the present study are less than 0.3. It is possible and useful to calculate the flow Reynolds number as well. The calculated stream-wise Reynolds number, Re_x for all experiments lies between 1.4×10^4 and 8×10^4 , where the characteristic length, x , is the distance between the nozzle exit and the hot jet virtual source [29]. For these cases, the jets are turbulent, because x/d is less than 25 and the Reynolds numbers are on the order of 10^4 .

Figure 10- Jet exit velocity based on various correlations for three nozzle diameters at $\phi=0.8$. None of the three correlations is consistently accurate over the entire range of the data.

Discussion and Conclusions

A series of tests was carried out to determine the influences of orifice diameter, d , and mixture composition, ϕ , on the jet inlet and tip speed in a turbulent jet ignition (TJI) system and the rate of flame spread in the main combustion chamber. Experiments performed in a RCM and experimental images of the combustion process have been presented in order to illustrate the progress of hot jet development and combustion initiation in the main chamber as well as the features of the flow structures within the turbulent jet. The results reveal two direct relationships for the orifice sizes and equivalence ratios tested here: (1) reducing the orifice exit area below a threshold produces a decrease in the hot jet penetration speed; (2) a decrease in the mixture equivalence ratio increases the hot jet penetration speed with the exception of the 3.0 mm orifice

diameter case. In general, the penetration speed of the burnt gas products exiting the orifice via the jet is higher for leaner mixtures relative to stoichiometric mixtures, which was explained in terms of density increases for the leaner mixtures. The changes in the hot jet densities are mainly due to the effect of leaner mixtures on the burned gas temperatures while the pressure differences are relatively minor. Due to the higher exit velocities enabled by the larger orifice diameters, the hot jets that exit the 3.0 and 2.5 mm orifices impinge on the far chamber wall in a shorter period of time. This generates faster lateral spread and higher burn rates in the combustion chamber.

Results show that as the hot jet evolves downstream of the orifice exit, it remains in the near and intermediate field x/d regimes, and does not evolve into the far field stage $x/d \geq 25$. For the range of Reynolds numbers calculated, $\sim 20,000$ and higher, the round jets are all turbulent. Having a turbulent jet in the main chamber increases the mixing rate and also yields a shorter burn duration and thus lower heat losses during the combustion process. The turbulent jet vorticity also improves combustion by increasing the flame surface area.

Measured results reveal that the factors governing the hot jet characteristics and the main chamber flow field are not similar in the near and the intermediate fields. In addition, there is a distinct transient location, $x/d \approx 10$, where the hot jet penetration speed dependence on time changes from a linear to a square root function. There is also a range of values, $15 \leq x/d \leq 23$ where the latter correlation breaks down as the jet impinges on the wall. The correlations for the larger nozzle break down earlier likely because the jets are wider and therefore reach the vicinity of the wall sooner than suggested by the centerline x/D value.

Due to the non-viability of velocity measurements during combustion in a high pressure (~ 50 bar) RCM, existing correlations have been utilized to estimate the orifice exit velocity. Data has been presented for the hot jet velocity at the nozzle exit, predicted by the similarity, inverse ratio and penetration correlations. The under prediction of the similarity analysis compared to the inverse ratio analysis may be due to the assumptions made for the jet density, among other factors that remain to be investigated. Although the inverse ratio analysis predictions are acceptable, they are highly dependent on empirical correlation constants determined from the experimental data, whereas the similarity analysis has no discernable connection to the experiments. The over prediction of the penetration correlation for $d=2$ mm orifice could be attributed to the uncertainty in the contraction coefficient. However, the uncertainty in the density ratio and dispersion angle are considerable too.

Although the correlations used in the current study have not precisely estimated the orifice exit velocity, they have provided acceptable results to calculate the Reynolds and Mach number of the hot jet, and they show, unequivocally, that the TJI jet and flow is turbulent. In addition, results show that with the conditions considered and the orifice sizes used the nozzle will most likely not be choked and, hence, no shock or expansion waves are expected.

Finally, the results suggest that as useful as previous literature correlations may be, the fact that the TJI jets impinge on an opposite wall while also demonstrating strong lateral entrainment and spread during injection requires new and improved analyses and correlations. Other major differences between previous studies are the transient injection velocity and the variation in the injection gas density and temperature. None of these variables is accounted for in the previous correlations nor in any of the PPJB studies.

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