3D Flame Topography and Curvature Measurements at 5 kHz on a Premixed Turbulent Bunsen Flame

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ABSTRACT

This work reports the measurements of three-dimensional (3D) flame topography and curvature of a premixed turbulent Bunsen flame at a rate of 5 kHz, using a technique combining chemiluminescence and tomography. Line-of-sight images of chemiluminescence (termed *projections*) of the target flame were recorded by six cameras from different orientations simultaneously at 5 kHz. Based on these projections, a tomography algorithm reconstructed the 3D flame structure, based on which 3D curvature was then calculated. Due to the 3D nature of the data, statistics of flame properties can then be extracted both in temporal and spatial domains. Probability density function (PDF) of flame topography was extracted from a series of 3D measurements, and the PDFs of the flame at different spatial locations were examined. Furthermore, the instantaneously measured 3D flame topography also enabled the calculation of 3D flame curvature (or 2D curvature along an arbitrary orientation). The PDFs of curvature in 2D and 3D were then extracted and compared. These results provide quantification of the flame surface shape in 3D (cylindrical, elliptic, or hyperbolic), illustrating the utility of 3D diagnostics to fully resolve the dynamics of turbulent flames.

1. Introduction

Turbulent premixed combustion has found widespread applications in practical combustion systems, and a fundamental understanding of such flames is essential to the efficient operation of these systems. However, the complicated governing processes and their intricate interactions are not fully understood yet, and many open questions still exist and hinder the development of predictive combustion models [1-4]. As result, significant research efforts, both numerical and experimental, have been invested to resolve the governing processes involved in turbulent premixed flames [1, 5, 6]. Due to the continued growth of computational power, DNS (direct numerical simulation) and LES (large eddy simulation) are becoming increasingly feasible to simulate the 3D (three-dimensional) and time-dependent structures of turbulent premixed flames [7-9]. Corresponding experimental data, 3D data with adequate temporal resolution, have therefore been long desired for comparison against numerical simulations and are ultimately needed to fully understand the underlying processes.

However, well-established experimental techniques have been targeted for measurements at a point, such as coherent anti-Stokes Raman scattering (CARS), along a line such as spontaneous Raman scattering, or across a plane such as planar laser-induced fluorescence (PLIF) [10]. With the recent advancements in lasers, cameras, computational technologies, and imaging processing algorithms, it has become feasible to contemplate 3D experimental measurements with the desired spatiotemporal resolutions to resolve turbulent flames. Several approaches are being actively pursued, and initial demonstration measurements have been reported. These approaches can be broadly divided into two categories. Approaches in the first category involve the scanning of a planar technique to obtain a series of 2D measurements sequentially, and these 2D measurements can then be combined to form an effectively instantaneous volumetric measurement when the scanning is performed rapidly. Demonstrations have been performed to obtain 3D measurements by the scanning of the laser sheet for PLIF [11-13], planar Mie scattering [14], and planar laser-induced incandescence [15]. With current high-repetition-rate lasers and scanning technologies, 3D measurements with temporal resolution near 1 kHz and a spatial resolution on the order of 1 mm in the direction of scanning [11-13] are feasible. Multiple lasers can potentially be used to improve the temporal and spatial resolution at the cost of complexity and capital investment [15]. Approaches in the second category involve tomography

to obtain 3D measurements volumetrically and instantaneously without scanning. Similar to the scanning approach, the tomography approach can be used in combination with a range of signal generation mechanisms, ranging from absorption spectroscopy [16, 17], Mie scattering [18, 19], chemiluminescence [20-22], and particle imaging velocimetry (PIV) [23]. In contrast to the scanning approach, higher temporal resolution (up to 20 kHz [24]) in larger measurement volumes (on the order of $10 \times 10 \times 10$ cm [20]) have been reported for the tomography approach because these techniques do not require the physical scanning of the illumination laser sheet. A comprehensive comparison of all the techniques in both categories, though highly valuable in the authors' opinion, is beyond the scope of the current paper. For more in-depth discussions about the capabilities and limitations of individual techniques, the readers are referred to these following references: sheet-scanning PLIF [25], tomographic PIV [23], tomographic chemiluminescence [26], sheet-scanning Mie scattering [14], and tomographic Mie scattering [18].

Within the above context, this work reports 3D flame measurements at a repetition rate of 5 kHz using tomographic chemiluminescence. The measured properties include the 3D topography of the flame front (defined as regions with strongest chemiluminescence emissions) and 3D flame curvature. The measurements were performed on a large Bunsen flame [27], a representative “envelope flame” (in which all the reactants must pass through the flame [1]) on which extensive measurements have been performed [27-30]. Measurements on this flame have been performed to characterize its velocity fields using low-and high-speed PIV [30], flame front topography using PLIF [27], and the subsequent inference of other properties such as stretch rate, flame surface density, and burning velocities [27-29]. However, due to the planar nature of the PLIF and PIV, only 2D data are available on this flame. It is the goal of this work to provide the first piece of 3D data of this representative flame. The data reported in this work included 3D flame topography and 3D flame curvature measured with a repetition rate of 5 kHz, an exposure time of 0.2 ms, and a nominal spatial resolution of 0.55 mm in all three spatial directions. Another unique feature of the results obtained in this work is that measurements were performed in a volume of ~ 70 mm (width) $\times 70$ mm (depth) $\times 105$ mm (height), sufficiently large to capture the entire flame in each tomographic measurement.

2. Experimental Arrangement

Fig. 1 illustrates the experimental setup schematically from the top view and the coordinate system used in this work. The setup used a total of six complimentary metal-oxide semiconductor (CMOS) cameras to record tomographic chemiluminescence (TC) from a Bunsen flame. The burner was designed to produce a 2D Bunsen flame (an envelope flame [1]). The specifics of this burner (e.g., the flow passage structures and turbulence generating grids) have been detailed previously [27-29], and only a brief summary is provided here. The burner consisted of three parts (part I, II, and III as shown). The central part (part II), with a width (W) of 25.4 mm and a length (L) of 50.8 mm, generated the target flame. The central rectangular tube was filled about half-way with stainless-steel beads, and a turbulence-generating grid was installed just upstream of a hexagonal matrix to wrinkle the target flame. The two parts on the side (parts I and III) generated the pilot flames to stabilize the target flame. Fed with premixed methane and air at an equivalence ratio of about 1 and a bulk velocity of 5 m/s, the burner stabilizes a flame about 100 mm in height. Hot wire anemometry was performed in the cold flow to characterize the turbulence. At a height 4 mm above the burner exit, the turbulent Reynolds number based on the integral scale was measured to be about 2,400 in the central region of the burner. The exit plane of the burner was defined as the x - y plane, with the origin defined as the center point of the exit plane, the x axis the central line along the width direction, and the y axis the central line along the length direction as shown. The z axis was defined along the flow direction (i.e., the height direction).

The TC diagnostics involved cameras (2 Photron SA-4, 2 Photron SA-5, and 2 Photron SA-Z cameras) arranged as shown in Fig. 1. All cameras were aligned in the x - y plane and their orientations were specified by θ , defined as the angle formed by the optical axis of a camera relative to the positive x direction as shown. The focal length and f -number of the lenses used on camera 1 through 6 were, respectively, 60 mm and 2.8, 35 mm and 1.4, 24 mm and 2.0; 50 mm and 1.2, 58 mm and 1.2, and 35 mm and 1.4. All these chemiluminescence cameras were used without optical filters for the consideration of signal level and temporal resolution. Thus, the chemiluminescence measurements did not discriminate CH chemiluminescence from that of other excited state species (though the signal was primarily due to CH* in the visible range, as the cameras did not have sensitivity in the UV range and the lenses were not UV transparent). To

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4 minimize the effects of emissions contributed from other species such as CO^* , this work
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6 extracted the inner flame edges from the 3D reconstructions to separate the chemiluminescence
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8 emissions caused by CH^* from those caused by combustion products. The operation of the
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10 cameras was synchronized using control electronics, and the camera control and image
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12 acquisition were centralized on a computer as shown. All six cameras were operated at a frame
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14 rate of 5 kHz and an exposure time of 0.2 ms, equaling the reciprocal of the framing rate.

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16 The next step in the experiments was to perform a view registration to determine the
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18 orientation and position of all the cameras [31, 32]. In the view registration process, a calibration
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20 target with known dimensions and patterns was placed at the location where the target flame was
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22 expected. Images of the calibration target were acquired by all the cameras, with a set of
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24 examples shown in Fig. 2. This work used a calibration plate with square patterns as shown, and
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26 the size of each grid is 2.5 mm (best seen on Fig. 2d, where the camera was aligned almost
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28 perpendicular to the calibration plate). As shown in Fig. 2, due to the different positions and
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30 orientations of each camera, each field of view showed a different degree of distortion and
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32 magnification. With the lens parameter known for each camera, a view registration program was
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34 applied to analyze such distortion and magnification and determine the location and orientation
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36 of each camera. The orientation of camera 1 through 6 was determined to be $\theta = 305.7^\circ, 50.1^\circ,$
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38 $90.4^\circ, 134.5^\circ, 227.1^\circ,$ and 269.7° , respectively, with an accuracy estimated to be within $\pm 0.5^\circ$
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40 [32].

41 Fig. 3 shows a set of example images (termed *projections*) captured simultaneously by
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43 cameras 1 through 6. As mentioned before, these projections were captured with a frame rate of
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45 5 kHz and an exposure time of 0.2 ms. All projections had a pixel resolution of 900×600 pixels,
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47 and each pixel corresponded to a physical dimension of $\sim 0.12 \times 0.12$ mm. Before leaving this
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49 section, it is worthwhile to comment on the factors considered in our choice of the temporal and
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51 spatial resolution of these measurements. The cameras used in this work can all operate at a
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53 maximum *frame size* of 1024×1024 pixels up to a certain frame rate (3.6 kHz for the SA-4, 7
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55 kHz for the SA-5, and 20 kHz for the SA-Z), and their frame size began to decrease when the
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57 cameras are operated at higher frame rates. Therefore, these measurements were designed
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59 primarily by balancing the tradeoff between the frame rate and the frame size achievable by the
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61 cameras. A secondary consideration involved obtaining an optimal match between the shape of
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the target flame (which had a height larger than width and length) with the imaging area on the camera chip.

3. 3D Tomographic Reconstruction

The projections shown in Fig. 3 were used as inputs to a tomographic reconstruction algorithm to obtain the 3D distributions of the chemiluminescence, which were then used to infer the flame-front locations and calculate other flame properties such as curvature. Performing the reconstruction frame-by-frame resulted in a series of 3D measurements to resolve the temporal dynamics of the flame at 5 kHz. The tomographic algorithm has been detailed and validated in our previous publications, and only a brief summary is provided here to facilitate the discussion. After discretizing the measurement volume into a series of Cartesian voxels, the projection measured by a camera (denoted as P) is related to the desired distribution of the chemiluminescence emissions (denoted as F) via the point spread function (PSF) as shown below [33]:

$$P(x_p, y_p) = \sum_{x_i=1}^I \sum_{y_j=1}^J \sum_{z_k=1}^K F(x_i, y_j, z_k) \cdot PSF(x_i, y_j, z_k; x_p, y_p) \quad (1)$$

where $P(x_p, y_p)$ represents the value of the projection at a pixel located at (x_p, y_p) ; x_i, y_j , and z_k the indices of the voxels in the x, y , and z directions, respectively; I, J , and K the total number of voxels in each direction; PSF the point spread function defined as the projection formed at (x_p, y_p) by a point source located at (x_i, y_j, z_k) with unity intensity. The PSF does not depend on F , but it depends on the lens used in the imaging system and the location and orientation of the imaging system. Physically, Eq. (1) states that the projection of F on pixel (x_p, y_p) is a weighted summation of signals contributed from all voxels on this pixel, and the weighting factor is the PSF . In our algorithm, the PSF was calculated by a combination of geometrical ray-tracing [34, 35] and Monte Carlo simulation [36, 37] using the specification of the lenses, and the orientation and location of the cameras as inputs. After the PSF for each camera was obtained, Eq. (1) was solved for F using the measured P s as inputs. The algorithm we used to solve Eq. (1) was a modified variation of the Algebraic Reconstruction Technique (ART) [21, 22, 35] and a simulated annealing algorithm [38]. This tomographic approach has been validated via numerical

simulations [35, 39] and controlled experiments in both non-reacting [19] and reacting flows [35, 39].

Here, the measurement volume had a dimension of 70 (x direction) $\times$ 70 (y direction) $\times$ 105 (z direction) mm, and it was discretized into $128 \times 128 \times 192$ voxels (i.e., $I = J = 128$ and $K = 192$), resulting in a total of 3,145,728 voxels. Under this discretization, the voxels were cubical with a dimension of 0.55 mm, representing the spatial resolution of the 3D measurements in all three directions. The choice of such a voxel dimension was due to two considerations. First, the voxel dimension was designed to be smaller than the spatial blurring during the 0.2 ms exposure time of the measurements to obtain the finest spatial resolution possible. An independent nanosecond CH PLIF measurement was performed on this flame, which showed that the maximum movement of the flame front was approximately 1 mm over 0.2 ms. With the magnification used in the projection measurements, 1 mm corresponded to approximately 9 pixels. Therefore, the 3D measurements reported here were blurred by up to 9 pixels (or equivalently 1 mm). Hence, a voxel dimension of 0.55 mm was smaller than such blurring in most of the regions. Measurements with a shorter integration time (using 6 SA-Z cameras or 6 intensified cameras, for example) could reduce the blurring and warrant the use of finer voxels. Second, the choice of the voxel dimension also needs to take into consideration the computational resources and the number of projections, both of which are elucidated by Eq. (1). The length of P was $\sim 3.2 \times 10^6$ (600×900 pixels/camera $\times$ 6 cameras), and the length of F $\sim 3.1 \times 10^6$ ($128 \times 128 \times 192$ voxels). As a result, the scale of the PSF matrix was on the order of 3.2×10^6 rows and 3.1×10^6 columns, and the total number of elements in the PSF was therefore on the order of 10^{13} . Even though the PSF is sparse [18, 19] and various techniques have been implemented to exploit such sparsity and reduce the memory requirement for its storage [21, 40], the memory requirement here was still on the order of 100 GB (and the reconstructions in this work were performed on two customized workstations, one with 256 GB of RAM and the other 512 GB). Furthermore, when the length of F (i.e., the number of unknowns) exceeds that of P (the number of equations), Eq. (1) becomes underdetermined. Several possible approaches could help to overcome these issues and enable finer voxels, including the use of *a priori* information in the reconstructions via regularization [21, 41, 42], the use of more cameras to provide more projections, or the application of the same cameras to target a smaller measurement volume.

Using the reconstruction scheme discussed above, Fig. 4 show a set of sample results of the 3D reconstructions obtained. Fig. 4a shows the 3D reconstruction obtained using the projections in Fig. 3 as inputs. Fig. 4a was formed based on an iso-surface of the flame determined by a thresholding method. The thresholding method determined the iso-surface in three steps. First, the 3D reconstruction was performed using projections. Second, a background signal level was determined from the projections. Third, a threshold value was set, and the 3D reconstruction obtained in step 1 was divided into two zones based on the threshold value: a zone with chemiluminescence signal higher than the threshold value (considered to be the flame zone) and another zone with chemiluminescence signal lower than the threshold value (considered to be the *no-flame* zone). This step was especially important because some chemiluminescence was not from the flame surface.

To present a closer examination of the reconstructed flame topography and the performance of the above thresholding method, Fig. 4b and c show the cross-sectional view of the 3D reconstruction at two different y locations overlapped with the flame front extracted by the thresholding method. Fig. 4b shows the 32nd layer at $y = 17.5$ mm and Fig. 4c shows the 96th layer at $y = 52.5$ mm, both overlapped with the corresponding flame fronts extracted, illustrating that closeness between the flame fronts extracted and the contour of the 3D reconstruction. The flame fronts are used for the determination of the flame location and curvature, as elaborated in Sections 3 and 4. Analysis of the systematic error and verification of TC algorithm has been thoroughly discussed in our previous work, and readers can refer to ref. [21].

To further illustrate the accuracy and the temporal resolution of the measurements, Fig. 5 shows five consecutive 3D measurements with comparison against the measured projections. Fig. 5a shows five consecutive frames of projections measured by camera 3 (at $\theta = 90.4^\circ$). The changes in the flame features were unappreciable by visual examination across these five images. As a comparison to the measured projections shown in Fig. 5a, Fig. 5b shows the corresponding view from the 3D reconstructions. Fig. 5b shows the cross-sectional view of the 3D reconstruction at the central plane (i.e. the 64th x - z layer at $y = 35$ mm) from the same view angle (i.e., $\theta = 90.4^\circ$) as the projections in Fig. 5a. As can be seen, both Fig. 5b and Fig. 5a exhibited the expected overall flame topography. However, the projections shown in Fig. 5a are line-of-sight measurements, and the results in Fig. 5b are a 2D cross-section of a 3D reconstruction. As a

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4 result, the flame topography features in Fig. 5b are generally sharper and more detailed than
5 those in Fig. 5a. As an example, Fig. 5b shows a small but clear feature with two dark rings at
6 the top of the flame highlighted in the oval, which cannot be observed in Fig. 5a due to the line-
7 of-sight integrated nature. These features were caused by pockets of reactants and have been
8 observed in independent measurements via CH PLIF. However, the 3D measurements performed
9 in this work elucidate such features more clearly than planar measurements. To provide a
10 validation to the 3D reconstructions, Fig. 5c shows projections computed via a ray-tracing
11 algorithm [36] based on the 3D reconstructions. These projections were computed along the
12 same orientation as camera 3, and therefore are directly comparable to those shown in Fig. 5a.
13 As seen, the comparison shows that measured projections in Fig. 5a agree with the computed
14 projections in Fig. 5c both qualitatively and quantitatively, supporting the validity of the 3D
15 measurements. Of course, such agreement is necessary but not sufficient to verify the correctness
16 of the tomography inversion.

30 4. 3D Flame Location Results

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33 After the above discussion of the 3D reconstruction, this section discusses the statistics
34 that can be derived from such reconstructions. Processing the 3D reconstruction frame by frame,
35 as illustrated in Section 3, essentially provides 4D measurements resolving the flame in all three
36 spatial directions and also in time, enabling statistics both in the spatial and temporal domains.
37 This section discusses the statistics of flame locations and the corresponding PDF (probability
38 density function [43]).

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41 Fig. 6 shows the statistics of the flame front at three locations derived from a series of 3D
42 reconstructions. The three locations were shown in Fig. 6a through Fig. 6c, respectively, and they
43 corresponded to the 32nd, 64th, and 96th layers in the reconstructions (i.e., at a location of $y = 17.5$,
44 35 and 52.5 mm). These three locations were chosen to represent the outer edge, central plane,
45 and inner edge of the flame. The statistics shown in each panel were obtained from 40
46 reconstructions chosen at random times out of a total of 1000 reconstructions. Each data point in
47 Fig. 6 represents the location of the flame front at a particular time obtained from the 3D
48 reconstruction. Plotting the statistics obtained from 40 reconstructions (instead of all 1000
49 reconstructions) improves the clarity of the figure while achieving reasonable convergence. The

regions with denser data points represent regions with higher possibilities for the flame front to occur. As can be seen, the results here reveal the overall shape of the flame [27]. However, the 4D nature of the data obtained in this work enabled the comparison of the statistics at different locations, and the measurements at these locations were obtained simultaneously. Comparing the results shown in Fig. 6 at three locations suggests similar overall trends but with some variations. At lower flame height (i.e., $z < 20$ mm), the flame fronts were relatively concentrated in two narrow branches at all three locations. The flame front became more corrugated with increasing height, and the two branches merged near $40 \text{ mm} < z < 60 \text{ mm}$. However, differences were also observed among these three locations. For example, the overall height of the flame gradually increased from the outer edge to the inner edge. Fig. 6a shows an overall flame height of ~ 60 mm at the outer edge, Fig. 6b an overall height of ~ 65 mm, and Fig. 6c an overall height near 80 mm. Such difference may be caused by the asymmetric flow conditions through the burner, and the results shown here provide an illustration for the utility of multi-dimensional diagnostics in practical combustion systems.

Based on data shown in Fig. 6, the PDF of the flame fronts can be obtained as shown in Fig. 7. Again, due to the availability of information in all three spatial directions, Fig. 7 shows the PDFs of flame fronts in three layers. Fig. 7a shows the PDF of the flame front at the 32nd and 64th layers at a height of $z = 25$ mm, and Fig. 7b that at the 64th and 96th layer at the same height. These statistics were performed using data in a range of $z \pm \Delta z$ with $\Delta z = 0.55$ mm (i.e., the thickness of one layer in the tomographic reconstruction). The intention here was to compare the PDF at the edges against that at the central plane. These results further elucidate the observations made above from Fig. 6, that the flame fronts had similar overall trend at different locations but with some variations.

In summary, in this section statistics of flame location derived from the 3D reconstructions were discussed. The results illustrate that the multi-dimensional nature of the data can enable statistics in both spatial and temporal domains. Admittedly, similar statistics can be obtained using 2D or planar diagnostics. For example, statistics at various flame locations can be obtained by repeating planar measurements at multiple locations (though such measurements are not simultaneous). The next section reports the measurement of 3D flame curvature, a quantity that is difficult to obtain with planar measurements.

5. 3D Curvature Results

Curvature of the flame front is an important parameter that quantifies corrugation of turbulent flame surfaces, and measurements of curvature are needed to understand the interaction of eddies [1]. However, measurement of curvature is difficult, and past efforts have been primarily limited to measurement of 2D curvature or the estimation of 3D curvature at a point of along a line. Measurement of 2D curvature is relatively straightforward with planar imaging techniques [44], for example, employing a crossed-plane configuration [45]; the 2D curvatures measured in these planes then can be used to estimate the 3D curvature along the intersection. This section reports the measurements of full-field 3D curvature, based on the 3D reconstructions obtained in this work.

We first define 2D and 3D curvature as used here. 2D curvature used is defined following ref. [46]. For example, the 2D curvature at point on the flame front in an x - z plane is defined as:

$$K_{xz} = \frac{\dot{z}\ddot{x} - \dot{x}\ddot{z}}{(\dot{x}^2 + \dot{z}^2)^{\frac{3}{2}}} \quad (2)$$

where x and z represent the coordinates of the target point along the flame front, $\dot{x}$ and $\dot{z}$ the first-order derivatives, and $\ddot{x}$ and $\ddot{z}$ the second-order derivatives (both defined along the flame front). The definition in Eq. (2) was discretized using a three-point finite-differencing scheme [46] into the following format:

$$K_{xz} = \frac{8[x_{i-1}(z_{i+1} - z_i) + x_i(z_{i-1} - z_{i+1}) + x_{i+1}(z_i - z_{i-1})]}{[(x_{i+1} - x_{i-1})^2 + (z_{i+1} - z_{i-1})^2]^{\frac{3}{2}}} \quad (3)$$

where the index i represents the i^{th} point along the flame front. The 2D curvature in other planes was defined and discretized in a similar way as shown in Eqs. (2) and (3). This work primarily examined the 2D curvature in the x - z plane (i.e., K_{xz} , also referred to as the vertical curvature) and in the x - y plane (i.e., K_{xy} , also referred to as the horizontal curvature). In contrast to the 2D curvature, the 3D curvature can be defined in multiple ways, and three of the definitions are examined: the principal curvatures, the mean curvature, and the Gaussian curvature. The principal curvatures at a given point on a 3D surface are defined in the following steps [47]. First,

the normal vector of the 3D surface at the given point is determined. Second, normal planes, defined as planes crossing the normal vector, can be determined. Each normal plane intersects the 3D surface to form a 2D curve, and a 2D curvature at the given point can be determined along the curve following Eq. (2). Finally, the principal curvatures are defined as the most positive and most negative 2D curvatures (i.e. K_{max} and K_{min}) found in step two among all normal planes. Here, the algorithm described in [48] was used to determine the principal curvatures from the 3D reconstructions. Once the principal curvatures are determined, the mean and Gaussian curvatures then are defined as the average and the product of the principal curvatures [45, 48] (i.e., $K_{mean}=1/2(K_{max}+K_{min})$ and $K_{gaussian} = K_{max}K_{min}$), respectively. Physically, the principal curvatures quantify the two steepest curvatures (both in the positive and negative directions) at a point, and the Gaussian curvature provides information on the overall 3D structure of the surface at the point. A Gaussian curvature greater than 0 indicates an elliptic (i.e., sphere-like) surface at the given point, smaller than 0 a hyperbolic (i.e., saddle-like) surface, and equal to 0 a parabolic (i.e., cylinder-like) or planar surface [45]. In the last case, when the Gaussian curvature is equal to 0, the principal curvatures are further examined to differentiate a parabolic surface from a planar surface: it is a parabolic surface when only one of the principal curvatures is equal to 0 and it is a planar surface when both are.

Based on the above definitions, 2D and 3D curvatures were calculated from the 3D reconstructions. Fig. 8a and Fig. 8b show the distribution of vertical (K_{xz}) and horizontal (K_{xy}) curvatures of the entire flame at a given moment. Here, a positive curvature value represents a convex point and a negative curvature a concave one (convex and concave were defined relative to the center y - z plane). First of all, Fig. 8 shows the overall increasing trend of curvature, both in the vertical and horizontal directions, in the streamwise direction as expected (as the flame became more corrugated at increasing height). However, differences can also be observed between the curvatures along different directions. For instance, Fig. 8 shows that the curvature in the vertical direction (K_{xz}) exhibited an overall magnitude larger than that in the horizontal direction (K_{xy}) (a more quantitative comparison is shown in Fig. 11 when the PDFs of K_{xz} and K_{xy} are compared). Such difference illustrates both that the flame front is more corrugated in the streamwise direction than the spanwise direction and also the limitations of 2D flame curvature measurements. Such different patterns again reflect the limitation of 2D curvature, because 2D

curvature only provides partial information of the 3D flame surface in a given plane and the information can show a different pattern in different directions.

To better illustrate the comparison between the 2D curvatures shown in Fig. 8, Fig. 9 shows the values of K_{xz} and K_{xy} at the central x - z plane of the flame (i.e., the 64th layer in the reconstruction). The horizontal axis (L) in Fig. 9 was defined as the coordinate along the flame front, and $L = 0$ was defined at the exit of the burner. To aid the visual examination of the data, results for the left branch and right branch of the flame are plotted separately. Fig. 9a shows the results along the left branch of the flame, and Fig. 9b shows the right branch. Fig. 9 elucidates the observations derived from Fig. 8 in a more quantitative manner. These results again show that the curvatures increased in the streamwise direction as the flame became more corrugated. Also, as observed in Fig. 8, the vertical curvature (K_{xz}) was generally larger than the horizontal curvature (K_{xy}) for both branches: for the left branch of the flame, the deviation of the magnitude of K_{xz} was calculated to be 0.0602 mm^{-1} based on the data shown in Fig. 9a, and that of K_{xy} was calculated to be 0.0544 mm^{-1} ; for the right branch of the flame, the deviation of the magnitude of K_{xz} was calculated to be 0.0281 mm^{-1} based on the data shown in Fig. 9b, and that of K_{xy} was calculated to be 0.0221 mm^{-1} . Furthermore, Fig. 9 illustrates the pattern of flame surface in terms of its alternating convex (indicated by a positive curvature) and concave shape (indicated by a negative curvature) in correspondence to the structures created by the shear layer and the eddies generated by the turbulence grid.

Below, comparison between 2D and 3D curvatures is made. First, Fig. 10 compares the principal curvatures against the vertical and horizontal curvatures shown in Fig. 9. Clearly, the principal curvatures encompass the vertical and horizontal curvatures as expected, because the principal curvatures are defined in the steepest directions while the vertical and horizontal curvatures are essentially curvatures along two random directions. Once the 2D and principal curvatures were obtained, their PDFs can be calculated and compared, as shown in Fig. 11. Here, a mean curvature (K_{mean}) was defined as the average of the two principal curvatures following refs. [45, 49]. The PDFs shown here were formed by curvatures measured on one flame (i.e., one frame of 3D flame reconstruction). More specifically, the 3D flame reconstruction at one chosen moment was analyzed in the following three steps to obtain these PDFs. First, the flame front was extracted by the 3D reconstruction of the flame front at a chosen moment. As mentioned

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4 before, the flame reconstruction was performed on a total of $128 \times 128 \times 128$ voxels (about
5 2×10^6). At this particular moment, the flame surface was identified on 25,344 voxels. Second,
6 the 2D, principal, and mean curvatures were then calculated at all these points on the flame
7 surface identified in the first step, resulting in a total of 25,344 measurements for each type of
8 curvature. Third, the PDFs of the vertical, horizontal, and mean curvatures were then extracted
9 from the results in step 2 and plotted in Fig. 11.
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16 Several observations can be made based on the results shown in Fig. 11. First, the PDFs
17 of all three curvatures are approximately symmetric with a zero mean, illustrating the stochastic
18 nature of the turbulent flame. Second, more than 99.9% of all the data points were bounded in
19 $\pm 0.5 \text{ mm}^{-1}$ for this flame, and the occurrence within $\pm 0.3 \text{ mm}^{-1}$ accounted for more than 99.4% of
20 all the data points (which was also the reason that the range of the color scale in Fig. 8 was
21 chosen to be $\pm 0.3 \text{ mm}^{-1}$). Third, the PDFs clearly show that the curvature in the vertical direction
22 (K_{xz}) exhibited an overall magnitude larger than that in the horizontal direction (K_{xy}), an
23 observation that agrees with the visual examination of the data shown in Fig. 8. Lastly, due to the
24 way K_{mean} is defined (as the average of the principal curvatures), the PDF of K_{mean} appears to be
25 between those of K_{xz} and K_{xy} .
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35 Fig. 12 and Fig. 13 show the 3D curvature under the Gaussian definition. As discussed
36 above, the Gaussian curvature indicates if the surface at a given point is elliptic, parabolic, or
37 hyperbolic, which is an *intrinsic* measure of the 3D flame surface [48]. Note that other similar
38 quantities can be defined to serve this same purpose. For instance, a shape factor, defined as the
39 ratio of the principal curvature as discussed in ref. [45], provides the same physical meaning as
40 the Gaussian curvature. Fig. 12 shows the Gaussian curvature distribution on the flame front (the
41 same flame surface shown in Fig. 8). From Fig. 12, it can be seen that the Gaussian curvature of
42 most points at low flame heights (e.g., below $z = 20 \text{ mm}$) was close to zero, which suggests that
43 the flame shape was close to cylindrical or planar, as expected for a 2D laminar Bunsen flame.
44 Such information cannot be obtained from 2D measurements (e.g., from the measurements of K_{xz}
45 shown in Fig. 8a), and such information is useful in practice. For example, whether a surface is
46 cylindrical determines if how accurately a 3D curvature can be inferred from 2D imaging
47 measurements. Fig. 13 shows the PDF of the Gaussian curvature is between -1.0 and 1.0 mm^{-2} , a
48 range that encompassed more than 99.9% of the data points. Similar to the PDFs shown in Fig.
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10, the PDF of the Gaussian curvature exhibits approximate symmetry due to the highly turbulent nature of the flame. Note that the Gaussian curvature is defined as the product of two principal curvatures: the vertical axis in Fig. 13 is shown in logarithmic scale to accommodate the range of the data. As can be seen, the PDF shows a distinct peak at zero ($-0.05 \text{ mm}^{-2} < K_{\text{gaussian}} < 0.05 \text{ mm}^{-2}$) with the distribution of cylindrical and planar points marked, illustrating that a significant portion of the surface on the entire flame is cylindrical and planar. Based on the results shown in Fig. 12, the flame surface at low z was the primary contributor into this portion. For a cylindrical surface, the 3D curvature (or shape of the 3D surface) can be determined from a 2D imaging measurements if the 2D imaging plane is properly oriented relative to the surface. Fig. 12 also shows that a significant portion of the flame surface exhibited a non-zero Gaussian curvature and therefore was either elliptic or hyperbolic. For these points, the 3D shape of the flame front cannot be fully determined by 2D measurements. A number of frames were processed (e.g., as shown in Figure 5), and the PDFs obtained from different frames were stable.

Before concluding this section, we briefly discuss the uncertainty analysis and validation of the curvature measurements. As aforementioned, it is a significant challenge to measure instantaneous 3D curvature of turbulent flames, and currently there is no established technique to our knowledge. Therefore, it is difficult to perform uncertainty analysis and validation via a direct experimental approach (i.e., which may require the use of another technique that can measure 3D curvature). To obtain some preliminary results, we have performed simultaneous measurements of CH-based PLIF and TC in flames generated by this same burner under the same conditions. The goal was to obtain 3D flame topography and curvature using the TC technique described in this work and also a 2D PLIF image simultaneously. Then, a 2D cross-section of the flame was extracted from the 3D TC measurements across the same plane at which the PLIF was performed, so that the flame topography and curvature (2D curvature though as only 2D curvature could be calculated from the PLIF measurements). Detailed description of these comparison experiments is provided in a separated publication [50]. The results show that the curvatures extracted from the 2D PLIF image and the 3D TC measurements were in reasonable agreement, both in terms of the instantaneous measurements and the statistics. The average difference between the magnitude of the curvatures obtained from PLIF and 3D TC was 1.3%, and 84% of the data points agree within one standard deviation (std), 95% within two std, and 98% within 3 std. Note that the difference was defined based on the *magnitude* of the

curvature (i.e., the absolute values) such that the positive and negative errors did not cancel each other. These results should provide a reasonable estimation to the uncertainty of the curvature measurements reported here.

6. Summary and Conclusions

In summary, this work described instantaneous, kHz-rate 3D measurements in a turbulent flame generated by a 2D Bunsen burner. The measurements were performed using a combination of computed tomography and chemiluminescence imaging of the flame; here, six CMOS cameras operating at 5 kHz (with an exposure time of 0.2 ms) recorded flame chemiluminescence from a volume measuring ~ 70 mm (width) $\times$ 70 mm (depth) $\times$ 105 mm (height).

Based on projection measurements of chemiluminescence emissions, 3D flame structures were reconstructed to extract 3D flame properties, and this work focused on the location and curvature of the flame front. For flame location analysis, the PDF of flame topography was extracted from a series of 3D measurements, and the PDFs of the flame at different spatial locations were examined. For flame curvature analysis, 2D curvature along selected orientations and 3D curvature under various definitions were calculated based on the measured 3D flame topography. The PDFs of curvature in 2D and 3D were then extracted and compared, and these results quantified the flame surface in 3D. The Gaussian curvature divided the flame surface into three categories (cylindrical, elliptic, or hyperbolic), and the PDFs provided relative portion of each category on the entire flame surface. These results illustrate the limitations of 2D measurements and the potential of 3D diagnostics to fully resolve the spatiotemporal behavior of turbulent flames.

Finally, this work serves as a demonstration of the technique of tomographic chemiluminescence. Limitations of the approach lie in the number of spatial resolution elements (which is in turn limited by the total number of camera pixels) and minimum feasible exposure time. This minimum exposure time affects the spatial resolution, through the light collected by each camera pixel and through the averaging effect of the flame movement from one frame to the next. Advances in CMOS camera technology, however, will improve the sensitivity of these

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4 systems, enabling shorter exposure times and higher framing rates. Indeed, it is believed that
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6 resolutions below 0.25 mm are possible with current CMOS camera technology framing at 20
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8 kHz. Of course, higher bulk velocities may require even higher framing rates.
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Figures

Fig. 1. Schematic of the experimental setup.

Fig. 2. Panel (a) - (d): calibration images obtained by camera 1, 2, 4 and 6, respectively.

Fig. 3. Panel (a) - (f): a set of example projections measured by camera 1 through 6, respectively.

Fig. 4. Panel (a): An example 3D tomographic reconstruction of the target flame. Panel (b): 2D cross-sectional view of the 32nd layer with extracted flame front overlapped. Panel (c): 2D cross-sectional view of the 96th layer with the extracted flame front overlapped.

Fig. 5. Panel (a): from left to right, five consecutive frames of measured projections from camera 3. Panel (b): from left to right, the 2D cross-sectional view at the central location of the corresponding reconstructions. Panel (c): from left to right, simulated projections at the same orientation as camera 3 via ray-tracing based on the 3D reconstructions.

Fig. 6. Panel (a) – (c): Statistics of flame front at layer 32nd, 64th and 96th, respectively.

Fig. 7. Panel (a): PDFs of flame front locations at layer 32nd and 64th and $z = 25$ mm. Panel (b): PDFs of flame front locations at layer 64th and 96th and $z = 25$ mm.

Fig. 8. Instantaneous measurements of flame curvature in the vertical (Panel a) and horizontal (Panel b) directions.

Fig. 9. Comparisons of the vertical and horizontal curvatures at the 64th layer the left (Panel a) and right (Panel b) branch of the flame.

Fig. 10. Comparisons of the vertical, horizontal, and principal curvatures of at the 64th x-z layer for the left (Panel a) and right (Panel b) branch of the flame.

Fig. 11. Comparison of the PDFs of 3D curvature (K_{mean}) and 2D curvatures (K_{xz} and K_{xy}).

Fig. 12. Instantaneous measurements of the Gaussian curvature.

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Fig. 13. PDF of the Gaussian curvature.