A numerical study of the diffusive-thermal instability of opposed nonpremixed tubular flames

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Abstract

The diffusive-thermal (D-T) instability of opposed nonpremixed tubular flames near extinction is investigated using two-dimensional (2-D) direct numerical simulations together with the linear stability analysis. Two different initial conditions (IC), i.e. the perturbed IC and the C-shaped IC are adopted to elucidate the effects of small and large amplitude disturbances on the formation of flame cells, similar to conditions found in linear stability analysis and experiments, respectively. The characteristics of the D-T instability of tubular flames are identified by a critical Damköhler number, Da_C , at which the D-T instability first occurs and the corresponding number of flame cells for three different tubular flames with different flame radii. It is found that Da_C predicted through linear stability analysis shows good agreement with that obtained from the 2-D simulations performed with two different ICs. The flame cell number, N_{cell} , from the 2-D simulations with the perturbed IC is also found to be equal to an integer close to the maximum wavenumber, k_{max} , obtained from the linear stability analysis. However, N_{cell} from the 2-D simulations with the C-shaped IC is smaller than k_{max} and N_{cell} found from the simulations with the perturbed IC. This is primarily because the strong reaction at the edges of the horseshoe-shaped cellular flame developed from the C-shaped IC is more likely to produce larger flame cells and reduce N_{cell} . It is also found that for cases with the C-shaped IC, once the cellular instability occurs, the number of flame cells remains constant until global extinction occurs by incomplete reaction

manifested by small Da . It is also verified through the displacement speed, S_d , analysis that the two edges of the horseshoe-shaped cellular flame are stationary and therefore do not merge due to the diffusion-reaction balance at the edges. Moreover, large negative S_d is observed at the local extinction points while small positive or negative S_d features in the movement of flame cells as they adjust their location and size towards steady state.

Keywords: Diffusive-thermal instability, nonpremixed tubular flame, hydrogen, linear stability analysis, displacement speed

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1. Introduction

In most practical engines, combustion instabilities need to be avoided because they may deteriorate the integrity of the mechanical systems in the engines. Occasionally, however, the cellular flame instability may play a positive role by enhancing the operation range of hydrogen-fueled engines by extending the extinction limit of hydrogen/air flames [1–3]. For meso-scale combustors, moreover, the flame instability can also be utilized to achieve stable combustion by increasing heat release per unit volume and consequently, compensating for increased heat losses associated with the high surface-to-volume ratios of such small-scale devices [4]. As such, the cellular flame instability has been extensively investigated in the combustion community not only because of its practical importance, but because of its most dynamical and beautiful features among combustion dynamics [1–6].

There exist three different modes of cellular flame instability, i.e. hydrodynamic instability, buoyancy-driven instability, and diffusive-thermal (D-T) instability [3]. The hydrodynamic instability is caused by a finite density drop across a premixed flame such that it is absolutely unstable to perturbations at all wavelengths. The buoyancy-driven instability occurs under the condition of negative density stratification in the direction of gravity and, as such, an upwardly-propagating premixed flame is buoyantly unstable. The D-T instability occurs due to the imbalance between diffusion of heat and mass at a flame front such

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that a flame can be stable or unstable depending on the mixture Lewis number upstream of the flame front [2, 3, 7].

Although these cellular flame instabilities are intrinsic characteristics of premixed flames, the D-T instability has also been observed in many nonpremixed flames with their effective Lewis number being less than unity near the extinction limit [1, 7–14]. The structure of nonpremixed flames becomes similar to that of premixed flames near extinction and, as such, they are considered to be in the “premixed flame regime” according to Liñán’s asymptotic study of a nonpremixed counterflow flame [3, 8, 15]. From this point of view, it is a natural extension that nonpremixed flames near extinction would also exhibit cellular flame structures like premixed flames when the effects of the preferential diffusion and/or curvature become significant [8].

Among the many different flow configurations, an opposed nonpremixed tubular flow is an ideal one to simultaneously investigate the effects of stretch rate and curvature on the characteristics of the D-T instability [1]. In this configuration the stretch rate and curvature are independently specified and hence their effects on the cellular flame instability can systematically be investigated. In this context, the characteristics of opposed nonpremixed tubular flames have extensively been investigated by Pitz and coworkers [1, 13, 14, 16–18]. As mentioned above the D-T instability of nonpremixed tubular flame also occurs near the extinction limit when the Lewis number of the fuel is less than unity [1, 13, 14]. Near extinction, small perturbations can induce alternating locally-weak and strong reaction regions of the tubular flame due to the focusing and defocusing effects of mass and heat diffusion, leading to the formation of flame cells. The resultant temperature at the edge of a flame cell is higher than it is in the central part due to the strong focusing of the fuel mass flux at the edge. By the same mechanism flame cells can survive beyond the one-dimensional (1-D) flame extinction limit [10, 11].

Although numerous experimental and theoretical/numerical studies of cellular flame instability in tubular and counterflow flames have been conducted [2, 3, 9–11, 16, 19–22], a numerical study of the D-T instability in opposed nonpremixed tubular flames has not been performed. Moreover, discrepancies still remain between experimental/numerical ob-

servations of cellular flame instability and theoretical prediction via linear stability analysis [11, 19] even though linear stability analysis predicts numerical simulation results with reasonable accuracy [22].

In the present study, therefore, the D-T instability of opposed nonpremixed tubular flames and the dynamics of flame cell formation are investigated using 2-D direct numerical simulations together with conventional linear stability analysis. By comparing previous experimental results with numerical/theoretical predictions in the present study, the implication of cellular flame instability found in the numerical simulations on the experimental results is also elucidated. In the present study, highly-diluted hydrogen is considered as the fuel, for which its Lewis number is much less than unity, and as such, the cellular flame characteristics induced by the D-T instability can be investigated.

2. Problem formulation

In the experiments by Pitz and coworkers [13, 14, 17, 18] the opposed tubular flow configuration is designed such that fuel and oxidizer issue from a porous inner nozzle and contoured outer nozzle, respectively, and a tubular flame develops between the two nozzles. Readers are referred to [13, 14, 17, 18] for details of the experimental setup. An idealization of this configuration adopted in the present numerical study is such that fuel and oxidizer are assumed to issue from two infinitely-long concentric cylinders, forming a tubular flame in between. To further simplify the problem, only variations in the plane of symmetry are considered as shown in Fig. 1. In particular, there is no axial variation in the solutions and the characteristics of nonpremixed flame cell formation induced by the D-T instability in the $r - \theta$ plane are elucidated. Therefore, these effects are investigated without performing expensive 3-D simulations.

2.1. 1-D axisymmetric tubular flames

Prior to performing 2-D simulations of nonpremixed tubular flames, the extinction characteristics of 1-D steady opposed nonpremixed tubular flames are investigated and compared

with previous experimental and numerical results. For this purpose, a 1-D axisymmetric model of opposed tubular flames, similar to the models for opposed counterflow flames [2, 11, 19, 23], is adopted:

$$\tilde{\rho}\tilde{u}_r\frac{d}{d\tilde{r}}\begin{pmatrix}\tilde{c}_p\tilde{T} \\ \tilde{Y}_F \\ \tilde{Y}_O\end{pmatrix} = \frac{1}{\tilde{r}}\frac{d}{d\tilde{r}}\left(\tilde{r}\frac{d}{d\tilde{r}}\right)\begin{pmatrix}\tilde{\lambda}\tilde{T} \\ \tilde{\rho}\tilde{D}_F\tilde{Y}_F \\ \tilde{\rho}\tilde{D}_O\tilde{Y}_O\end{pmatrix} + \tilde{B}\tilde{Y}_F\tilde{Y}_Oe^{-\tilde{E}/\tilde{R}\tilde{T}}\begin{pmatrix}\tilde{Q} \\ -\alpha_F \\ -\alpha_O\end{pmatrix}, \quad (1)$$

where $\tilde{T}$, $\tilde{Y}_F$, and $\tilde{Y}_O$ are the dimensional temperature, fuel mass fraction, and oxidizer mass fraction, respectively, with the density, $\tilde{\rho}$, radial velocity, $\tilde{u}_r$, specific heat, $\tilde{c}_p$, thermal conductivity, $\tilde{\lambda}$, fuel diffusivity, $\tilde{D}_F$, oxidizer diffusivity, $\tilde{D}_O$, pre-exponential factor, $\tilde{B}$, activation energy, $\tilde{E}$, universal gas constant, $\tilde{R}$, heat release rate, $\tilde{Q}$, stoichiometric coefficients, α_F and α_O , and independent variable, $\tilde{r}$, denoting the radial direction.

In the present study, the D-T instability is numerically investigated by adopting the constant density model not only because it is common practice in many previous studies [2, 11, 19, 22, 23], but also because the density variation or thermal expansion has a minor effect on the D-T instability of nonpremixed flames compared to that of premixed flames as found in [12]. Assuming that there is no density variation in the flow, the radial velocity $\tilde{u}_r$ is analytically given by [24]:

$$\tilde{u}_r(\tilde{r}) = \begin{cases} \tilde{u}_{r,1}\frac{\tilde{r}_1}{\tilde{r}}\sin\left(\frac{\tilde{u}_{r,2}\tilde{r}_2}{\tilde{u}_{r,1}\tilde{r}_1}\sqrt{\frac{Q}{4}}\left(\frac{\tilde{r}}{\tilde{r}_2}\right)^2 + \frac{\pi}{2} - \frac{\tilde{u}_{r,2}\tilde{r}_1}{\tilde{u}_{r,1}\tilde{r}_2}\sqrt{\frac{Q}{4}}\right) & : \tilde{r}_1 \leq \tilde{r} < \tilde{r}_s, \\ \tilde{u}_{r,2}\frac{\tilde{r}_2}{\tilde{r}}\sin\left(\sqrt{\frac{Q}{4}}\left(\frac{\tilde{r}}{\tilde{r}_2}\right)^2 + \frac{\pi}{2} - \sqrt{\frac{Q}{4}}\right) & : \tilde{r}_s \leq \tilde{r} \leq \tilde{r}_2, \end{cases} \quad (2)$$

where $\tilde{r}_s$ and Q denotes the location of the stagnation plane and pressure eigenvalue, respectively:

$$\tilde{r}_s = \tilde{r}_2\left[1 - \frac{(\tilde{r}_2/\tilde{r}_1 - \tilde{r}_1/\tilde{r}_2)}{(\tilde{r}_2/\tilde{r}_1 - \tilde{u}_{r,1}/\tilde{u}_{r,2})}\right]^{1/2}, \quad (3)$$

$$\sqrt{Q} = \pi\left(\frac{\tilde{r}_2/\tilde{r}_1 - \tilde{u}_{r,1}/\tilde{u}_{r,2}}{\tilde{r}_2/\tilde{r}_1 - \tilde{r}_1/\tilde{r}_2}\right), \quad (4)$$

where the subscripts 1 and 2 denote the fuel and oxidizer inlets, respectively. Note that Wang et al. [24] originally derived Eqs. 2–4 by analytically solving the constant density model equations of the opposed tubular flame and found that the stretch rates derived from

the constant density model are appropriate enough to describe those from the corresponding full numerical simulations with detailed chemistry and transport.

The 1-D governing equations are subject to Dirichlet boundary conditions at the fuel and oxidizer inlets:

$$\begin{cases} \tilde{T} = \tilde{T}_1, \tilde{Y}_F = \tilde{Y}_{F,1}, \tilde{Y}_O = 0, \tilde{u}_r = \tilde{u}_{r,1}, & \text{at } \tilde{r} = \tilde{r}_1, \\ \tilde{T} = \tilde{T}_2, \tilde{Y}_F = 0, \tilde{Y}_O = \tilde{Y}_{O,2}, \tilde{u}_r = \tilde{u}_{r,2}, & \text{at } \tilde{r} = \tilde{r}_2. \end{cases} \quad (5)$$

By normalizing the dimensional variables with appropriate reference values, i.e. $r = \tilde{r}/\sqrt{\tilde{\lambda}/\tilde{\rho}\tilde{c}_p\tilde{\kappa}}$, $u_r = \tilde{u}_r/\sqrt{\tilde{\kappa}\tilde{\lambda}/\tilde{\rho}\tilde{c}_p}$, $T = \tilde{T}/\tilde{T}_{\text{ref}}$, $Y_F = \tilde{Y}_F/\tilde{Y}_{F,1}$, and $Y_O = \tilde{Y}_O/\tilde{Y}_{O,2}$, Eq. 1 can be written in non-dimensional form:

$$u_r \frac{d}{dr} \begin{pmatrix} T \\ Y_F \\ Y_O \end{pmatrix} = \frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) \begin{pmatrix} T \\ Y_F/Le_F \\ Y_O/Le_O \end{pmatrix} + Da Y_F Y_O e^{-T_a/T} \begin{pmatrix} q \\ -\alpha_F \tilde{Y}_{O,2} \\ -\alpha_O \tilde{Y}_{F,1} \end{pmatrix}, \quad (6)$$

where $Le_F (= \tilde{\lambda}/\tilde{\rho}\tilde{c}_p\tilde{D}_F)$ is the fuel Lewis number, $Le_O (= \tilde{\lambda}/\tilde{\rho}\tilde{c}_p\tilde{D}_O)$ is the oxidizer Lewis number, $Da (= \tilde{B}/\tilde{\rho}\tilde{\kappa})$ is the Damköhler number, $q (= \tilde{Y}_{F,1}\tilde{Y}_{O,2}\tilde{\dot{Q}}/\tilde{c}_p\tilde{T}_{\text{ref}})$ is the heat release rate, and $T_a (= \tilde{E}/\tilde{R}\tilde{T}_{\text{ref}})$ is the activation temperature. Note that $\tilde{\kappa}$ is the reference stretch rate and is inversely proportional to Da by definition. The corresponding non-dimensional boundary conditions at the two inlets are given by:

$$\begin{cases} T = T_1, Y_F = 1, Y_O = 0, u_r = u_{r,1}, & \text{at } r = r_1, \\ T = T_2, Y_F = 0, Y_O = 1, u_r = u_{r,2}, & \text{at } r = r_2. \end{cases} \quad (7)$$

To simulate cases similar to the experiments of opposed hydrogen/air nonpremixed tubular flames [13, 17], we specify $q = 1.2$, $T_a = 8$, $Le_F = 0.3$, $Le_O = 1.0$, $\alpha_F \tilde{Y}_{O,2} = 1.0$, $\alpha_O \tilde{Y}_{F,1} = 0.36$, $r_1 = 20$, $r_2 = 100$, and $T_1 = T_2 = 0.2$. Note that these values represent nonpremixed tubular flames formed by opposing nitrogen-diluted hydrogen and air flows [11, 23], rendering the maximum flame temperature, T_{max} , to be approximately unity near extinction. If $T_1 = T_2 = 0.2$ is assumed to be 300 K, $T_{\text{max}} \approx 1$ near extinction represents approximately 1500 K [11].

For a specific stagnation plane, only the ratio of $u_{r,1}$ to $u_{r,2}$ can be determined from Eq. 3. To specify each value of $u_{r,1}$ and $u_{r,2}$, another condition is required. For this purpose and for fair comparison between tubular flames with different stagnation planes, we adopt an identical stretch rate, κ , defined as $\kappa = -(u_{r,2}/r_2)\sqrt{Q}$ at the stagnation plane [16, 24]. Since the flame is located close to the stagnation plane for our choice of parameters, it is reasonable to adopt a constant κ at the stagnation plane as the condition for determining $u_{r,1}$ and $u_{r,2}$. For all of the 1-D and 2-D simulations, the same stretch rate, $\kappa = 0.03927$, is used, which is evaluated from $u_{r,1} = -u_{r,2} = 1.0$. Once the stagnation plane is specified, then $u_{r,1}$ and $u_{r,2}$ are uniquely determined from the stretch rate of 0.03927. Note that the identical stretch rate is used for all of the different tubular flames and hence, the effect of stretch rate on the flames is solely incorporated in the Da of the governing equations (see Eq. 6).

2.2. 1-D steady solutions

Figure 2 shows the so-called “C-curve” of $T_{\max}$ as functions of Da and the ratio of $u_{r,1}$ to $u_{r,2}$. The C-curves are obtained by solving Eq. 6 using the Newton-Raphson method with a simple continuation algorithm [23]. The upper branch of the C-curves represents the intensely-burning state and the lower branch is unstable and physically unrealistic. Note that the corresponding parameters, $(u_{r,1}, u_{r,2}, r_s)$, for the three different cases are $(1.0, -1.0, 44.72)$, $(1.7143, -0.8571, 56.06)$, and $(2.6667, -0.6667, 68.31)$, respectively. The extinction Damköhler number, Da_E , for the 1-D tubular flames with $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 are found to be 13950, 11977, and 10542, respectively.

Two points are noted from the figure. First, Da_E decreases with increasing $|u_{r,1}/u_{r,2}|$ and r_s . Second, for a given Da , $T_{\max}$ also increases with increasing $|u_{r,1}/u_{r,2}|$ and r_s . These results are consistent with those found in a previous 1-D numerical study of opposed tubular flames [16], i.e. “negatively-curved” nonpremixed tubular flames become weaker as the stretch rate is increased when the fuel Lewis number is less than unity. Note that a negatively-curved flame is formed in this configuration where fuel and oxidizer issue from the inner and outer nozzles, respectively, as in the present study. The negative curvature weakens the

preferential diffusion of fuel with small Le_F and, as such, the tubular flame becomes more vulnerable to stretch rate ($\sim 1/Da$) as r_s becomes smaller. More generally, curvature plays a critical role in the extinction of negatively-curved nonpremixed tubular flames when the fuel Lewis number is much less than unity and the flame radius becomes the order of the flame thickness [16].

To further quantify the effect of curvature on the extinction of steady tubular flames, the variation of Da_E and $T_{\max,E}$ as a function of r_s is shown in Fig. 3. $T_{\max,E}$ denotes the maximum flame temperature at the extinction point. It is readily observed from the figure that if the flame lies in the middle of the domain (i.e. $40 \lesssim r_s \lesssim 90$), Da_E decreases and $T_{\max,E}$ increases with increasing r_s as can be expected from Fig. 2 and the previous result [16].

On the contrary, as the tubular flame approaches the inner/outer cylinder ($r_s \lesssim 40$ or $r_s \gtrsim 90$), Da_E decreases/increases and $T_{\max,E}$ increases/decreases. Near the two boundaries, the steady solutions are directly affected by the Dirichlet boundary conditions; when the tubular flame lies very close to the inner cylinder, the fuel mass flux into the flame becomes significantly larger compared to the mass flux into the flame in the center of the domain. Since a highly-diluted hydrogen jet is assumed to issue from the inner cylinder in the present study, the flame becomes stronger despite the weakened preferential diffusion of the fuel due to the small r_s . This results in smaller Da_E and larger $T_{\max,E}$ for flames lying near the inner cylinder. However, when the tubular flame is formed very close to the outer cylinder, the oxidizer mass flux into the flame increases and the flame becomes weaker, subsequently leading to larger Da_E and smaller $T_{\max,E}$. In the present study, therefore, only tubular flames formed in the center of the domain ($40 \lesssim r_s \lesssim 90$) are investigated, where such artefacts of the Dirichlet boundary conditions on the solution are minimal.

2.3. 2-D tubular flames

To investigate the characteristics of the D-T instability of nonpremixed opposed tubular flames, it is common practice to extend the governing equations describing the 1-D steady tubular flames to time-dependent partial differential equations including variations in the

azimuthal direction, θ [2, 11, 19, 23]:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} \right) \begin{pmatrix} T \\ Y_F \\ Y_O \end{pmatrix} &= \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \begin{pmatrix} T \\ Y_F / Le_F \\ Y_O / Le_O \end{pmatrix} \\ &+ Da Y_F Y_O e^{-T_a/T} \begin{pmatrix} q \\ -\alpha_F \tilde{Y}_{O,2} \\ -\alpha_O \tilde{Y}_{F,1} \end{pmatrix}, \end{aligned} \quad (8)$$

which are also subject to the same boundary conditions of Eq. 7 in the r -direction as in the 1-D problems. Periodic boundary conditions are imposed in the θ -direction. Note that the dimensional time, $\tilde{t}$, is normalized as $t = \tilde{t}\tilde{\kappa}$. The 2-D direct numerical simulations of tubular flames are performed using a six-stage fourth-order explicit Runge-Kutta method for time integration and an eighth-order central finite-difference scheme for evaluating spatial derivatives [25, 26] as in [23]. The message passing interface (MPI) is used for scalable parallelism. After a series of grid convergence tests, a uniform grid system of 400×1200 is adopted to discretize the r - and θ -directions for $|u_{r,1}/u_{r,2}| = 1.0$ and 2.0 , while a grid system of 800×2400 is used for $|u_{r,1}/u_{r,2}| = 4.0$. A timestep, Δt , of 0.001 or 0.004 is required for accurate time integration.

For 2-D simulations, two different initial conditions (IC) are adopted to investigate the effects of small ($\sim O(\varepsilon)$) and large ($\sim O(1)$) amplitude disturbances on the formation of flame cells. The first IC is constructed from the 1-D steady solution of a tubular flame at a given Da , and is replicated along the θ -direction. A small sinusoidal perturbation of temperature ($\sim O(\varepsilon)$) is superimposed on the steady solution field:

$$T_{2-D,init}(r, \theta) = T_{1-D}(r) + \varepsilon \cdot \sin \theta \cdot (T_{1-D}(r) - T_1) \quad : 0 \leq \theta < 2\pi, \quad (9)$$

where $T_{1-D}(r)$ represents the 1-D steady temperature solution given by Eq. 6. The perturbation amplitude, ε , of 10^{-5} is selected through a series of 2-D simulations from which it is confirmed that there is no change in the number of flame cells and extinction behavior for $\varepsilon \leq 10^{-5}$. Henceforth, this initial condition is referred to as the ‘‘perturbed IC’’.

The second IC is devised by combining intensely-burning and frozen-flow solutions through a smooth transition using the hyperbolic tangent function, representing $\sim O(1)$ disturbance [11, 23]:

$$\begin{aligned} \mathbf{x}_{2-D,init}(r, \theta) = \mathbf{x}_{1-D,F}(r) + 1 \cdot [(\tanh(\theta - \theta_1) - \tanh(\theta - \theta_2)) / \sigma] \\ \cdot (\mathbf{x}_{1-D,B}(r) - \mathbf{x}_{1-D,F}(r)) / 2 \quad : 0 \leq \theta < 2\pi, \end{aligned} \quad (10)$$

where $\mathbf{x}$ is the solution vector including T , Y_F , and Y_O ; $\theta_1 = 0.5\pi$; $\theta_2 = 1.5\pi$; $\sigma = 0.1\pi$. The subscripts B and F represent intensely-burning and frozen-flow solutions. Note that this type of initial condition was adopted in [11, 23] to investigate the edge flame dynamics near the extinction limit. Henceforth, it is referred to as the ‘‘C-shaped IC’’. From the C-shaped IC, a ‘‘horseshoe-shaped’’ cellular flame is formed at relatively-large Da , similar to a large flame cell induced by local quenching in previous experimental studies [13, 14]. Starting from the horseshoe-shaped cellular flame the dynamics of flame cell formation is investigated by gradually reducing Da , similar to the experimental procedure in [13, 14]. Figure 4 shows isocontours of temperature for the two different ICs. Note that the perturbation amplitude in the perturbed IC is small enough that it cannot be clearly observed in the figure.

3. Stability analysis

Prior to performing 2-D direct numerical simulations, a linear stability analysis of opposed nonpremixed tubular flames near extinction is conducted to gain insight into the cellular instability characteristics as in [11, 19]. For the stability analysis, the steady solution of the 1-D tubular flame, denoted by an overbar, is perturbed with small harmonic perturbations:

$$T = \bar{T}(r) + \epsilon T'(r)e^{ik\theta + \lambda t}, \quad Y_F = \bar{Y}_F(r) + \epsilon Y'_F(r)e^{ik\theta + \lambda t}, \quad Y_O = \bar{Y}_O(r) + \epsilon Y'_O(r)e^{ik\theta + \lambda t}, \quad (11)$$

where λ is a complex number whose the real part, $\text{Re}(\lambda)$, represents the growth rate, k is the wavenumber, and ϵ is the small perturbation amplitude similar to ε in the perturbed

IC. Substituting Eq. 11 into Eq. 8 and linearizing the equation yields:

$$\begin{aligned} \frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) \begin{pmatrix} T' \\ Y'_F / Le_F \\ Y'_O / Le_O \end{pmatrix} - u_r \frac{d}{dr} \begin{pmatrix} T' \\ Y'_F \\ Y'_O \end{pmatrix} - \begin{pmatrix} k^2 / r^2 + \lambda \\ k^2 / r^2 Le_F + \lambda \\ k^2 / r^2 Le_O + \lambda \end{pmatrix} \begin{pmatrix} T' \\ Y'_F \\ Y'_O \end{pmatrix} \\ + Da e^{-T_a / \bar{T}} \left(\bar{Y}_O Y'_F + \bar{Y}_F Y'_O + \frac{\bar{Y}_F \bar{Y}_O}{\bar{T}^2} T_a T' \right) \begin{pmatrix} q \\ -\alpha_F \tilde{Y}_{O,2} \\ -\alpha_O \tilde{Y}_{F,1} \end{pmatrix} = 0, \end{aligned} \quad (12)$$

where the boundary conditions are $T' = Y'_F = Y'_O = 0$ at both $r = r_1$ and $r = r_2$. An eigenvalue problem of $(\mathbf{A} - \lambda \mathbf{B})\mathbf{x}' = 0$ is then obtained by discretizing Eq. 12, where $\mathbf{x}'$ is the discretized solution vector including T' , Y'_F , and Y'_O .

By solving the eigenvalue problem for fixed k and Da , the largest growth rate, λ_R , can be obtained among many $\text{Re}(\lambda)$. By finding λ_R for different k and Da values, the variation of λ_R as functions of k and Da is ultimately obtained. Note that if all λ_R exhibit negative values over the entire range of k for a given Da , small perturbations would decay in time and, as such, no flame instability occurs at that Da . However, if at least one λ_R exhibits a positive value over a certain range of k , even infinitesimally-small disturbances would grow in time, ultimately leading to the occurrence of flame instability manifested by the formation of flame cells.

Figure 5 shows the largest growth rate, λ_R , as a function of wavenumber for three different $|u_{r,1}/u_{r,2}|$ with several Damköhler numbers near their extinction limits. Several points are noted from the figure. First, it is readily observed that for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 , positive λ_R first appears at approximately $Da_C = 14412, 12360$, and 10865 , respectively and the corresponding k is found to be $7.0, 9.0$, and 11.0 . Henceforth, Da_C denotes the critical Damköhler number at which the first positive λ_R appears, and hence, the D-T instability occurs. This result implies that the D-T instability of opposed nonpremixed tubular flames can occur over the range, $Da_E \leq Da \leq Da_C$, near the 1-D extinction limit, consistent with previous theoretical and numerical studies of nonpremixed counterflow flames [9–11].

Second, the wavenumber, $k_{\max}$, at which λ_R attains its maximum value for a given Da ,

slightly increases as Da decreases from Da_C to Da_E ; for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 , $k_{\max} = 7.0 \rightarrow 7.8, 9.0 \rightarrow 9.9$, and $11.0 \rightarrow 12.1$, respectively, as $Da = Da_C \rightarrow Da_E$. Furthermore, it is also observed that the maximum λ_R and the range of k exhibiting positive λ_R increase with decreasing Da . This implies that opposed nonpremixed tubular flames are more vulnerable to flame instability at smaller Da as can be expected. It can also be expected that the D-T instability can occur more rapidly in time at smaller Da near Da_E due to its relatively large λ_R there. Note also that if the wavenumber is related to the number of flame cells, N_{cell} , induced by the D-T instability, it can be expected that N_{cell} increases with increasing $|u_{r,1}/u_{r,2}|$ and r_s , which is further discussed in the following section.

4. The diffusive-thermal instability with the perturbed IC

In this section, 2-D direct numerical simulations of tubular flames are performed to investigate the characteristics of the D-T instability of tubular flames with the perturbed IC. As mentioned above, the perturbed IC is used to compare the results of the numerical simulations based on the full nonlinear equations (Eq. 8) with those based on the linearized equations (Eq. 12).

Figure 6 shows the temperature isocontours of flame cells induced by the D-T instability for three different values of $|u_{r,1}/u_{r,2}|$ at their corresponding Da_E , obtained by numerically integrating Eq. 8 with the perturbed IC of Eq. 9. As shown in the figure, N_{cell} for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 is found to be 7, 10, and 12, respectively, quite similar to the values of $k_{\max}$ obtained from the linear stability analysis (see Fig. 5): for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 , $k_{\max}$ is 7.8, 9.9, and 12.1 at their Da_E , respectively. From these results, it can be conjectured that the number of flame cells induced by the D-T instability in the 2-D simulations is equal to an integer close to $k_{\max}$.

The dynamics of flame cell formation is identified by examining sequential snapshots in time of the temperature field for $|u_{r,1}/u_{r,2}| = 1.0$ at $Da = 13950$ as shown in Fig. 7. Small initial perturbations continue to grow in time due to the D-T instability, inducing alternating weak and strong reaction regions (Fig. 7a). This leads to local extinction at the weak reaction regions between two strong reaction regions, forming small flame cells (Fig. 7b

and c). Ultimately, a finite number of steady flame cells are formed as shown in Fig. 7d. This sequential dynamics of flame cell formation is qualitatively similar to that observed in nonpremixed counterflow flames [19].

To further identify the general characteristics of the D-T instability by the perturbed IC, the variation of $T_{\max}$ and N_{cell} as a function of Da is shown in Fig. 8 for three different values of $|u_{r,1}/u_{r,2}|$. For the purpose of comparison, $T_{\max}$ from the 1-D solutions and $k_{\max}$ from the linear stability analysis are also shown in the figure. Note that since the perturbed ICs are based on the 1-D steady solutions, 2-D simulations are performed only for $Da \geq Da_E$. Several points are to be noted from the figure. First, for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 , the critical Damköhler number with the perturbed IC at the onset of the D-T instability, $Da_{C,\text{PI}}$, is approximately 14350, 12330, and 10830, respectively, which are slightly smaller than the corresponding Da_C from the linear stability analysis: i.e. $Da_C = 14412, 12360$, and 10865 . The corresponding number of flame cells in the 2-D simulations is 7, 9, and 11, which is quite similar to the correspond $k_{\max}$ from the stability analysis. Therefore, the linear stability analysis is able to predict the characteristics of nonpremixed tubular flame instability including the number of flame cells and the Damköhler number at the onset of cellular flame instability with reasonable accuracy.

Second, when the D-T instability occurs, $T_{\max}$ of the flame cells bifurcates from that of the corresponding 1-D steady tubular flame and exhibits a much larger value. This is primarily the result of focusing effects, that is, the fuel-mass diffusion into the edge of the flame cells strengthens overall reaction and hence, the flame temperature near the edges increases significantly compared to the temperature of the corresponding 1-D steady tubular flame as shown in Figs. 6 and 7.

Third, it is also readily observed from the figure that $T_{\max}$ of the flame cells continues to increase as Da decreases from $Da_{C,\text{PI}}$ to Da_E . This is the result of the flame cell length, l_{cell} , decreasing with decreasing Da , and hence, the focusing effect of fuel-mass diffusion into the flame cell increases, resulting in the increase of $T_{\max}$ with decreasing Da . These results are also consistent with previous studies of the D-T instability of nonpremixed counterflow flames [11]. In fact, l_{cell} decreases from 2.68 to $2.48l_D$ when Da decreases from $Da_{C,\text{PI}}$ to Da_E

for $|u_{r,1}/u_{r,2}| = 1.0$, where l_D is the diffusion flame thickness of the corresponding 1-D tubular flame. Note that l_D is measured by the full-width-half-maximum (FWHM) temperature, which remains nearly identical regardless of Da near extinction. From a previous study of the D-T instability of planar diffusion flames [7, 27], it was also reported that the characteristic flame cell length at the onset of the instability is in the range of $3 \sim 12l_D$, which is similar to the results of the present study.

It is of interest to note that the slight discrepancies between the 2-D simulations and the linear stability analysis may stem from the inherent differences between the nonlinear governing equations of Eq. 8 for the 2-D simulations and the linearized equations of Eq. 12 for the stability analysis. The discrepancies may also be attributed to the slight difference between the perturbed IC for the 2-D simulations and the perturbed variables in Eq. 11 for the stability analysis. The numerical dissipation of the solutions in the 2-D simulations may also be another reason for the discrepancies.

5. The diffusive-thermal instability with the C-shaped IC

As discussed in Section 4, the prediction of the linear stability analysis shows good agreement with the characteristics of the D-T instability of 2-D tubular flames with the perturbed IC in terms of Da_C and the resultant N_{cell} . In experiments, however, a noncellular tubular flame is first formed between the two nozzles with low stretch rate and subsequently, the formation of flame cells is observed by incrementally increasing the stretch rate [13]. From a noncellular tubular flame, a horseshoe-shaped cellular flame first appears by local extinction at intermediate stretch rate and subsequently, divides into several flame cells as the stretch rate is further increased (see Fig. 6 in [13]).

In an attempt to replicate this experimental procedure, a horseshoe-shaped cellular flame is first generated from the C-shaped IC by reducing Da , similar to the increase in stretch rate in the experiments. The detailed simulation procedure is as follows: to locate the onset of a horseshoe-shaped cellular flame, a new simulation is started with the C-shaped IC by gradually decreasing Da until a horseshoe-shaped cellular flame is established. For the case with $|u_{r,1}/u_{r,2}| = 1.0$, for instance, the edge flames develop from the C-shaped IC, ultimately

merging into a noncellular tubular flame for $Da > 18900$. At $Da = 18900$, however, the two edges propagate towards each other but do not merge. Rather they form a horseshoe-shaped cellular flame as shown in Fig. 9. Once a solution exhibiting a horseshoe-shaped flame at a specific Da is obtained, successive simulations at successively smaller Da are restarted from each of the previous steady solutions, resetting the simulation time to the beginning upon restarting. For $|u_{r,1}/u_{r,2}| = 1.0$, the horseshoe-shaped cellular flame divides into five flame cells at approximately $Da = 14600$. From the 2-D simulations performed using this procedure three different types of flames are observed for different values of $|u_{r,1}/u_{r,2}|$ within the entire range of Da as shown in Fig. 10: i.e. a noncellular tubular flame at relatively-large Da (Fig. 10a), a horseshoe-shaped cellular flame at intermediate Da (Fig. 10b), and several flame cells at relatively-small Da (Figs. 10c and d). This result is qualitatively similar to the experimental result by Shopoff et al. [13] which identified the sequence of flame cell dynamics evolving from a noncellular flame to a horseshoe-shaped cellular flame to several flame cells with increasing stretch rate.

To further identify the characteristics of the D-T instability, the variation of $T_{\max}$ and N_{cell} as a function of Da for three different values of $|u_{r,1}/u_{r,2}|$ is shown in Fig. 11. Several points are noted from the figure. First, it is readily observed that the flame cellularity starts to occur at much larger Da than the corresponding critical Damköhler number, Da_C , from the stability analysis for all cases: i.e. for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 , the horseshoe-shaped cellular flame first occurs at $Da = 18900, 16300$, and 14350 , and the corresponding Da_C is $14412, 12360$, and 10865 , respectively. The implication is that the horseshoe-shaped cellular flame is induced not by small perturbations ($\sim O(\varepsilon)$) but by large amplitude disturbances ($\sim O(1)$). From the experimental perspective, it can be conjectured that inevitable asymmetry in the tubular flame experiments may induce locally-high stretch rate, leading to the local extinction of a noncellular tubular flame and the formation of a horseshoe-shaped cellular flame even at intermediate values of mean stretch rate.

Second, although the horseshoe-shaped cellular flame starts to occur at intermediate Da , the formation of several small flame cells induced by the D-T instability starts to occur at the critical Damköhler number with the C-shaped IC, $Da_{C,\text{CI}}$, close to Da_C from the linear

stability analysis: i.e. $Da_{C,CI} = 14600, 12520, \text{ and } 10980$ for $|u_{r,1}/u_{r,2}| = 1.0, 2.0, \text{ and } 4.0$, respectively. In fact, the $Da_{C,CI}$ values are slightly larger than the corresponding Da_C from the linear stability analysis. This is due to the large amplitude disturbance ($\sim O(1)$) between the two edges of the horseshoe-shaped cellular flame which may induce the onset of flame cells at slightly larger Da than Da_C . This result also confirms that the linear stability analysis can predict Da_C for the 2-D simulations with both perturbed and C-shaped ICs with reasonable accuracy.

Third, however, the number of flame cells at $Da_{C,CI}$ for $|u_{r,1}/u_{r,2}| = 1.0, 2.0, \text{ and } 4.0$, is found to be 5, 7, and 8, respectively, which is smaller than that found from the linear stability analysis and the 2-D simulations with the perturbed IC. To identify the characteristics of N_{cell} , the dynamics of flame cell formation is examined as shown in Fig. 12. When the horseshoe-shaped cellular flame starts to divide into flame cells, the first local extinction occurs farther away from the flame edge (Figs. 12a and b) due to a relatively-large flame edge with strong reaction manifested by high temperature. As such, the flame cell length becomes larger compared to that found in the 2-D simulations with the perturbed IC (see Fig. 7). Following the first local extinction, another flame edge develops and subsequently, the second local extinction occurs (Figs. 12b and c), ultimately leading to the formation of five flame cells. These results imply that the number and size of flame cells are affected by the size and strength of the edge flames of the horseshoe-shaped cellular flame. From this perspective, the number of flame cells observed in experiments cannot be the same as that from the linear stability analysis because the flame cells in experiments are usually generated from the horseshoe-shaped cellular flame [13, 14].

Fourth, once the horseshoe-shaped cellular flame divides into several flame cells, the number of flame cells remains constant until global extinction occurs for all $|u_{r,1}/u_{r,2}|$ as shown in Fig. 11. In the 2-D simulations with the perturbed IC, N_{cell} changes according to k_{max} as discussed above. For the cases with the C-shaped IC, however, the flame cells generated from the horseshoe-shaped cellular flame are strong enough to survive small Da and hence, no further local extinction occurs within the flame cells up to the point of global extinction.

Fifth, it is also readily observed from the figure that the flame cells can survive beyond the 1-D extinction Damköhler number, Da_E ; i.e. for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 the extinction Damköhler number for the global extinction of the flame cells, $Da_{E,2D}$, is found to be 7690, 6860, and 6000, respectively. In addition, $T_{\max}$ increases with decreasing Da , similar to the results with the perturbed IC. As Da decreases, the flame cell length, l_{cell} , is significantly reduced from 4.0 to $1.4l_D$ as shown in Fig. 10, leading to an enhanced focusing effect of fuel-mass diffusion into the flame cells, consequently increasing $T_{\max}$. This result is qualitatively consistent with previous studies of the D-T instability of nonpremixed counterflow flames [11]. However, $T_{\max}$ decreases slightly with decreasing Da near the 2-D extinction Damköhler number, $Da_{E,2D}$. This is primarily attributed to incomplete reaction by small Damköhler number near the point of global extinction which becomes significant compared to the enhanced focusing effect of fuel-mass diffusion.

6. Dynamics of flame cell formation - displacement speed analysis

In this section, the dynamics of flame cell formation is further quantified by evaluating the propagation speed of the flame cell edges. The propagation speed of the flame edges is obtained by evaluating the displacement speed which measures the velocity of a scalar isocontour (e.g. the fuel mass fraction) relative to the local flow velocity. In the present study, the azimuthal flow velocity is assumed to be zero and, as such, the displacement speed represents directly the propagation speed of the flame edges along the azimuthal direction. In the context of constant density and Lewis number, the displacement speed, S_d , can be defined by [23, 28–36]:

$$S_d = S_d^R + S_{d,r}^D + S_{d,\theta}^D = \pm \frac{1}{|\nabla Y_k|} \left[\dot{\omega}_k + \frac{1}{Le_k} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial Y_k}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 Y_k}{\partial \theta^2} \right) \right], \quad (13)$$

where S_d^R , $S_{d,r}^D$, and $S_{d,\theta}^D$ represent the reaction, r – and θ –directional diffusion components of S_d , respectively. The sign in Eq. 13 is positive for products and negative for reactants [30]. In this form, therefore, each component of the displacement speed is a measure of its corresponding contribution to the edge flame propagation.

First, the formation of the horseshoe-shaped cellular flame is investigated. Figure 13 shows the temporal evolution of S_d and its components of the edge flame during the formation of the horseshoe-shaped cellular flame from the C-shaped IC for $|u_{r,1}/u_{r,2}| = 1.0$ at $Da = 18900$ (see Fig. 9). For this case, S_d is evaluated at the intersection of the $Y_F = 0.0765$ isoline and the stagnation plane. This particular intersection coincides approximately with the location of maximum heat release rate of the edge flames. Initially, S_d exhibits a very large value due to the formation of strong edge flames at the two edges of the C-shaped IC, qualitatively similar to the ignition characteristics of S_d in previous studies [23, 31–36]. After the formation of the edge flames, they propagate towards each other with constant positive speed. Note that the flame edges are convex towards both fuel and oxidizer, and hence, the reaction at the flame edges become stronger due to the focusing effect of fuel mass diffusion with $Le_F < 1$. As the two edge flames approach each other, S_d decreases because its reaction term decreases as the θ –directional fuel diffusion into the edge flame decreases. Ultimately S_d vanishes due to the balance that is established between its reaction and diffusion terms and hence, two stationary edge flames form in the absence of merging, forming a horseshoe-shaped cellular flame.

In other words the two edges of the horseshoe-shaped cellular flame maintain their structure by consuming equal amounts of fuel and oxidizer issuing from the nozzles, while reaction at the edges is not strong due to the relatively-small Damköhler number enough to unite them and form a noncellular tubular flame. This result also implies that the horseshoe-shaped cellular flame is generated not by the D-T instability induced by $O(\varepsilon)$ perturbation but rather by the balance between diffusion and reaction at its edges initiated with $O(1)$ disturbance. In experiments, the $O(1)$ disturbance may be induced by the local extinction of a noncellular tubular flame due to the occurrence of locally-high stretch rate aforementioned.

In addition to the formation of the horseshoe-shaped cellular flame, the dynamics of flame cell formation by the D-T instability is also investigated by examining the displacement speed of the flame cells. Figure 14 shows the temperature isocontours and the corresponding 1-D profiles of the displacement speed and its components along the stagnation plane at $Da = 14600$ during the flame cell formation from the horseshoe-shaped cellular flame at the

same times as in Fig. 12. Note that in the context of nonpremixed flames, the displacement speed can be defined only at the location of the flame edges which exhibit propagation characteristics similar to premixed flames. As such S_d at the nonpremixed flame between the two flame edges of the horseshoe-shaped cellular flame has no significance except that it represents the balance between diffusion and reaction in the nonpremixed flame, manifested by $S_d \approx 0$. In the present study, however, the profile of S_d along the stagnation plane is examined to elucidate the characteristics of flame cell formation by the D-T instability.

As shown in Fig. 14a, the shape of the strong flame edges induces a locally more concave region towards the fuel next to the edges and therefore the defocusing effect of fuel mass diffusion with $Le_F < 1$ in the negatively-curved flame is enhanced, leading to local flame extinction similar to experiments [13]. This is manifested in S_d which exhibits relatively-large negative values due to the significant reduction of the reaction term at the extinction location. Following the local extinction two small flame cells and a large cellular flame featuring a nonpremixed flame between the two flame edges are formed as shown in Fig. 14b. At this instant, the small flame cells grow while the large cellular flame shrinks, and as such, S_d exhibits relatively-small positive/negative values at the edges of the small flame cells/the large cellular flame. A short while later, this process repeats with another local flame extinction induced by the shape of the flame edges occurring in the large cellular flame (Fig. 14c). Ultimately, five flame cells develop approaching a steady state by adjusting their size and location (Fig. 14d).

In summary, when local flame extinction occurs, the corresponding S_d exhibits relatively-large negative values at the extinction locations (see Figs. 14a and c). Following the local flame extinction, the flame cells adjust their size and location, featuring relatively-small positive or negative S_d . As the size of the flame cell increases/decreases, S_d at the edges of the flame cell exhibits relatively-small positive/negative values as shown in Figs. 14b and d.

7. Conclusions

The diffusive-thermal instability of opposed nonpremixed tubular flames near extinction is investigated using two-dimensional direct numerical simulations together with a linear

stability analysis based on their 1-D steady solutions. The characteristics of the D-T instability of tubular flames are identified by a critical Damköhler number, Da_C , where the D-T instability first occurs, and the corresponding number of flame cells, N_{cell} , for three different tubular flames with different flame radii. To elucidate the effect of small and large perturbations on the D-T instability, the perturbed and C-shaped ICs are adopted for the 2-D simulations. The following results were obtained from the 2-D simulations together with the linear stability analysis:

1. The predicted Da_C through the linear stability analysis shows good agreement with that obtained from the 2-D simulations for the two different ICs.
2. The number of flame cell from the 2-D simulations with the perturbed IC is also found to be equal to an integer close to the maximum wavenumber, k_{max} , obtained from the linear stability analysis.
3. However, N_{cell} from the 2-D simulations with the C-shaped IC is smaller than k_{max} and N_{cell} found from cases with the perturbed IC. This is primarily due to the strong reaction at the edge of the horseshoe-shaped cellular flame which is more likely to create larger flame cells and reduce N_{cell} within the limited space along the azimuthal direction.
4. Once the cellular instability occurs in the 2-D simulations with the C-shaped IC, N_{cell} remains constant until global extinction occurs by incomplete reaction. The flame cells can survive beyond the 1-D extinction Da due to the focusing effect of fuel mass diffusion into the flame cells, for which their size continually decreases as Da gradually decreases.
5. The horseshoe-shaped cellular flame is induced not by the $O(\varepsilon)$ perturbation but by the $O(1)$ disturbance, which may be a consequence of local flame extinction induced by the locally-high stretch rate occurring in experiments.
6. The dynamics of the flame cell formation was also quantified through the displacement speed analysis. The two flame edges of the horseshoe-shaped cellular flame remain stationary in the absence of any merging due to the balance between diffusion and

reaction at the flame edges, manifested by $S_d = 0$.

7. Local flame extinction in the horseshoe-shaped cellular flame occurs at the concave region towards the fuel side, which can be identified by relatively-large negative values of S_d . After local extinction, the resultant flame cells with small S_d approach a steady state by adjusting their size and location.

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Figure 1: Schematic of (a) a noncellular tubular flame with large Damköhler number and (b) flame cells induced by the diffusive-thermal instability with small Damköhler number near the extinction limit.

Figure 2: The maximum flame temperature, $T_{\max}$, as a function of Damköhler number, Da , for three different $|u_{r,1}/u_{r,2}|$ of 1.0, 2.0, and 4.0.

Figure 3: The extinction Damköhler number, Da_E , and maximum flame temperature, $T_{\max,E}$, at the extinction point as a function of the stagnation plane location, r_s .

Figure 4: Initial temperature fields for 2-D simulations of opposed tubular flames: (a) the perturbed IC and (b) the C-shaped IC.

Figure 5: The largest growth rate, λ_R , as functions of the wavenumber, k , and the Damköhler number, Da , from the linear stability analysis for different $|u_{r,1}/u_{r,2}|$ of (a) 1.0, (b) 2.0, and (c) 4.0. Thick segments and thin dotted segments represent the imaginary part of $\lambda = 0$ and $\neq 0$, respectively. The solid circles represent the points of $\lambda_{R,\max}$ and $k_{\max}$.

Figure 6: Temperature isocontours of steady flame cells from the perturbed IC for different $|u_{r,1}/u_{r,2}|$ of (a) 1.0, (b) 2.0, and (c) 4.0 at their $Da_E = 13950$, 11977, and 10542, respectively.

Figure 7: Temporal evolution of flame cells from the perturbed IC for $|u_{r,1}/u_{r,2}| = 1.0$ at $Da_E = 13950$. From left to right, $t =$ (a) 1070, (b) 1090, (c) 1110, and (d) 3800.

Figure 8: The variation of (a) $T_{\max}$ and (b) the number of flame cells as a function of Da from the perturbed IC for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 . The solid lines in (a) and (b) represent $T_{\max}$ of 1-D solutions and $k_{\max}$, respectively.

Figure 9: Temporal evolution of the horseshoe-shaped cellular flame from the C-shaped IC at $Da = 18900$ for $|u_{r,1}/u_{r,2}| = 1.0$. From left to right, $t =$ (a) 0, (b) 300, (c) 600, and (d) 1200.

Figure 10: Temperature isocontours of steady flame cells from the C-shaped IC for $|u_{r,1}/u_{r,2}| = 1.0$ at $Da =$ (a) 20000, (b) 16000, (c) 13000, and (d) 8000.

Figure 11: The variation of (a) the maximum flame temperature, $T_{\max}$, and (b) the number of flame cells as a function of Damköhler number, Da , from the C-shaped IC for $|u_{r,1}/u_{r,2}| = 1.0, 2.0$, and 4.0 .

Figure 12: Temporal evolution of flame cells from a horseshoe-shaped cellular flame at $Da = 14600$ for $|u_{r,1}/u_{r,2}| = 1.0$. From left to right, $t =$ (a) 2400, (b) 2800, (c) 5400, and (d) 7200.

Figure 13: Temporal evolution of the displacement speed, S_d , and its components from the C-shaped IC at $Da = 18900$ for $|u_{r,1}/u_{r,2}| = 1.0$.

Figure 14: Temperature isocontours (top) and the corresponding 1-D profiles of the displacement speed, S_d , and its components along the stagnation plane (bottom) from a horseshoe-shaped cellular flame at $Da = 14600$ for $|u_{r,1}/u_{r,2}| = 1.0$: (a) $t = 2400$, (b) 2800, (c) 5400, and (d) 7200